4 Coherence in neutrino oscillations

A project about the subtle quantum mechanics of neutrino oscillations, including several potential brain-teasers.

Focus: Neutrino oscillations
Requirements: Analytical calculation
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Neutrino oscillations are an interference effect. They can occur only if different neutrino mass eigenstates evolve coherently (i.e. their relative phases are well-defined and evolve smoothly with time) and if the detector is unable to distinguish different mass eigenstates. In understanding the conditions for coherence, it is often useful to picture the propagating neutrino as a superposition of three Gaussian wave packets, one for each mass eigenstate.

1. Consider pion decay at rest, $\pi^+ \to \mu^+ + \nu_\mu$. Compute the energy and momentum of the emitted neutrino. You will find that the result depends on the neutrino mass.

2. Oscillations of ν_μ produced in pion decay can occur if the detector is unable to distinguish the different momentum eigenstates corresponding to the mass eigenstates ν_1, ν_2, ν_3. Use the Heisenberg principle to derive a condition on the size of the detector.

3. Because of their different masses and momenta, different neutrino mass eigenstates propagate with different group velocities. Eventually, the wave packets corresponding to different mass eigenstates will be separated in space and time. They will no longer overlap, and coherence will be lost. Estimate the propagation distance L^{coh} (as a function of the neutrino mass, energy, and the wave packet width σ) at which wave packets become separated. For which neutrino sources is decoherence relevant? How far away does a source need to be for two mass eigenstates $m_1 = 0$ eV and $m_2 = 0.05$ eV to arrive at the detector with a separation of 1 sec?

4. Supernova neutrinos arrive at Earth as an incoherent superposition of mass eigenstates. Derive an expression for the probability for observing a ν_2 as a ν_e, assuming neutrinos do *not* travel through significant amounts of Earth matter prior to detection.

5. What changes if neutrinos travel through the Earth before being detected?

6. Discuss whether neutrinos produced in Z boson decays oscillate, and what it would take to observe these oscillations.