Lecture 2: Neutrino oscillations

Aims

Develop neutrino oscillations formalism and apply it to exp.

Outline

Neutrino oscillations in vacuum: 3 neutrino mixing
Neutrino oscillations in matter
- constant density case
- varying density
  - adiabatic condition
  - solar neutrino oscillations and the MSW effect

Brief review of current and past neutrino oscillation experiments
- solar neutrinos
- reactor neutrinos
- atmospheric neutrinos
- accelerator neutrinos

RECAP

Neutrino mixing

Mixing is described by the *Pontecorvo-Maki-Nakagawa-Sakata* matrix: $\nu_\alpha = \sum_i U_{\alpha i} \nu_i$

Flavour field (α) — Mass field (i)

which enters in the CC interactions

$$\mathcal{L}_{CC} = -\frac{g}{\sqrt{2}} \sum_{k\alpha} (U^*_{\alpha k} \bar{\nu}_{kL} \gamma^\rho l_{\alpha L} W_\rho + \text{h.c.})$$

This implies that in an interaction with an electron, the corresponding (anti-)neutrino will be produced, as a superposition of different mass eigenstates.

Implications of the existence of neutrino oscillations

The oscillation probability implies that

$$P(\nu_\alpha \to \nu_\beta) = \left| \sum_i U^*_{\alpha i} U_{\beta i} e^{-i \frac{\Delta m^2_{i1}}{2E} L} \right|^2$$

- **neutrinos have mass** (as the different components of the initial state need to propagate with different phases)

- **neutrinos mix** (as U needs not be the identity. If they do not mix the flavour eigenstates are also eigenstates of the propagation Hamiltonian and they do not evolve)

3-neutrino oscillations

They depend on two mass squared-differences

7.5 ×10⁻⁵ eV² ~ $\Delta m^2_{21} \ll \Delta m^2_{31}$ ~ 2.5 ×10⁻³ eV²

In general the formula is quite complex

$$P(\nu_\alpha \to \nu_\beta) = \left| U^*_{\alpha 1} U_{\beta 1} + U^*_{\alpha 2} U_{\beta 2} e^{-i \frac{\Delta m^2_{21}}{2E} L} + U^*_{\alpha 3} U_{\beta 3} e^{-i \frac{\Delta m^2_{31}}{2E} L} \right|^2$$

2-neutrino limits relevant for experiments

For a given L, the neutrino energy determines the impact of a mass squared difference. Various limits are of interest in concrete experimental situations. Using the current values

$$\Delta m^2_{21} = 7.5 \times 10^{-5} \mathrm{eV}^2 \qquad \Delta m^2_{31} = 2.5 \times 10^{-3} \mathrm{eV}^2$$

Type of neutrinos	Average E	(Typical) L	$\frac{\Delta m_{21}^2}{4E}L$	$\frac{\Delta m_{31}^2}{4E}L$
Atmospheric neutrinos	(0.1-100 GeV) 10 GeV	(10-12000 km) 6000 km	0.06	1.9
Reactor neutrinos I	3 MeV	1 km	0.03	1.1
Reactor neutrinos II	3 MeV	100 km	3.2	106
Accelerator (T2K)	0.7 GeV	300 km	0.04	1.36
Accelerator (DUNE)	3 GeV	1300 km	0.04	1.37
SBL	0.8 GeV	500 m	0.00006	0.001

"Atmospheric regime"

$$\frac{\Delta m^2_{21}}{4E} L \ll 1$$

The first limit applies to atmospheric, reactor with short baseline, accelerator neutrinos (K2K, MINOS, T2K, NOvA, DUNE, T2HK) at leading order.

$$\alpha \neq \beta$$

$$P(\nu_\alpha \to \nu_\beta) \simeq \Big| \underbrace{U^*_{\alpha 1} U_{\beta 1} + U^*_{\alpha 2} U_{\beta 2}} + U^*_{\alpha 3} U_{\beta 3} e^{-i \frac{\Delta m^2_{31} L}{2E}} \Big|^2$$

$$\underbrace{U^*_{\alpha 1} U_{\beta 1} + U^*_{\alpha 2} U_{\beta 2}} + U^*_{\alpha 3} U_{\beta 3} = \delta_{\alpha\beta}$$

$$\simeq \Big| U^*_{\alpha 3} U_{\beta 3} - U^*_{\alpha 3} U_{\beta 3} e^{-i \frac{\Delta m^2_{31} L}{2E}} \Big|^2$$

$$\simeq |U_{\alpha 3} U_{\beta 3}|^2 \left| 1 - e^{-i \frac{\Delta m^2_{31} L}{2E}} \right|^2$$

$$\simeq 4 |U_{\alpha 3} U_{\beta 3}|^2 \sin^2 \frac{\Delta m^2_{31} L}{4E}$$

The same we have encountered in the 2-neutrino case

$$P(\nu_\alpha \to \nu_\beta) \simeq 4 |U_{\alpha 3} U_{\beta 3}|^2 \sin^2 \frac{\Delta m^2_{31} L}{4E}$$

Exercise
Derive

Exp relevant probabilities:

$$P(\nu_\mu \to \nu_\mu) \simeq 1 - 4|U_{\mu 3}|^2 (1 - |U_{\mu 3}|^2) \sin^2 \left(\frac{\Delta m^2_{31} L}{4E} \right) \simeq 1 - \sin^2 2\theta_{23} \sin^2 \left(\frac{\Delta m^2_{31} L}{4E} \right) + \mathcal{O}(s^2_{13})$$

$$P(\nu_\mu \to \nu_e) \simeq 4|U_{e3}|^2 |U_{\mu 3}|^2 \sin^2 \left(\frac{\Delta m^2_{31} L}{4E} \right) \simeq s^2_{23} \sin^2 2\theta_{13} \sin^2 \left(\frac{\Delta m^2_{31} L}{4E} \right) + \cdots$$

$$P(\overline{\nu}_e \to \overline{\nu}_e) \simeq 1 - \sin^2 2\theta_{13} \sin^2 \left(\frac{\Delta m^2_{31} L}{4E} \right)$$

Exercise
Derive

“Small Dm2 regime”

$$\frac{\Delta m^2_{31}}{4E} L \gg 1$$

The oscillations due to the atmospheric mass squared differences get averaged out.

$$P(\bar{\nu}_e \to \bar{\nu}_e) = \left| |U_{\alpha 1}|^2 + |U_{\alpha 2}|^2 e^{-i\frac{\Delta m^2_{21} L}{2E}} + |U_{\alpha 3}|^2 e^{-i\frac{\Delta m^2_{31} L}{2E}} \right|^2$$

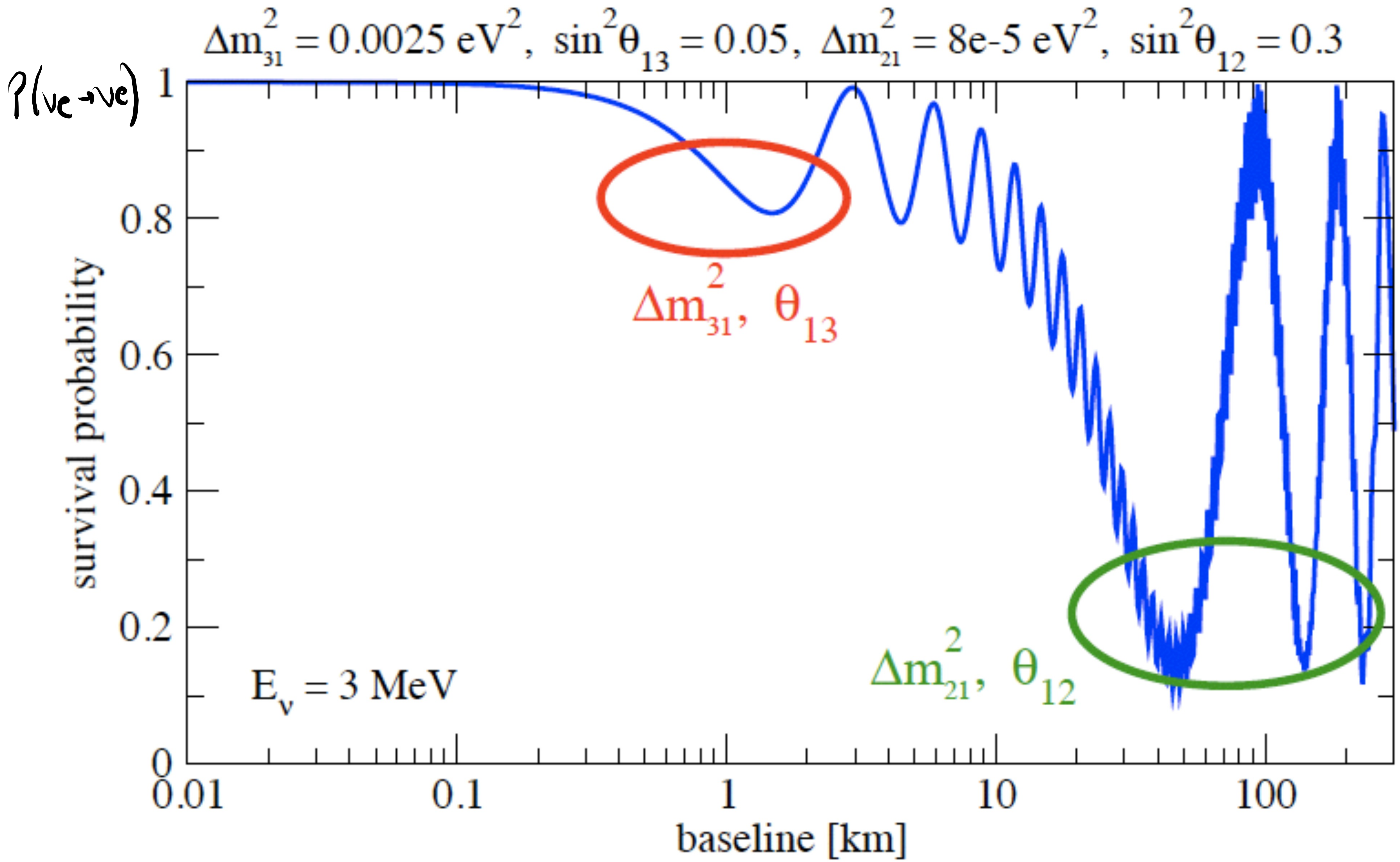
$$= \left(|U_{\alpha 1}|^2 + |U_{\alpha 2}|^2 \cos\frac{\Delta m^2_{21} L}{2E} + |U_{\alpha 3}|^2 \cos\frac{\Delta m^2_{31} L}{2E} \right)^2$$

$$+ \left(\quad \mathrm{Im} \quad \right)^2$$

$$= |U_{\alpha 1}|^4 + |U_{\alpha 2}|^4 + |U_{\alpha 3}|^4 + 2 |U_{\alpha 1}|^2 |U_{\alpha 2}|^2 \cos\frac{\Delta m^2_{21} L}{2E}$$

$$+ \ldots$$

$$\cong c_{13}^4 \left(1 - \sin^2 2\theta_{12} \sin^2 \frac{\Delta m^2_{21} L}{4E} \right) + s_{13}^4$$



Thanks to T. Schwetz

"CPV effects"

The oscillations due to the atmospheric mass squared differences get averaged out.

CP-violation will manifest itself in neutrino oscillations, due to the delta phase. Let's consider the CP-asymmetry:

$$P(\nu_\alpha \to \nu_\beta; t) - P(\bar{\nu}_\alpha \to \bar{\nu}_\beta; t) =$$

$$= \left| U_{\alpha 1} U^*_{\beta 1} + U_{\alpha 2} U^*_{\beta 2} e^{-i\frac{\Delta m^2_{21} L}{2E}} + U_{\alpha 3} U^*_{\beta 3} e^{-i\frac{\Delta m^2_{31} L}{2E}} \right|^2 - (U \to U^*)$$

$$= U_{\alpha 1} U^*_{\beta 1} U^*_{\alpha 2} U_{\beta 2} e^{i\frac{\Delta m^2_{21} L}{2E}} + U^*_{\alpha 1} U_{\beta 1} U_{\alpha 2} U^*_{\beta 2} e^{-i\frac{\Delta m^2_{21} L}{2E}} - (U \to U^*) + \cdots$$

$$= 4 s_{12} c_{12} s_{13} c^2_{13} s_{23} c_{23} \sin\delta \left[\sin\left(\frac{\Delta m^2_{12} L}{2E}\right) + \sin\left(\frac{\Delta m^2_{23} L}{2E}\right) + \sin\left(\frac{\Delta m^2_{31} L}{2E}\right) \right]$$

Exercise**
Derive

Notice:

- CP-violation requires all angles to be nonzero. It is a genuinely 3-mixing effect.

- It is proportional to the sine of the delta phase. It can be expressed in terms of the Jarlskog invariant.

- If one can neglect Δm^2_{21}, the asymmetry goes to zero as we have seen that effective 2-neutrino probabilities are CP-symmetric.

This implies that searching for leptonic CPV requires P which are sensitive to both Dm^2.

Neutrinos oscillations in matter

- When neutrinos travel through a medium, they interact with the background of electron, proton and neutrons and acquire an effective mass.

- This modifies the mixing between flavour states and propagation states and the eigenvalues of the Hamiltonian, leading to a different oscillation probability w.r.t. vacuum.

- Typically the background is CP and CPT violating, e.g. the Earth and the Sun contain only electrons, protons and neutrons, and the resulting oscillations are CP and CPT violating.

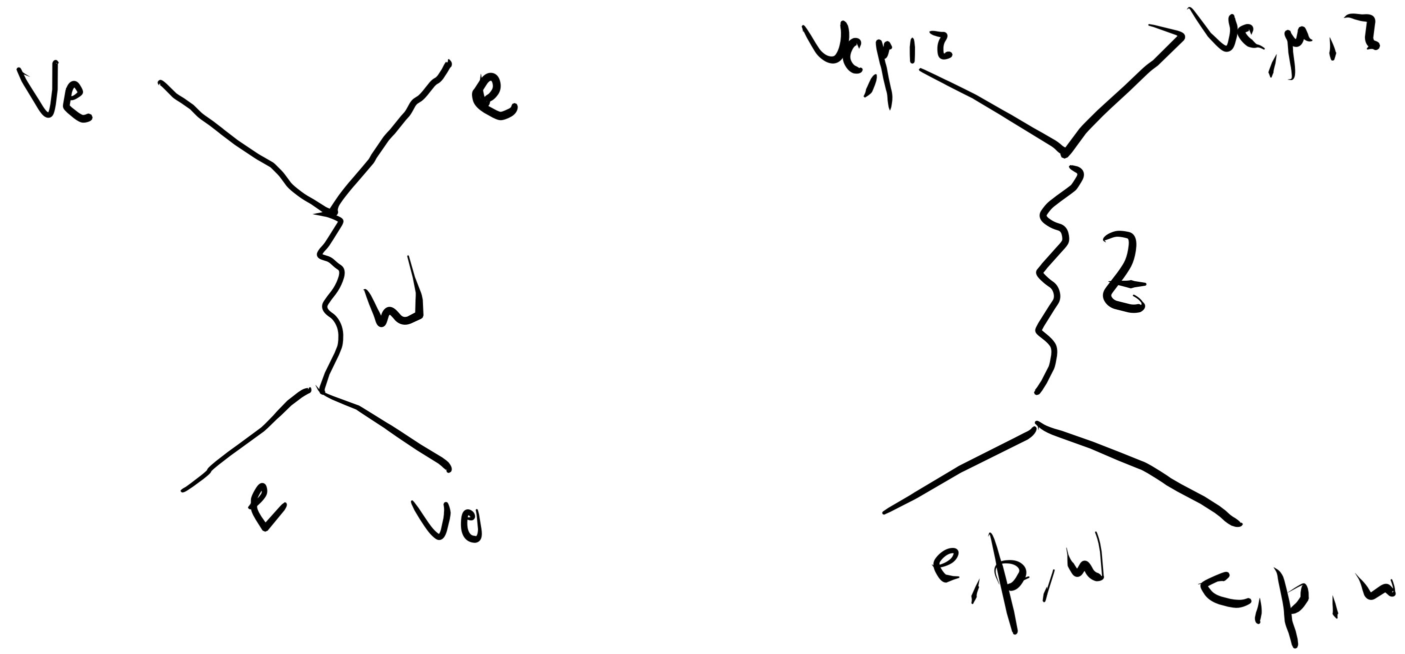
Effective potentials

Inelastic scattering and absorption processes go as G_F^2 and are typically negligible. An order of magnitude for the interaction cross section is

For the Earth density, the mean free path of a 1 GeV neutrino is 10^{14} cm. The Earth become opaque to neutrinos only above 100 TeV.

Neutrinos undergo also forward elastic scattering, in which they do not change momentum. [L. Wolfenstein, Phys. Rev. D 17, 2369 (1978); ibid. D 20, 2634 (1979), S. P. Mikheyev, A. Yu Smirnov, Sov. J. Nucl. Phys. 42 (1986) 913.]

Electron neutrinos have CC and NC interactions, while muon and tau neutrinos only the latter.



For a useful discussion, see E. Akhmedov, hep-ph/0001264; A. de Gouvea, hep-ph/0411274.

The effective Hamiltonian density for CC interactions is

$$H_{cc} = 2\sqrt{2}\, G_F \left[\bar{e}\, \gamma_\mu P_L\, e\right]\left[\bar{\nu}_e\, P_L \gamma^\mu \nu_e\right]$$

We treat the electrons as a background, averaging over it and we take into account that neutrinos see only the left-handed component of the electrons.

$$\langle \bar{e}\gamma_0 e\rangle = N_e \quad \langle \bar{e}\vec{\gamma} e\rangle = \langle \vec{v}_e\rangle \quad \langle \bar{e}\gamma_0\gamma_5 e\rangle = \langle \frac{\vec{\sigma}_e \cdot \vec{p}_e}{E_e}\rangle \quad \langle \bar{e}\vec{\gamma}\gamma_5 e\rangle = \langle \vec{\sigma}_e\rangle$$

For an unpolarised at rest background, the only term is the first one. N_e is the electron density.

The neutrino dispersion relation can be found by solving the Dirac eq with plane waves, in the ultrarelativistic limit

$$\left(i\partial^\mu \gamma_\mu - \sqrt{2}\, G_F N_e \gamma_0 \right) |\nu_e\rangle = 0$$

$$E = |\vec{p}| \pm \sqrt{2}\, G_F N_e \qquad \begin{matrix} + & \nu \\ - & \bar{\nu} \end{matrix}$$

medium	V_{CC} (A_{CC})	V_{NC}
e	$\pm\sqrt{2}\, G_F N_e$	
p		$\Big\}\ \mp\sqrt{2}\, G_F \frac{N_n}{2}$
n		

The Hamiltonian

Let's start with the vacuum Hamiltonian for 2-neutrinos

$$i\frac{d}{dt}\begin{pmatrix}|\nu_1\rangle\\|\nu_2\rangle\end{pmatrix}=\begin{pmatrix}E_1&0\\0&E_2\end{pmatrix}\begin{pmatrix}|\nu_1\rangle\\|\nu_2\rangle\end{pmatrix}$$

Recalling that $|\nu_\alpha\rangle=\sum_i U^*_{\alpha i}|\nu_i\rangle$, one can go into the flavour basis

$$U=\begin{pmatrix}\cos\theta&-\sin\theta\\\sin\theta&\cos\theta\end{pmatrix}$$

$$i\frac{d}{dt}\begin{pmatrix}|\nu_\alpha\rangle\\|\nu_\beta\rangle\end{pmatrix}=U\begin{pmatrix}E_1&0\\0&E_2\end{pmatrix}U^\dagger\begin{pmatrix}\nu_\alpha\\\nu_\beta\end{pmatrix}$$

$$=\begin{pmatrix}-\frac{\Delta m^2}{4E}\cos2\theta&\frac{\Delta m^2\sin2\theta}{4E}\\\ldots&\frac{\Delta m^2}{4E}\cos2\theta\end{pmatrix}\begin{pmatrix}\nu_\alpha\\\nu_\beta\end{pmatrix}$$

We have neglected common terms on the diagonal as they amount to an overall phase in the evolution.

The full Hamiltonian in matter can then be obtained by adding the potential terms, diagonal in the flavour basis.

$$\mathrm{H}^m = \mathrm{H}^0 + \mathrm{diag}(V_e, V_\mu, V_\tau)$$

V_e: V_{CC} + V_{NC}; V_μ: V_{NC}; V_τ: V_{NC}

For electron and muon neutrinos

$$\begin{pmatrix} \ldots + \sqrt{2} G_F N_e & \ldots \\ \ldots & \ldots \end{pmatrix}$$

For antineutrinos the potential has the opposite sign.

In general the evolution is a complex problem but there are few cases in which analytical or semi-analytical results can be obtained.

2-neutrino case in constant density

$$i\frac{d}{dt}\left(\begin{array}{c}|\nu_e\rangle\\|\nu_\mu\rangle\end{array}\right) = \left(\begin{array}{cc}-\frac{\Delta m^2}{4E}\cos 2\theta+\sqrt{2}G_F N_e & \frac{\Delta m^2}{4E}\sin 2\theta\\ \frac{\Delta m^2}{4E}\sin 2\theta & \frac{\Delta m^2}{4E}\cos 2\theta\end{array}\right)\left(\begin{array}{c}|\nu_e\rangle\\|\nu_\mu\rangle\end{array}\right)$$

If the electron density is constant (a good approximation for oscillations in the Earth crust), it is easy to solve. We need to diagonalise the Hamiltonian.

2-neutrino case in constant density

$$i\frac{d}{dt}\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix} = \begin{pmatrix} -\frac{\Delta m^2}{4E}\cos 2\theta + \sqrt{2}G_F N_e & \frac{\Delta m^2}{4E}\sin 2\theta \\ \frac{\Delta m^2}{4E}\sin 2\theta & \frac{\Delta m^2}{4E}\cos 2\theta \end{pmatrix}\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix}$$

If the electron density is constant (a good approximation for oscillations in the Earth crust), it is easy to solve. We need to diagonalise the Hamiltonian.

- Eigenvalues:

$$E_A - E_B = \sqrt{\left(\frac{\Delta m^2}{2E}\cos(2\theta) - \sqrt{2}G_F N_e\right)^2 + \left(\frac{\Delta m^2}{2E}\sin(2\theta)\right)^2}$$

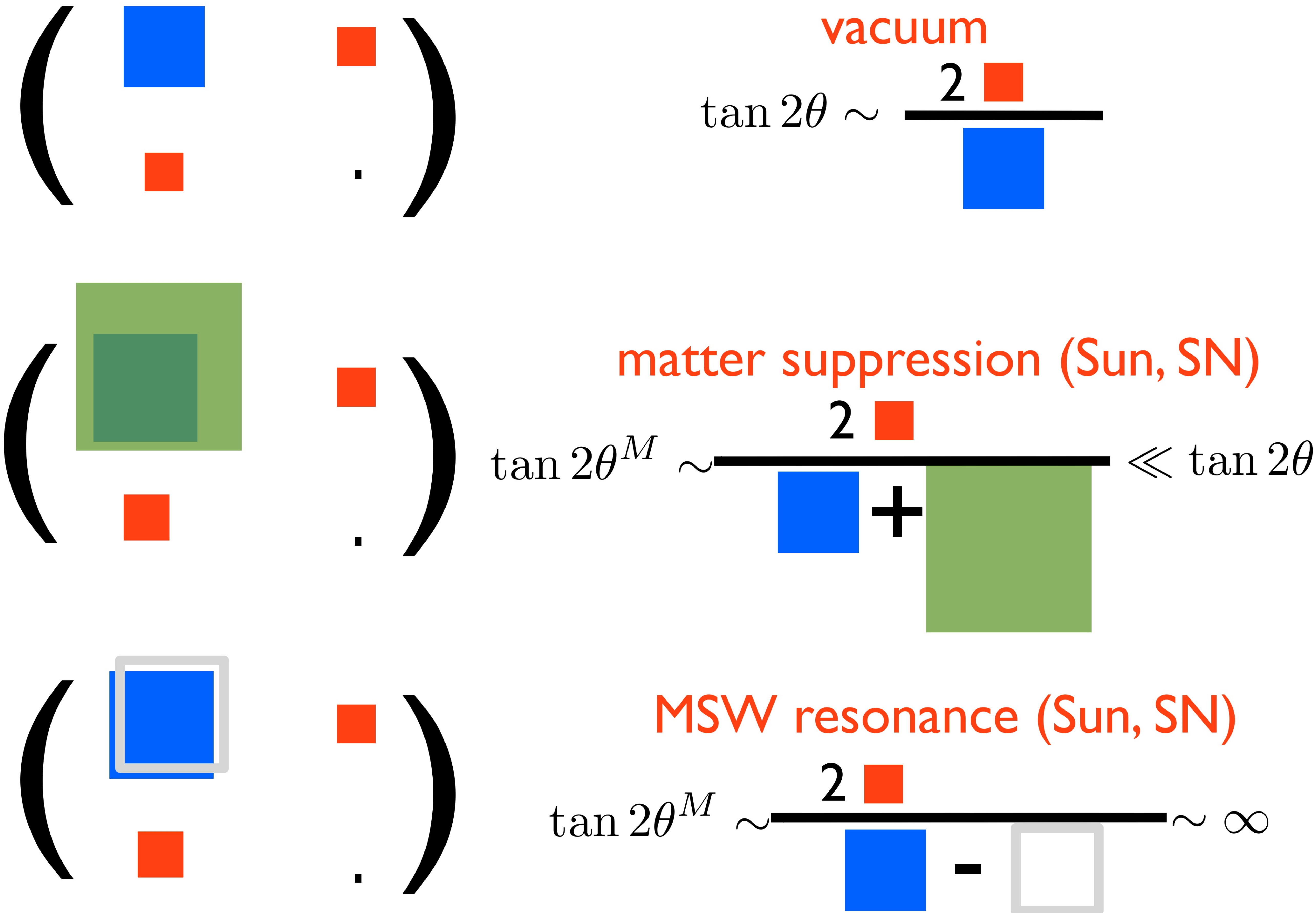
- The diagonal basis and the flavour basis are related by a unitary matrix with angle in matter

Exercise Derive

$$\tan(2\theta_m) = \frac{\frac{\Delta m^2}{2E}\sin(2\theta)}{\frac{\Delta m^2}{2E}\cos(2\theta) - \sqrt{2}G_F N_e}$$

Effective Hamiltonian

Mixing angle



- If $\sqrt{2}G_F N_e \ll \frac{\Delta m^2}{2E}\cos 2\theta$, we recover the vacuum case and $\theta_m \simeq \theta$

- If $\sqrt{2}G_F N_e \gg \frac{\Delta m^2}{2E}\cos(2\theta)$, matter effects dominate and oscillations are suppressed.

- If $\sqrt{2}G_F N_e = \frac{\Delta m^2}{2E}\cos 2\theta$: resonance and maximal mixing $\theta_m = \pi/4$

- The resonance condition can be satisfied for
  - neutrinos if $\Delta m^2 > 0$
  - antineutrinos if $\Delta m^2 < 0$

$$P(\nu_e \to \nu_\mu; t) = \sin^2(2\theta_m)\sin^2\frac{(E_A - E_B)L}{2}$$

2-neutrino oscillations with varying density

Let's consider the case in which N_e depends on time. This happens, e.g., if a beam of neutrinos is produced and then propagates through a medium of varying density (e.g. Sun, supernovae).

$$i\frac{d}{dt}\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix} = \begin{pmatrix} -\frac{\Delta m^2}{4E}\cos 2\theta + \sqrt{2}G_F N_e(t) & \frac{\Delta m^2}{4E}\sin 2\theta \\ \frac{\Delta m^2}{4E}\sin 2\theta & \frac{\Delta m^2}{4E}\cos 2\theta \end{pmatrix}\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix}$$

At a given instant of time t, the Hamiltonian can be diagonalised by a unitary transformation as before. We find the instantaneous matter basis and the instantaneous values of the energy. The expressions are exactly as before but with the angle which depends on time, $\theta(t)$.

We have

$$|\nu_\alpha\rangle = U(t)|\nu_I\rangle, \qquad U^\dagger(t) H_{m,fl} U(t) = \mathrm{diag}(E_A(t), E_B(t))$$

Starting from the Schroedinger equation, we can express it in the instantaneous basis

$$i \frac{d}{dt} \begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix} = \begin{pmatrix} \cdots \end{pmatrix} \begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix}$$

$$\downarrow |\nu_I\rangle$$

$$i \frac{d}{dt} U(t) \begin{pmatrix} |\nu_A\rangle \\ |\nu_B\rangle \end{pmatrix} = \begin{pmatrix} \cdots \end{pmatrix} U_m(t) \begin{pmatrix} |\nu_A\rangle \\ |\nu_B\rangle \end{pmatrix}$$

$$i \frac{d}{dt} \begin{pmatrix} |\nu_A\rangle \\ |\nu_B\rangle \end{pmatrix} = U^\dagger(t) \left[-i \dot{U}_m(t) + \begin{pmatrix} \cdots \end{pmatrix} U_m(t) \right] \begin{pmatrix} |\nu_A\rangle \\ |\nu_B\rangle \end{pmatrix}$$

$$i \frac{d}{dt} \begin{pmatrix} |\nu_A\rangle \\ |\nu_B\rangle \end{pmatrix} = \begin{pmatrix} E_A & -i\dot{\theta}_m \\ i\dot{\theta}_m & E_B \end{pmatrix} \begin{pmatrix} |\nu_A\rangle \\ |\nu_B\rangle \end{pmatrix}$$

Exercise

The evolution of ν_A and ν_B are not decoupled. In general, it is very difficult to find an analytical solution to this problem.

Adiabatic case

In the adiabatic case, each component evolves independently. In the non adiabatic one, the state can "jump" from one to the other.

If the evolution is sufficiently slow (adiabatic case):

$$|\dot{\theta}(t)| \ll |E_A - E_B|$$

we can follow the evolution of each component independently.

Adiabaticity condition

$$\gamma^{-1} \equiv \frac{2|\dot{\theta}|}{|E_A - E_B|} = \frac{\sin(2\theta)\frac{\Delta m^2}{2E}}{|E_A - E_B|^3}|\dot{V}_{CC}| \ll 1$$

In the Sun, typically we have $\gamma \sim \frac{\Delta m^2}{10^{-9}\mathrm{eV}^2}\frac{\mathrm{MeV}}{E_\nu}$

Solar neutrinos: MSW effect

The oscillations in matter were first discussed by L. Wolfenstein, S. P. Mikheyev, A. Yu Smirnov.

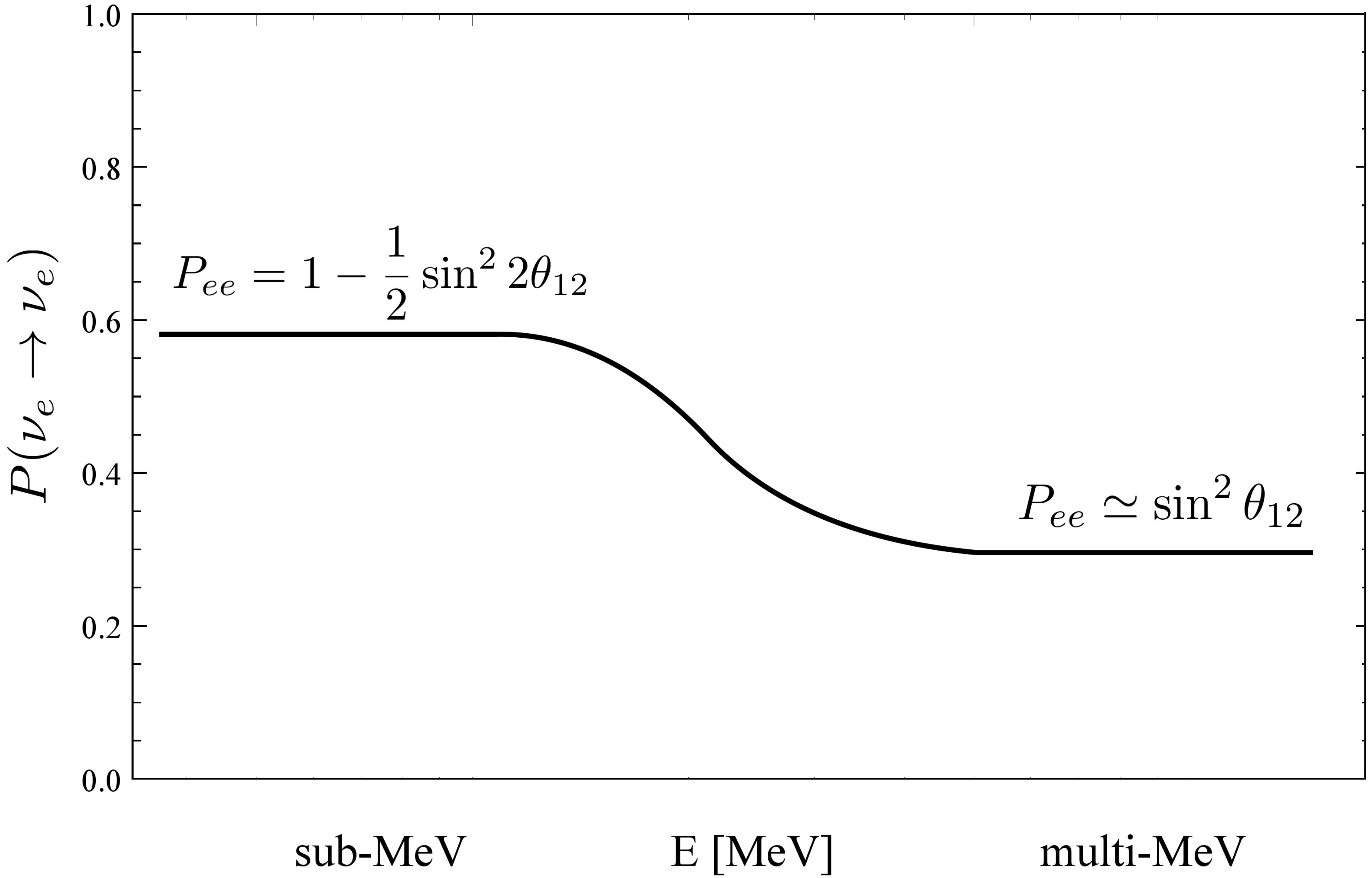
- Production in the centre of the Sun: matter effects dominate at high energy, negligible at low energy.

The probability of ν_e to be

$$\nu_A \quad \text{is} \quad \cos^2\theta_m$$
$$\nu_B \quad \text{is} \quad \sin^2\theta_m$$

If matter effects dominate, $\sin^2\theta_m \simeq 1$

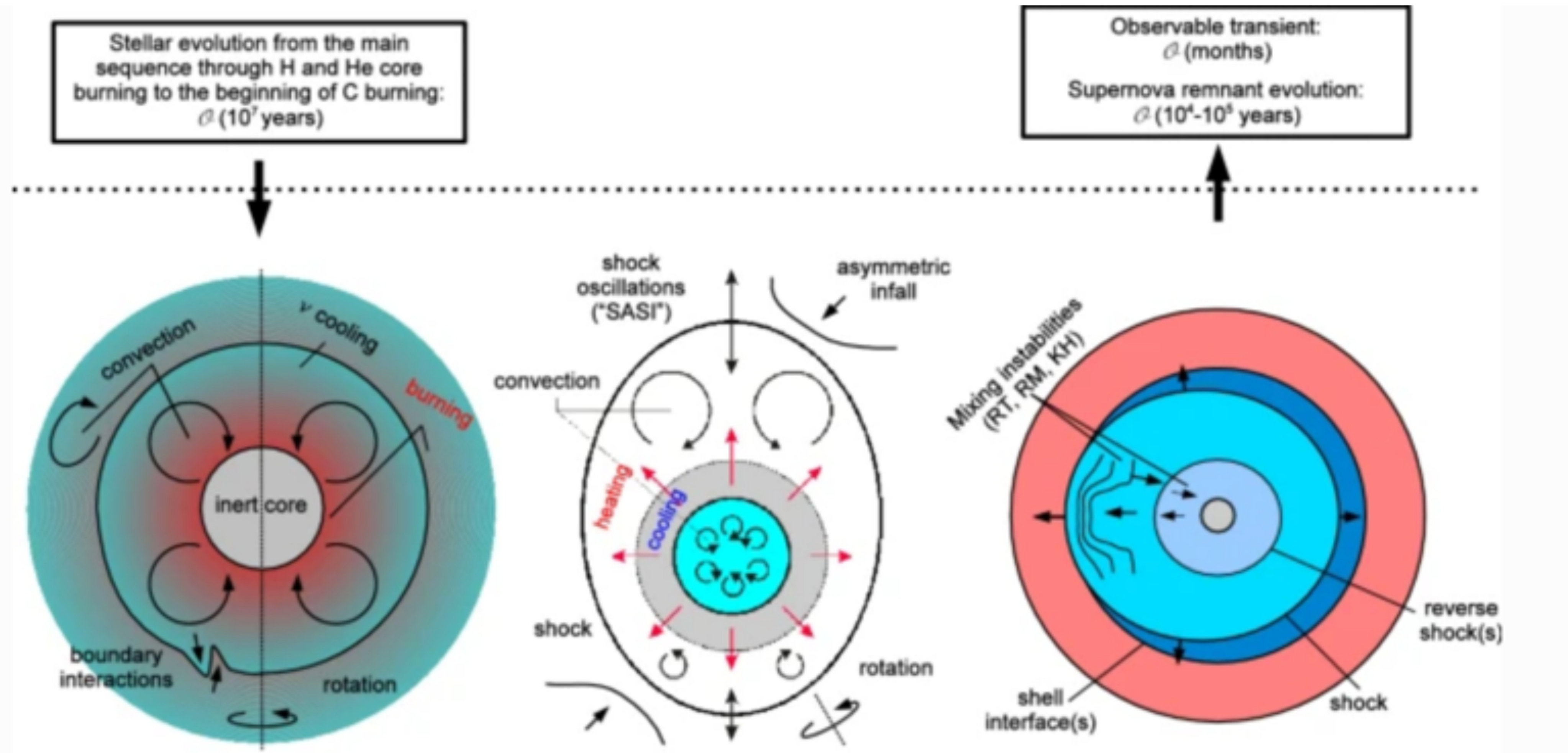
- $P(\nu_e \to \nu_e) = 1 - \frac{1}{2}\sin^2(2\theta)$ (averaged vacuum oscillations), when matter effects are negligible (low energies)
- $P(\nu_e \to \nu_e) = \sin^2\theta$ (dominant matter effects and adiabaticity) (high energies)


1.0
0.8
0.6
0.4
0.2
0.0
$P(\nu_e \to \nu_e)$
$P_{ee} = 1 - \frac{1}{2}\sin^2 2\theta_{12}$
$P_{ee} \simeq \sin^2 \theta_{12}$
sub-MeV
E [MeV]
multi-MeV

*SN neutrino oscillations****

At end of the star evolution, the core of the star is made mainly of onion shells with an iron core at the centre.

See I. Tamborra's lectures



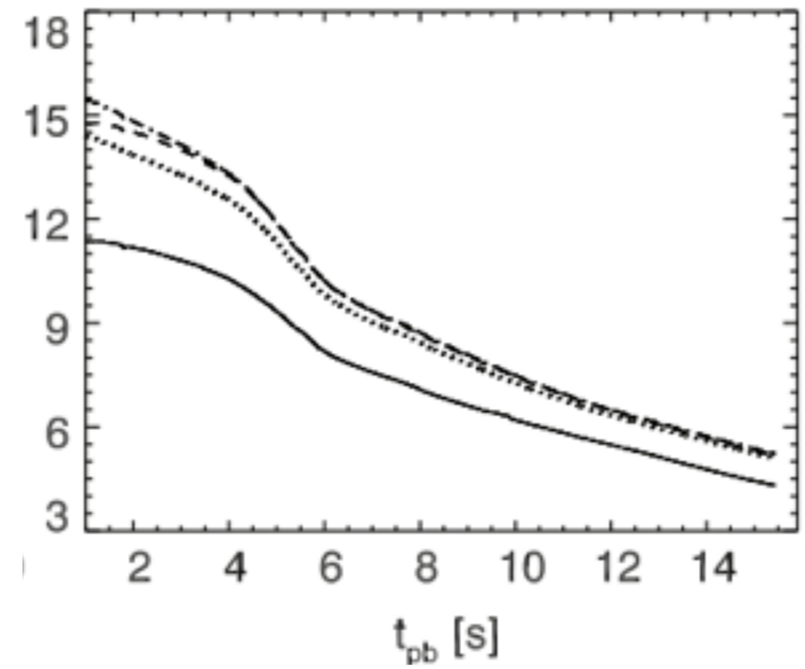
(a) C, Ne, O, and Si core and shell burning (years to hours)
(b) Core collapse and shock revival (hundreds of milliseconds)
(c) Explosion phase up to shock breakout (hours to days)

B. Muller

See I. Tamborra's lectures

Most of the SN energy (~99%) is carried away by neutrinos which can be emitted by the inner parts of the SN core.

Neutrinos are mainly emitted in three phases:
the neutronisation burst,
the accretion phase and
the cooling phases.



I. Tamborra, 1604.07332

Neutrinos undergo oscillations in matter while going from the inner parts of the core to the free space. These effects are very complex but can tell us information on the dynamics of the SN evolution.

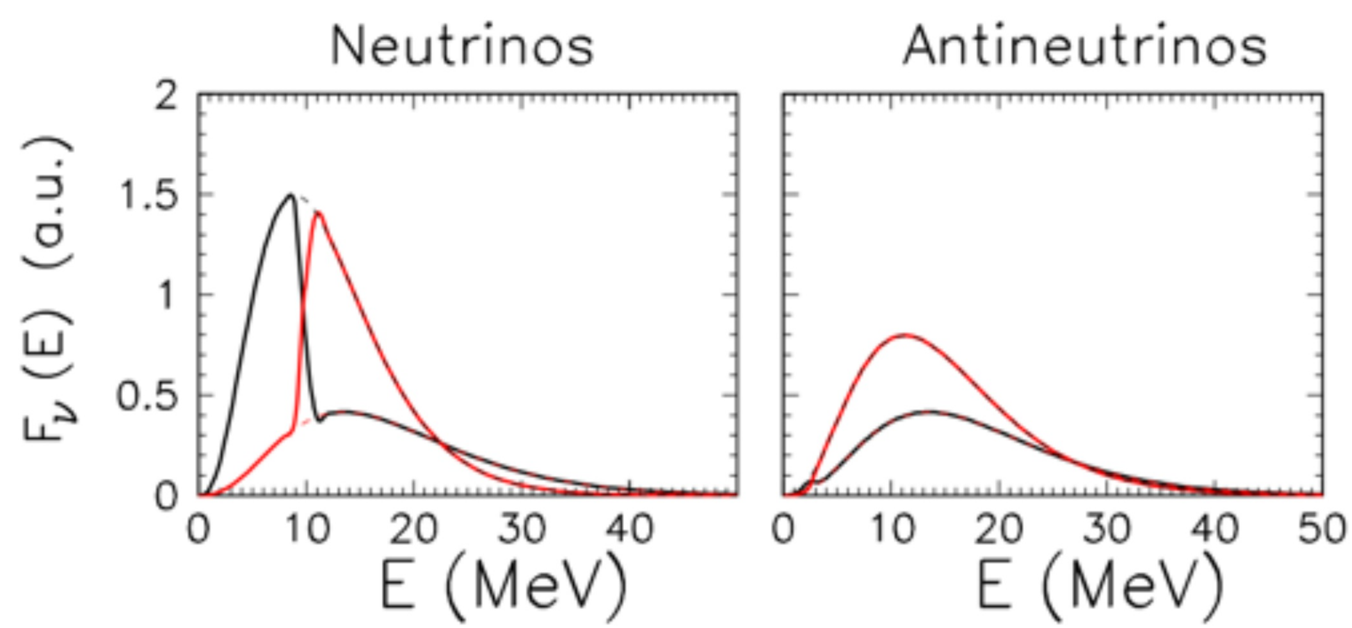


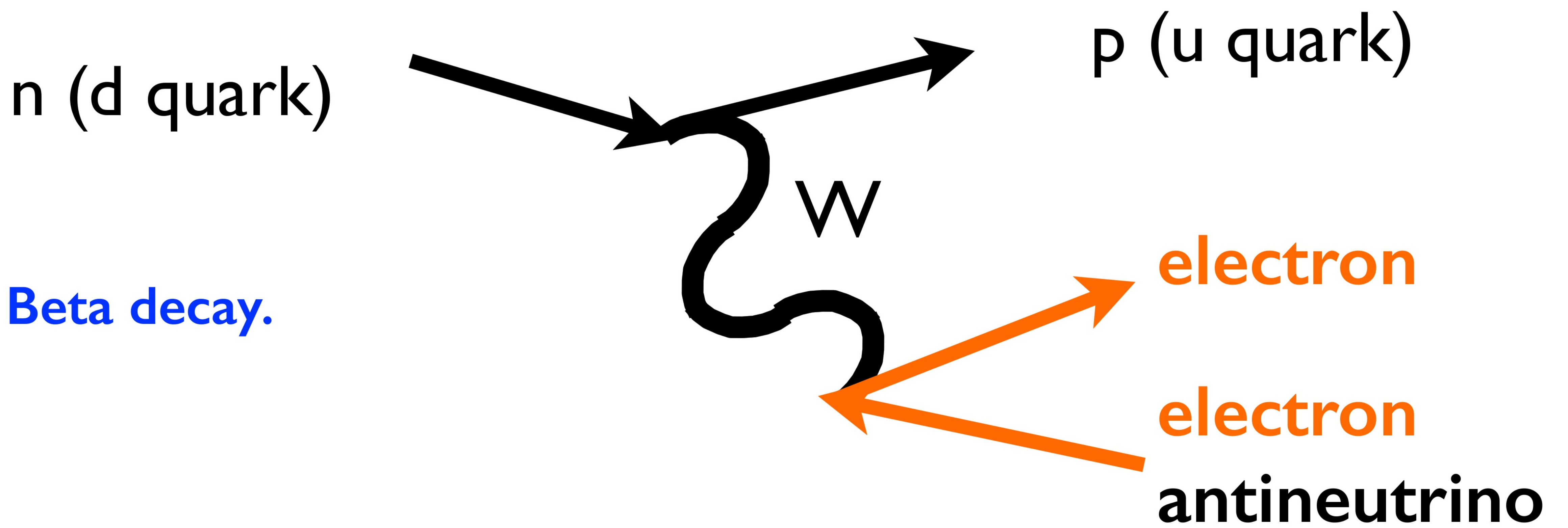
Fig. 25. – Two-flavor single-angle simulation in inverted hierarchy for an initial flux ordering $F^0_{\nu_e} : F^0_{\bar{\nu}_e} : F^0_{\nu_x} = 2.40 : 1.60 : 1.0$. Initial energy spectra for ν_e (black dashed curves) and ν_x (light dashed curves) and after collective oscillations for ν_e (black continuous curves) and ν_x (light continuous curves) at $r = 350$ km.

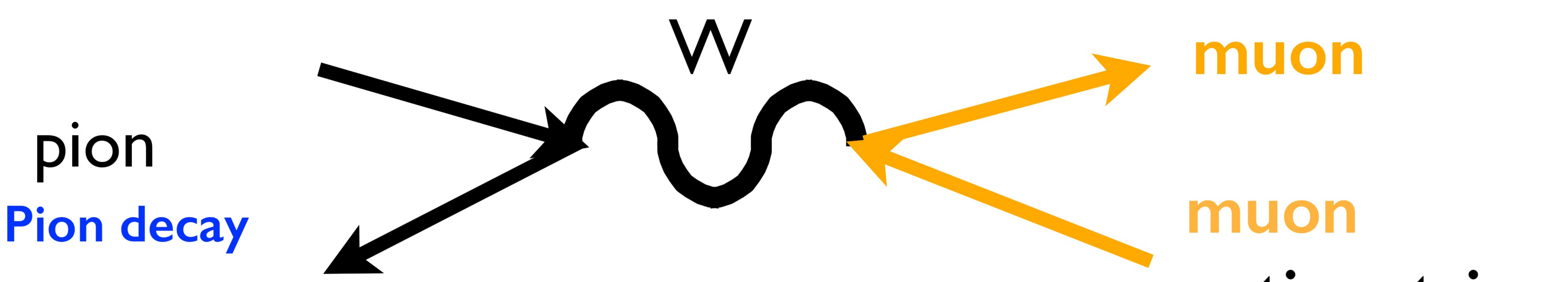
A. Mirizzi et al., 1508.00785

Neutrinos oscillations in experiments

Neutrino production

In CC (NC) SU(2) interactions, the W boson (Z boson) will be exchanged leading to the production of neutrinos.

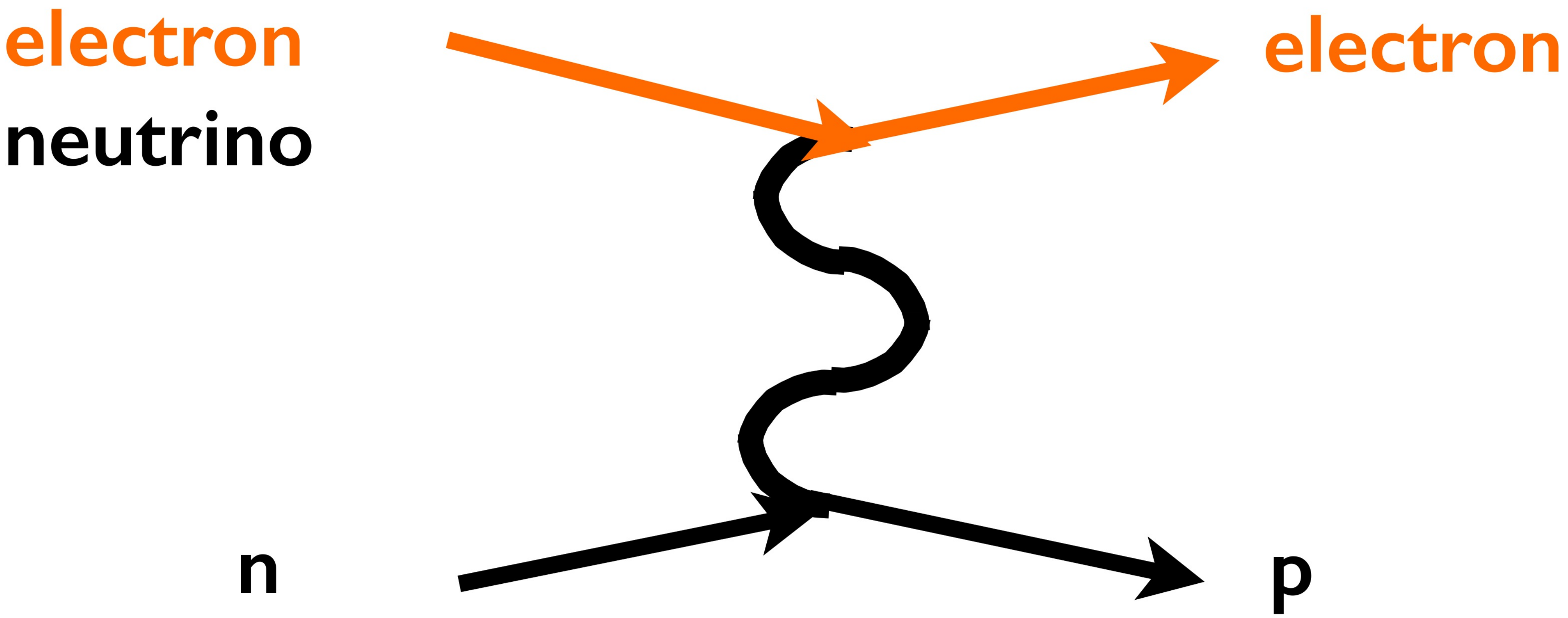




Decay into electrons is suppressed.

Neutrino detection

Neutrino detection proceeds via CC (and NC) SU(2) interactions. Example:

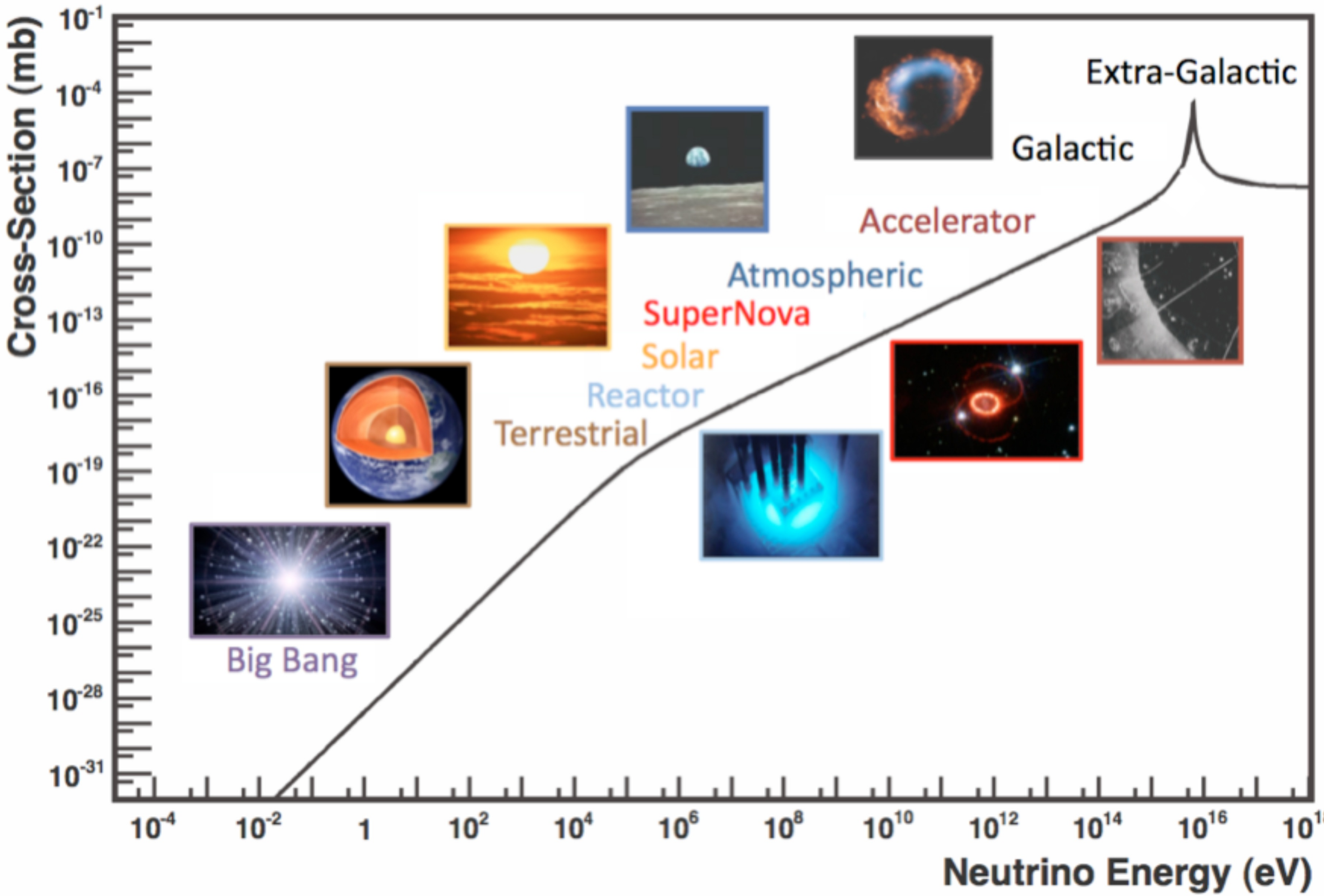


Notice that the leptons have different masses:

m_e = 0.5 MeV < m_{mu} = 105 MeV < m_{tau}= 1700 MeV

A certain lepton will be produced in a CC only if the neutrino has sufficient energy.

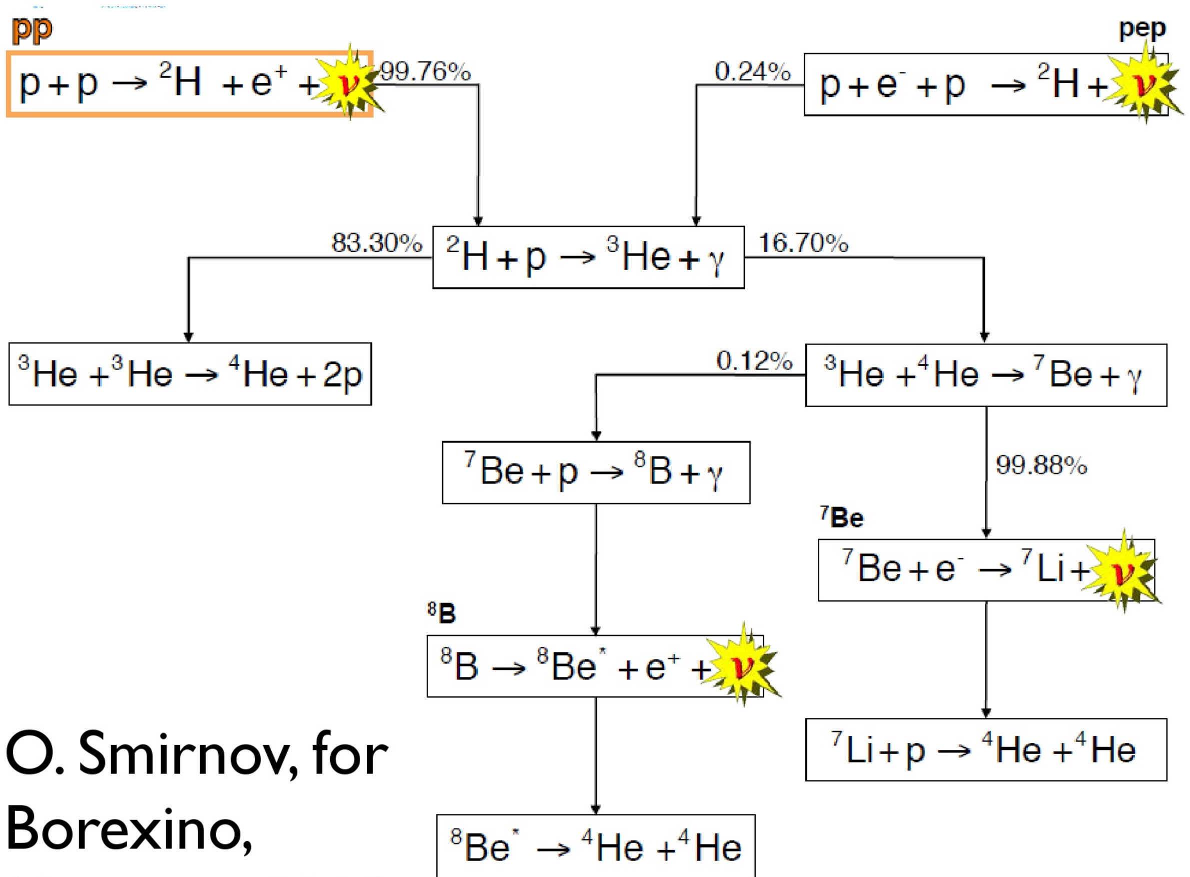
Neutrino sources



J. Formaggio and S. Zeller, 1305.7513

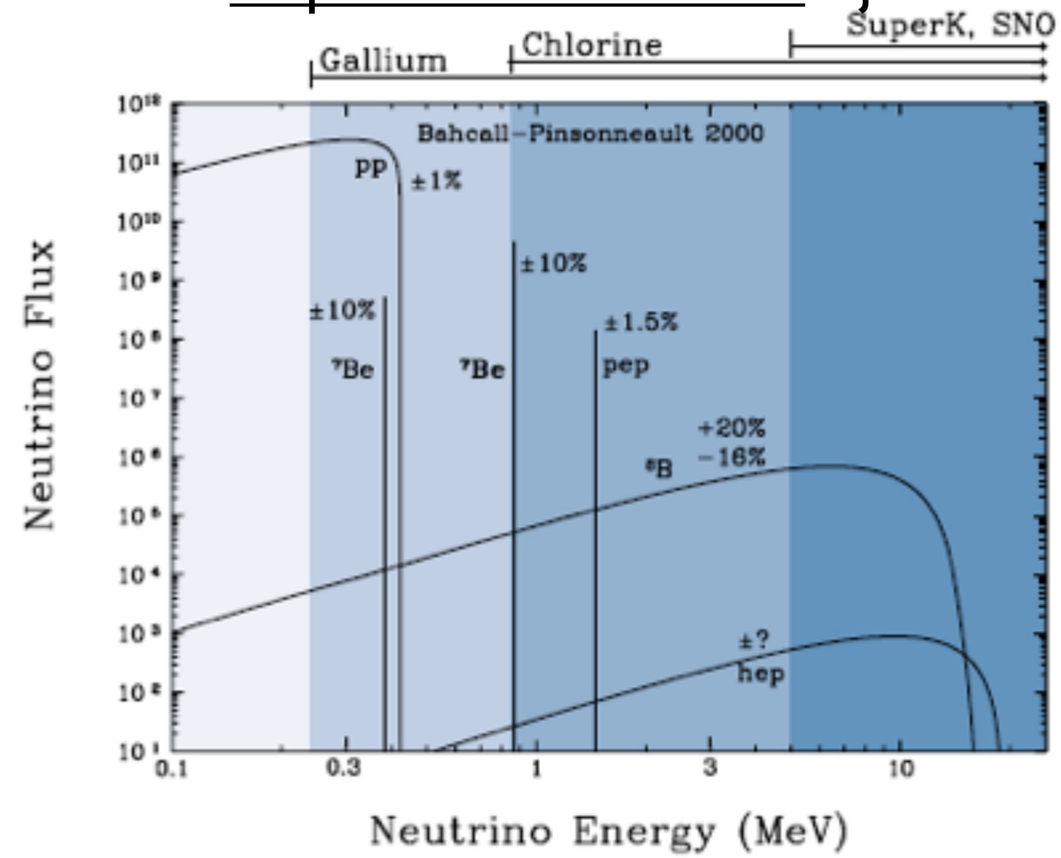
Solar neutrinos

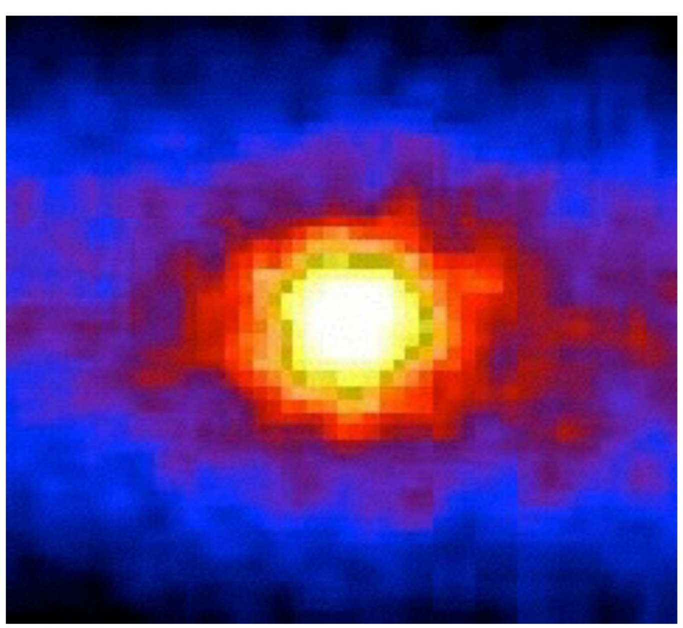
Electron neutrinos are copiously produced in the Sun.



O. Smirnov, for Borexino, Neutrino 2018

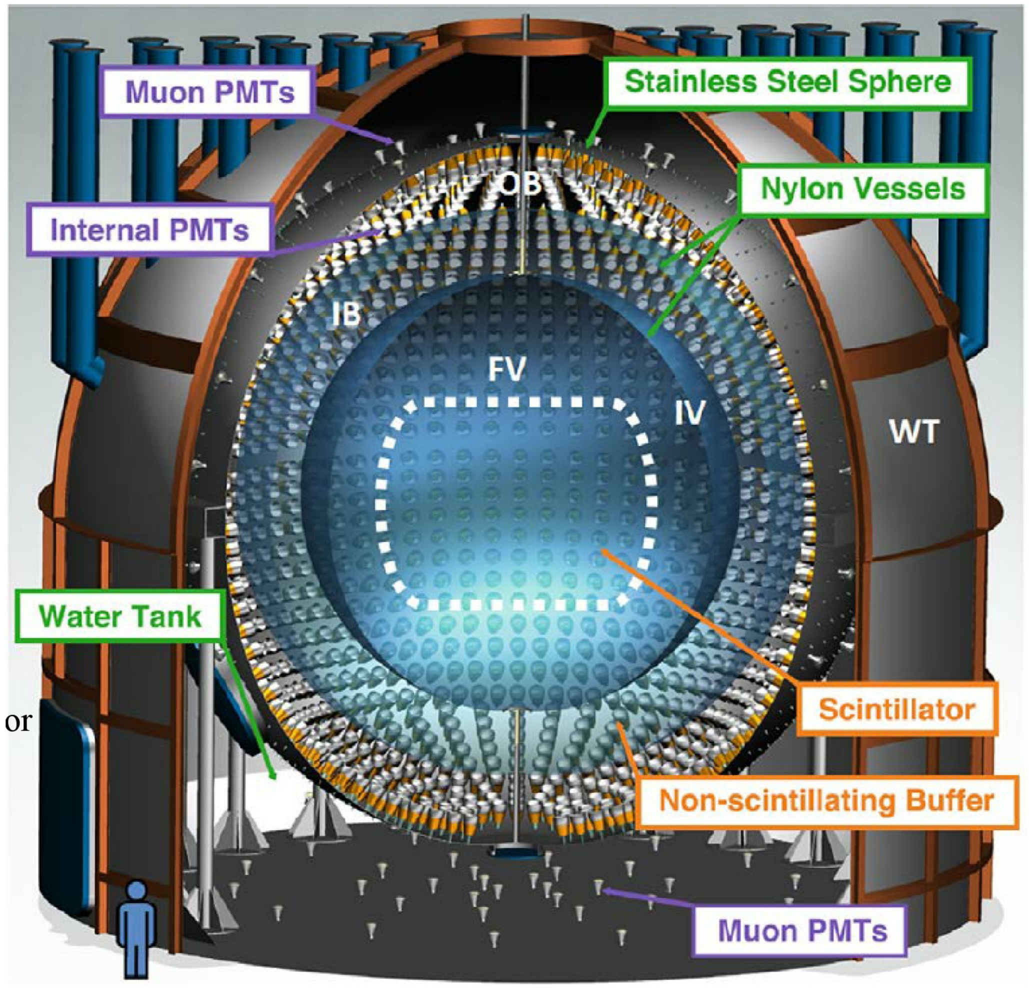
http://www.sns.ias.edu/~jnb/

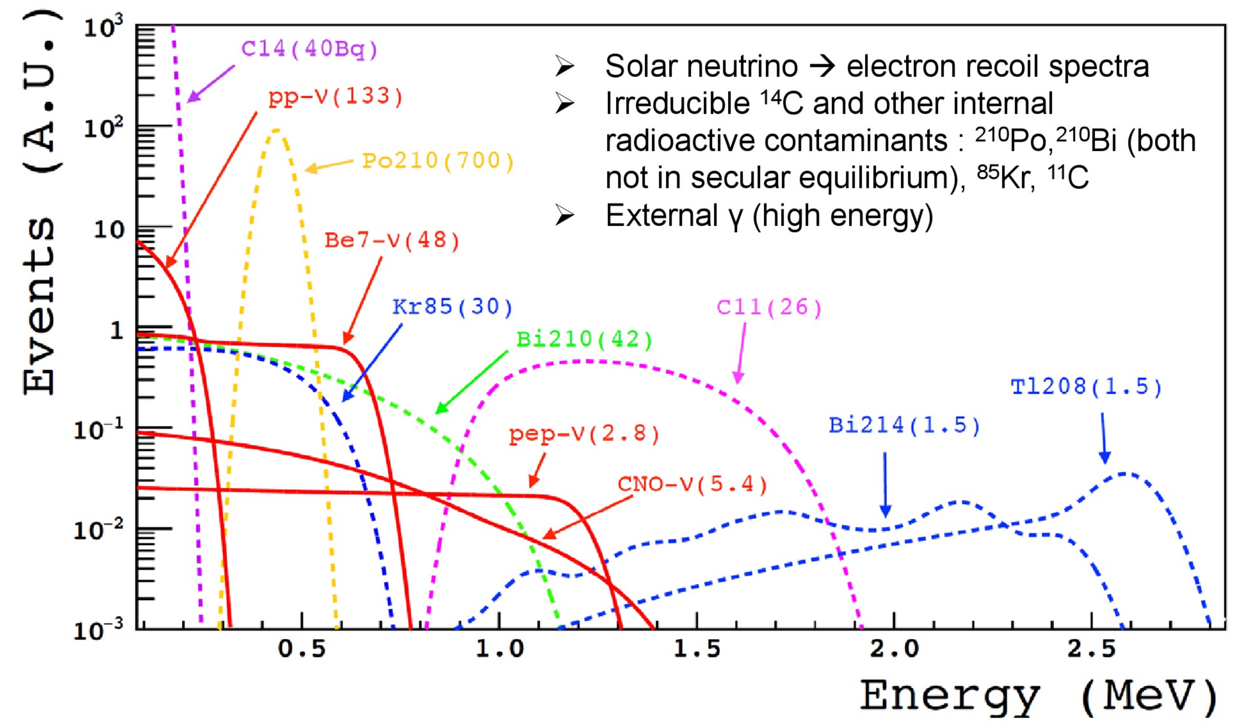




Super-Kamiokande

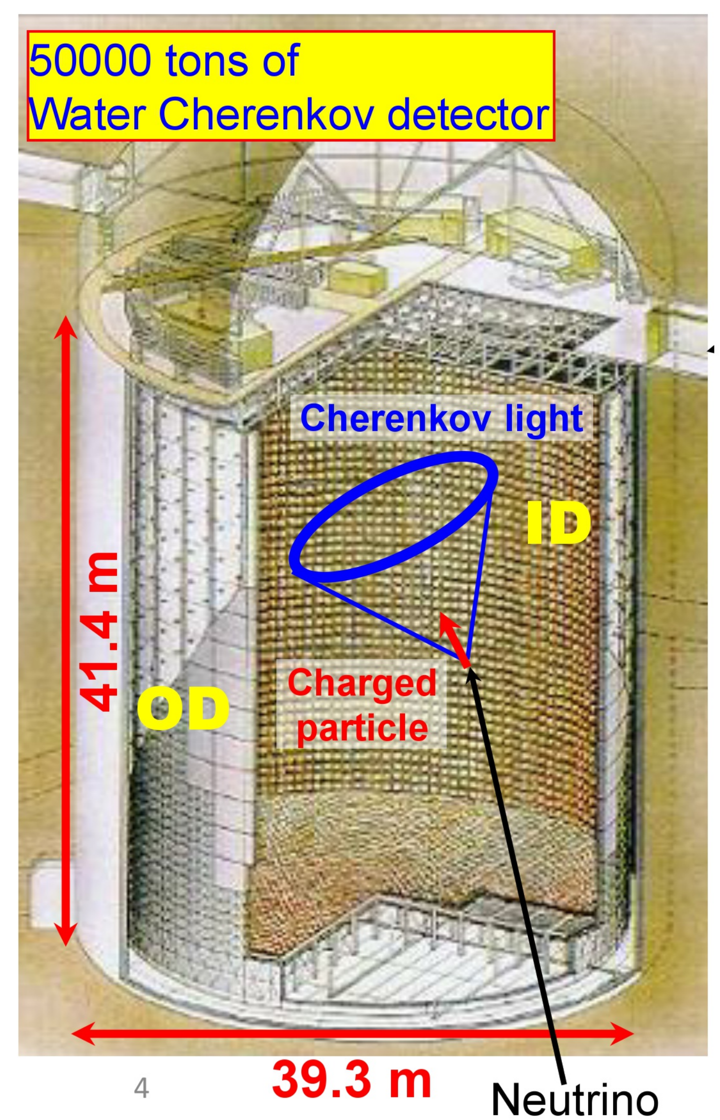
- Energies: 0.1-10 MeV.
- One can observed CC ν_e and NC: measuring the oscillation disappearance and the overall flux.





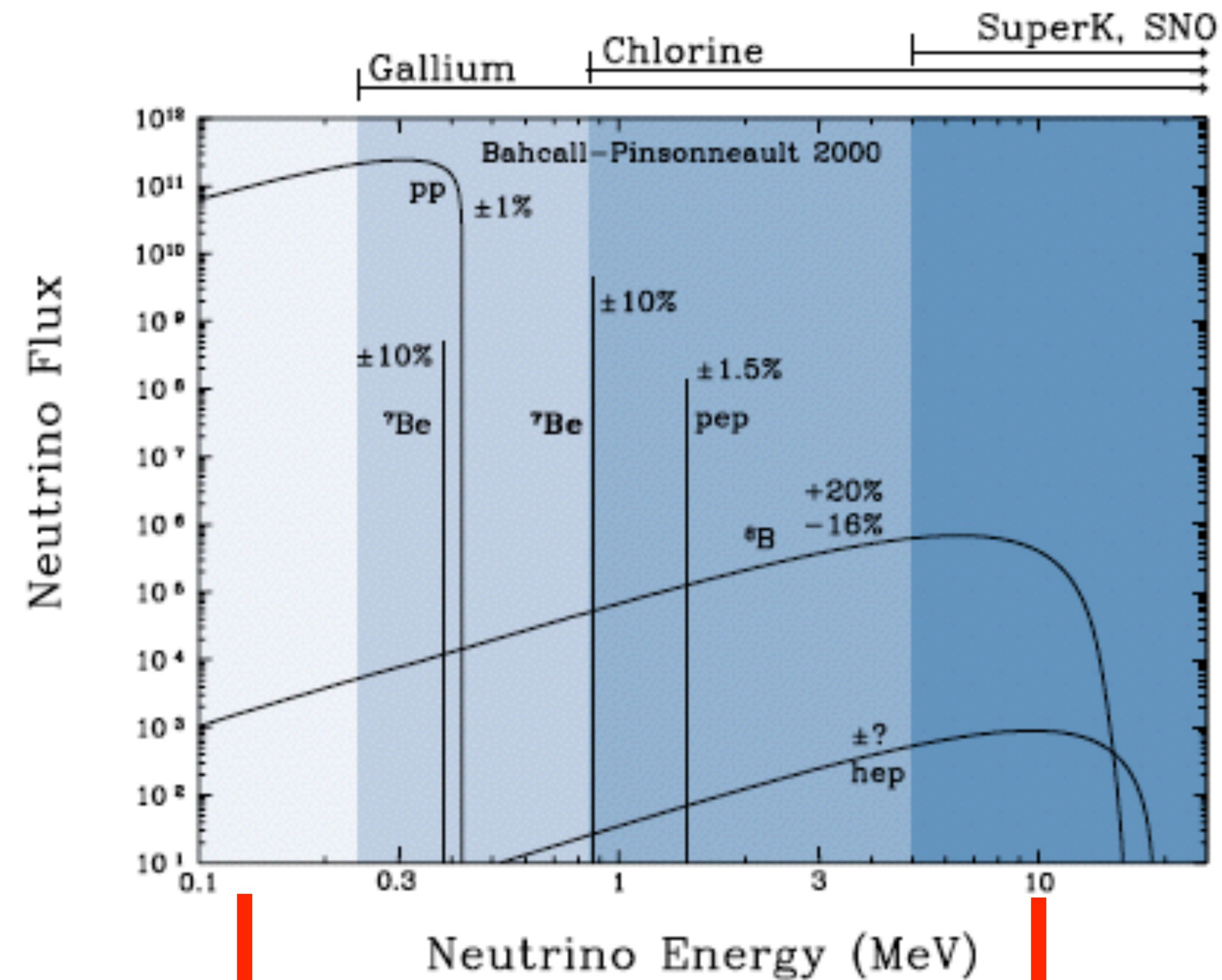
G. Ranucci, for Borexino, Neutrino 2020

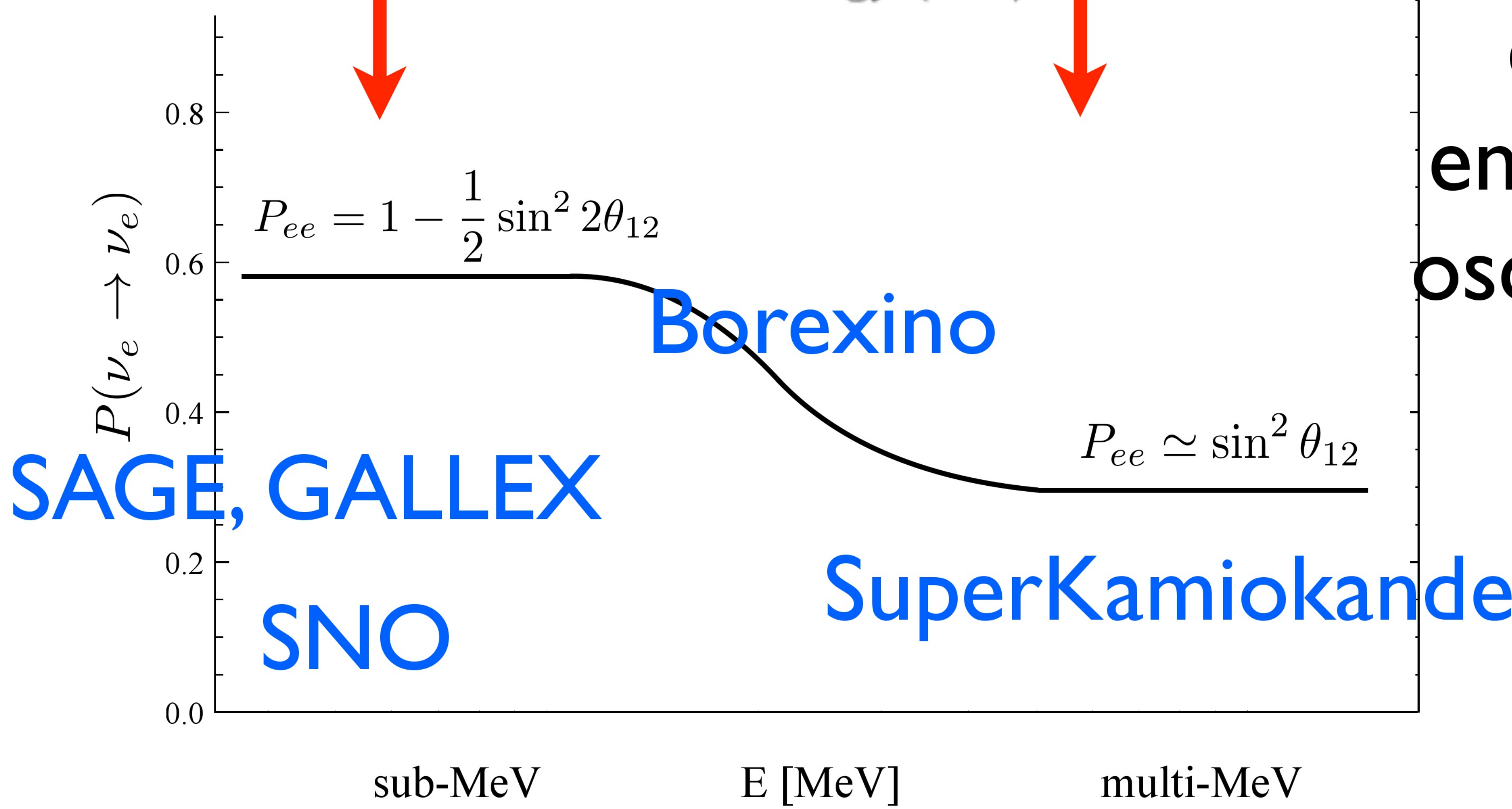
Solar ν	Flux $[\mathrm{cm}^{-2}\mathrm{s}^{-1}]$
pp	$(6.1 \pm 0.5\ ^{+0.3}_{-0.5}) \times 10^{10}$
^{7}Be	$(4.99 \pm 0.13\ ^{+0.07}_{-0.10}) \times 10^{9}$
pep (HZ)	$(1.27 \pm 0.19\ ^{+0.08}_{-0.12}) \times 10^{8}$
pep (LZ)	$(1.39 \pm 0.19\ ^{+0.08}_{-0.13}) \times 10^{8}$
CNO	$< 7.9 \times 10^{8}$ (95% C.L.)



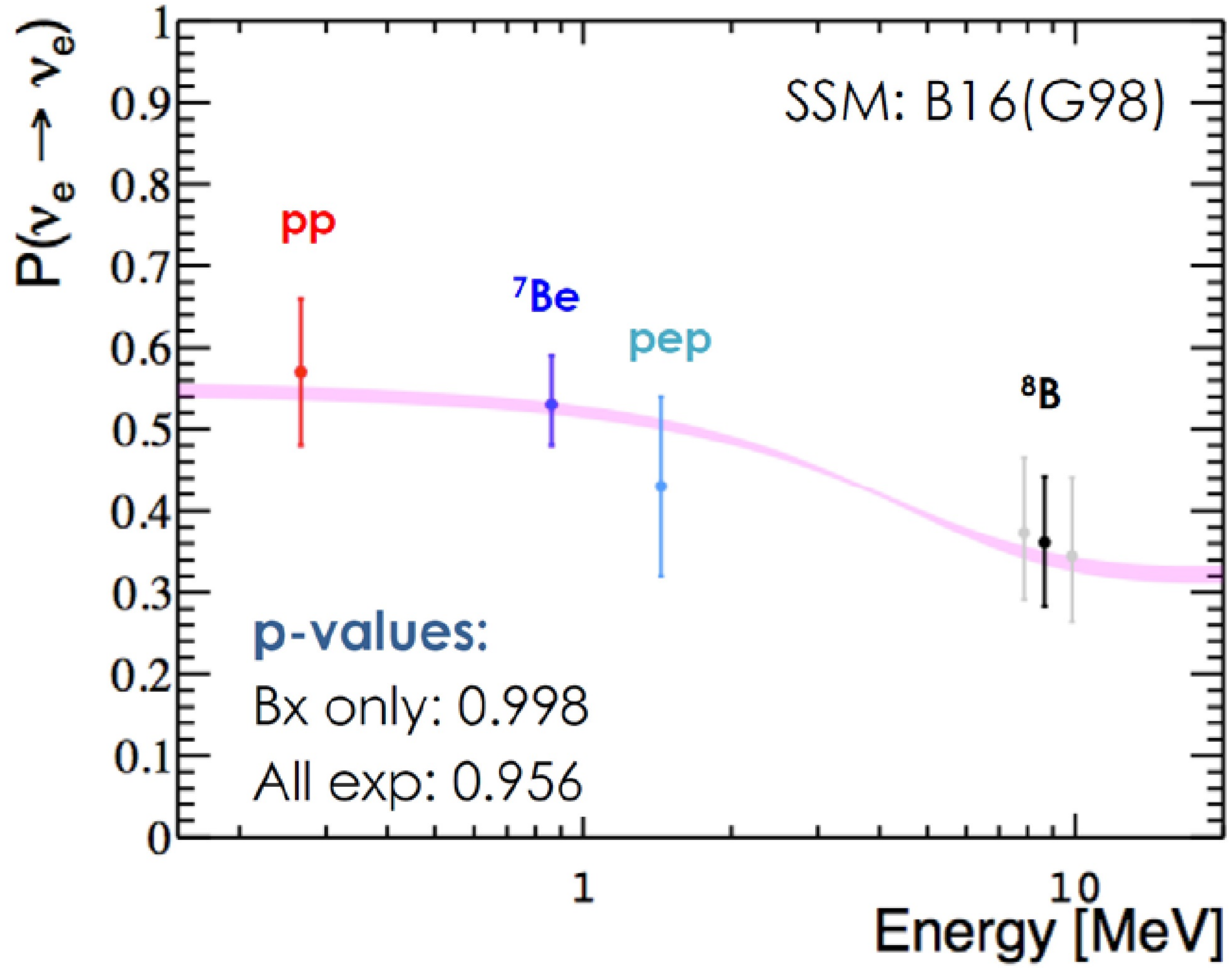
Elastic neutrino scattering in WC: directional information

M. Ikeda, for Super-Kamiokande, Neutrino 2018



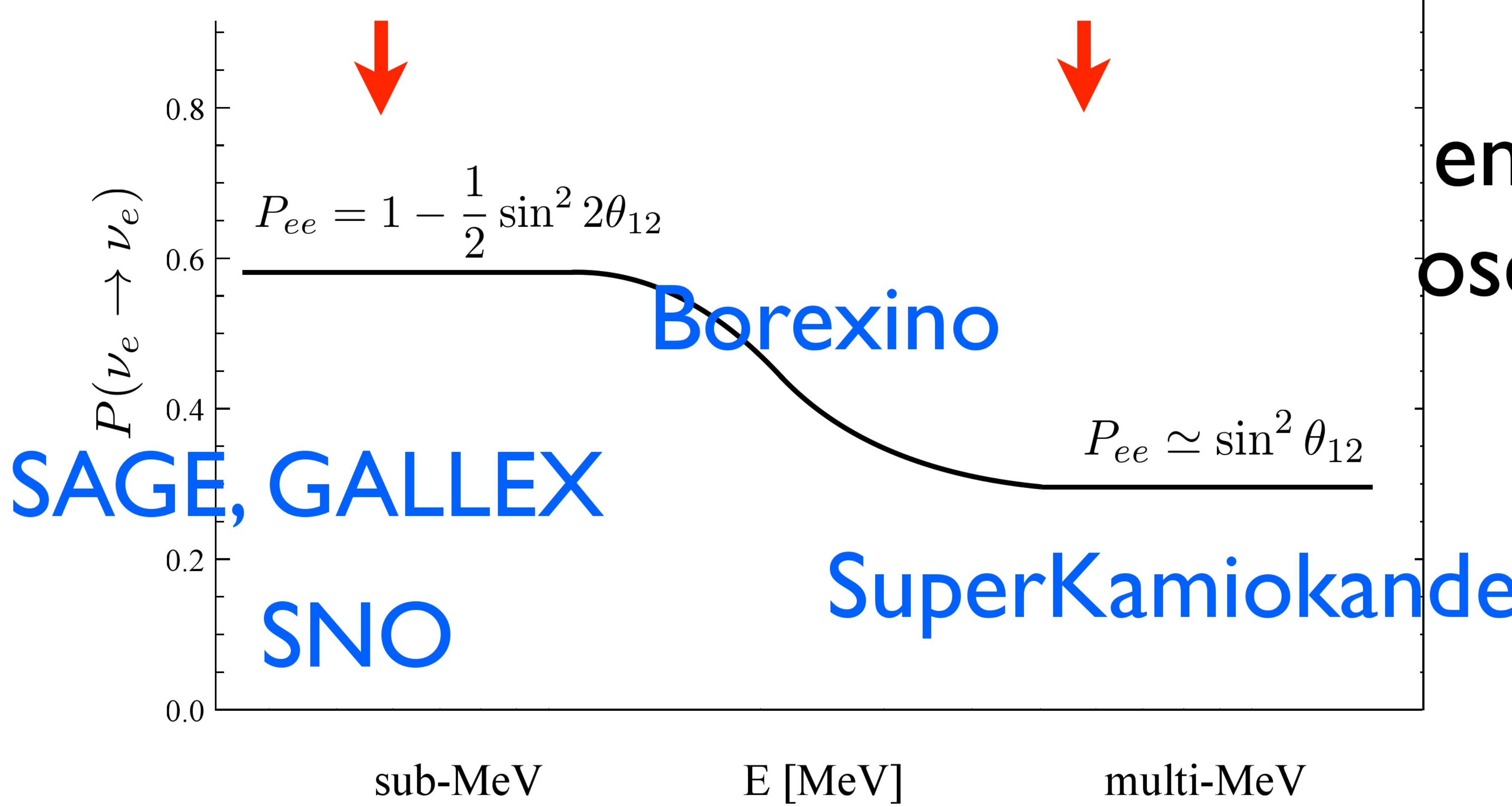


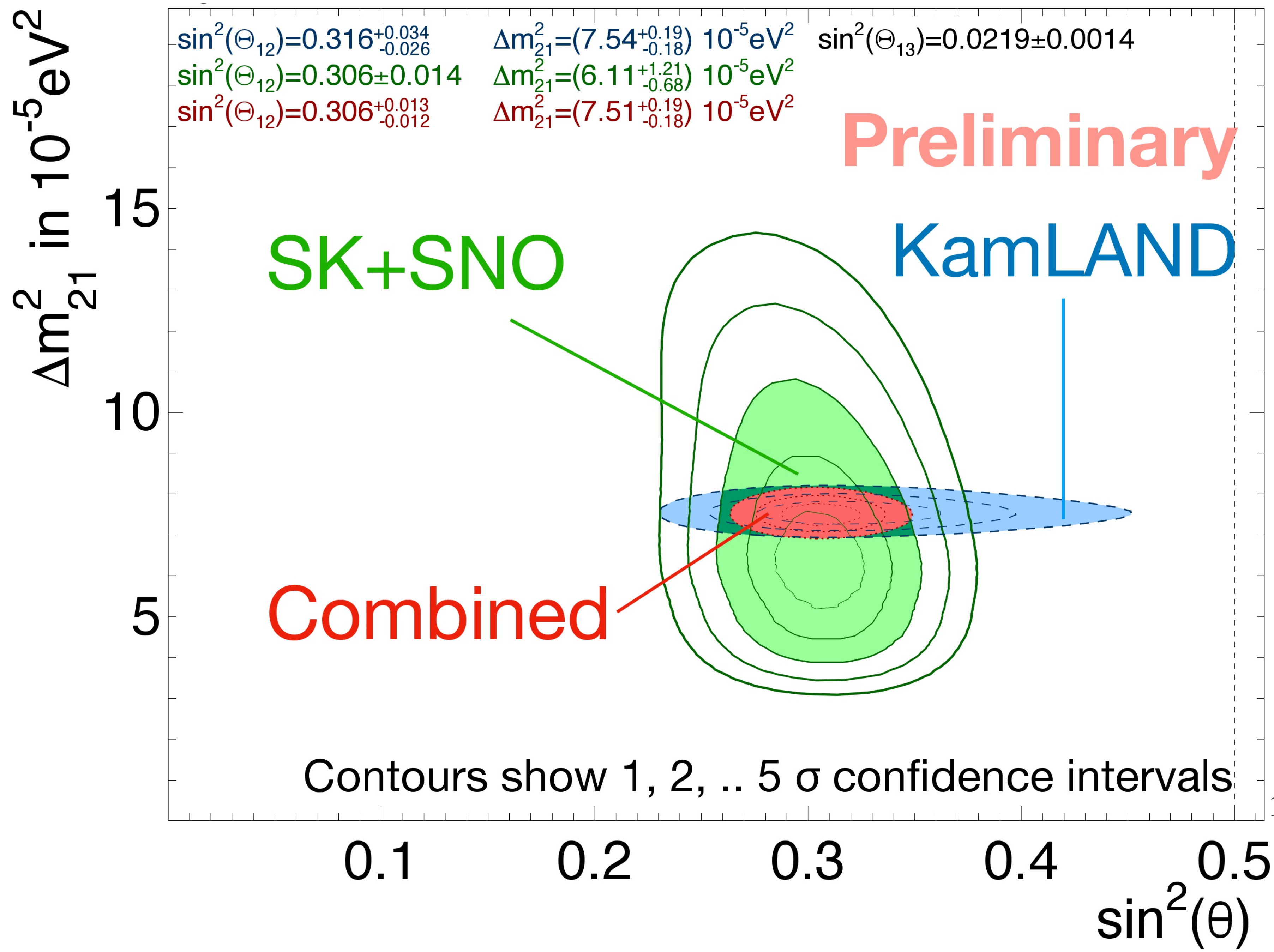
Solar neutrinos have energies which go from vacuum oscillations to adiabatic resonance. MSW effect at high energies, vacuum oscillations at low energy.



O. Smirnov, for Borexino, Neutrino 2018

Solar neutrinos have energies which go from vacuum oscillations to adiabatic resonance. MSW effect at high energies, vacuum oscillations at low energy.





Y. Nakajima, for Super-Kamiokande, Neutrino 2020

Solar experiments best constrain the "solar mixing" angle of theta$_{12}$ to be large (but non-maximal). The mass squared difference is around 7×10^{-5} eV^2.

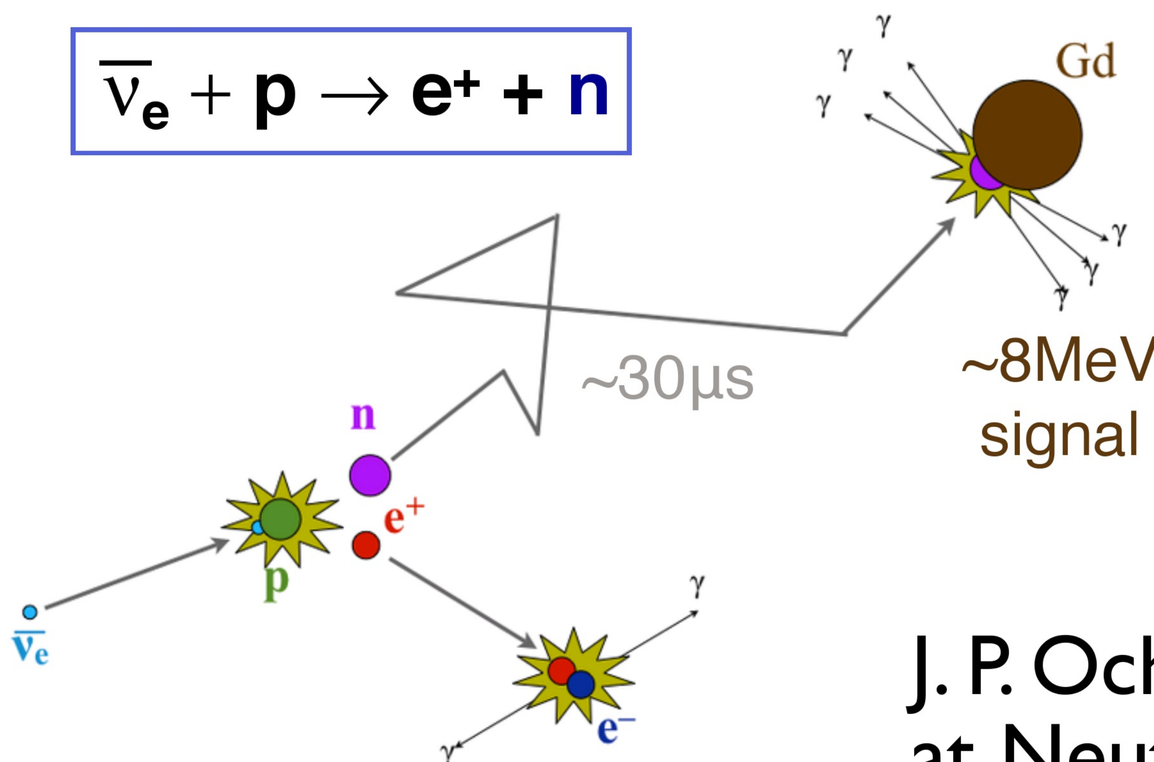
Reactor neutrinos

Copious amounts of electron antineutrinos are produced from reactors.

- Typical energy: 1-3 MeV;
- Typical distances: 1 km (Daya Bay, DCHOOZ, RENO) —100 km (KamLAND).

- At these energies inverse beta decay interactions dominate.

$$\overline{\nu}_e + p \rightarrow e^+ + n$$



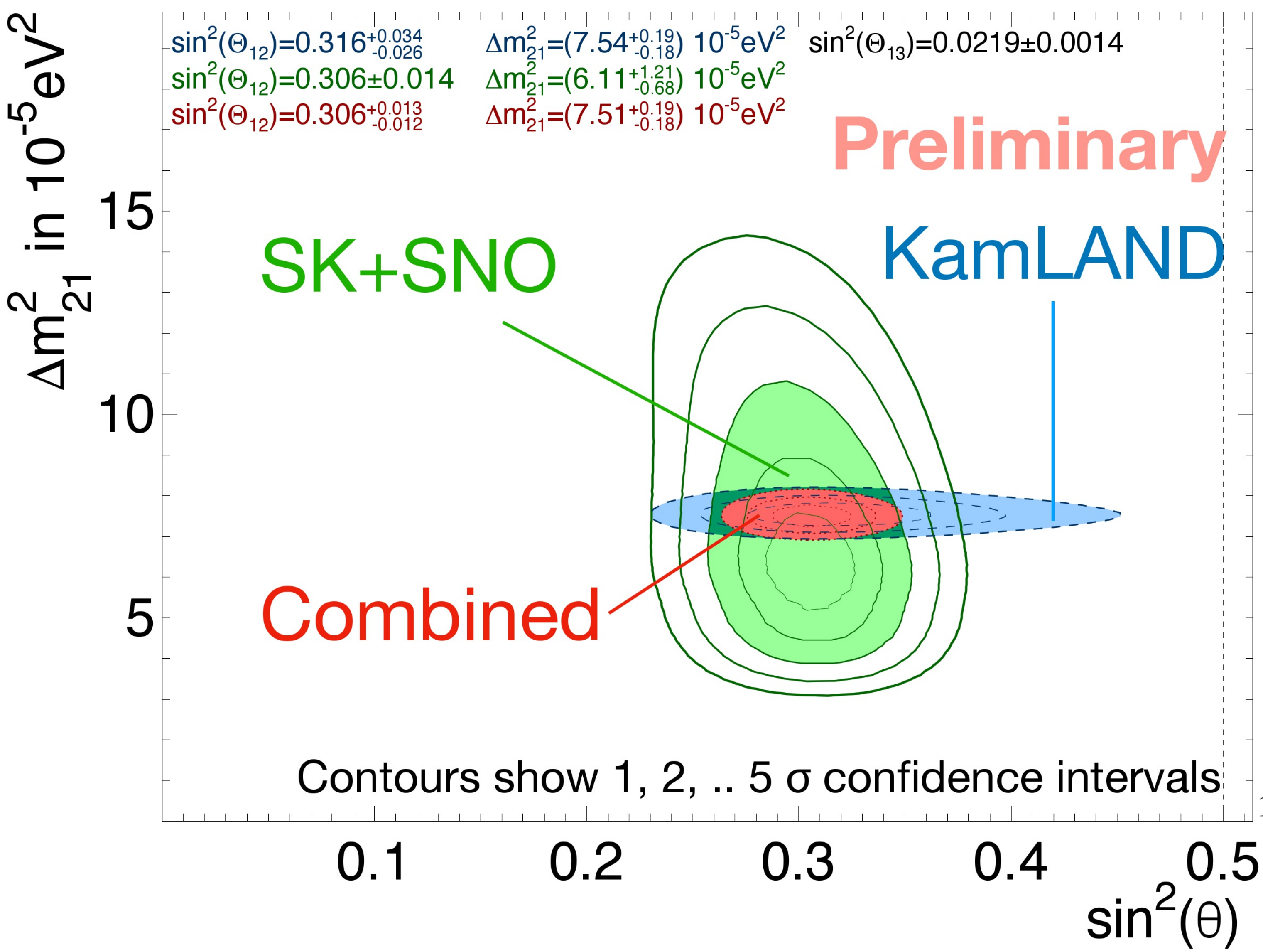
These detectors use liquid scintillator.

J. P. Ochoa-Ricoux, for Daya Bay, at Neutrino 2018

At ~100 km, the disappearance probability is

$$P(\bar{\nu}_e \to \bar{\nu}_e; t) \simeq c_{13}^4 \left(1 - \sin^2(2\theta_{12}) \sin^2 \frac{\Delta m^2_{21} L}{4E}\right) + s_{13}^4$$

Sensitivity to Delta m^2_{21}.

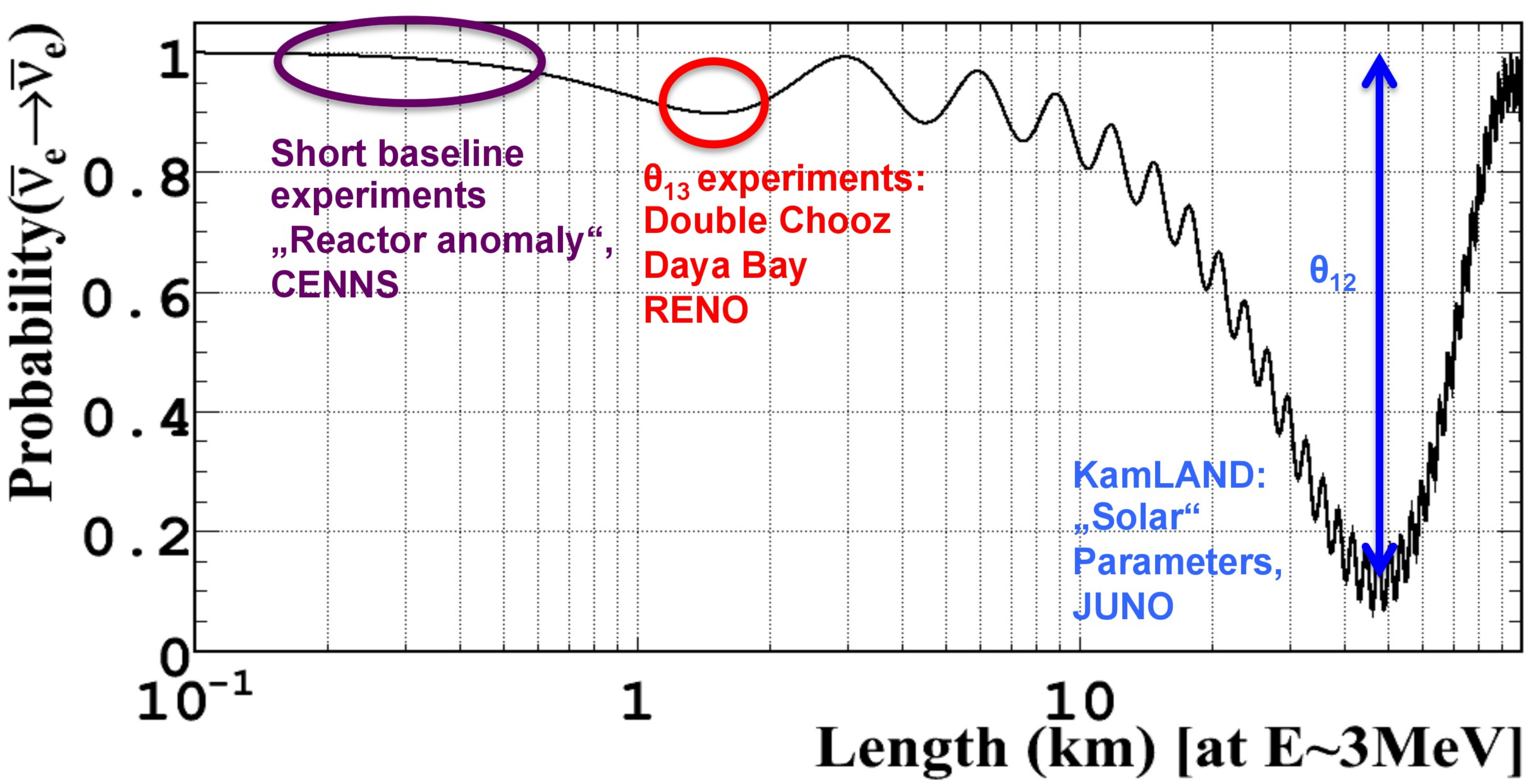


Y. Nakajima, for Super-Kamiokande, Neutrino 2020

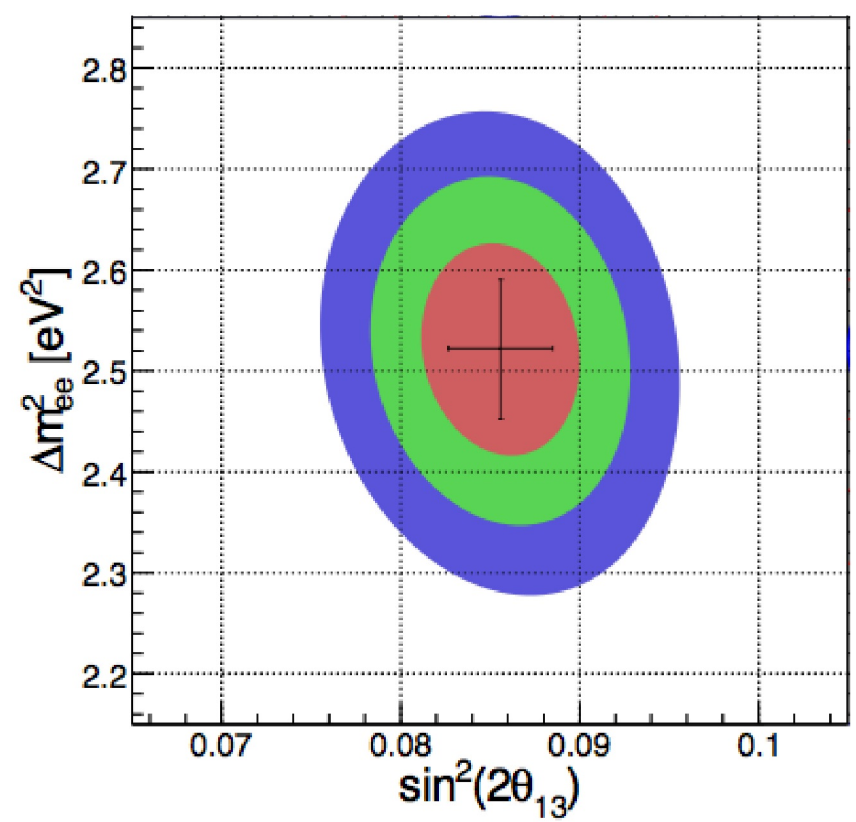
At ~few km, the disappearance probability is

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e; t) \quad = \quad 1 - \sin^2(2\theta_{13}) \sin^2 \frac{\Delta m^2_{31} L}{4E}$$

Sensitivity to θ_{13}. Reactors played an important role in the discovery of θ_{13} and in its precise measurement.



C. Buck, for DoubleCHOOZ Coll., at Neutrino 2018

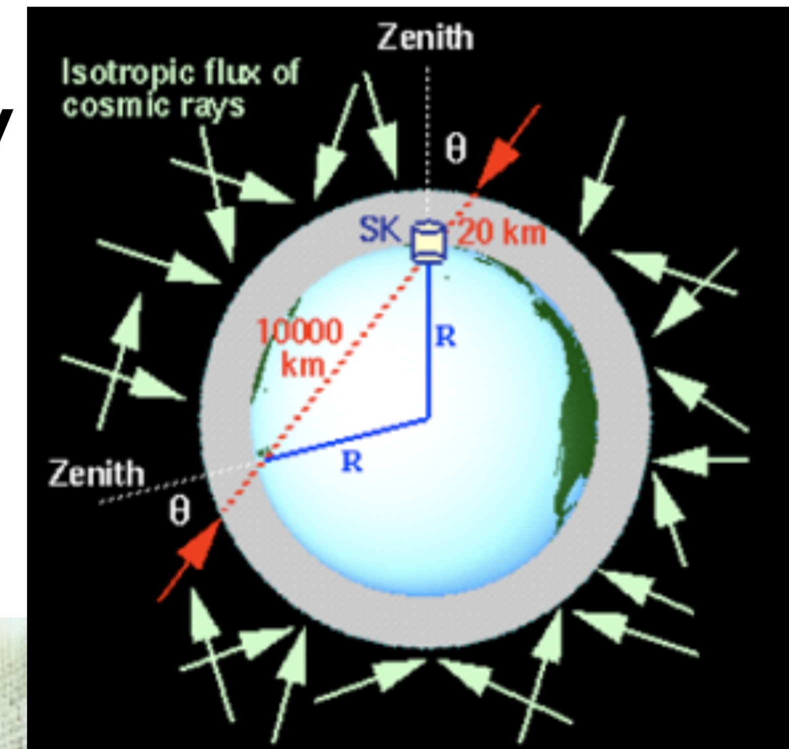


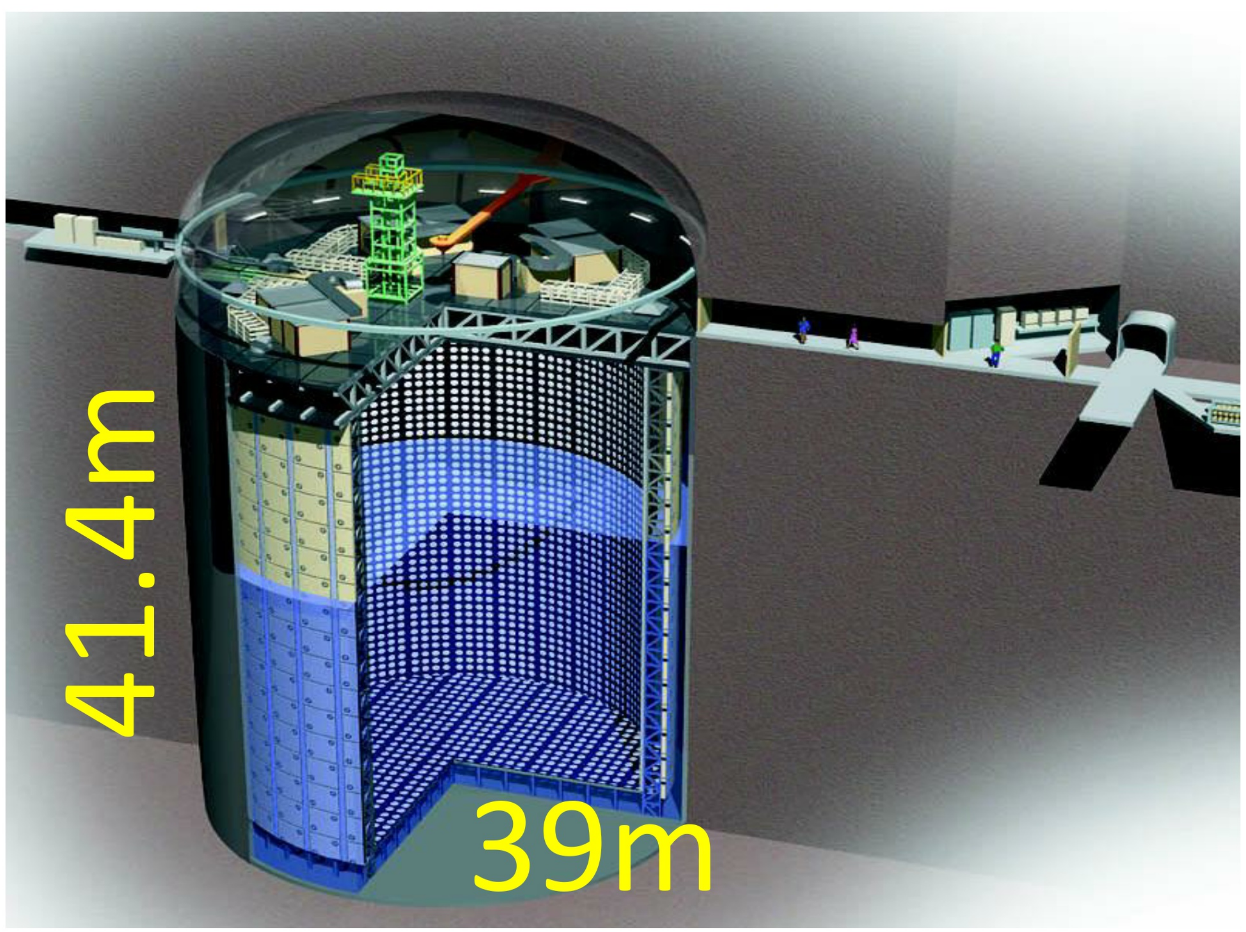
Days Bay coll., PRL 121 (2018)

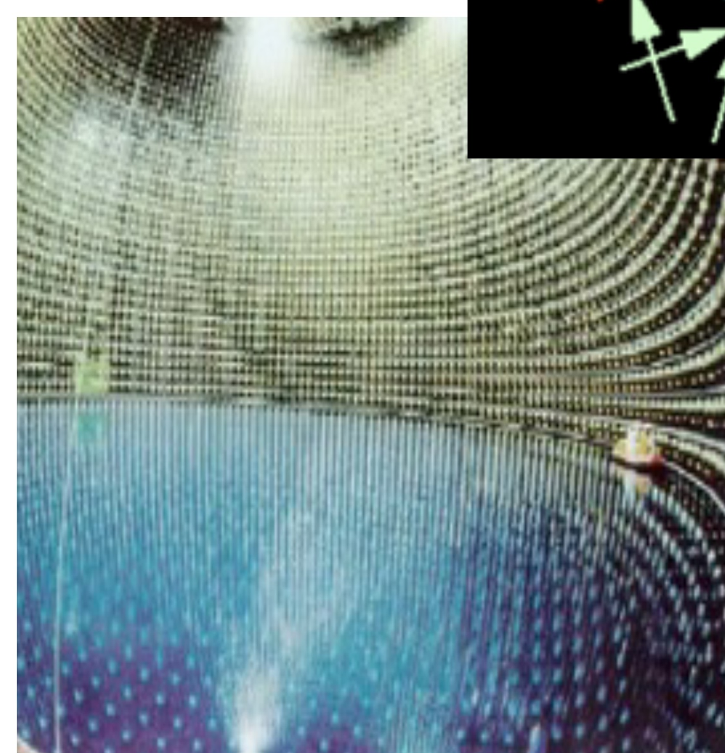
Atmospheric neutrinos

Cosmic rays hit the atmosphere and produce pions (and kaons) which decay producing lots of muon and electron (anti-) neutrinos.

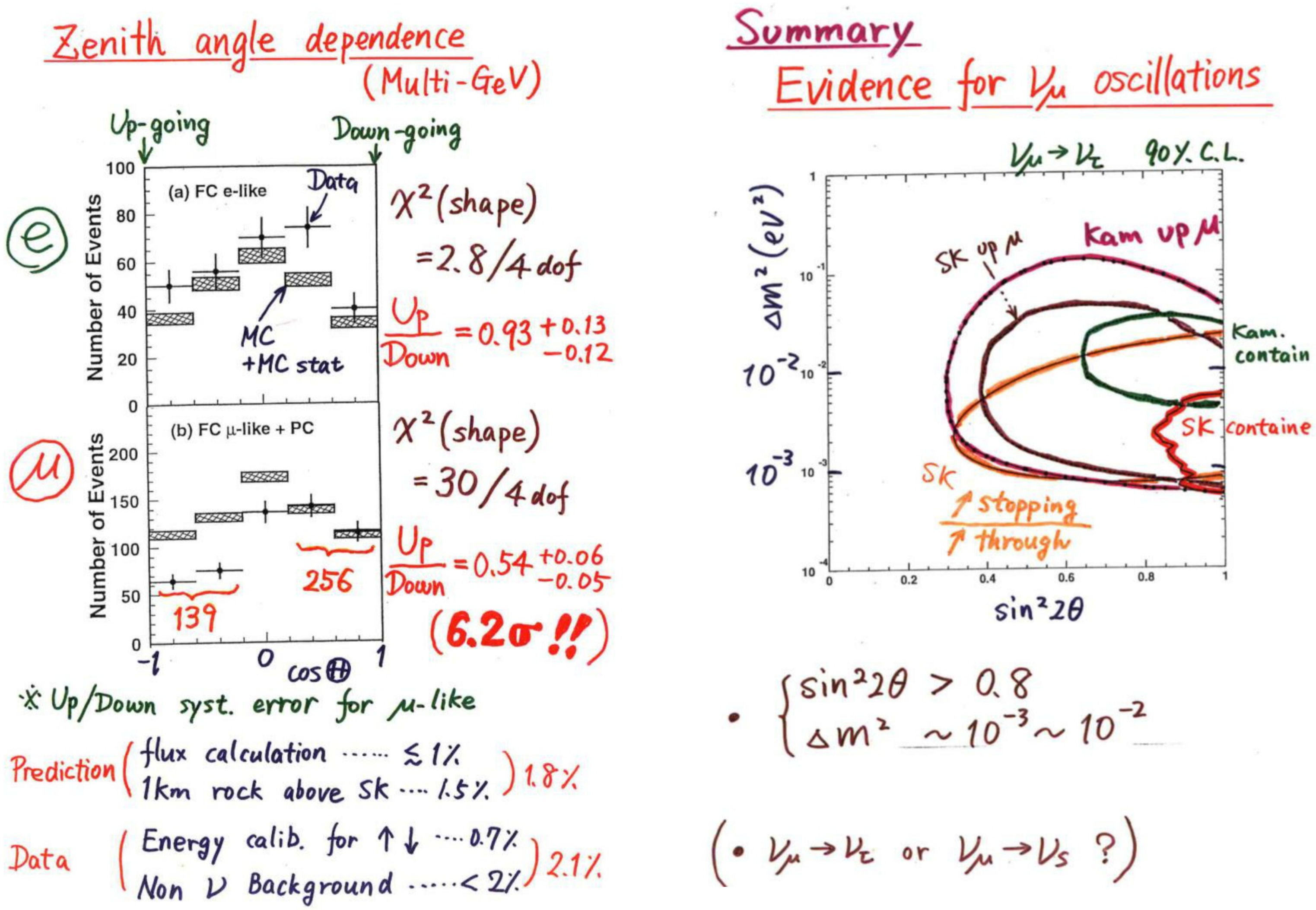
- Typical energies: 100 MeV - 100 GeV
- Typical distances: 100-10000 km.







SuperKamiokande Coll.



http://www-sk.icrr.u-tokyo.ac.jp/nu98/

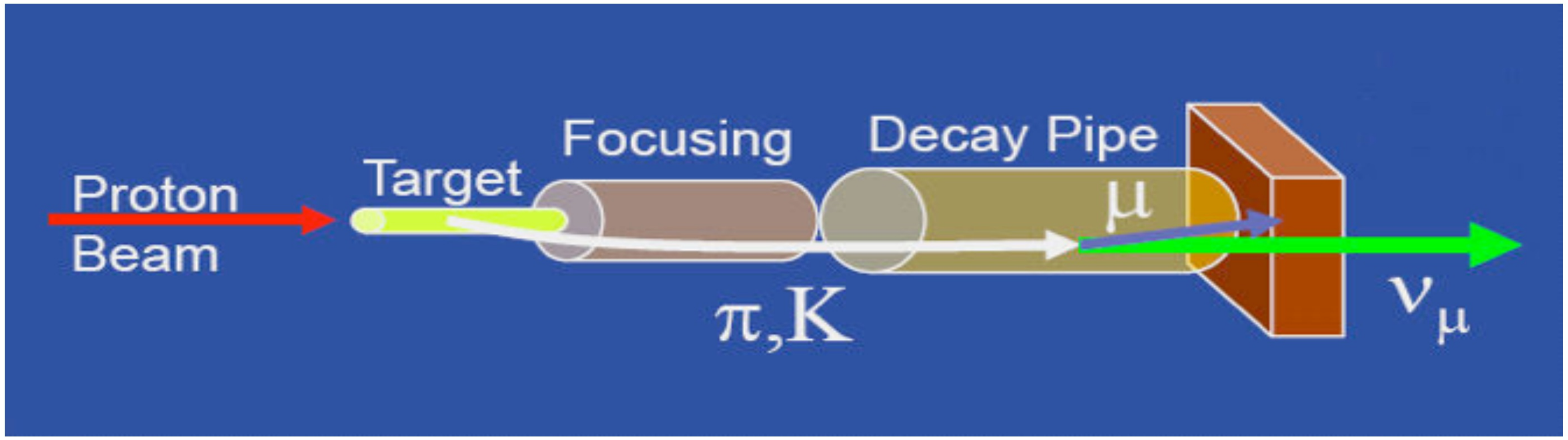
T. Kajita's talk at Neutrino 1998

SK reported the first evidence of neutrino oscillations in 1998 with atmospheric neutrinos (nu_{mu}->nu_{tau}).

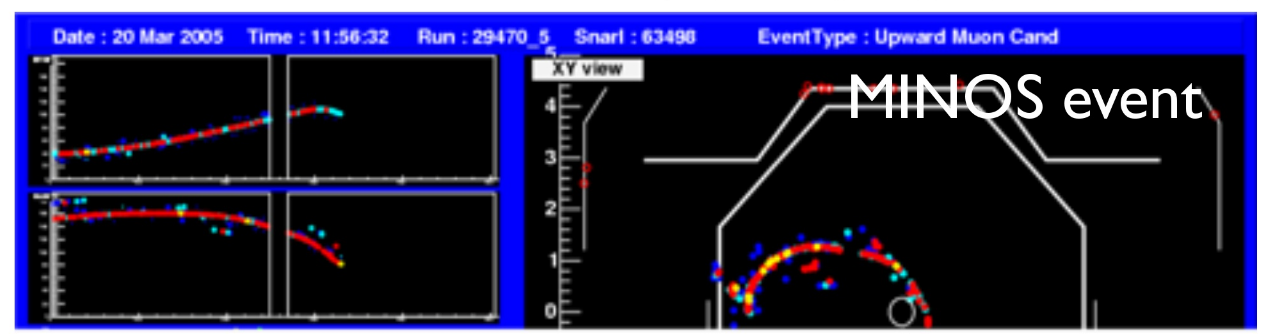
SK and MINOS went on to measure the atmospheric mixing angle to be large (mainly maximal) and the atm mass squared difference at ~2.5x 10^{-3} eV^2.

Accelerator neutrinos

Conventional beams: muon neutrinos from pion decays



Neutrino production.
Credit: Fermilab



- Typical energies:

MINOS: E~4 GeV; T2K: E~700 MeV; NOvA: E~2 GeV.
OPERA and ICARUS: E~20 GeV.

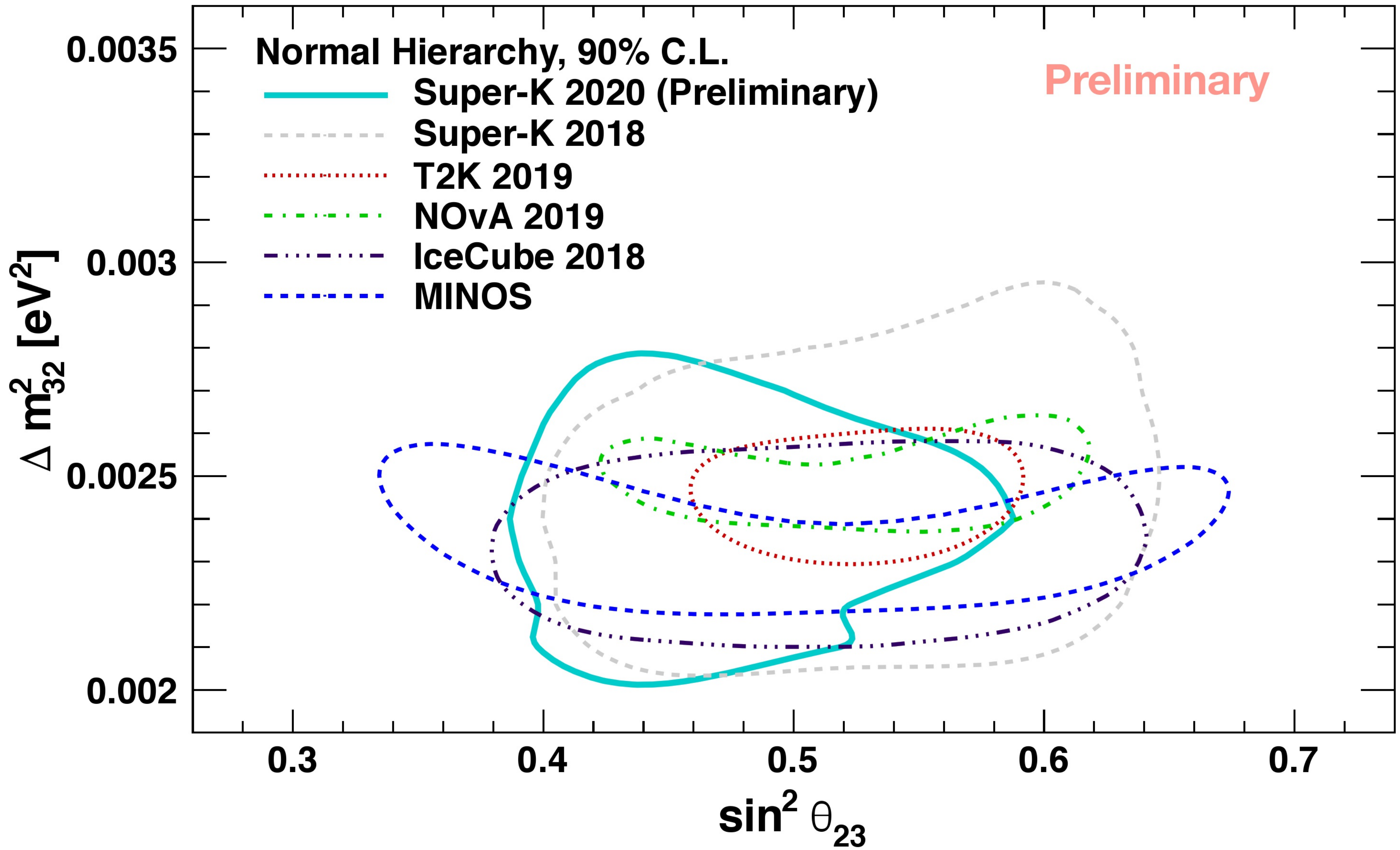
- Typical distances: 100 km - 2000 km.

MINOS: L=735 km; T2K: L=295 km; NOvA: L=810 km.
OPERA and ICARUS: L=700 km.

$$P(\nu_\mu \to \nu_\mu) \simeq \sin^2 2\theta_{23} \sin^2 \frac{\Delta m^2_{31} L}{4E}$$

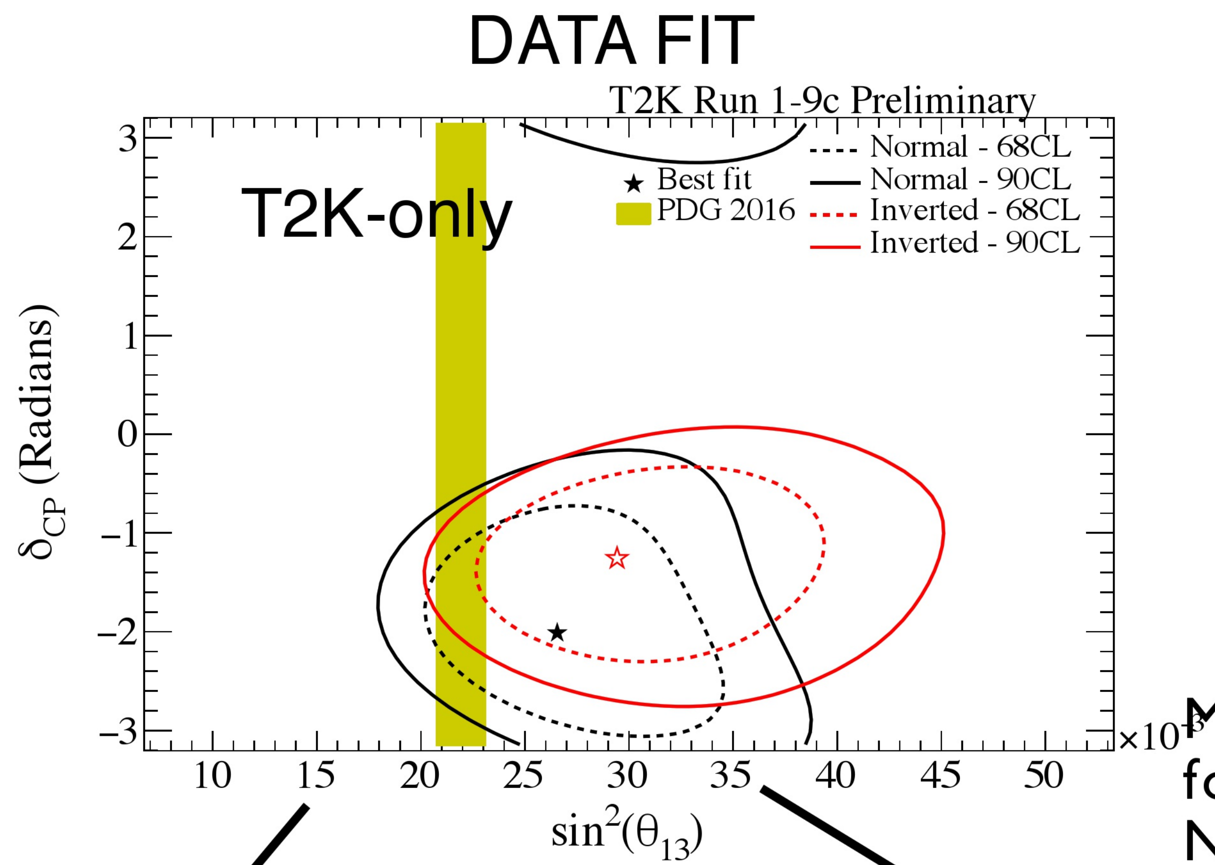
Sensitivity to theta$_{23}$ and Delta m$^2{}_{31}$.

Δm^2_{32} vs $\sin^2\theta_{23}$ constraints

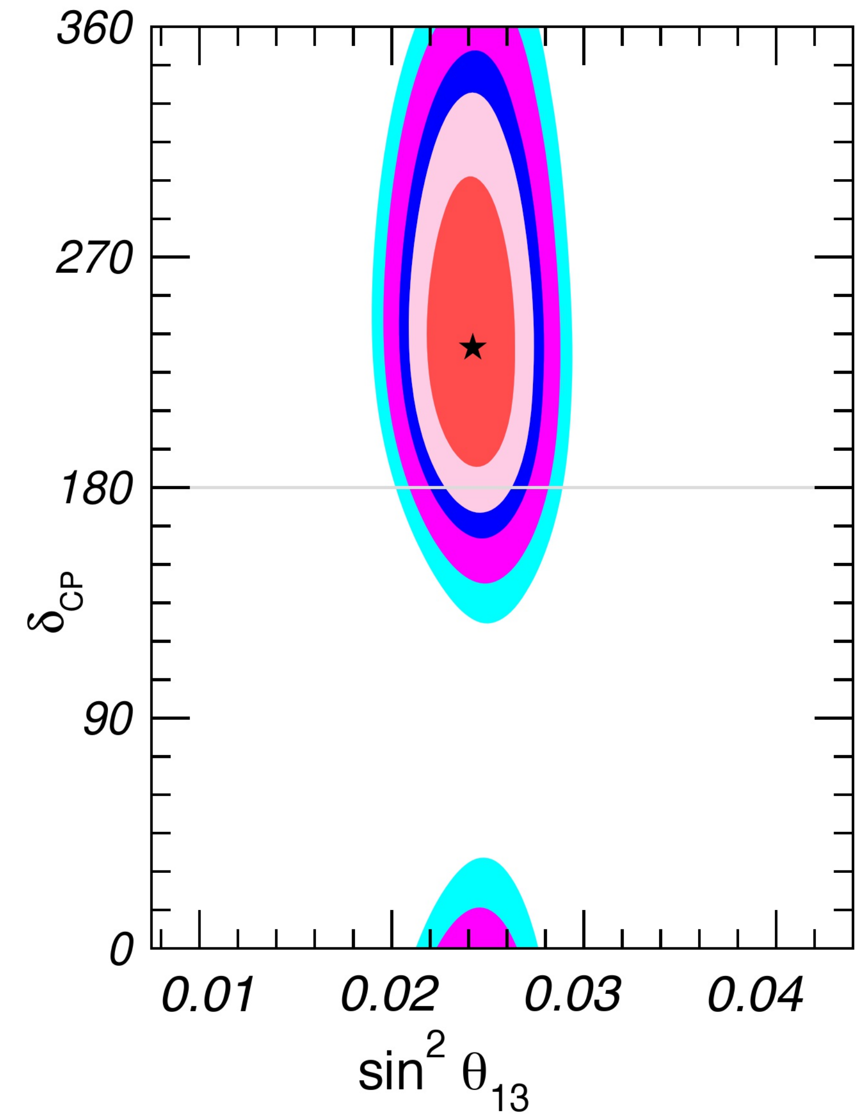


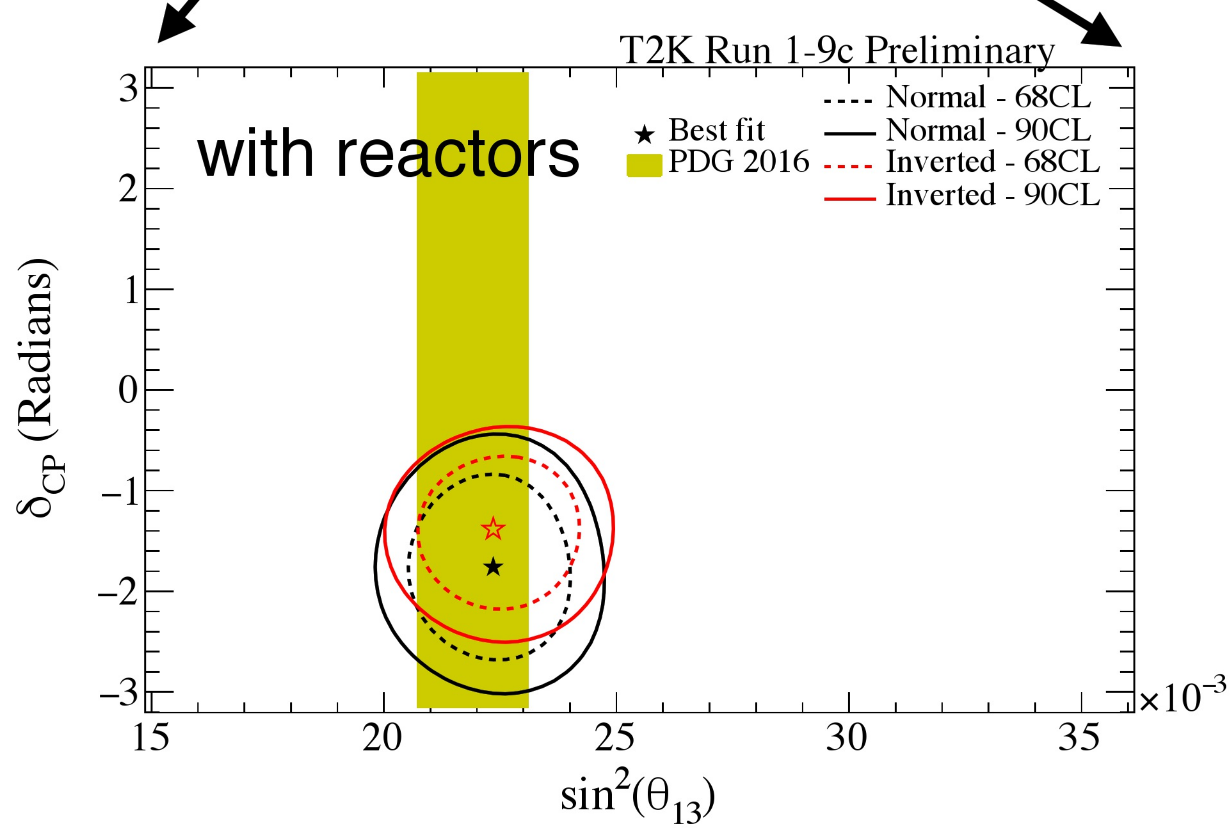
Y. Nakajima, for Super-Kamiokande, Neutrino 2020

Hints for CP violation?



M. Wasko, for T2K, Neutrino 2018

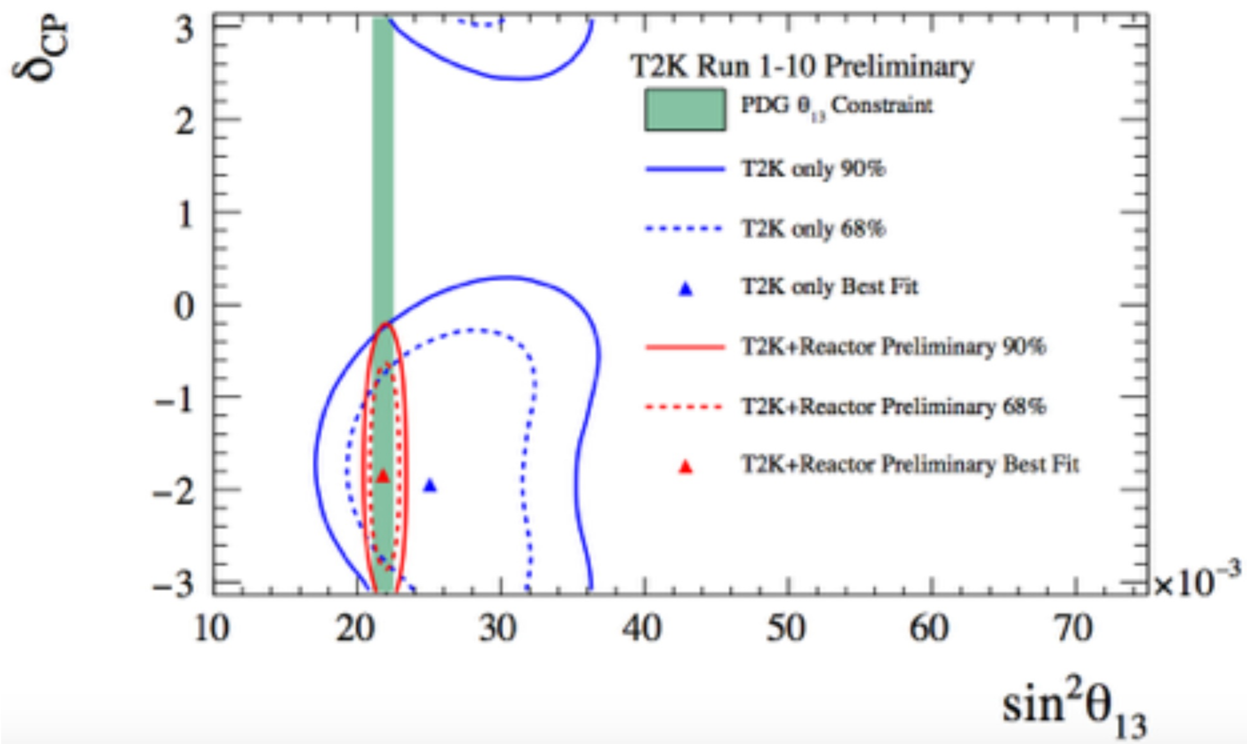




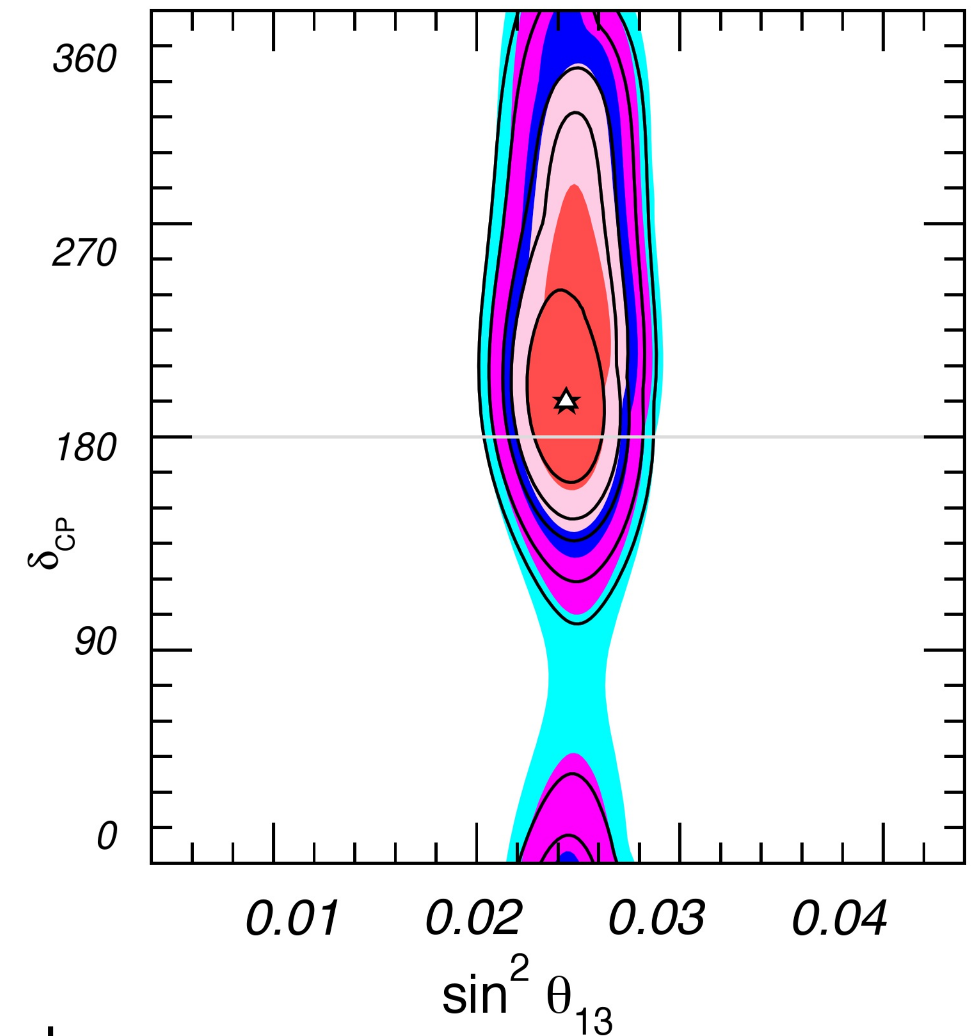
M. C. Gonzalez-Garcia et al., NuFit, 1611.01514
Pre-Neutrino 2018

Some hints for CP-violation, mainly due to combining T2K (NOvA) with reactor neutrino data.

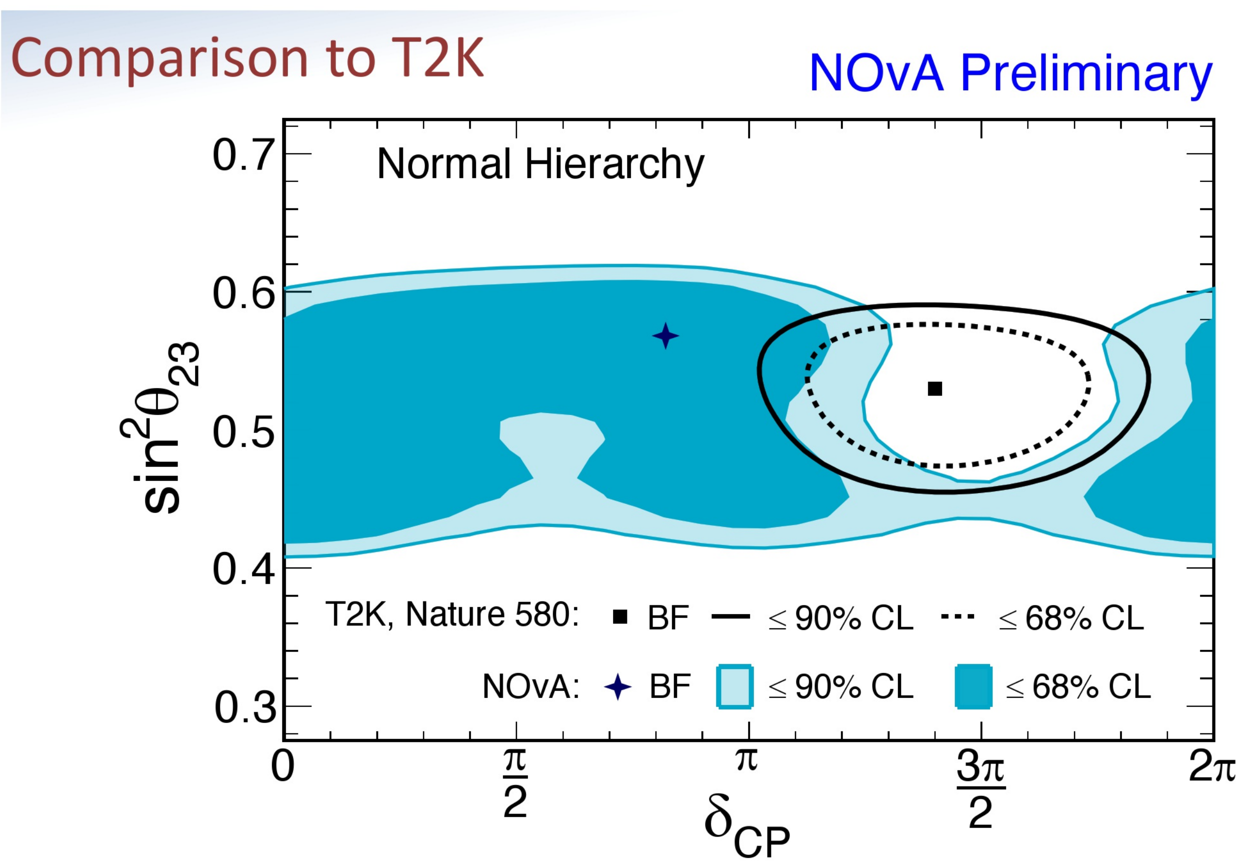
Hints for CP violation?



M. Guigue, for T2K, NeuTel 2021



M. C. Gonzalez-Garcia et al., NuFit, 2007.14792



A. Himmel, for NOvA, Neutrino 2020

Some mild preference for CP-violation, mainly due to combining T2K (NOvA) with reactor neutrino data.

Current knowledge of neutrino properties

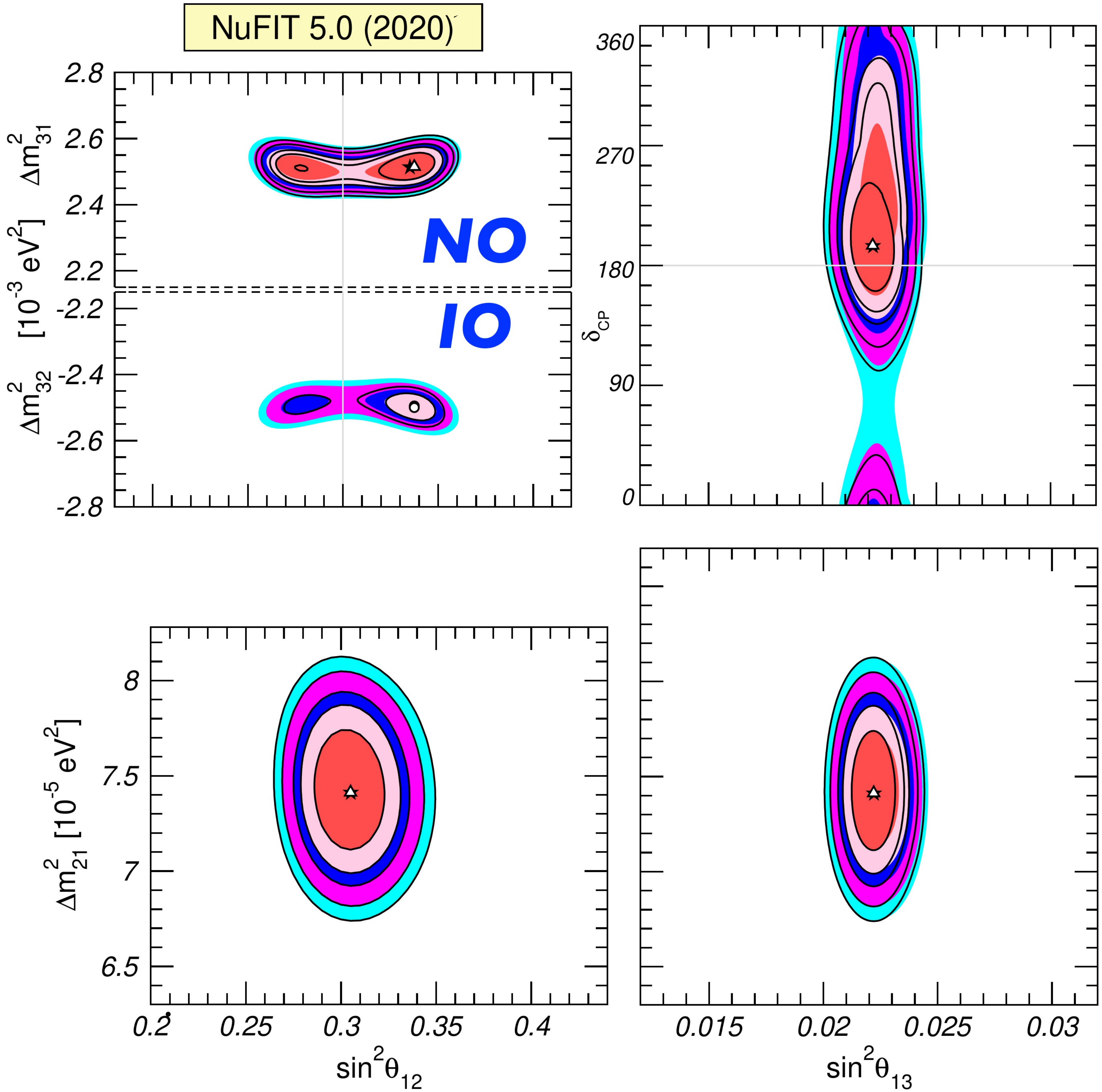
Current status of neutrino parameters

NuFIT 5.0 (2020)

without SK atmospheric data

	Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 2.7$)	
	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
$\sin^2\theta_{12}$	$0.304^{+0.013}_{-0.012}$	$0.269 \to 0.343$	$0.304^{+0.013}_{-0.012}$	$0.269 \to 0.343$
$\theta_{12}/^\circ$	$33.44^{+0.78}_{-0.75}$	$31.27 \to 35.86$	$33.45^{+0.78}_{-0.75}$	$31.27 \to 35.87$
$\sin^2\theta_{23}$	$0.570^{+0.018}_{-0.024}$	$0.407 \to 0.618$	$0.575^{+0.017}_{-0.021}$	$0.411 \to 0.621$
$\theta_{23}/^\circ$	$49.0^{+1.1}_{-1.4}$	$39.6 \to 51.8$	$49.3^{+1.0}_{-1.2}$	$39.9 \to 52.0$
$\sin^2\theta_{13}$	$0.02221^{+0.00068}_{-0.00062}$	$0.02034 \to 0.02430$	$0.02240^{+0.00062}_{-0.00062}$	$0.02053 \to 0.02436$
$\theta_{13}/^\circ$	$8.57^{+0.13}_{-0.12}$	$8.20 \to 8.97$	$8.61^{+0.12}_{-0.12}$	$8.24 \to 8.98$
$\delta_{CP}/^\circ$	195^{+51}_{-25}	$107 \to 403$	286^{+27}_{-32}	$192 \to 360$
$\frac{\Delta m^2_{21}}{10^{-5}\ \mathrm{eV}^2}$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$
$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \mathrm{eV}^2}$	$+2.514^{+0.028}_{-0.027}$	$+2.431 \to +2.598$	$-2.497^{+0.028}_{-0.028}$	$-2.583 \to -2.412$

M. C. Gonzalez-Garcia et al., 2007.14792



Current knowledge of neutrino properties:

- 2 mass squared differences
- 3 sizable mixing angles,
- some mild preference for CPV
- mild indications in favour of NO

http://www.nu-fit.org/ M. C. Gonzalez-Garcia et al., 2007.14792

Summary

Neutrino oscillations in vacuum

Neutrino oscillations in matter

Application to past and current experimental programme (except appearance channel in LBL exp)

Current knowledge of neutrino oscillation parameters