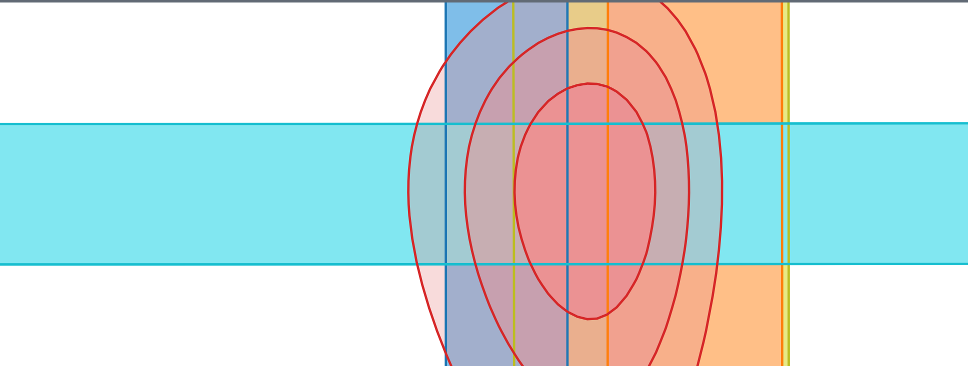
Heavy flavour aspects in EFT fits, LHC EFT WG, 12 April 2021

Fitting tools: SMEFT interpretation of anomalies using Flavio

Peter Stangl AEC & ITP University of Bern



Tools

- **flavio**: Theory predictions, Database of measurements, Likelihoods
- **wilson**: RG evolution in SMEFT and WET, matching from SMEFT to WET
- **Wilson coefficient exchange format (WCxf)**
- **smelli** - the **SME**FT **Like LI**hood: WET and SMEFT likelihood function

Tools

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- **smelli** - the **SME**FT **Like****LI**hood: WET and SMEFT likelihood function

flavio: what can it do for me?

flavio: main features

1. Computing theory predictions

for a huge number of observables (flavour physics, electroweak precision observables, Higgs physics, ...)

- **Standard Model** (SM) predictions
- Predictions in the presence of **new physics** (NP) (parameterized by Wilson coefficients)
- Theory **uncertainties** for SM and NP

flavio: main features

2. Database of experimental data

for all implemented observables that have been measured

- provided in terms of YAML file
- easy to update and extend

flavio: main features

3. Likelihoods

Combining predictions with experimental data allows constructing likelihoods

- Likelihoods in parameters (e.g. CKM parameters) or Wilson coefficients
- Possibility to use Gaussian approximation for **fast likelihood** estimates
- Use external fitters to perform Bayesian or frequentist statistics with `flavio` likelihoods
- Basis for **smelli** - the **SME**FT **Like****LI**hood

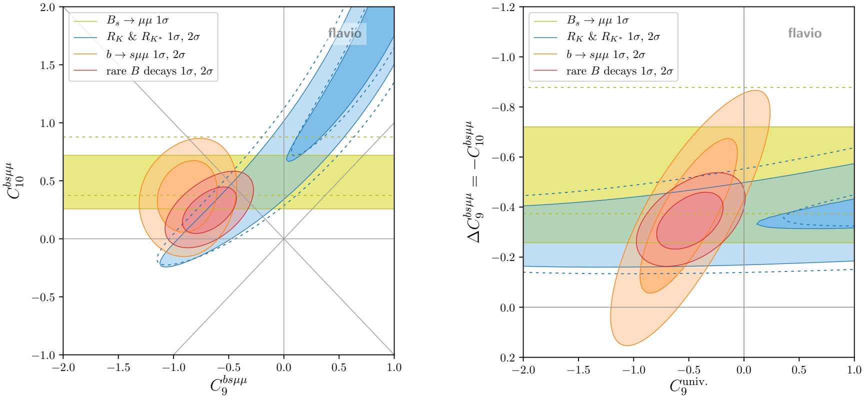
flavio: main features

4. Plots

- Visualize experimental measurements & theory predictions
- Visualize your likelihoods

flavio: showcase

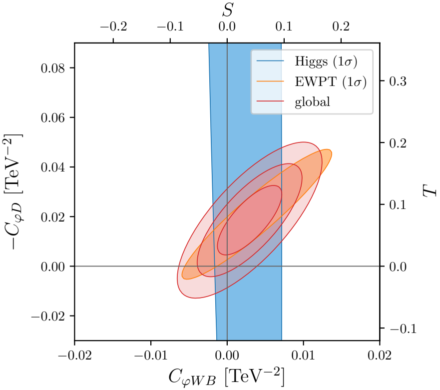
New physics in *B*-decays in Weak effective theory Wilson coefficients @ 4.8 GeV



Altmannshofer, PS, arXiv:2103.13370

flavio: showcase

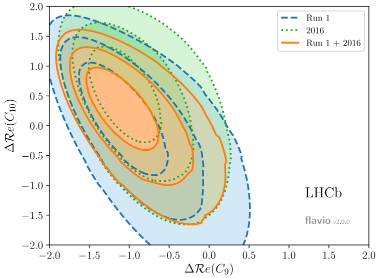
S-T fit using combined Higgs and electroweak likelihood in SMEFT

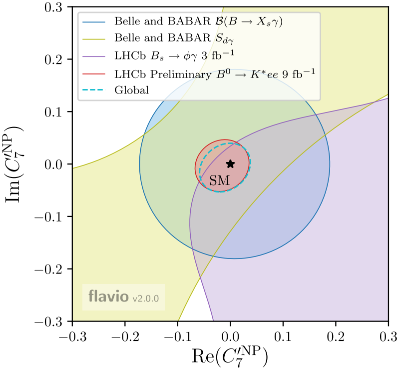


Falkowski, Straub, arXiv:1911.07866

flavio: showcase

Fits to new physics Wilson coefficients from recent LHCb analyses





LHCb-PAPER-2020-002
LHCb-TALK-2020-155

flavio: observables

See `https://flav-io.github.io/docs/observables.html`

- B physics: $B \to (V, P, X)\ell\ell$, $B \to \ell\ell$, $B \to (V, X)\gamma$, $\Lambda_b \to \Lambda\ell\ell$, $B \to (V, P, X)\ell\nu$, $B \to \ell\nu$, mixing
- K physics: $K \to \pi\nu\nu$, $K \to \ell\ell$, $K \to \ell\nu$, $K \to \pi\ell\nu$, ε_K, ε'/ε
- D physics: $D \to \ell\nu$, CPV in mixing
- μ physics: $\mu \to e\gamma$, $\mu \to 3e$, μ-e conversion, ν trident
- τ physics: $\tau \to 3\ell$, $\tau \to \ell\gamma$, $\tau \to (P, V)\ell$, $\tau\nu$, $\tau\ell\nu\nu$
- EWPT: All LEP-1 Z and W pole observables
- Dipole moments: $(g-2)_{e,\mu,\tau}$, d_n
- Higgs production and decay *new flavio v2* Falkowski, Straub, arXiv:1911.07866
- Nuclear and neutron β decays *new flavio v2*
- Atomic and molecular EDMs *new flavio v2*

flavio: setup & documentation

- Requires **Python 3.6** and pip (Python package manager)
- Installation

```
python3 -m pip install flavio --user
```

  (automatically downloads `flavio` and all dependencies)
- **Introductory documentation**: `https://flav-io.github.io/`
- Detailed **API documentation** of all functions and classes: `https://flav-io.github.io/apidoc/flavio/`
- **GitHub repository**: `https://github.com/flav-io/flavio`
- Paper: D. Straub, arXiv:1810.08132 (not a manual)

wilson & WCxf

wilson & WCxf

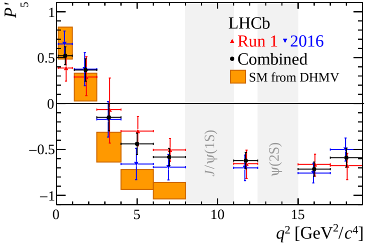
flavio depends on:

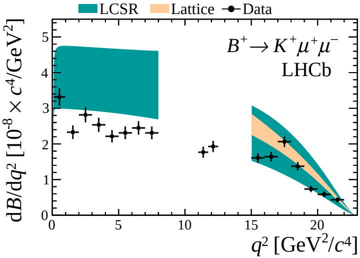
- **wilson** `https://wilson-eft.github.io` Aebischer, Kumar, Straub, arXiv:1804.05033
  - RG evolution above* and below** the EW scale arXiv:1308.2627, arXiv:1310.4838 arXiv:1312.2014, arXiv:1704.06639, arXiv:1711.05270
  - Matching from SMEFT to the weak effective theory (WET) arXiv:1709.04486, arXiv:1908.05295
  - Basis translation
- **Wilson coefficient exchange format (WCxf)** `https://wcxf.github.io/` Aebischer et al., arXiv:1712.05298
  - Representing and exchanging Wilson coefficient values
  - Different EFTs, different bases
  - Interface between codes

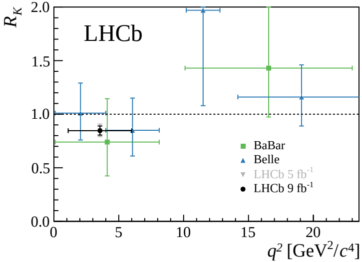
* based on DsixTools Celis, Fuentes-Martin, Vicente, Virto, arXiv:1704.04504
** see talk by Peter Stoffer

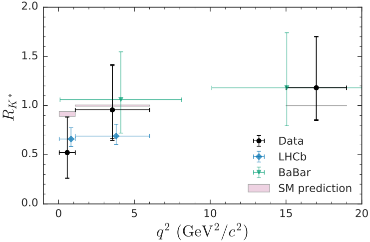
smelli - the **SME**FT **Like LI**hood

Motivation



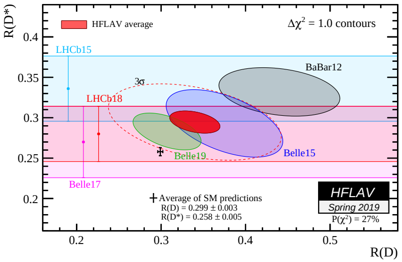


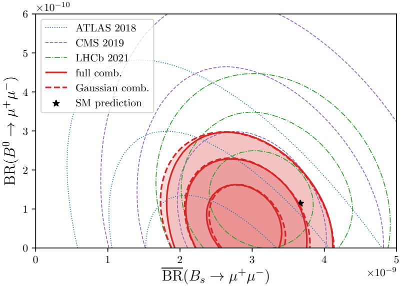




LHCb, arXiv:2003.04831, arXiv:1403.8044, arXiv:1506.08777, arXiv:1606.04731, arXiv:1705.05802, arXiv:1903.09252, arXiv:2103.11769
Belle, arXiv:1904.02440, arXiv:1908.01848

Motivation





BaBar, arXiv:1205.5442, arXiv:1303.0571
LHCb, arXiv:1506.08614, arXiv:1708.08856
Belle, arXiv:1507.03233, arXiv:1607.07923, arXiv:1612.00529, arXiv:1904.08794
ATLAS, arXiv:1812.03017; CMS, arXiv:1910.12127; LHCb seminar 23 March 2021; Altmannshofer, PS, arXiv:2103.13370

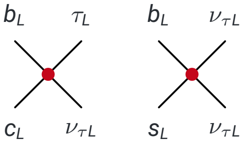
Model building - lessons learned

- Model explaining $R_{D^{(*)}}$ using $b_L \to c_L \tau_L \nu_{\tau L}$

$$b_L \to c_L \tau_L \nu_{\tau L} \xrightarrow{\text{SU(2)}_L} b_L \to s_L \nu_{\mu L} \nu_{\tau L}$$

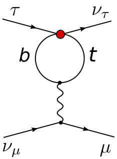
  Constrained by $B \to K\nu\bar{\nu}$ searches

  Buras, Girrbach-Noe, Niehoff, Straub, arXiv:1409.4557



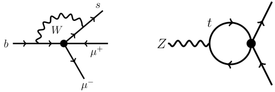
- Model explaining $R_{D^{(*)}}$ and $R_{K^{(*)}}$ using mostly 3rd gen. couplings
  Modifies LFU in τ and Z decays, strongly constrained

  Feruglio, Paradisi, Pattori, arXiv:1705.00929



- Model explaining $b \to s\mu\mu$ using $tt\mu\mu$ interaction
  Modifies $Z \to \mu\mu$, constrained by LEP

  Camargo-Molina, Celis, Faroughy, arXiv:1805.04917



What one would have to do

- Compute **all relevant observables** $\vec{O}$ (flavour, EWPO, ...) in terms of Lagrangian parameters $\vec{\xi}$

$$\mathcal{L}_{\text{NP}}(\vec{\xi}) \to \vec{O}(\vec{\xi})$$

- Take into account loop / RGE effects

$$\mathcal{L}_{\text{NP}}(\vec{\xi}) \xrightarrow{\Lambda_{\text{NP}} \to \Lambda_{\text{IR}}} \vec{O}(\vec{\xi})$$

- Compare to experiment

$$\vec{O}(\vec{\xi}) \to \underbrace{\mathcal{L}_{\text{exp}}(\vec{O}(\vec{\xi}))}_{\text{Likelihood}}$$

Tedious to do this for each model...

Effective field theories to the rescue

- Assuming $\Lambda_{\text{NP}} \gg v$, NP effects in flavour, EWPO, Higgs, top,... can be expressed in terms of Standard Model Effective Field Theory (SMEFT) Wilson coefficients

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \sum_{n>4} \sum_{i} \frac{C_i}{\Lambda^{n-4}} O_i$$

Buchmuller, Wyler, Nucl. Phys. B 268 (1986) 621
Grzadkowski, Iskrzynski, Misiak, Rosiek, arXiv:1008.4884

- Powerful tool to connect model-building to phenomenology without needing to recompute hundreds of observables in each model
  - Model building:

$$\mathcal{L}_{\text{NP}}(\vec{\xi}) \to \vec{C}(\vec{\xi}) \;\; @ \;\; \Lambda_{\text{NP}}$$

  - *Model-independent* pheno:

$$\vec{C} \xrightarrow{\Lambda_{\text{NP}} \to \Lambda_{\text{IR}}} \vec{O}(\vec{C}) \to \mathcal{L}_{\text{exp}}(\vec{O}(\vec{C}))$$

Effective field theories to the rescue

- **SMEFT likelihood function** $\mathcal{L}(\vec{C})$ can tremendously simplify analyses of NP models
- Several likelihood functions have been considered

$$\mathcal{L}(\vec{C}) = \mathcal{L}_{\text{EW + Higgs}}(\vec{C}_{\text{EW + Higgs}}) \times \dots$$
$$\mathcal{L}(\vec{C}) = \mathcal{L}_{\text{top physics}}(\vec{C}_{\text{top physics}}) \times \dots$$
$$\mathcal{L}(\vec{C}) = \mathcal{L}_{B\text{ physics}}(\vec{C}_{B\text{ physics}}) \times \dots$$
$$\mathcal{L}(\vec{C}) = \mathcal{L}_{\text{LFV}}(\vec{C}_{\text{LFV}}) \times \dots$$

cf. eg. Falkowski, Mimouni, arXiv:1511.07434
Falkowski, González-Alonso, Mimouni, arXiv:1706.03783
Ellis, Murphy, Sanz, You, arXiv:1803.03252
Biekötter, Corbett, Plehn, arXiv:1812.07587
Hartland et al., arXiv:1901.05965
...

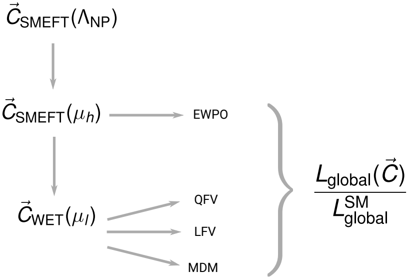
- But these likelihood functions should **not considered separately** since RG (loop) effects mix different sectors
- We need to consider the **global** SMEFT likelihood

smelli: implementation

- Based on **flavio**, **wilson**, and **WCxf**, we have implemented a **SMEFT likelihood function** in a Python package:
  - **smelli** - the **SME**FT **LikeLI**hood `https://github.com/smelli/smelli`

Aebischer, Kumar, PS, Straub, arXiv:1810.07698

- More than 400 observables included
  - Rare B decays
  - Semi-leptonic B and K decays
  - Meson-antimeson mixing
  - FCNC K decays
  - (LFV) tau and muon decays
  - Z and W pole EWPOs
  - $g-2$
  - beta decays *new*
  - Higgs physics *new* Falkowski, Straub arXiv:1911.07866



smelli: setup & documentation

- Requires **Python 3.6** and pip (Python package manager)
- Installation

```
python3 -m pip install smelli --user
```

  (automatically downloads `smelli` and all dependencies)
- Detailed **API documentation** of all functions and classes: `https://smelli.github.io/`
- **GitHub repository**: `https://github.com/smelli/smelli`
- Original paper: Aebischer, Kumar, PS, Straub, arXiv:1810.07698 (containing brief user manual)
- Recent article: PS, arXiv:2012.12211 (up-to-date usage examples)

smelli: EFT fits

smelli: EFT fits

- Global likelihood from **smelli**
- Quantify agreement between theory and experiment by likelihood L, $\Delta\chi^2$, and pull

$$\text{pull}_{\text{1D}} = 1\sigma \cdot \sqrt{\Delta\chi^2}\,, \qquad \text{where } -\frac{1}{2}\Delta\chi^2 = \ln L(\vec{0}) - \ln L(\vec{C}_{\text{best fit}})\,.$$

$$\text{pull}_{\text{2D}} = 1\sigma, 2\sigma, 3\sigma, \ldots \quad \text{for} \quad \Delta\chi^2 \approx 2.3, 6.2, 11.8, \ldots$$

- New physics scenarios **Weak Effective Theory (WET)** at scale 4.8 GeV

$b \to s\ell\ell$ in the weak effective theory

- Effective Hamiltonian at scale m_b: $\mathcal{H}_{\text{eff}}^{bs\ell\ell} = \mathcal{H}_{\text{eff, SM}}^{bs\ell\ell} + \mathcal{H}_{\text{eff, NP}}^{bs\ell\ell}$

$$\mathcal{H}_{\text{eff, NP}}^{bs\ell\ell} = -\mathcal{N} \sum_{\ell=e,\mu} \sum_{i=9,10,S,P} \left(C_i^{bs\ell\ell} O_i^{bs\ell\ell} + C_i'^{bs\ell\ell} O_i'^{bs\ell\ell} \right) + \text{h.c.}$$

- Operators considered here ($\ell = e, \mu$)

$$\begin{aligned}
O_9^{bs\ell\ell} &= (\bar{s}\gamma_\mu P_L b)(\bar{\ell}\gamma^\mu \ell)\,, & O_9'^{bs\ell\ell} &= (\bar{s}\gamma_\mu P_R b)(\bar{\ell}\gamma^\mu \ell)\,,\\
O_{10}^{bs\ell\ell} &= (\bar{s}\gamma_\mu P_L b)(\bar{\ell}\gamma^\mu \gamma_5 \ell)\,, & O_{10}'^{bs\ell\ell} &= (\bar{s}\gamma_\mu P_R b)(\bar{\ell}\gamma^\mu \gamma_5 \ell)\,,\\
O_S^{bs\ell\ell} &= m_b(\bar{s} P_R b)(\bar{\ell}\ell)\,, & O_S'^{bs\ell\ell} &= m_b(\bar{s} P_L b)(\bar{\ell}\ell)\,,\\
O_P^{bs\ell\ell} &= m_b(\bar{s} P_R b)(\bar{\ell}\gamma_5 \ell)\,, & O_P'^{bs\ell\ell} &= m_b(\bar{s} P_L b)(\bar{\ell}\gamma_5 \ell)\,.
\end{aligned}$$

- Not considered here
  - Dipole operators: strongly constrained by radiative decays. e.g. [arXiv:1608.02556]
  - Four quark operators: dominant effect from RG running above m_B.
    Jäger, Leslie, Kirk, Lenz [arXiv:1701.09183]

Scenarios with a single Wilson coefficients

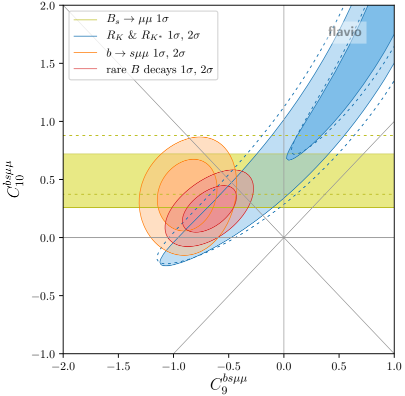
Coefficient	type	best fit	1σ	$\text{pull}_{1D} = \sqrt{\Delta\chi^2}$
$\boldsymbol{C_9^{bs\mu\mu}}$	$\boldsymbol{L \otimes V}$	-0.82	$[-0.96, -0.68]$	**6.2σ**
$\boldsymbol{C_{10}^{bs\mu\mu}}$	$\boldsymbol{L \otimes A}$	$+0.56$	$[+0.44, +0.68]$	**4.9σ**
$C_9^{\prime bs\mu\mu}$	$R \otimes V$	-0.09	$[-0.22, +0.04]$	0.7σ
$C_{10}^{\prime bs\mu\mu}$	$R \otimes A$	$+0.01$	$[-0.08, +0.11]$	0.1σ
$C_9^{bs\mu\mu} = C_{10}^{bs\mu\mu}$	$L \otimes R$	-0.06	$[-0.17, +0.05]$	0.5σ
$\boldsymbol{C_9^{bs\mu\mu} = -C_{10}^{bs\mu\mu}}$	$\boldsymbol{L \otimes L}$	-0.43	$[-0.50, -0.36]$	**6.2σ**

Only small pull for

- Coefficients with $\ell = e$ (cannot explain $b \to s\mu\mu$ anomaly and $B_s \to \mu\mu$)
- Scalar coefficients (can only reduce tension in $B_s \to \mu\mu$)

see also similar fits by other groups:
Algueró et al., arXiv:1903.09578 Ciuchini et al., arXiv:2011.01212 Datta et al., arXiv:1903.10086
Kowalska et al., arXiv:1903.10932 Arbey et al., arXiv:1904.08399 Geng et al., arXiv:2103.12738

Scenarios with two Wilson coefficients



WET at 4.8 GeV

- BR($B_s \to \mu\mu$): moved closer to SM, better agreement with $b \to s\mu\mu$ observables, smaller uncertainty
- R_K: same central value, reduced uncertainty
- Global fit in $C_9^{bs\mu\mu}$-$C_{10}^{bs\mu\mu}$ plane prefers negative $C_9^{bs\mu\mu}$ or negative $C_9^{bs\mu\mu} = -C_{10}^{bs\mu\mu}$

smelli: Fit of a new physics model

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Model setup:

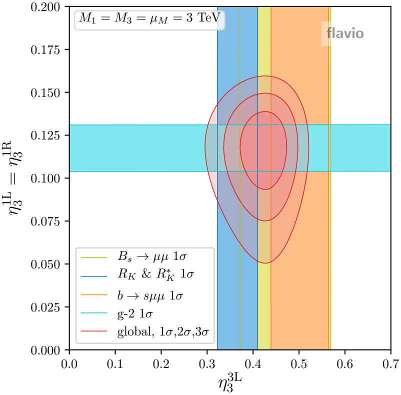
- Effect in $C_9^{bs\mu\mu} = -C_{10}^{bs\mu\mu}$ can be generated at tree level by scalar leptoquark $S_3 \sim (\bar{\mathbf{3}}, \mathbf{3}, 1/3)$
  Hiller, Schmaltz, arXiv:1408.1627
- Generic S_3 couples to all lepton generations $\Rightarrow$ **Lepton Flavour Violation** (LFV)
- Generic S_3 has di-quark couplings $\Rightarrow$ **proton decay**
- **Strong experimental constraints** on LFV and proton decay

Idea:

- Charge S_3 and muon under **new U**$(\mathbf{1})$ **gauge symmetry** such that
  - S_3 cannot couple to two quarks $\Rightarrow$ prevents proton decay
  - Muon is only lepton that couples to S_3 $\Rightarrow$ prevents LFV

  Hambye, Heeck, arXiv:1712.04871
  Davighi, Kirk, Nardecchia, arXiv:2007.15016
- Second leptoquark $S_1 \sim (\bar{\mathbf{3}}, \mathbf{1}, 1/3)$ charged under **same U**$(\mathbf{1})$ gauge symmetry receives same protection (only coupling to muons, no LFV, no proton decay)
  $\Rightarrow$ "**Muoquark**" models **explaining** $\mathbf{R}_{\mathbf{K}^{(*)}}$ and $(\mathbf{g}-\mathbf{2})_\mu$
  Greljo, PS, Thomsen, arXiv:2103.13991

smelli: Fit of a new physics model



$$\eta_i^{3L} = (V_{td},\, V_{ts},\, 1)\, \eta_3^{3L}$$
$$\eta_i^{1L} = (V_{td},\, V_{ts},\, 1)\, \eta_3^{1L}$$
$$\eta_i^{1R} = (0,\, 0,\, 1)\, \eta_3^{1R}$$

- Model for muon anomalies:
  $$\mathcal{L} \supset \eta_i^{3L}\, \overline{q}_L^{c\,i} \ell_L^2\, S_3 + \eta_i^{1L} \overline{q}_L^{c\,i} \ell_L^2\, S_1 + \eta_i^{1R} \overline{u}_R^{c\,i} \mu_R S_1$$
- One-loop matching to SMEFT
  Gherardi, Marzocca, Venturin, arXiv:2003.12525
- Interface to **smelli** using SMEFT Wilson coefficients
- Likelihood in space of model parameters
- Excellent fit to data with best fit point at $(\eta_3^{3L}, \eta_3^{1L} = \eta_3^{1R}) \simeq (0.43, 0.12)$ and $\Delta\chi^2 \simeq 62$ compared to SM point $(0, 0)$
- Compatible with all measurements included in smelli

Conclusions

Conclusions

flavio provides

- **theory predictions** for a huge number of flavour and other precision observables in the SM and beyond
- a database of experimental **measurements**
- different ways to construct **likelihoods** combining the theory predictions and the experimental measurements

smelli - the SMEFT likelihood

- provides **likelihood function** in space of WET and SMEFT Wilson coefficients
- includes more than **400 observables**
- can be used to perform fits in the WET and the SMEFT
- can be interfaced to **NP models** matched to SMEFT

Developement is open source and open for everyone:

`https://github.com/flav-io/flavio`

`https://github.com/smelli/smelli`

Backup slides

The likelihood

Construct **likelihood** that quantifies the agreement between **experimental data** and **theoretical predictions**

- Experimental data of measurement *i* yields **experimental likelihood** for **observables $\vec{O}$**

$$\mathcal{L}^i_{\text{exp}}(\vec{O})$$

  - non-trivial likelihood function for one or several correlated observables
  - uniform likelihood for observables not measured by measurement *i*

- In SM or NP model, **theory predictions** in terms of theory parameters $\vec{C}$ and $\vec{\theta}$

$$\vec{O}_{\text{th}}(\vec{C}, \vec{\theta})$$

  $\vec{C}$: NP Wilson coefficients, defined such that SM is given by $\vec{C} = \vec{0}$
  $\vec{\theta}$: model-independent theory parameters (e.g. particle masses, hadronic form factors, ...)

The likelihood

- Define individual likelihoods in theory parameters

$$\mathcal{L}^i_{\text{exp}}(\vec{C},\vec{\theta}) = \mathcal{L}^i_{\text{exp}}(\vec{O} = \vec{O}_{\text{th}}(\vec{C},\vec{\theta}))$$

- Define full likelihood taking into account parametric theory uncertainties

$$\mathcal{L}(\vec{C},\vec{\theta}) = \prod_i \mathcal{L}^i_{\text{exp}}(\vec{C},\vec{\theta}) \times \mathcal{L}_{\text{th}}(\vec{\theta})$$

- Assumptions:
  - Measurements are independent of each other
  - Measurements do not explicitly depend on theory parameters (only through $\vec{O}_{\text{th}}$)

The New Physics likelihood

In the New Physics likelihood, all parameters $\vec{\theta}$ are **nuisance parameters**

- How do we get a "nuisance-free" likelihood?

$$\mathcal{L}(\vec{C},\vec{\theta}) = \prod_i \mathcal{L}^i_{\text{exp}}(\vec{C},\vec{\theta}) \times \mathcal{L}_{\text{th}}(\vec{\theta}) \quad \overset{?}{\rightarrow} \quad \mathcal{L}(\vec{C})$$

- **Bayesian approach**:
  Interpret $\mathcal{L}_{\text{th}}(\vec{\theta})$ as *prior* and $\mathcal{L}(\vec{C})$ as *posterior*, marginalise over nuisance parameters
- **Frequentist approach**:
  Interpret $\mathcal{L}_{\text{th}}(\vec{\theta})$ as *likelihood of pseudo-experiments* and $\mathcal{L}(\vec{C})$ as *profiled likelihood*

For large numbers of nuisance parameters $\vec{\theta}$ and NP parameters $\vec{C}$, both approaches are **computationally expensive**.

What special cases exist that allow obtaining a "nuisance-free" likelihood **computationally *in*expensive** and that could serve as reasonable approximations?

Approximations: Case 1

$$\mathcal{L}(\vec{C},\vec{\theta}) = \prod_i \mathcal{L}^i_{\text{exp}}(\vec{C},\vec{\theta}) \times \mathcal{L}_{\text{th}}(\vec{\theta}) \quad \overset{?}{\to} \quad \mathcal{L}(\vec{C})$$

Special case 1:

$$\mathcal{L}^i_{\text{exp}}(\vec{C},\vec{\theta}) \approx \mathcal{L}^i_{\text{exp}}(\vec{C},\hat{\vec{\theta}}) \qquad \text{for } \vec{\theta} \text{ sampled from } \mathcal{L}_{\text{th}}(\vec{\theta})$$

this is the case for **small parametric uncertainty of theory prediction** compared to experimental uncertainty e.g.

- Ratios of branching ratios like $R_{K^{(*)}}$, $R_{D^{(*)}}$
- Electroweak precision observables
- LFV decays
- ...

$$\Rightarrow \qquad \mathcal{L}(\vec{C}) \approx \prod_{i \in \text{case 1}} \mathcal{L}^i_{\text{exp}}(\vec{C},\hat{\vec{\theta}}) \times \mathcal{L}'(\vec{C})$$

Approximations: Case 2

$$\mathcal{L}'(\vec{C},\vec{\theta}) = \prod_{i\notin \text{case 1}} \mathcal{L}^i_{\text{exp}}(\vec{C},\vec{\theta}) \times \mathcal{L}_{\text{th}}(\vec{\theta}) \quad \overset{?}{\to} \quad \mathcal{L}'(\vec{C})$$

Special case 2:

- Theoretical **prediction likelihood** of subset of observables $\vec{O}^k$ can be approximated as multivariate **normal distribution** for given $\vec{C}$

$$-2\ln \mathcal{L}_{\text{th}}(\vec{O}^k,\vec{C}) = \left(\vec{O} - \vec{O}^k_{\text{th}}(\vec{C},\hat{\vec{\theta}})\right)^T \Sigma_{\text{th}}^{-1} \left(\vec{O} - \vec{O}^k_{\text{th}}(\vec{C},\hat{\vec{\theta}})\right) ,$$

with **covariance matrix** Σ_{th} determined for $\vec{C} = \vec{0}$ and (approximately) **independent of $\vec{C}$**

- Approximate **experimental likelihoods** for measurements of observables $\vec{O}^k$ as multivariate **normal distributions**

$$-2\ln \mathcal{L}^i_{\text{exp}}(\vec{O}^k) = (\vec{O}^k - \hat{\vec{O}}^{k,i})^T (\Sigma^i_{\text{exp}})^{-1} (\vec{O}^k - \hat{\vec{O}}^{k,i}) ,$$

$\hat{\vec{O}}^{k,i}$ exp. central value, Σ^i_{exp} covariance matrix

Approximations: Case 2

- Combine $\mathcal{L}^i_{\text{exp}}(\vec{O}^k)$ ($i \in$ case 2) in terms of **weighted averaged** covariance matrix Σ_{exp} and mean $\hat{\vec{O}}^k$

- Define **modified experimental likelihood** $\tilde{\mathcal{L}}_{\text{exp}}(\vec{O}^k)$

$$-2\ln\tilde{\mathcal{L}}_{\text{exp}}(\vec{O}^k) = (\vec{O}^k - \hat{\vec{O}}^k)^T(\Sigma_{\text{exp}} + \Sigma_{\text{th}})^{-1}(\vec{O}^k - \hat{\vec{O}}^k)\,,$$

  Takes into account theoretical uncertainties and correlations in terms of covariance matrix Σ_{th}, treated as additional experimental uncertainties

- Express in terms of $\vec{C}$ and $\hat{\vec{\theta}}$

$$-2\ln\tilde{\mathcal{L}}_{\text{exp}}(\vec{C},\hat{\vec{\theta}}) = \left(\vec{O}^k_{\text{th}}(\vec{C},\hat{\vec{\theta}}) - \hat{\vec{O}}^k\right)^T(\Sigma_{\text{exp}} + \Sigma_{\text{th}})^{-1}\left(\vec{O}^k_{\text{th}}(\vec{C},\hat{\vec{\theta}}) - \hat{\vec{O}}^k\right)\,,$$

$$\Rightarrow \qquad \mathcal{L}'(\vec{C}) \approx \tilde{\mathcal{L}}_{\text{exp}}(\vec{C},\hat{\vec{\theta}}) \times \mathcal{L}''(\vec{C})$$

The New Physics likelihood

The (approximative) **global New Physics likelihood** Aebischer, Kumar, PS, Straub, arXiv:1810.07698

$$\mathcal{L}(\vec{C}) \approx \prod_{i \in \text{case 1}} \mathcal{L}^{i}_{\text{exp}}(\vec{C}, \hat{\vec{\theta}}) \times \tilde{\mathcal{L}}_{\text{exp}}(\vec{C}, \hat{\vec{\theta}})$$

- $\prod_{i \in \text{case 1}} \mathcal{L}^{i}_{\text{exp}}(\vec{C}, \hat{\vec{\theta}})$: negligible parametric theory uncertainties

  e.g. EFT fits to electroweak precision tests:
  Efrati, Falkowski, Soreq, arXiv:1503.07872
  Falkowski, González-Alonso, Mimouni, arXiv:1706.03783

- $\tilde{\mathcal{L}}_{\text{exp}}(\vec{C}, \hat{\vec{\theta}})$: theoretical and experimental uncertainties combined at $\vec{C} = \vec{0}$ (SM)

  EFT fits of rare B decays first in: Altmannshofer, Straub, arXiv:1411.3161
  also used by other groups, e.g. Descotes-Genon, Hofer, Matias, Virto, arXiv:1510.04239

Advantages and disadvantages of approximations

Disadvantages

- Theory uncertainties only weakly dependent on New Physics $\vec{C}$:
  strong assumption, validity has to be **checked explicitly**
  (e.g. by computing $\Sigma_{\text{th}}(\vec{C} \neq \vec{0})$)
- **Not able to include certain observables**, e.g. electric dipole moments afflicted by sizable hadronic uncertainties for $\vec{C} \neq \vec{0}$ but negligible ones for $\vec{C} = \vec{0}$

Advantages

- Computationally expensive determination of Σ_{th}
  - has to be done **only once**
  - is **independent of experimental data**
  - computing time is **independent of number of nuisance parameters**
- Computation of global likelihood **fast** enough for **phenomenological analysis of New Physics** models ($\sim$ 5 sec. per point on laptop)