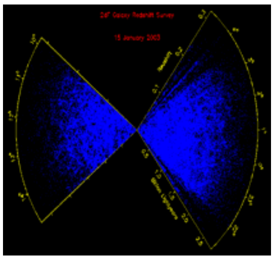
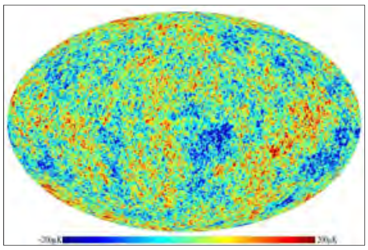
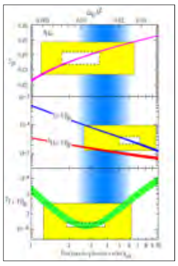
NEUTRINOS AND COSMOLOGY

Gabriela Barenboim
U.Valencia and IFIC

International Neutrino Summer School 2021
August 2021
CERN

some time ago at U Mass...

The Redbook, a manual for faculty members that explained what a university was, and what it wasn't. It cited two courses one wouldn't find in a curriculum of higher education: witchcraft and cosmology.

WITCHCRAFT AND COSMOLOGY

N[redacted]NOS AND COSMOLOGY

Gabriela Barenboim
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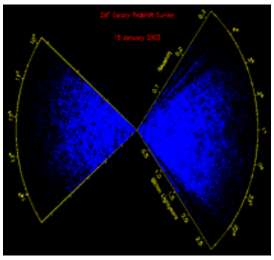

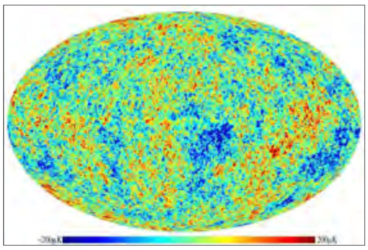

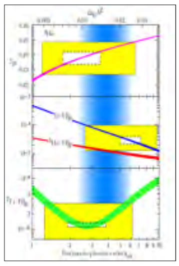

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1. **INTRODUCTION: FUNDAMENTAL INGREDIENTS & THERMAL HISTORY OF THE UNIVERSE**

2. NEUTRINO DECOUPLING IN THE EARLY UNIVERSE

3. BIG BANG NUCLEOSYNTHESIS & N_{eff}

4. COSMOLOGY & N_{eff}

5. COSMOLOGY & NEUTRINO MASSES

6. TAKE HOME MESSAGES

Standard Cosmology refers to FLRW Cosmology (FRIEDMANN LEMAITRE ROBERTSON WALKER) and it is based on two basic elements:

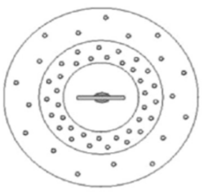
- FLRW Geometry (i.e. the metric, which determines the geodesics)
- FLRW Dynamics (Friedmann Equations, which determine the curvature of the space-time)

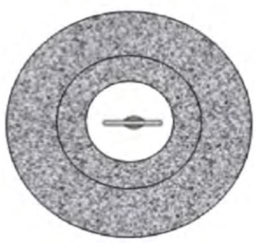
FLRW GEOMETRY

The FLRW geometry asumes that at large scales the universe is homogeneous and isotropic.

Homogeneous → same everywhere

Isotropic → same in all directions

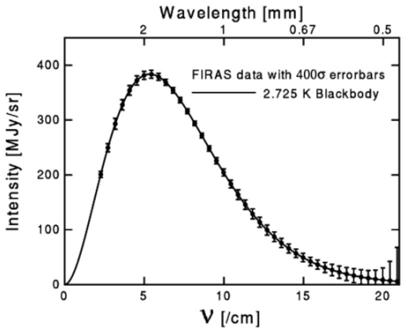


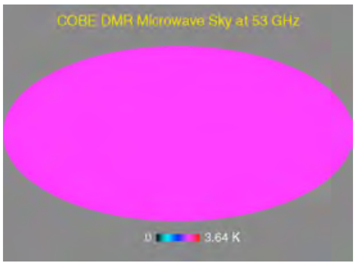


Sufficiently large scales

A parsec amounts to go and come back from the Sun … 100000 times

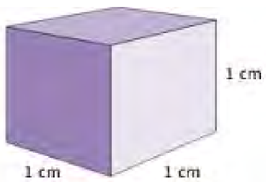
The most robust confirmation of the isotropy of the universe at large scales is provided by the CMB, the Cosmic Microwave Background radiation (Penzias & Wilson'64).
When one measures the sky temperature in any direction, one notices that the photons have a thermal black body spectrum with a temperature of 2.725 K. This has been measured with high accuracy by the spectrophotometer FIRAS on the NASA COBE satellite. There are small fluctuations in the temperature across the sky at the level of about 1 part in 100,000 ~(10^{-5})



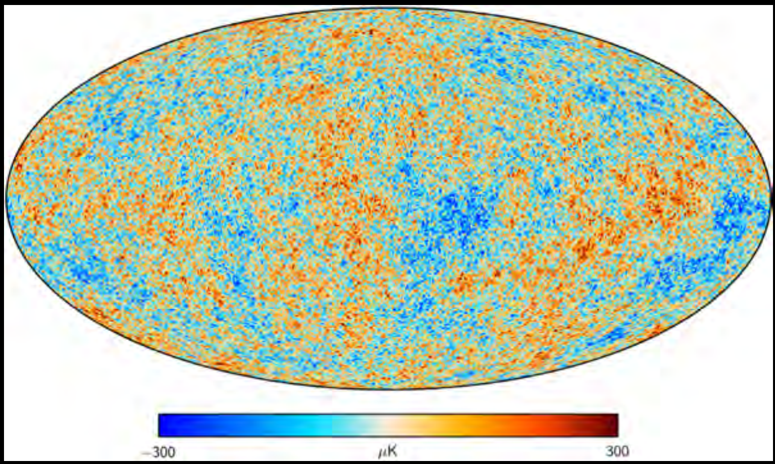


The existence of a CMB, that is, a relic photon bath, was predicted by Alpher & Herman in 1948 while working on BBN. Penzias & Wilson, in 1965, discovered accidentally the CMB while working with a very sensitive radio telescope at Bell Labs in New Jersey. In 1978, Penzias and Wilson were awarded the Nobel Prize for Physics for their joint discovery of the CMB.

410 photons/cm^3

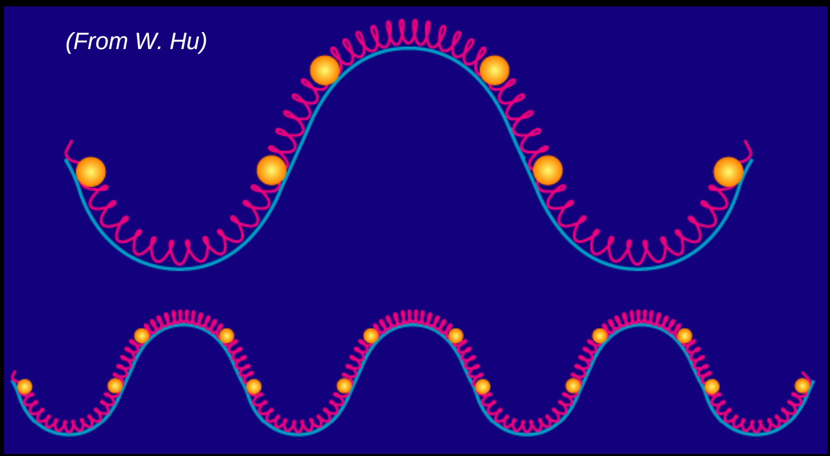


The radiation in the universe has a mean T≃ 2.725 K!



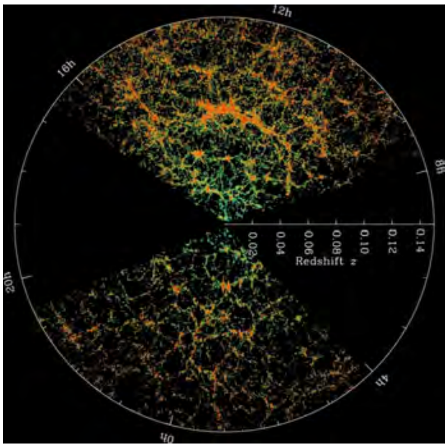
This map is just telling us how the CMB temperature varies with the angular size of patches in the sky…

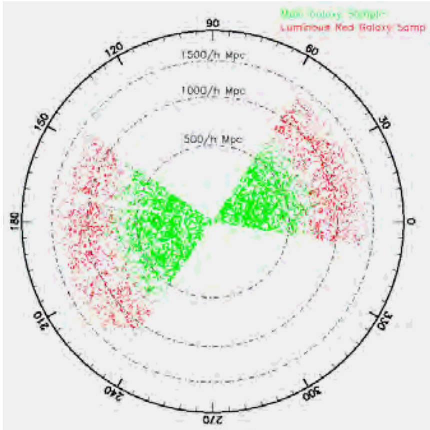
The CMB fluctuations are due to the acoustic oscillations in the baryon-photon fluid before recombination.



Potential wells ⟶ High density ⟶ COLD SPOTS in CMB maps

Potential hills ⟶ Low density ⟶ HOT SPOTS in CMB maps





At distances larger than 100 Mpc, galaxy survey observations indicate that the universe is homogeneous, that is, galaxies and clusters of galaxies are equally distributed in the sky in all possible directions.

The metric $g_{\mu\nu}$ connects the values of the coordinates to the more physical measure of the interval (proper time):

$$ds^2 = \sum_{\mu,\nu=0}^{3} g_{\mu\nu} dx^\mu dx^\nu$$

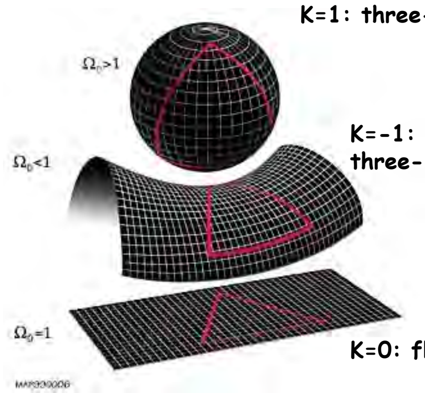
- dx^0 refers to the time-like component, the last three are spatial coordinates.
- $g_{\mu\nu}$ is the metric, necessarily symmetric.
- In special relativity, $g_{\mu\nu}=\eta_{\mu\nu}$ (Minkowski metric)
- In an expanding, homogeneous and isotropic universe the metric is the FLRW one:

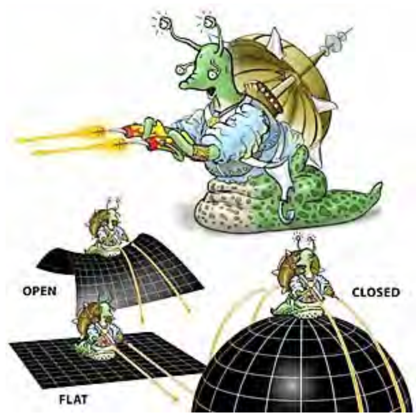
$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-Kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right)$$

- If the universe is flat (K=0), the FLRW metric, with a(t) the scale factor:

$$g_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & a^2(t) & 0 & 0 \\ 0 & 0 & a^2(t) & 0 \\ 0 & 0 & 0 & a^2(t) \end{pmatrix}$$

The spatial geometry depends on the curvature, K:



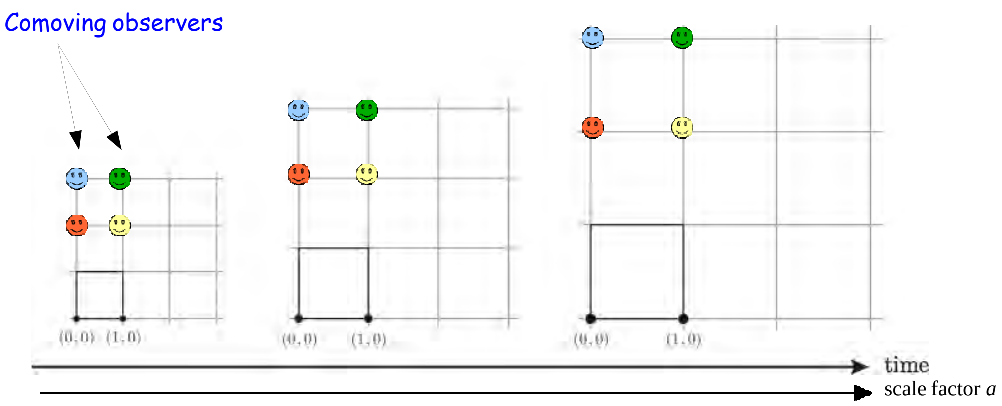


The FLRW metric tells us how to measure distances in each of these possible geometries.

An observer at rest with the FLRW spatial coordinates is a **comoving observer**.

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-Kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right)$$

Factor by which a physical length scale increases between time t_1 and t_2.



The **physical distance** between two comoving observers increases with time, but the coordinate distance between them remains unchanged.

Hubble parameter

- It provides the expansion rate of the universe as a function of time:

$$H \equiv \frac{1}{a}\frac{da}{dt} = \frac{d\ln a}{dt}$$

- The cosmic time reads as:

$$t = \int dt = \int \frac{1}{a}\frac{da}{H(a)}$$

- The conformal time is given by:

$$\eta = \int \frac{dt}{a} = \int \frac{1}{a^2}\frac{da}{H(a)}$$

Cosmological redshift

- Doppler Effect:

- Cosmic time: The photon wavelength is stretched with the scale factor as the universe expands.

$$\lambda = \frac{\lambda_0}{a} = (1+z)\lambda_0$$

$$a = \frac{1}{1+z}$$

- If we interpret the redshift z as the Doppler effect, galaxies recede (i.e. they move further away) in an expanding universe.

In an FLRW universe, there is a one-to-one correspondence between t, a, and z:

redshift z ←
→ time t
→ scale factor a

We use them interchangeably as a measure of time.

Hubble law

- The comoving distance to an object located at redshift z reads as:

$$D(a) = \int_a^1 \frac{da'}{a'^2 H(a')} \qquad D(z) = \int_0^z \frac{dz'}{H(z')}$$

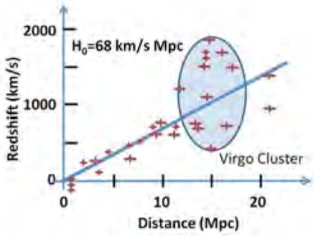
- At small redshifts, $z \simeq v/c$.

The Hubble law can be written as: $$lim_{z\to 0} D(z) = \frac{z}{H(z=0)} = \frac{z}{H_0}$$

with the Hubble constant, H_0: $$H_0 = 100h \text{ km/s/Mpc}$$

- Cosmological observations have determined that $h \simeq 0.7$

1929: Edwin Hubble measures the spectra of hundred of galaxies and notices that they are redshifted, meaning that they are moving away from our galaxy. Furthermore, the further the galaxy is located, the faster it moves away from our galaxy.



DYNAMICS FLRW

General Relativity relates the metric with the matter and energy content in the universe. The sale factor a(t) will evolve in time accordingly to the matter-energy content of the universe.

In other words, matter and energy will tell us how the geometry of the space-time is curved via the Einstein equations.

Einstein Equations

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\mathcal{R} = 8\pi T_{\mu\nu}$$

- $R_{\mu\nu}$ is the Ricci tensor, depending on the metric $g_{\mu\nu}$ and its derivatives:

$$R_{\mu\nu} = \Gamma^{\alpha}_{\mu\nu,\alpha} - \Gamma^{\alpha}_{\mu\alpha,\nu} + \Gamma^{\alpha}_{\beta\alpha}\Gamma^{\beta}_{\mu\nu} - \Gamma^{\alpha}_{\beta\nu}\Gamma^{\beta}_{\mu\alpha}$$

$$\Gamma^{\mu}_{\alpha\beta} = \frac{g^{\mu\nu}}{2}\left(\frac{\partial g_{\alpha\nu}}{\partial x^{\beta}} + \frac{\partial g_{\beta\nu}}{\partial x^{\alpha}} - \frac{\partial g_{\alpha\beta}}{\partial x^{\nu}}\right)$$

Einstein Equations

$$\Gamma^{\mu}_{\alpha\beta} = \frac{g^{\mu\nu}}{2}\left(\frac{\partial g_{\alpha\nu}}{\partial x^{\beta}} + \frac{\partial g_{\beta\nu}}{\partial x^{\alpha}} - \frac{\partial g_{\alpha\beta}}{\partial x^{\nu}}\right) \qquad g_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & a^2(t) & 0 & 0 \\ 0 & 0 & a^2(t) & 0 \\ 0 & 0 & 0 & a^2(t) \end{pmatrix}$$

$$\Gamma^{0}_{\alpha\beta} = -\frac{1}{2}\left(\frac{\partial g_{\alpha 0}}{\partial x^{\beta}} + \frac{\partial g_{\beta 0}}{\partial x^{\beta}} - \frac{\partial g_{\alpha\beta}}{\partial x^{0}}\right) = \frac{1}{2}\left(\frac{\partial g_{\alpha\beta}}{\partial x^{0}}\right)$$

•EXERCISE, Check that:

$$\Gamma^{0}_{00} = 0 \qquad \Gamma^{0}_{0i} = \Gamma^{0}_{i0} = 0$$

$$\boxed{\Gamma^{0}_{ij} = \delta_{ij}\dot{a}a \qquad \Gamma^{i}_{0j} = \Gamma^{i}_{j0} = \delta^{i}_{j}\frac{\dot{a}}{a}}$$

Einstein Equations

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(there are only two components different from 0, the 00 and the ii ones)

•R is the Ricci scalar, $R=g^{\mu\nu}R_{\mu\nu}$.

•$T_{\mu\nu}$ is the energy-momentum tensor.

Einstein Equations

$$\Gamma^0_{ij} = \delta_{ij}\dot{a}a$$

$$\Gamma^i_{0j} = \Gamma^i_{j0} = \delta^i_j \frac{\dot{a}}{a}$$

- Lets compute the 00 component for the Einstein equations:

$$R_{00} - \frac{1}{2}g_{00}\mathcal{R} = 8\pi G T_{00}$$

$$R_{00} = \Gamma^\alpha_{00,\alpha} - \Gamma^\alpha_{0\alpha,0} + \Gamma^\alpha_{\beta\alpha}\Gamma^\beta_{00} - \Gamma^\alpha_{\beta 0}\Gamma^\beta_{0\alpha}$$

- But we know that $\Gamma^\alpha_{00}=0$, therefore:

$$R_{00} = -\Gamma^i_{0i,0} - \Gamma^i_{j0}\Gamma^j_{0i} = -\frac{\partial}{\partial t}\left(\frac{\dot{a}}{a}\right)\delta_{ii} - \left(\frac{\dot{a}}{a}\right)^2\delta_{ii} = -3\left(\frac{\ddot{a}}{a} - \left(\frac{\dot{a}}{a}\right)^2\right) - 3\left(\frac{\dot{a}}{a}\right)^2 = -3\frac{\ddot{a}}{a}$$

- EXERCISE, Check that: $R_{ij} = \delta_{ij}\left(2\dot{a}^2 + a\ddot{a}\right)$

- And consequently: $\mathcal{R} \equiv g^{\mu\nu}R_{\mu\nu} = -R_{00} + \frac{1}{a^2}R_{ii} = 6\left(\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2\right)$

- Finally we find that:

$$\boxed{R_{00} - \frac{1}{2}g_{00}\mathcal{R}} = 8\pi G T_{00}$$

$$3\left(\frac{\dot{a}}{a}\right)^2$$

Einstein Equations

$$3\left(\frac{\dot{a}}{a}\right)^2$$

$$\boxed{R_{00} - \frac{1}{2}g_{00}\mathcal{R}} = 8\pi G T_{00}$$

- $T_{\mu\nu}$ is the energy-momentum tensor, that in the case of a isotropic perfect fluid:

$$T^\mu{}_\nu = \begin{pmatrix} -\rho & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix}$$

$$3\left(\frac{\dot{a}}{a}\right)^2 = 8\pi G\rho \qquad H^2(a) = \frac{8\pi G}{3}\rho$$

Friedmann Equation (1)

$$H^2(a) = H_0^2\frac{\rho(a)}{\rho_{crit}} \qquad \rho_{crit} \equiv \frac{3H_0^2}{8\pi G}$$

- EXERCISE $R_{ij} - \frac{1}{2}g_{ij}\mathcal{R} = 8\pi G T_{ij}$

- Derive the Friedmann Equation (2): $\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)$

In order to have accelerated expansion we need $\rho + 3p < 0$

Energy-momentum tensor conservation

- Time evolution of the $T_{\mu\nu}$ components

- In the absence of external forces, the energy momentum tensor is conserved.
- In an expanding universe, the energy momentum tensor conservation implies that its covariant derivative equals zero.

$$T^{\mu}{}_{\nu;\mu} \equiv \frac{\partial T^{\mu}{}_{\nu}}{\partial x^{\mu}} + \Gamma^{\mu}{}_{\alpha\mu} T^{\alpha}{}_{\nu} - \Gamma^{\alpha}{}_{\nu\mu} T^{\mu}{}_{\alpha}$$

$$T^{\mu}{}_{0;\mu} = 0 \qquad T^{\mu}{}_{\nu;\mu} = 0$$

$$T^{\mu}{}_{\nu} = \begin{pmatrix} -\rho & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix} \qquad \boxed{\Gamma^{i}_{0j} = \Gamma^{i}_{j0} = \delta^{i}_{j}\frac{\dot a}{a}}$$

$$\frac{\partial T^{\mu}{}_{0}}{\partial x^{\mu}} + \Gamma^{\mu}{}_{\alpha\mu} T^{\alpha}{}_{0} - \Gamma^{\alpha}{}_{0\mu} T^{\mu}{}_{\alpha}$$

$$-\frac{\partial \rho}{\partial t} - \Gamma^{\mu}{}_{0\mu}\rho - \Gamma^{\alpha}{}_{0\mu} T^{\mu}{}_{\alpha}$$

$$\frac{\partial \rho}{\partial t} + \frac{\dot a}{a}(3\rho + 3p) = 0$$

$$\frac{\partial \rho}{\partial t} + 3H\rho(1 + w) = 0$$

Equation of state

Energy-momentum tensor conservation

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$$\frac{\partial \rho}{\partial t} + 3H\rho(1 + w) = 0$$

Equation of state

- Matter (either cold dark matter or baryonic one) has zero pressure: $\rho_m \propto a^{-3}$

$$\frac{d\rho_m}{dt} = 3H\rho_m$$

$$H = \frac{\dot a}{a} = \frac{da}{dt}\frac{1}{a}$$

$$\frac{d\rho_m}{\rho_m} = 3Hdt$$

$$\frac{d\rho_m}{\rho_m} = 3\frac{da}{a}$$

Energy-momentum tensor conservation

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$$\frac{\partial \rho}{\partial t} + 3H\rho(1 + w) = 0$$

Equation of state

- Matter (either cold dark matter or baryonic one) has zero pressure: $\rho_m \propto a^{-3}$
- Radiation is characterised by p=ρ/3: $\rho_r \propto a^{-4}$
- While dark energy should behave as: $\rho + 3p < 0$

$$w < -1/3 \qquad \rho_{de} \propto \exp\left[-3\int_1^a \frac{da}{a}(1 + w(a))\right]$$

Friedmann Equations

- The first Friedmann equation can be written as:

$$H^2(a) = H_0^2 \frac{\rho(a)}{\rho_{crit}} \qquad \rho_{crit} \equiv \frac{3H_0^2}{8\pi G}$$

$$H_0 = 100h \text{ km/s/Mpc}$$

$$\rho_{crit} = 1.879h^2 \times 10^{-29} \text{g} \quad \text{cm}^{-3}$$

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$$\rho_{crit} = 1.879h^2 \times 10^{-29}\mathrm{g\ \ cm^{-3}}$$

$$H^2(a) = H_0^2 \left(\Omega_m(a) + \Omega_r(a) + \Omega_{de}(a)\right)$$

$$\Omega_m(a) = \rho_m(a)/\rho_{crit} = \rho_{m,0}a^{-3}/\rho_{crit} = \Omega_{m,0}a^{-3} = (\Omega_{dm,0} + \Omega_{b,0})a^{-3}$$

$$\Omega_r(a) = \rho_r(a)/\rho_{crit} = \rho_{r,0}a^{-4}/\rho_{crit} = \Omega_{r,0}a^{-4} = (\Omega_{\gamma,0} + \Omega_{\nu,0})a^{-4}$$

$$\Omega_{de}(a) = \rho_{de}(a)/\rho_{crit} = \rho_{de,0}a^{-3(1+w)}/\rho_{crit} = \Omega_{de,0}a^{-3(1+w)}$$

- These expressions are valid for a FLAT universe. In case the universe is not flat:

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-Kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right)$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3} - \frac{K}{a^2}$$

$$H^2(a) = H_0^2 \left(\Omega_m(a) + \Omega_r(a) + \Omega_{de}(a) + \Omega_K(a)\right)$$

$$\Omega_K(a) = -Ka^{-2}/H_0^2 = \Omega_{K,0}a^{-2}$$

"Cosmic sum rule"

$$\Omega_{m,0} + \Omega_{r,0} + \Omega_{de,0} + \Omega_{K,0} = 1$$

- In a flat universe, K=0, therefore:

$$\Omega_{m,0} + \Omega_{r,0} + \Omega_{de,0} = 1$$

- In an open universe, K=-1, therefore the curvature contribution is positive:

$$\Omega_{m,0} + \Omega_{r,0} + \Omega_{de,0} < 1$$

- In a close universe, K=+1, therefore the curvature contribution is negative:

$$\Omega_{m,0} + \Omega_{r,0} + \Omega_{de,0} > 1$$

Current cosmological observations indicate that the universe as a geometry very, very <u>close to the FLAT one</u>:

$$\Omega_K = -0.037^{+0.043}_{-0.049}$$

Radiation: photons and neutrinos

$$\int_0^\infty \frac{x^3\,dx}{e^x-1} = \frac{\pi^4}{15}$$

•**Photons:** The cosmic microwave background radiation temperature is 2.725 K, measured with a precision of 50 parts in a million. The energy of such a photon bath is given by the integral of the Bose-Einstein distribution times E=p (massless):

$$\rho_\gamma = 2\int \frac{d^3p}{(2\pi)^3}\frac{1}{e^{p/T}-1}p \qquad x \equiv p/T \qquad \rho_\gamma = \frac{8\pi T^4}{(2\pi)^3}\int_0^\infty \frac{x^3dx}{e^x-1} = \frac{\pi^2}{15}T^4$$

$$\Omega_\gamma(a) = \frac{\rho_\gamma}{\rho_{crit}} = \frac{\pi^2}{15}\left(\frac{2.725\text{ K}}{a}\right)^4\frac{1}{\rho_{crit}} = \frac{2.47\times10^{-5}}{a^4h^2} = \frac{4.75\times10^{-5}}{a^4}$$

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•**Neutrinos:** Neutrinos are fermions and therefore follow the Fermi-Dirac statistics. As we shall soon see, neutrinos decouple from the thermal bath before electron-positron annihilation and therefore they did not share in the entropy release, being their temperature lower than that of photons:

$$\int_0^\infty \frac{x^3\,dx}{e^x+1} = \frac{7\pi^4}{120}$$

$$\Omega_\nu(a) = \frac{\rho_\nu}{\rho_{crit}} = \frac{1.68\times10^{-5}}{a^4h^2} \quad (m_\nu = 0) \qquad \left(\frac{T_\nu}{T_\gamma}\right) = \left(\frac{4}{11}\right)^{1/3}$$

$$\int_0^\infty \frac{x^2\,dx}{e^x-1} = 2\,\zeta(3)$$

$$\int_0^\infty \frac{x^2\,dx}{e^x+1} = \frac{3}{2}\,\zeta(3)$$

But neutrinos are massive particles!

$$n_\nu(T) = \int \frac{d^3p}{(2\pi)^3}\frac{1}{e^{p/T}-1} = \frac{3}{22}n_\gamma(T)$$

$$\Omega_\nu(a) = \frac{m_\nu n_\nu}{\rho_{crit}} = \frac{\sum m_\nu}{94\text{ eV}h^2}\frac{1}{a^3}$$

Data tell us….

$$0.0006 \lesssim \Omega_{\nu,0}h^2 \lesssim 0.0025$$

We integrate the Fermi-Dirac distribution for the (anti)neutrinos, with 0 chemical potential

$$\rho_\nu = m_\nu n_\nu$$

$$n_{\nu_i}(T_\nu) = n_{\nu_i^c}(T_\nu) = \frac{1}{(2\pi)^3}\int d^3p \, \frac{1}{\exp(p/T_\nu)+1} = \frac{3\zeta(3)}{4\pi^2} T_\nu^3,$$

This was undertood already in the 60's by Gerstein and Zel'dovich who realized that the simple requirement that neutrinos do not overclose the Universe $\Omega_\nu < 1$ implies

$$\sum m_\nu < 100 \text{ eV}$$

is was far better than bounds on e sum of neutrino masses ovided by laboratory experiments the times !!

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Left-Handed Neutrino Disappearance Probe of Neutrino Mass and Character

G. Barenboim,[1,2] J. Bernabéu,[1] and O. Vives[1,2]
[1]Department de Física Teòrica, Universitat de València, Valencia, Spain
[2]Instituto de Física Corpuscular, Centre Mixte Universitat de València-Consejo Superior de Investigaciones Científicas, E-46100 Burjassot, Valencia, Spain
(Received 3 June 1996)

We explore the sensitivity to a nonvanishing neutrino mass offered by dynamical observables, i.e., branching ratios and polarizations. The longitudinal polarization in the c.m. frame decreases by a 4% for $D^+ \to \tau^+ \nu_\tau$ and $m_{\nu_\tau} = 24$ MeV. Taking advantage of the fact that the polarization is a Lorentz variant quantity, we study the polarization effects in a boosted frame. By means of a neutrino beam, produced by a high velocity boosted parent able to flip the neutrino helicity, we find that an enhanced left-handed neutrino deficit, induced by a Wigner rotation, appears. [S0031-9007(96)01406-8]

PACS numbers: 14.60.Pq, 13.15.+g, 13.20.Fc, 13.88.+e

$$\Omega_\nu h^2 = \frac{\sum m_\nu}{93 \text{ eV}}$$

Radiation: photons and neutrinos

$$\int_0^\infty \frac{x^3\, dx}{e^x - 1} = \frac{\pi^4}{15}$$

•**Photons:** The cosmic microwave background radiation temperature is 2.725 K, measured with a precision of 50 parts in a million. The energy of such a photon bath is given by the integral of the Bose-Einstein distribution times E=p (massless):

$$\rho_\gamma = 2\int \frac{d^3p}{(2\pi)^3}\frac{1}{e^{p/T}-1}p \qquad x \equiv p/T \qquad \rho_\gamma = \frac{8\pi T^4}{(2\pi)^3}\int_0^\infty \frac{x^3 dx}{e^x-1} = \frac{\pi^2}{15}T^4$$

$$\Omega_\gamma(a) = \frac{\rho_\gamma}{\rho_{crit}} = \frac{\pi^2}{15}\left(\frac{2.725\text{ K}}{a}\right)^4 \frac{1}{\rho_{crit}} = \frac{2.47\times 10^{-5}}{a^4 h^2} = \frac{4.75\times 10^{-5}}{a^4}$$

•**Neutrinos:** Neutrinos are fermions and therefore follow the Fermi-Dirac statistics. As we shall soon see, neutrinos decouple from the thermal bath before electron-positron annihilation and therefore they did not share in the entropy release, being their temperature lower than that of photons:

$$\int_0^\infty \frac{x^3\, dx}{e^x + 1} = \frac{7\pi^4}{120}$$

$$\Omega_\nu(a) = \frac{\rho_\nu}{\rho_{crit}} = \frac{1.68\times 10^{-5}}{a^4 h^2} \quad (m_\nu = 0) \qquad \left(\frac{T_\nu}{T_\gamma}\right) = \left(\frac{4}{11}\right)^{1/3}$$

$$\int_0^\infty \frac{x^2\, dx}{e^x - 1} = 2\,\zeta(3)$$

$$\int_0^\infty \frac{x^2\, dx}{e^x + 1} = \frac{3}{2}\,\zeta(3)$$

But neutrinos are massive particles!

$$n_\nu(T) = \int \frac{d^3p}{(2\pi)^3}\frac{1}{e^{p/T}-1} = \frac{3}{22}n_\gamma(T)$$

Data tell us....

$$\Omega_\nu(a) = \frac{m_\nu n_\nu}{\rho_{crit}} = \frac{\sum m_\nu}{94\text{ eV}h^2}\frac{1}{a^3} \longrightarrow 0.0006 \lesssim \Omega_{\nu,0}h^2 \lesssim 0.0025$$

Matter: baryons and dark matter

•**Baryons:** The baryon density can not be inferred from temperature measurements. Currently we know that:

$$\Omega_b h^2 = 0.02205^{+0.00056}_{-0.00055}$$

from the CMB anisotropies. Other methods to extract the present baryonic mass-energy density are light element abundances, quasar spectra or the gas population in galaxies.

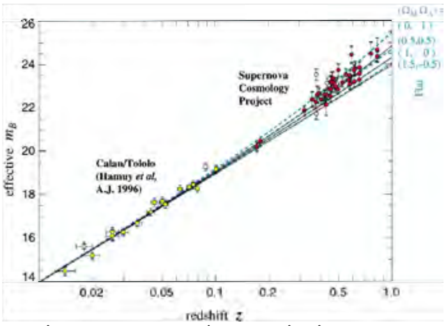
•**Dark matter**

A number of observations (galaxy rotation curves, galaxy clusters, gravitational lensing, large scale structure and the CMB anisotropies) indicate that the majority of the matter in the universe is unknown: dark matter!

$$\Omega_{dm} h^2 = 0.1199^{+0.0053}_{-0.0052}$$

Furthermore, observations of the large scale structure of our universe tell us that a COLD dark matter component provides an excellent fit to data.

Dark energy

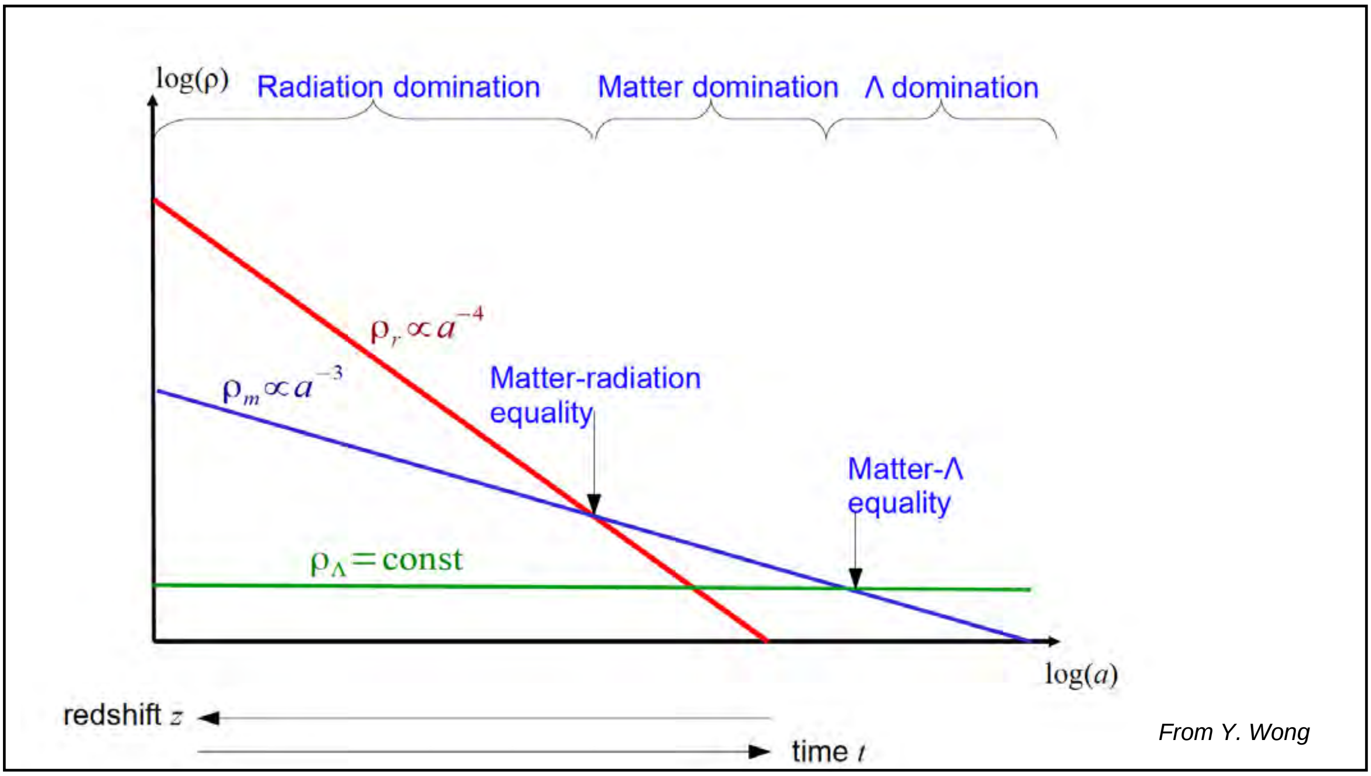


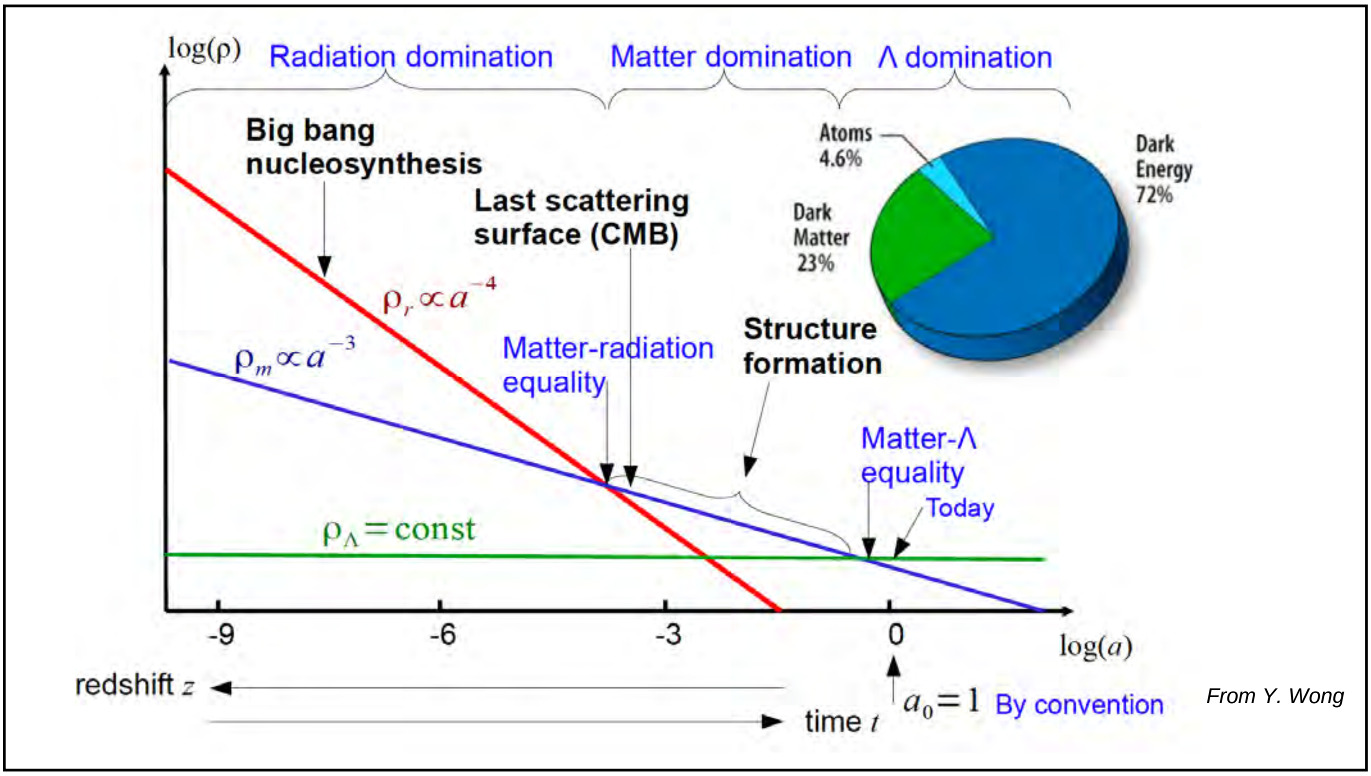
In 1998, two independent groups, observed that type Ia Supernovae were much fainter than what one would expect in a universe with only matter.

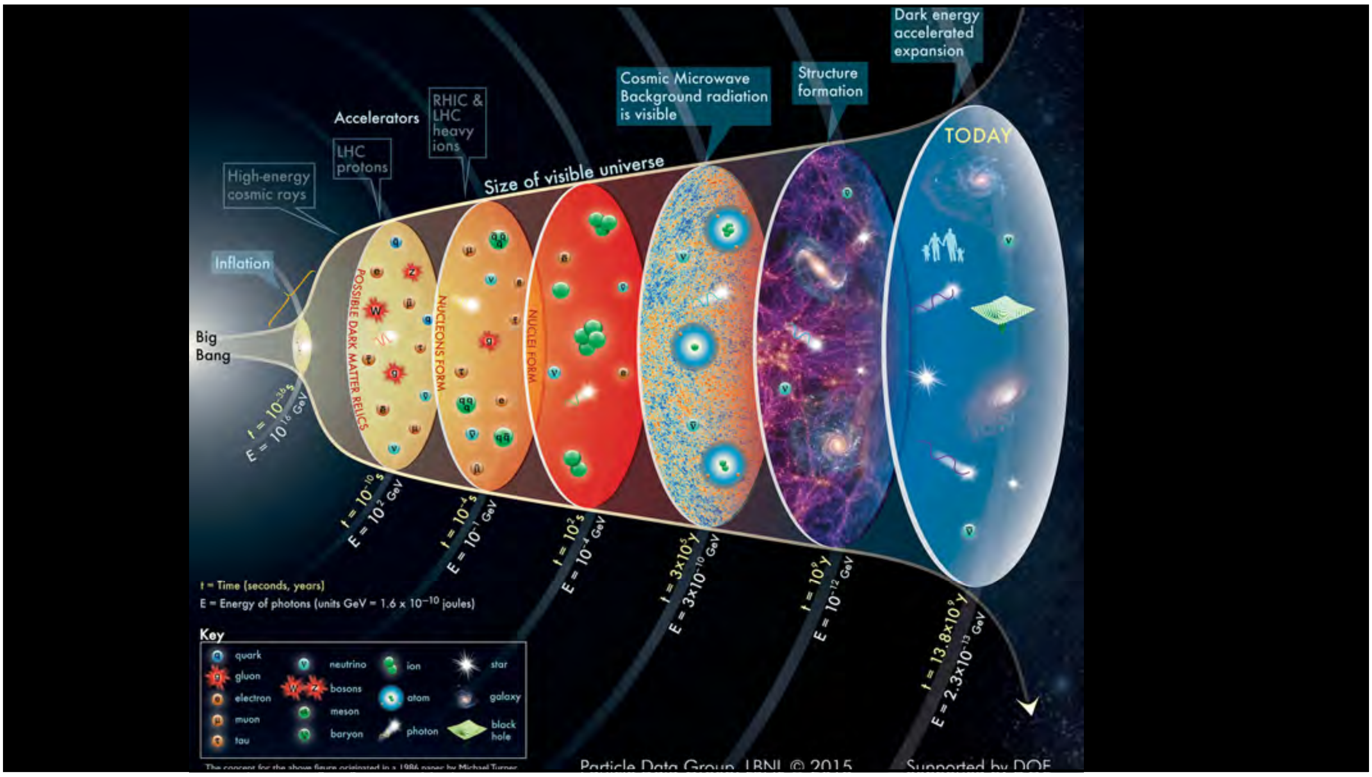
An additional ingredient was needed to make the universe to expand in an accelerated way!

Today the evidence for an accelerated expansion of the universe is 4.2σ-4.6σ with JLA SNIa data alone, and 11.2σ in a flat universe.

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) \qquad \rho + 3p < 0$$


log(ρ)
Radiation domination
Matter domination
Λ domination
$\rho_r \propto a^{-4}$
$\rho_m \propto a^{-3}$
Matter-radiation equality
Matter-Λ equality
$\rho_\Lambda = \mathrm{const}$
log(a)
redshift z
time t
From Y. Wong


log(ρ)
Radiation domination
Matter domination
Λ domination
Big bang nucleosynthesis
Last scattering surface (CMB)
Atoms 4.6%
Dark Energy 72%
Dark Matter 23%
Structure formation
$\rho_r \propto a^{-4}$
$\rho_m \propto a^{-3}$
Matter-radiation equality
Matter-Λ equality
Today
$\rho_\Lambda = \mathrm{const}$
-9
-6
-3
0
log(a)
redshift z
time t
$a_0 = 1$ By convention
From Y. Wong



BOLTZMANN EQUATIONS

- Throughout the universe's history, particles remain in thermal equilibrium until their interaction rate is equal or larger than the expansion rate of the universe. Then, the particle will decouple from the thermal bath. Of course this is an approximation:

$$\Gamma \lesssim H$$

- The accurate calculation requires to solve the Boltzmann equation:

$$Lf = Cf$$

- where f is the distribution function, L is the Liouville operator, and C contains all the collision terms.

- In classical mechanics: $\hat{L} = \frac{\partial}{\partial t} + \mathrm{v} \cdot \nabla_{\mathrm{x}} + \frac{\mathrm{F}}{m} \cdot \nabla_{\mathrm{v}}\,,$

- The relativistic version is: $\hat{L} = p^\alpha \frac{\partial}{\partial x^\alpha} - \Gamma^\alpha_{\beta\gamma} p^\beta p^\gamma \frac{\partial}{\partial p^\alpha}$ $\quad P^\alpha = (E, \vec{P}) \quad P^\alpha = \frac{dx^\alpha}{d\lambda}$

- FRW geometry: $\hat{L}f = E\frac{\partial f}{\partial t} - Hp^2 \frac{\partial f}{\partial E}$ $\quad \boxed{\frac{dn}{dt} + 3Hn = \frac{g}{(2\pi)^3} \int Cf \frac{d^3p}{E}}$

BOLTZMANN EQUATIONS

- Simplifying the possible processes (1+2 ↔3+4):

In an expanding universe, the number of particles gets diluted!

In the absence of interactions, $n \propto a^{-3}$

$$\frac{dn}{dt} + 3Hn = \int \frac{d^3p_1}{(2\pi)^3 2E_1} \int \frac{d^3p_2}{(2\pi)^3 2E_2} \int \frac{d^3p_3}{(2\pi)^3 2E_3} \int \frac{d^3p_4}{(2\pi)^3 2E_4}$$

Particle Physics

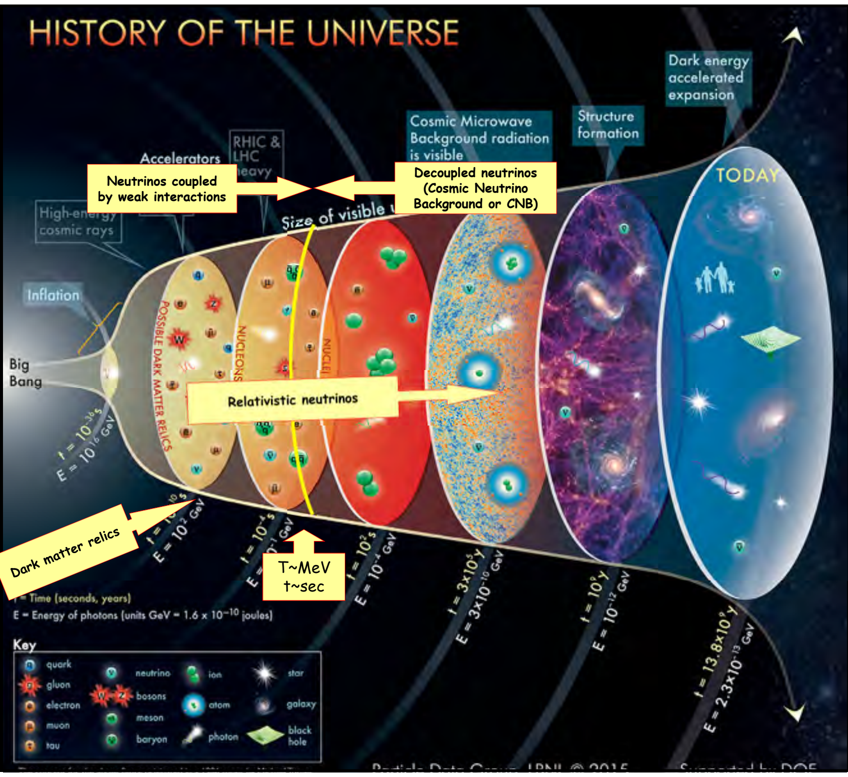
$$\times (2\pi)^4 \delta^3(p^1 + p^2 - p^3 - p^4)\, \delta(E^1 + E^2 - E^3 - E^4)\, |\mathcal{M}|^2$$

Energy-momentum tensor conservation

$$\times (f_3 f_4 - f_1 f_2)$$

Loss rate of 1 is proportional to the occupation numbers of 1 and 2

Production rate of 1 is proportional to the occupation numbers of 3 and 4



Neutrino cosmology is interesting because Relic neutrinos are very abundant:

- The CNB contributes to radiation at early times and to matter at late times (info on the number of neutrinos and their masses)
- Cosmological observables can be used to test non-standard neutrino properties

YOUR TURN !!

- PLEASE GO TO THE WEB PAGE:

http://www.astro.ucla.edu/%7Ewright/CosmoCalc.html

- COMPUTE THE AGE OF THE UNIVERSE TODAY AS A FUNCTION OF THE HUBBLE CONSTANT. HOW IS THE CORRELATION? CAN YOU EXPLAIN THAT IN TERMS OF THE ANALYTICAL EXPRESSIONS?

- COMPUTE THE AGE OF THE UNIVERSE TODAY AS A FUNCTION OF THE MATTER DENSITY. HOW IS THE CORRELATION? CAN YOU EXPLAIN THAT IN TERMS OF THE ANALYTICAL EXPRESSIONS?

- COMPUTE THE DIFFERENT OBSERVABLES IN OPEN AND FLAT COSMOLOGIES, AT A REDSHIFT OF Z=1000. EXPLAIN THE DIFFERENCES!

1. INTRODUCTION: FUNDAMENTAL INGREDIENTS & THERMAL HISTORY OF THE UNIVERSE ✓
2. **NEUTRINO DECOUPLING IN THE EARLY UNIVERSE**
3. BIG BANG NUCLEOSYNTHESIS & N_{eff}
4. COSMOLOGY & N_{eff}
5. COSMOLOGY & NEUTRINO MASSES
6. TAKE HOME MESSAGES

Particle decoupling in the early universe: Neutrinos

- We have seen that a very easy and straightforward hand-waving rule to compute a particle decoupling time in the early universe is:

$$\Gamma \lesssim H$$

- Neutrinos only interact via weak interactions, with a rate:

$$\Gamma_\nu = n\sigma v \simeq T^3 G_F^2 T^2 \sim G_F^2 T^5$$

- While the expansion rate of the universe is given by the Hubble factor:

$$H^2 = \frac{8\pi G}{3}\rho \sim T^4/m_{pl}^2$$

$$\Gamma_\nu / H \sim \left(\frac{T}{1\ \mathrm{MeV}}\right)^3$$

- Therefore neutrinos decouple from the thermal bath around 1 MeV.

Particle content & interactions at 0.1 < T < 100 MeV...

QED plasma

$$e^+, e^-, \gamma$$

3 families of neutrinos + antineutrinos

$$\nu_e, \bar{\nu}_e, \nu_\mu, \bar{\nu}_\mu, \nu_\tau, \bar{\nu}_\tau$$

EM interactions:

$$e^+e^- \leftrightarrow e^+e^-$$
$$e\,e \leftrightarrow e\,e$$
$$\gamma\, e \leftrightarrow \gamma\, e$$
$$\gamma\,\gamma \leftrightarrow e^+e^-$$
etc.

Weak interactions @ T > O(1) MeV:

$$\nu_i \nu_j \leftrightarrow \nu_i \nu_j$$
$$\nu_i \bar{\nu}_j \leftrightarrow \nu_i \bar{\nu}_j$$
$$\nu_i \nu_i \leftrightarrow \nu_i \nu_i$$
$$\nu_i \bar{\nu}_i \leftrightarrow \nu_j \bar{\nu}_j$$
etc.

Decoupled

$$\nu\, e \leftrightarrow \nu\, e$$
$$\nu\,\bar{\nu} \leftrightarrow e^+ e^-$$

Weak interactions @ T > O(1) MeV:

Not efficient
T< O(1) MeV

Particle decoupling in the early universe: Neutrinos

¿How are related the photon and the neutrino temperatures?

- The entropy density is: $s \equiv \dfrac{\rho + p}{T}$

$e^- e^+ \rightarrow \gamma\,\gamma$ becomes "one-way"

- Electron positron annihilation takes place AFTER neutrino decoupling.

- In an expanding universe the entropy density per comoving volume is conserved:

  - Massless boson's entropy contribution: $2\pi^2 T^3/45$

  - Massless fermion's entropy contribution: $7/8 \times 2\pi^2 T^3/45$

- Before electron/positron annihilation= electrons (g=2), positrons (g=2), neutrinos (3x g=1), antineutrinos (3x g=1) and photons (g=2) therefore:

$$s(a_1) = 2\pi^2 T_1^3/45(2 + 7/8(2 + 2 + 3 + 3))$$

- After, only neutrinos, antineutrinos and photons but at different temperature!

$$s(a_2) = 2\pi^2/45(2T_\gamma^3 + 7/8(3+3)T_\nu^3)$$

Particle decoupling in the early universe: Neutrinos

$$s(a_1)a_1^3 = s(a_2)a_2^3$$

$$\frac{43}{2}(a_1 \cancel{T_1})^3 = 4\left[\left(\frac{T_\gamma}{T_\nu}\right)^3 + \frac{21}{8}\right](a_2 \cancel{T_\nu})^3$$

$$a_1 T_1 = a_2 T_\nu \longrightarrow \left(\frac{T_\nu}{T_\gamma}\right) = \left(\frac{4}{11}\right)^{1/3}$$

Number of neutrinos: N_{eff}

The total radiation in the universe can be written as:

In the idealized pictured we discussed N_{eff} =3

$$\Omega_r h^2 = \left(1 + \frac{7}{8}\left(\frac{4}{11}\right)^{4/3} N_{\rm eff}\right)\Omega_\gamma h^2$$

N_{eff} = 3.0440 ± 0.0002 standard scenario: electron, muon and tau neutrinos   *Bennett et al, 2012.02726*

N_{eff} < 3.044 (less neutrinos): Neutrino decays ?

N_{eff} > 3.044 (more neutrinos): Sterile neutrino species ?

N_{eff} ,the "effective number of neutrino species", is a measure of the energy density of light species, other than photons in the early Universe (normalized to that of a single massles neutrino).

All particle species behave as ideal gases   *Bennett et al, 1911.04504*

(ideal gas approximation)

The neutrino decoupling process is localized at $T=T_\nu=T_d$
the neutrino and QED sectors transit from a state of tight thermal contact to a state of zero thermal contact at the neutrino decoupling temperature

(instantaneous decoupling approximation)

The electron/positron sector is fully ultra-relativistic at the time of neutrino decoupling

$T_d/m_e \to \infty$ (ultra-relativistic approximation)

Bennett et al, 2012.02726

Standard-model corrections to $N_{\rm eff}^{\rm SM}$	Leading-digit contribution
m_e/T_d correction	$+0.04$
$\mathcal{O}(e^2)$ FTQED correction to the QED EoS	$+0.01$
Non-instantaneous decoupling+spectral distortion	-0.005
$\mathcal{O}(e^3)$ FTQED correction to the QED EoS	-0.001
Flavour oscillations	$+0.0005$
Type (a) FTQED corrections to the weak rates	$\lesssim 10^{-4}$

Bennett et al, 1911.04504

10^{-4} Uncertainty due to measurement errors on the solar mixing angle

They do not inherit any of the energy associated to $e^+ e^-$ annihilations, being colder than photons:

$$T_{\nu 0} = \left(\tfrac{4}{11}\right)^{1/3} T_0 = 1.945\ \mathrm{K} \rightsquigarrow 1.697 \times 10^{-4}\ \mathrm{eV}$$

If these neutrinos are massive, their energy density, at T<<m is

$$\rho_\nu = m_\nu n_\nu \qquad n_{\nu_i}(T_{\nu 0}) \approx 56\ \mathrm{cm}^{-3} \qquad \Omega_\nu h^2 = \frac{\sum m_\nu}{93\ \mathrm{eV}}$$

Then, demanding that massive neutrinos do not over-close the universe, $\sum m_\nu \lesssim 45\ \mathrm{eV}$

If the relic neutrinos are relativistic today m_ν < 0.1 meV)

$$\Delta m^2_{12} = (7.05 - 8.14) \times 10^{-5}\mathrm{eV}^2$$
$$\Delta m^2_{13} = (2.41 - 2.60) \times 10^{-3}\mathrm{eV}^2$$
$$\Delta m^2_{13} = -(2.31 - 2.51) \times 10^{-3}\mathrm{eV}^2$$

We are sure then that two neutrinos have a mass above:

$$\sqrt{\Delta m^2_{12}} \simeq 0.008\ \mathrm{eV}$$

and that at least one of these neutrinos has a mass larger than

$$\sqrt{|\Delta m^2_{13}|} \simeq 0.05\ \mathrm{eV}$$

They do not inherit any of the energy associated to e⁺ e⁻ annihilations, being colder than photons:

$$T_{\nu 0} = \left(\tfrac{4}{11}\right)^{1/3} T_0 = 1.945\ \mathrm{K} \rightsquigarrow 1.697 \times 10^{-4}\ \mathrm{eV}$$

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If the relic neutrinos are relativistic today m_ν < 0.1 meV)

Observations indicate Ω_{DM}~0.25. Can it be explained by neutrino dark matter? NO

The obvious reason: a neutrino mass of ~ 10 eV is needed (not allowed by current tritium β-decay experiments).

The deeper reason: relic neutrinos come with large thermal motion, with a characteristic **thermal speed**

$$\frac{\langle p_\nu \rangle}{m_\nu} \simeq 150(1+z)\left(\frac{\mathrm{eV}}{m_\nu}\right)\mathrm{km/s}$$

Thermal motion counters the effect of gravitational instability.
Neutrino gas **does not collapse** because neutrinos fly away!

Collapse time scale:

$$\Delta t_{\text{collapse}} \equiv (4\pi G \rho a)^{-1/2}$$

How long does it take for the overdense region to collapse to a point

Escape time scale:

$$\Delta t_{\text{escape}} \equiv \frac{\lambda}{v_{\text{thermal}}}$$

How long does it take for the neutrinos to fly out of the region

Scenario 1: Growth

Collapse happens **faster** than escape

$$\Delta t_{\text{collapse}} \ll \Delta t_{\text{escape}}$$

- Density perturbation collapses before neutrinos can fly away.
- Perturbations grow.

Scenario 2: Erasure

Collapse happens **slower** than escape

$$\Delta t_{\text{collapse}} \gg \Delta t_{\text{escape}}$$

- Neutrinos fly away before gravity can capture them.
- Perturbations are erased.

Suppose relic neutrinos make up **all** of the dark matter…

Growth or erasure? Define the free-streaming scale at redshift z:

$$\lambda_{\mathrm{FS}}(z) \equiv v_{\mathrm{thermal}} \Delta t_{\mathrm{collapse}}$$
$$= 0.41 \Omega_{m,0}^{-1/2} (1+z)^{1/2} \left(\frac{\mathrm{eV}}{m_\nu}\right) h^{-1} \mathrm{Mpc}$$

Equivalent to
$\Delta t_{\mathrm{collapse}} = \Delta t_{\mathrm{escape}}$

Unless density perturbations are regenerated by other means, at any redshift z, structures of length scale $\lambda < \lambda_{\mathrm{FS}}(z)$ **cannot be formed** out of relic neutrinos.

Suppose relic neutrinos make up **all** of the dark matter…

The maximum free-streaming scale is that at the time when neutrinos become nonrelativistic:

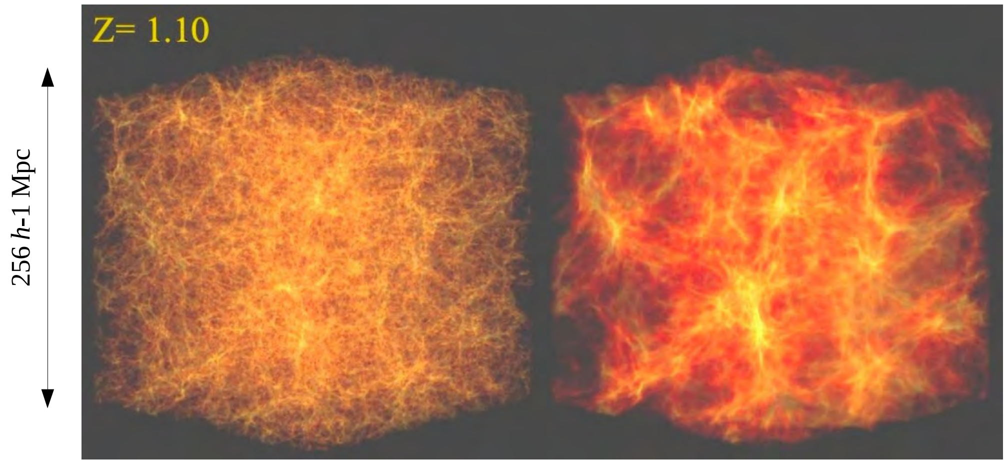
$$\lambda_{\mathrm{FS,max}} \equiv \lambda_{\mathrm{FS}}(z_{\mathrm{nr}}) = 31.8 \Omega_{m,0}^{-1/2} \left(\frac{\mathrm{eV}}{m_\nu}\right)^{1/2} h^{-1} \mathrm{Mpc}$$

Using
$1 + z_{\mathrm{nr}} \simeq \frac{m_\nu}{T_{\nu,0}}$

$\lambda_{\mathrm{FS,max}}$ corresponds to the maximum size of objects that **could not have been formed** in a neutrino dark matter-only universe.

If a 10 eV-mass neutrino was the dark matter, $\lambda_{\mathrm{FS,max}} \sim 25$ Mpc , we would not have galaxies (λ ~10 kpc) and galaxies clusters (λ ~1 Mpc)!

Cold dark matter Massive neutrinos 7 eV



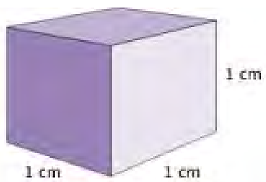
Simulations by Troels Haugbølle

According to standard cosmology, there is a cosmic neutrino background, equivalent to the CMB photon background, albeit slightly colder T≃ 1.94 K

… **is a difficult business.**

cf WIMP detection, ~ 10^{-46} cm^2

340 neutrinos/cm^3



410 photons/cm^3

Small interaction cross-section $\sigma_{\nu N} \sim \frac{G_F^2 m_\nu^2}{\pi} \simeq 10^{-56} \left(\frac{m_\nu}{\text{eV}}\right)^2 \text{cm}^2$

Neutrino energy too small to cross most detection thresholds.

$$\Delta p \sim m_\nu v_{\text{earth}} \simeq 10^{-3} m_\nu$$

Conventional WIMP detection techniques via nuclear recoil don't work here.

This cosmic relic neutrino background has never been detected directly.