NEUTRINO MASS MODELS (André de Gouvêa)

OUTLINE FOR THE 2 LECTURES

1 - FERMIONS, FERMION MASSES, CHIRAL FERMIONS } REVIEW
2 - FERMION MASSES IN THE STANDARD MODEL
3 - NEUTRINO MASSES (OVERVIEW)
- WHAT WE KNOW
- WHY WE CARE (CHOICES)

4 - MODELS, INCLUDING THE "FLAVOR PUZZLE"

REF'S
- AdG, Ann. Rev. Nucl. Part. Sci 66, 197 (2016)
- Willenbrock, hep-ph/0410370 [TASI LECTURES]
- Giunti + Kim [TEXTBOOK]

1) FERMIONS, FERMION MASSES, CHIRAL FERMIONS

DIRAC LAGRANGIAN: $\mathcal{L} = \bar{\Psi}(i\not{\partial} - m)\Psi$

$\Psi \equiv$ 4-COMP., ANTI-COMMUTING OBJECT $\begin{cases} \text{SPIN} \uparrow + \text{SPIN} \downarrow \\ \otimes \\ \text{PARTICLE + ANTIPAR.} \end{cases}$

$\not{\partial} = \gamma^\mu \partial_\mu$ $\qquad \{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$

$\hookrightarrow$ DIRAC GAMMA MATRICES

$\bar{\Psi} = \Psi^\dagger \gamma^0$

[DIRAC REP. $\gamma^0 = \begin{pmatrix} \mathbb{1} & 0 \\ 0 & -\mathbb{1} \end{pmatrix}$

WEYL-REPRESENTATION

$$\gamma^0 = \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix}; \quad \vec{\gamma} = \begin{pmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{pmatrix}; \quad \gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}$$

$\hookrightarrow$ PAULI

$\sigma^\mu = (\mathbb{1}, \vec{\sigma})$; $\bar{\sigma}^\mu (\mathbb{1}, -\vec{\sigma})$

IT TURNS OUT $\exists\ \gamma_5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3 \ |\ \{\gamma_5, \gamma^\mu\} = 0$
$\forall \mu = 0,1,2,3$

$$\gamma_5 = \begin{pmatrix} -\mathbb{1} & 0 \\ 0 & \mathbb{1} \end{pmatrix}$$ [WEYL REP]

$\gamma_5 \psi_L = -\psi_L \rightarrow$ LEFT-CHIRAL FERMION

$\gamma_5 \psi_R = +\psi_R \rightarrow$ RIGHT-CHIRAL FERMION

$$\psi_L = \begin{pmatrix} \chi_L \\ 0 \end{pmatrix} \qquad \psi_R = \begin{pmatrix} 0 \\ \xi_R \end{pmatrix}$$

$$P_L = \frac{1-\gamma_5}{2} \qquad \psi = \psi_L + \psi_R = \begin{pmatrix} \chi_L \\ \xi_R \end{pmatrix}$$

$$P_R = \frac{1+\gamma_5}{2} \qquad P_L\psi = \begin{pmatrix} \chi_L \\ 0 \end{pmatrix}; \quad P_R = \begin{pmatrix} 0 \\ \xi_R \end{pmatrix}$$

χ_L, ξ_R TWO-COMP. ANTI-COMMUTING OBJECT [WEYL FERMION]

$P_L^2 = P_L$; $P_R^2 = P_R$; $P_L P_R = P_R P_L = 0$ (PROJECTION OPERATORS)

Rewrite $\mathcal{L}$ using χ_L, ξ_R:

$$\mathcal{L} = i\,\xi_R^\dagger \sigma^\mu \partial_\mu \xi_R + i\,\chi_L^\dagger \bar{\sigma}^\mu \partial_\mu \chi_L - m\left[\xi_R^\dagger \chi_L + \chi_L^\dagger \xi_R\right]$$

E.O.M's

$$\begin{cases} i\bar{\sigma}^\mu \partial_\mu \chi_L - m\,\xi_R = 0 \\ i\sigma^\mu \partial_\mu \xi_R - m\,\chi_L = 0 \end{cases}$$

(2 coupled EQ's)

Why do we care?

a) χ_L, ξ_R are the irreducible rep. of Lorentz group

$$\chi_L \to \chi_L'$$

$$\xi_R \to \xi_R'$$

$SO(3,1) = SU(2) \times SU(2)$

b) IF $m = 0$ (AND THERE ARE ONLY GAUGE INT.)

χ_L, ξ_R "DECOUPLE":

CHIRAL SYMMETRY: $\begin{cases} \chi_L \to e^{i\theta_L} \chi_L \\ \xi_R \to e^{i\theta_R} \xi_R \end{cases}$

χ_L, ξ_R DIFFERENT, UNRELATED OBJECTS

$\chi_L \to$ LEFT-HELICITY "PARTICLE", RIGHT-HELICITY "ANTI-P."

$\xi_R \to$ RIGHT-HELICITY "PARTICLE", LEFT-HELICITY "ANTI-P."

c) THE $SU(2) \times U(1)$ GAUGE GROUP IS CHIRAL!

$$\mathcal{L} \supset \chi_L^\dagger \, i \bar{\sigma}_\mu D^\mu \chi_L + \xi_R^\dagger \, i \sigma_\mu D^\mu \xi_R$$

$\{ D_\mu = \partial_\mu + i g A_\mu$

CAN BE (AND ARE!) DIFFERENT

EXAMPLE : THE ELECTRON, MASSLESS EDITION

$e_L \rightarrow$ LEFT-HANDED ELECTRON / RIGHT-HANDED POSITRON $\left\{ \begin{array}{l} e^-_\ell \\ e^+_r \end{array} \right.$

ν_L

TALKS TO γ, Z^0, W, ν

$e_R \rightarrow$ RIGHT-HANDED ELECTRON / LEFT-HANDED POSITRON $\left\{ \begin{array}{l} e^-_r \\ e^+_\ell \end{array} \right.$

ν_R?

TALKS TO γ, Z^0 NOT W, ν

↳ DIFFERENT Z COUPLING

THE CHARGE-CONJUGATED FIELD : $\psi^c = \gamma^0 C \psi^*$,

$C = i\gamma^2\gamma^0 =$

$= \begin{pmatrix} i\sigma_2 & 0 \\ 0 & -i\sigma_2 \end{pmatrix}$

IF $\psi = \begin{pmatrix} \chi_L \\ \xi_R \end{pmatrix}$

$i\sigma_2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

$\psi^c = \begin{pmatrix} -i\sigma_2 \, \xi_R^* \\ i\sigma_2 \, \chi_L^* \end{pmatrix} = \begin{pmatrix} \chi_L^c \\ \xi_R^c \end{pmatrix}$

$\begin{cases} -i\sigma_2 \, \xi_R^* = \chi_L^c \\ i\sigma_2 \, \chi_L^* = \xi_R^c \end{cases}$

WE CAN EXCHANGE

$\xi_R \longleftrightarrow (\chi_L^c)^*$

$\Rightarrow \psi = \begin{pmatrix} \chi \\ (i\sigma_2)\chi^{c*} \end{pmatrix}$

BOTH χ, χ^c LEFT-CHIRAL

χ^c NOT DIRECTLY RELATED TO χ IN SPITE OF NAME!

HENCE $\quad m\bar{\Psi}\Psi = m\left[\underbrace{\chi^{cT}(i\sigma_2)\chi}_{=\chi^c\cdot\chi} + \underbrace{\chi^\dagger(i\sigma_2)\chi^{c*}}_{(\chi^c\cdot\chi)^\dagger}\right]$

DOT PRODUCT (χ, ξ LEFT-CHIRAL)

$$\chi\cdot\xi = \chi_\uparrow\xi_\downarrow - \chi_\downarrow\xi_\uparrow \qquad \chi = \begin{pmatrix}\chi_\uparrow\\ \chi_\downarrow\end{pmatrix}$$

$\rightarrow$ SPIN-0 OUT OF 2 SPIN-$\frac{1}{2}$!

TWO TYPES OF MASS-TERMS (FERMION BILINEARS)

$m\,\chi\cdot\xi$ $\qquad\times\qquad$ $m\,\chi\chi \begin{cases}\chi_\uparrow\chi_\downarrow\\ -\chi_\downarrow\chi_\uparrow\end{cases}$

$\chi \neq \xi$, DIRAC MASS $\qquad$ $\chi = \chi$, MAJORANA MASS

SIMPLEST MASSIVE FREE-FERMION LAGRANGIAN

$$\mathcal{L} = i\,\chi^{\dagger}\bar{\sigma}^{\mu}\partial_{\mu}\chi - \frac{1}{2}\left(m\,\chi\chi + h.c.\right)$$

χ = LEFT-CHIRAL FIELD

2 STATES : (SPIN UP + SPIN DOWN) x (PARTICLE = ANTI-PAR)

ASIDE : 4-COMPONENT MAJORANA FIELD $\boxed{\psi_M = \begin{pmatrix} \chi \\ (i\sigma_2)\chi^* \end{pmatrix}}$; $\psi = \psi^c$

IF 2 WEYL FERMIONS χ_i, ξ_j

e^-_L e^+_L

$$\mathcal{L} \supset \frac{1}{2}\left[m_1^{ij}\chi^i\xi^j + m_2^{ij}\chi^i\chi^j + m_3^{ij}\xi^i\xi^j \right]$$

$$e^+_L e^-_L \qquad \cancel{e^-_L e^-_L} \qquad \cancel{e^+_L e^+_L}$$

WHETHER $m_1, m_2, m_3 \neq 0$ DEPENDS ON "SYMMETRIES" OF χ, ξ

2) FERMIONS IN THE STANDARD MODEL

	$SU(3)$	$SU(2)_L$	$U(1)_Y$
Q	3	$2 \begin{pmatrix} u \\ d \end{pmatrix}$	$1/6$
u^c	$\bar{3}$	1	$-2/3$
d^c	$\bar{3}$	1	$+1/3$
L	1	$2 \begin{pmatrix} \nu \\ e \end{pmatrix}$	$-1/2$
e^c	1	1	$+1$

$u_\uparrow^R$, $u_\downarrow^R$, $u_\uparrow^B$, $u_\downarrow^B$, $u_\uparrow^Y$, $u_\downarrow^Y$, $d_\uparrow^R$, $d_\downarrow^R$, ...

(x 3 GENERATIONS)

↳ ALL LEFT-CHIRAL WEYL FERMIONS

SCALAR

$$H = \begin{pmatrix} h^+ \\ h^0 \end{pmatrix}; \quad (1, 2, +1/2)$$

$$\tilde{H} = \begin{pmatrix} h^{0*} \\ h^- \end{pmatrix}; \quad (1, 2, -1/2)$$

$(h^+)^*$

ALL FERMION BILINEARS FORBIDDEN BY GAUGE INV.

$\Rightarrow$ ALL FERMION MASS ARE ZERO ...

... WERE IT NOT FOR EWSB

$\langle H \rangle = \begin{pmatrix} 0 \\ v \end{pmatrix} 1/\sqrt{2}$

$\langle \tilde{H} \rangle = \begin{pmatrix} v \\ 0 \end{pmatrix} 1/\sqrt{2}$

$v = 246\,\text{GeV}$

YUKAWA LAGRANGIAN

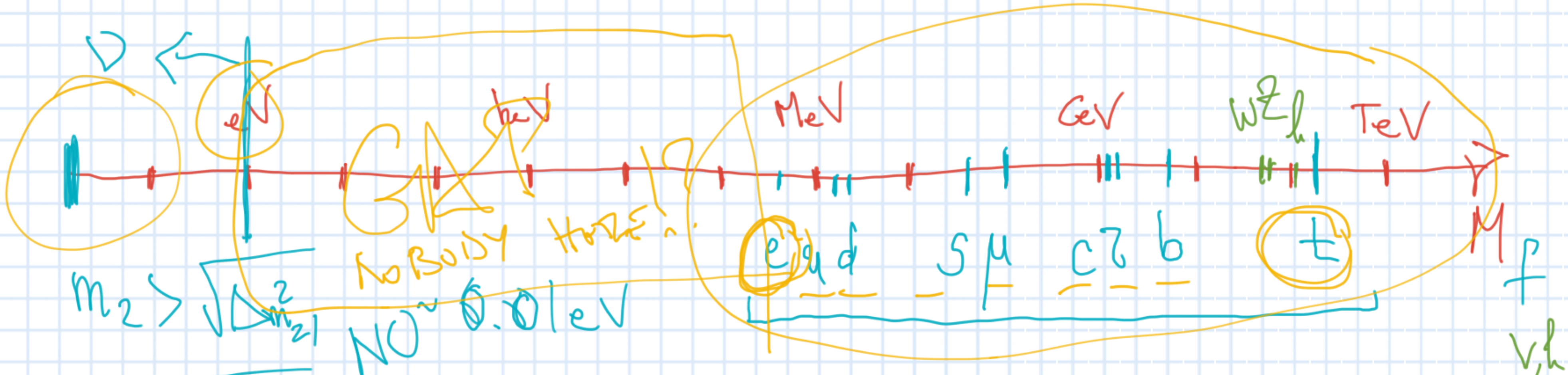
$$-\mathcal{L} \supset \lambda_u \underbrace{\underline{Q}\,\underline{u^c}\,\underline{H}}_{1/6 - 2/3 + 1/2 = 0} + \lambda_d \underbrace{\underline{Q}\,\underline{d^c}\,\underline{\tilde{H}}}_{1/6 + 1/3 - 1/2 = 0} + \lambda_e \underbrace{\underline{L}\,\underline{e^c}\,\underline{\tilde{H}}}_{-1/2 + 1 - 1/2 = 0}$$

NEW D.O.F.

$m_\nu ? \neq 0$

$\langle H \rangle + \begin{pmatrix} 0 \\ h \end{pmatrix} = H$

$$-\mathcal{L} \supset \underbrace{m_u}_{\lambda_u v/\sqrt{2}} u u^c + \underbrace{m_d}_{\lambda_d v/\sqrt{2}} d d^c + \underbrace{m_e}_{\lambda_e v/\sqrt{2}} e e^c$$

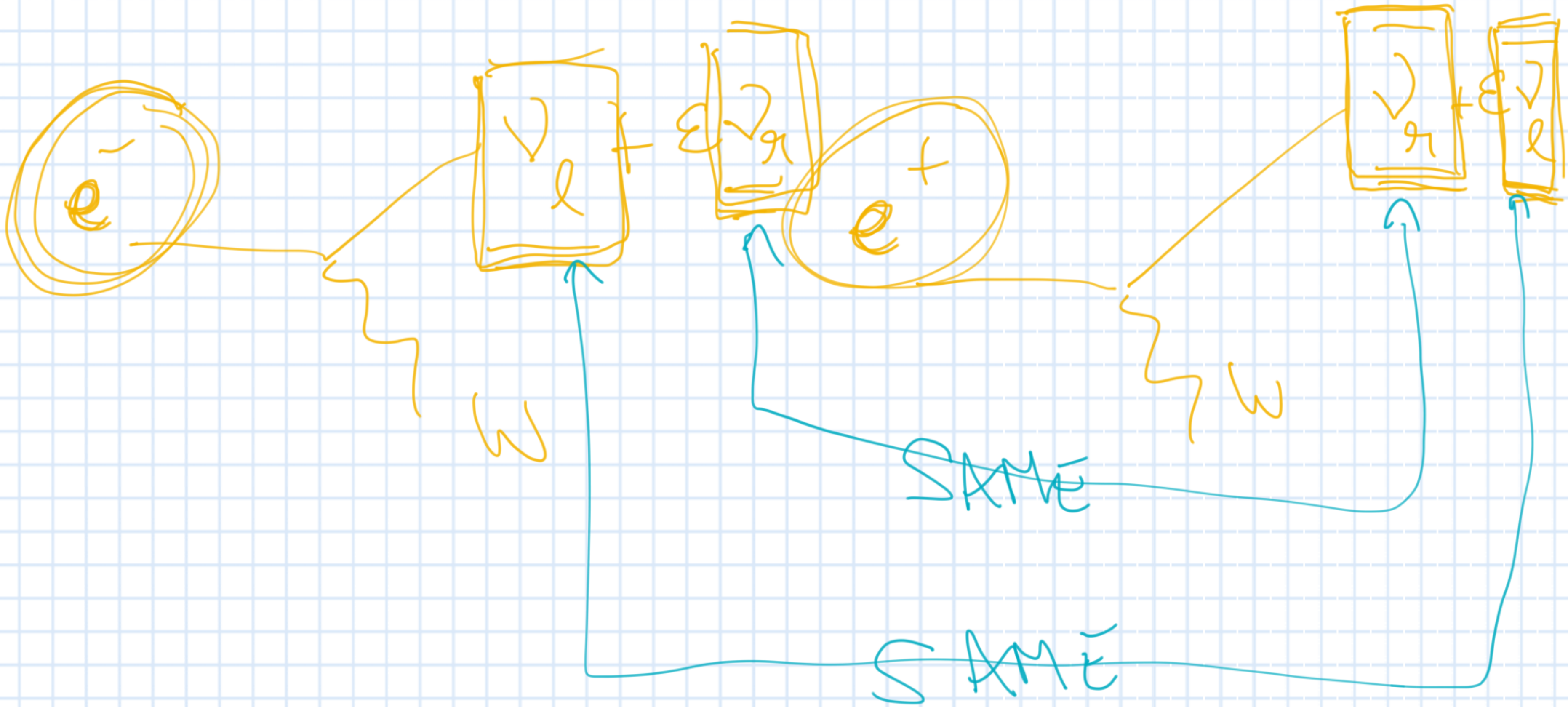


$m_2 > \sqrt{\Delta m^2_{21}}$ ~ 0.01 eV
NO

$m_3 > \sqrt{\Delta m^2_{31}} \sim 0.05\,eV$

- NEUTRINO MASSES ARE VERY SMALL!
- NEUTRINO MASSES ARE "DIFFERENT" (?)

+ FLAVOR PUZZLE (?)



IF ν's MAJORANA!