Coherence in neutrino oscillations

Subject suggested by J. Kopp

Gustavo Alves, Yoxara Sánchez Villamizar, Alexander Trettin, Bouke Jung, Abhishek Kumar Jha, Thomas Kosc

0. Motivation

Neutrino oscillations is a very subtle quantum phenomenon. In the literature very strong assumptions are made when deriving the oscillation probability formula, the most famous ones are:

- All the mass eigenstates have equal momentum.
- All the mass eigenstates have equal energy.

More problems and the discussion of their solutions can be found in here

Are those at least consistent? Even though they allow to reach the final result in a simple and quick way, there is no reason for these to hold in general. For specific processes, e.g pion decay, those requirements are not satisfied.

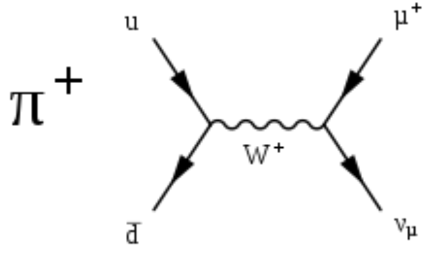
Even more problematic are the conceptual problems associated with such requirements. If the neutrinos have the same momentum, they are described by a plane wave which has flat distribution probability to be found in any point of the space which is in disagreement with the need for well defined production and detection regions for the oscillations to take place.

To handle such problems (and other ones) a wave packet approach is mandatory and a careful study of the coherence properties of the wave packet.

1. Compute the momentum and energy of a neutrino emitted in a pion decay at rest:

$$E_\nu = \frac{m_\pi^2 + m_\nu^2 - m_\mu^2}{2m_\pi}$$

$$P_\nu = \sqrt{\frac{(m_\pi^2 + m_\nu^2 - m_\mu^2)^2 - 4m_\pi^2 m_\nu^2}{4m_\pi^2}} \approx \frac{m_\pi^2 - m_\mu^2}{2m_\pi} - \frac{(m_\mu^2 + m_\pi^2)m_\nu^2}{2m_\pi(m_\pi^2 - m_\mu^2)}$$



This reflects in a momentum resolution that must be achieved in order to discriminate the different mass eigenstates:

$$\sigma_{p_{ij}} = \frac{(m_\mu^2 + m_\pi^2)}{2m_\pi(m_\pi^2 - m_\mu^2)}\Delta m_{ji}^2 \quad \begin{cases} \sigma_{p13} = 3.3 \times 10^{-11} \text{ eV} \\ \sigma_{p12} = 9.79 \times 10^{-13} \text{ eV} \end{cases}$$

We can convert this in an uncertainty in the position of production or detection via the Heisenberg principle. If the required resolution is achieved the localization uncertainty will be much bigger than the oscillation length, washing out the oscillation pattern.

2. The oscillatory behavior can be observed if the detector is unable to distinguish between the different mass eigenstates. Use the Heisenberg principle to derive a condition on the size of the detector.

- The detector can tell the difference between different mass eigenstates if it can achieve a resolution bigger than the mass squared splitting.

- If we use the dispersion relation for massive neutrinos we can find the correlation between errors in energy, momentum and mass measurements. This restricts the momentum resolution necessary to observe the oscillatory pattern:

$$\Delta p > \frac{|M_i^2 - M_j^2|}{2p}$$

- We can then convert this result into a inequality for the position of detection or production uncertainty:

$$\Delta x < \frac{2p}{|M_i^2 - M_j^2|}$$

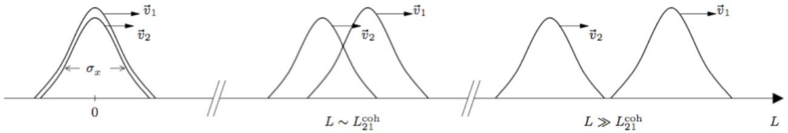
See also: KAYSER, Boris. *Physical Review D*, 1981, 24.1: 110.

3. Coherence length in neutrino oscillations

In order to estimate the coherence length we can calculate the difference between the velocities of different mass eigenstates:

$$v_k \simeq 1 - \frac{m_k^2}{2E}$$

Giunti & Kim, 8.2.2



Assume 2 neutrino waves packets v_k, wave packet dispersion reaches:

$$|v_j - v_k| T \geq \sigma \iff T = L \geq \frac{2E^2}{\Delta m_{jk}^2} \sigma$$

Wave packet treatment of neutrino oscillations adds a factor that penalizes oscillations for large L

$$p(\nu_\alpha \to \nu_\beta) \propto \sum_{j,k} \exp\left(-(L/L_{jk}^{(coh)})^2\right) \times \mathrm{Osc.\ \ terms} \qquad L_{jk}^{(coh)} = \frac{4\sqrt{2}E^2}{|\Delta m_{jk}^2|}\sigma$$

Coherence lengths

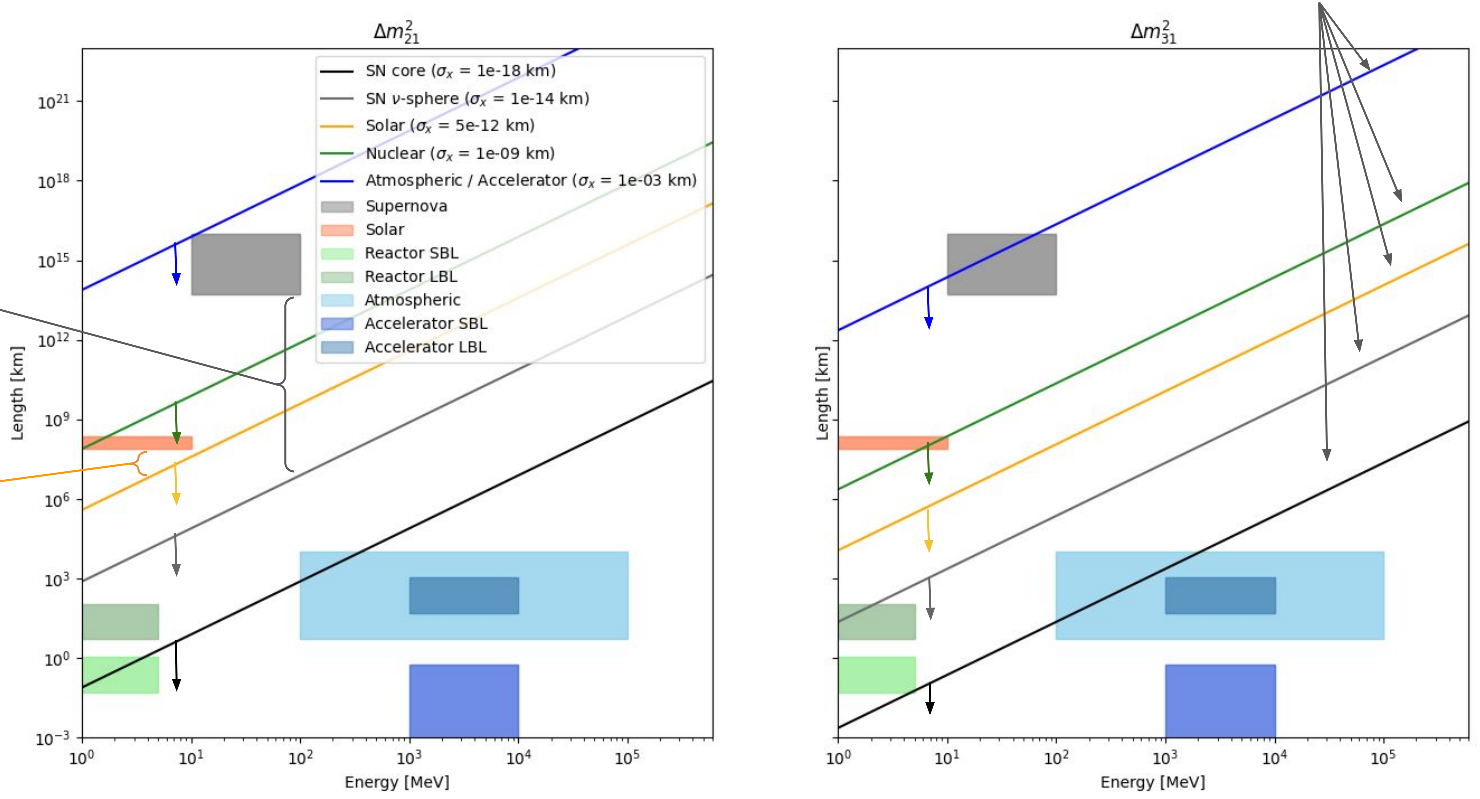
wave packet sizes from [arXiv:hep-ph/9607391v1]

Coherence lengths:

$$L_{k,j}^{\mathrm{coh}} \approx \frac{4\sqrt{2}E^2}{\left|\Delta m^2_{k,j}\right|}\sigma_x$$

SN ν's decoherent

solar ν's potentially decoherent



4. & 5. Decoherent Neutrino States in Matter (two flavor approximation)

Probability of observing a ν_2 state as ν_e:

$$P_{\nu_2 \to \nu_e} = \left|\langle \nu_e | \nu_2 \rangle\right|^2 = \left|\sum_\alpha \langle \nu_e | U_{\alpha 2} | \nu_\alpha \rangle\right|^2 = |U_{e2}|^2$$

Evolution through layer with effective θ_m and Δm^2_m :

$$\mathcal{U} = U_m \mathrm{diag}\left[\exp\left(i \frac{\Delta m^2_m L_m}{2E}\right), 1\right] U_m^\dagger, \quad U_m = \begin{pmatrix} \cos(\theta_m) & \sin(\theta_m) \\ -\sin(\theta_m) & \cos(\theta_m) \end{pmatrix}$$

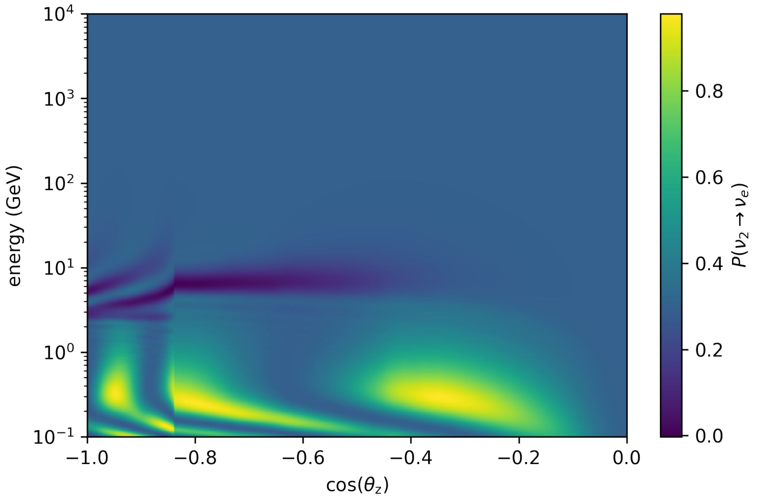
flavor-to-flavor transition matrix

Simple Earth with inner core:

$$\mathcal{U}^{\mathrm{Earth}} = \mathcal{U}^m \mathcal{U}^c \mathcal{U}^m$$

$$P_{\nu_2 \to \nu_e} = \left|\langle \nu_e | \nu_2 \rangle\right|^2 = \left|\sum_\alpha \sum_\beta \langle \nu_e | \mathcal{U}^{\mathrm{Earth}}_{\alpha\beta} U_{\beta 2} | \nu_\alpha \rangle\right|^2 = \left|U_{e2}\mathcal{U}^{\mathrm{Earth}}_{ee} + U_{\mu 2}\mathcal{U}^{\mathrm{Earth}}_{e\mu}\right|^2$$

need to rotate to flavor basis before plugging into $\mathcal{U}^{\mathrm{Earth}}$



Full three flavor oscillation probability from nuSQuIDS.

Joern Kersten et al. Eur. Phys. J. C76 (2016), 339

6. Discuss whether neutrinos produced in Z boson decays oscillate, and what it would take to observe these oscillations.

$\nu_\beta \to \nu_\ell$

Noncoherent fluxes

$$I_\beta(r) = \sum_\alpha I_\alpha^0 P_{\alpha\beta}(r) = I^0$$

$\beta = e, \mu, \tau$

conservation of total probability

initial fluxes: $I_e^0 = I_\mu^0 = I_\tau^0 \equiv I^0$

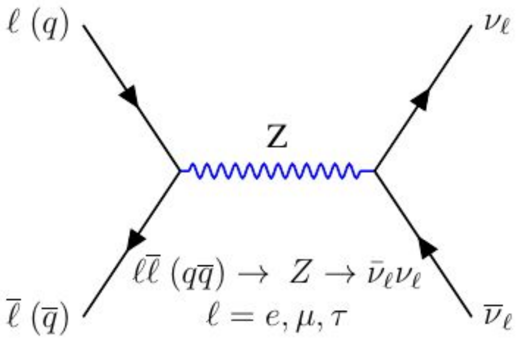
Z -decay neutrinos do not oscillate.

Coherent fluxes

Neutrino state oscillations produced at Z-Decay

$$|\nu_Z(r,\overline{r})\rangle = \frac{1}{\sqrt{3}} \sum_{i=1,2,3} |\nu_i\rangle\langle\overline{\nu_i}| e^{i\phi_i(r,\overline{r})}$$

$$\nu_\ell = \sum_{i=1,2,3} U_{i\alpha}\nu_i$$



Z-decay Feynman diagram

Amplitude of probability: $\langle \nu_\ell, \nu_\beta | \nu_Z(r,\overline{r})\rangle = \frac{1}{\sqrt{3}} \sum_{i=1,2,3} U_{i\ell}^* U_{i\beta} e^{i\phi_i(r,\overline{r})}$, $\phi_i(r,\overline{r}) \cong \frac{m_i^2}{2E} r + \frac{m_i^2}{2\overline{E}}\overline{r}$

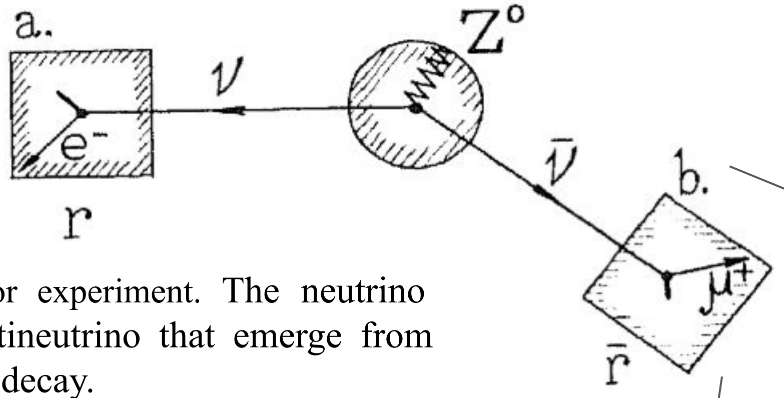
Probability to find ν_ℓ at r and $\overline{\nu_\beta}$ at $\overline{r}$: $W_{\ell\beta} = \frac{1}{3}|U_{i\ell}|^2 . |U_{i\beta}|^2 + \frac{2}{3}\sum_{i>j} |U_{i\ell}^* U_{i\beta} U_{j\ell} U_{j\beta}^*| \cos(\phi_i - \phi_j - \xi_{\ell\beta ij})$, $\xi_{\ell\beta ij} = arg\,(U_{i\ell}^* U_{i\beta} U_{j\ell} U_{j\beta}^*)$

$$W_\ell \equiv \sum_\beta W_{\ell\beta} = \frac{1}{3}$$

YU. SMIRNOV, A. and Zatsepin, G.T., 1992. Can the neutrinos from Z0-decay oscillate?. Modern Physics Letters A, 7(14), pp.1273-1280.

Pavlichenkov, I.M., 2012. Neutrino and antineutrino oscillations from decay of Z boson. arXiv preprint arXiv:1201.3103.

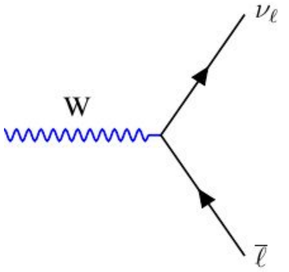
(Effect: entanglement between two oscillating leptons).

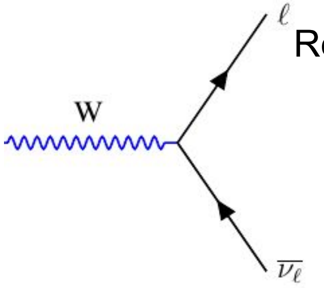


Two detector experiment. The neutrino and the antineutrino that emerge from the same Z-decay.

Neutrino and antineutrino oscillations in coincidence

Independent oscillations of neutrinos and antineutrinos could be observed by synchronising the time in two detectors.





Registration of charge-defined leptons

$$W_{\ell\beta} \propto event\ numbers$$

YU. SMIRNOV, A. and Zatsepin, G.T., 1992. Can the neutrinos from Z0-decay oscillate?. Modern Physics Letters A, 7(14), pp.1273-1280.

EXTRA

Mass eigenstate separation

The relative velocity between a massless and massive neutrino:

$$\Delta v = 1 - |\vec{v}_2| \approx \frac{m_2^2}{2E^2}$$

The travel distance needed for a 1 MeV v to end up with a 1 second separation:

$$D \approx \Delta T = \frac{\Delta x}{\Delta v} = \frac{2E^2}{m_2^2}\Delta t$$

$$D = \frac{2E^2}{m_2^2}\Delta t \approx 2.4 \times 10^{20}\ \mathrm{km} \approx O(1)\ \mathrm{Mpc}$$

The wave packet size

Dependent upon production process:

1. Decay ($\Longleftrightarrow$ Natural Linewidth)

$$\sigma_x^\nu \sim \gamma \tau_X$$

2. Scattering ($\Longleftrightarrow$ Collision Broadening)

$$\sigma_x^\nu \sim \mathrm{Min}_X \left[\frac{\lambda_X}{v_X} \right]$$

$$\lambda_X \sim \mathrm{Max} \left[\frac{1}{\pi b^2 n}, \frac{1}{n^{1/3}} \right]$$

$$b \approx \frac{4\pi\alpha Z}{E_k}$$

For a medium in thermal equilibrium:

$$E_k \approx \frac{3}{2} k_B T$$

E.g.: SN neutrinos

In neutronization phase: $\mathrm{n} \to \mathrm{p} + \mathrm{e}^- + \bar{\nu}_e$

$$T \sim 10^{10} K$$

$$\rho \sim 10^{10}\,\mathrm{g\,cm^{-3}}$$

$$n \sim \frac{\rho}{M} N_A \sim 10^{32}\,\mathrm{cm^{-3}}$$

$$\sigma_x \sim 2.4 \cdot 10^8 \frac{\sqrt{\frac{m_n}{\mathrm{MeV}}} \left(\frac{T}{1\,\mathrm{K}}\right)^{\frac{3}{2}}}{n Z^2}\,\mathrm{cm} \approx 10^{-10}\,\mathrm{cm}$$

C. Giunti & C. W. Kim *Fundamentals of Neutrino Physics and Astrophysics.* 8.3