## EFT below the electroweak scale: running, matching, and hadronic effects

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in collaboration with E. Jenkins and A. Manohar: JHEP 03 (2018) 016, JHEP 01 (2018) 084, with W. Dekens, E. Jenkins, and A. Manohar: JHEP 01 (2019) 088, with W. Dekens: JHEP 10 (2019) 197

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LHC EFT WG Meeting "Heavy flavour aspects in EFT fits"


## Outline

# 1 Introduction 

2 EFT below the electroweak scale

## 3 Matching SMEFT to LEFT

4 Non-perturbative effects in $\mu \rightarrow e \gamma$

5 Summary

## Overview

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## Effective field theories (EFTs)

- based on a small set of assumptions
- generic and systematic quantum field theories, can be used "stand-alone" for fits to experiments or in connection with a broad range of specific models
- work only with the relevant particles at a particular energy $\Rightarrow$ simplify calculations
- EFT parameters depend on energy scale $\Rightarrow$ running \& mixing
- connect different energy regimes (renormalization group, avoid large logarithms)


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EFT for new physics above the weak scale
"Standard Model EFT" (SMEFT) assumptions:
$\rightarrow$ Buchmüller, Wyler (1986)

- new physics at high energies $\Lambda \gg v \approx 246 \mathrm{GeV}$
- underlying theory respects the same symmetry principles as the SM
- Higgs particle part of electroweak doublet (as in SM)
- dimensional power counting, expansion in $v / \Lambda, p / \Lambda$


## EFT below the electroweak scale

"Low-energy EFT" (LEFT):

- only light SM particles (no Higgs, weak bosons, or top quark)
- basically the old Fermi theory of weak interaction
- complete and systematic treatment up to dimension 6 recently worked out
$\rightarrow$ Jenkins, Manohar, Stoffer, JHEP 01 (2018) 084, JHEP 03 (2018) 016
- operator basis now known up to dimension 9
$\rightarrow$ Li et al. (2020), C. W. Murphy (2020)
- dimensional power counting, expansion in $m / v, p / v$
(2) EFT below the electroweak scale


## EFTs for new physics



- partial LEFT operator basis and running previously studied in detail
- first complete treatment up to dimension six:
$\rightarrow$ Jenkins, Manohar, Stoffer (2018)


## EFT below the electroweak scale

LEFT Lagrangian:

$$
\mathcal{L}_{\mathrm{LEFT}}=\mathcal{L}_{\mathrm{QED}+\mathrm{QCD}}+\sum_{i} L_{i} \mathcal{O}_{i}+\ldots
$$

Additional effective operators:

- dimension 3: Majorana-neutrino masses ( $\Delta L= \pm 2$ )
- dimension 5: $\Delta B=\Delta L=0$ dipole operators for $\psi=u, d, e$ and $\Delta L= \pm 2$ neutrino-dipole operators
- dimension 6: $C P$-even and $C P$-odd three-gluon operators, as well as four-fermion operators


## LEFT operators

$\rightarrow$ Jenkins, Manohar, Stoffer, JHEP 03 (2018) 016

- in total 5963 operators at dimensions three, five, and six: $3099 C P$-even and $2864 C P$-odd
- basis free of redundancies (EOM, Fierz, etc.)
- cross-checked with Hilbert series


## Power counting and RGE

$\rightarrow$ Jenkins, Manohar, Stoffer, JHEP 01 (2018) 084

- calculation of complete one-loop RGE up to dimension-six effects in LEFT
- graph with insertions of higher-dimensional operators $\left(d_{i} \geq 5\right): \quad d=4+\sum_{i}\left(d_{i}-4\right)$
- up to dimension six:
- single-operator insertions of dimension five and six
- double-operator insertions of dimension five
- results have later been implemented in public code:
wilson $\rightarrow$ Aebischer et al. (2018)
DsixTools $\rightarrow$ Celis et al. (2017), Fuentes-Martin et al. (2020)


## Full set of one-loop diagrams for LEFT running

$\rightarrow$ Jenkins, Manohar, Stoffer, JHEP 01 (2018) 084
$x \times x \times \times x \times x \times x \times 2$





dbAAAdd\&\&dA\& $A d A \perp A A A A A \Delta A A$ $A \Delta A \Delta A A b \alpha d A A A$ $\Delta A A A A A A A A A h \alpha$


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## Matching between the EFTs



- tree-level matching from SMEFT to LEFT
$\rightarrow$ Jenkins, Manohar, Stoffer JHEP 03 (2018) 016
- complete one-loop matching
$\rightarrow$ Dekens, Stoffer
JHEP 10 (2019) 197
- leads to relations between LEFT operator coefficients


## SMEFT in the broken phase

- Higgs in unitary gauge:

$$
H=\frac{1}{\sqrt{2}}\binom{0}{\left[1+c_{H, \mathrm{kin}}\right] h+v_{T}}
$$

where

$$
c_{H, \text { kin }}:=\left(C_{H \square}-\frac{1}{4} C_{H D}\right) v^{2}, \quad v_{T}:=\left(1+\frac{3 C_{H} v^{2}}{8 \lambda}\right) v
$$

- modifications from SM due to dimension-six Higgs operators in SMEFT


## SMEFT in the broken phase

- dimension-six modifications of fermion masses and Yukawa couplings $\Rightarrow$ no longer proportional
- modifications of gauge-boson mass terms
- weak charged and neutral currents modified as well, e.g. coupling of $\mathcal{W}^{+}$to right-handed current $\bar{u}_{R} \gamma^{\mu} d_{R}$
- after rotation to mass eigenstates, modified weak currents lead to non-unitary effective CKM quark-mixing matrix


## Integrating out weak-scale SM particles

consider Higgs-exchange diagram:


$$
\left[\mathcal{Y}_{\psi}\right]_{r s}=\frac{1}{v_{T}}\left[M_{\psi}\right]_{r s}\left[1+c_{H, \mathrm{kin}}\right]-\frac{v^{2}}{\sqrt{2}} C_{\psi H}^{*}
$$

$\mathcal{Y}^{2}$ has terms of order $(m / v)^{2}, m v / \Lambda^{2}, v^{4} / \Lambda^{4}$
$\Rightarrow$ diagram $\mathcal{Y}^{2} / m_{h}^{2}$ is of same order as dimension-7 or 8 contributions in LEFT or dimension-8 in SMEFT

## Integrating out weak-scale SM particles

- for SMEFT $\Rightarrow$ LEFT matching: rewrite terms

$$
\cdots \frac{1}{\Lambda^{n}}=\underbrace{\ldots \frac{1}{v^{n}}}_{\text {LEFT counting }} \times \underbrace{\frac{v^{n}}{\Lambda^{n}}}_{\text {SMEFT counting }}
$$

- tree-level matching simple: fix Higgs field to vev and compute $\mathcal{W} / \mathcal{Z}$-exchange diagrams

One-loop matching
$\rightarrow$ Dekens, Stoffer, JHEP 10 (2019) 197

- simplify matching calculation:

$$
\operatorname{tree}_{\mathrm{LEFT}}+\left.\mathrm{loop}_{\mathrm{LEFT}}\right|_{\mu_{W}}=\operatorname{tree}_{\mathrm{SMEFT}}+\left.\operatorname{loop}_{\mathrm{SMEFT}}\right|_{\mu_{W}}
$$

- expand loops in all low scales

$$
\operatorname{tree}_{\mathrm{LEFT}}+\left.\operatorname{loop}_{\mathrm{LEFT}}^{\exp }\right|_{\mu_{W}}=\operatorname{tree}_{\mathrm{SMEFT}}+\left.\operatorname{loop}_{\mathrm{SMEFT}}^{\exp }\right|_{\mu_{W}}
$$

- 754 SMEFT diagrams including finite parts


## One-loop matching

$\rightarrow$ Dekens, Stoffer, JHEP 10 (2019) 197
$--\quad=-\bigcirc+\cdots$,




## Gauge fixing

- using background-field gauge for SMEFT
$\rightarrow$ Helset, Paraskevas, Trott (2018)

$$
F \mapsto F+\hat{F}
$$

- external fields and tree-level propagators: classical fields $\hat{F}$, unitary gauge
- loops: quantum fields $F, R_{\xi}$ gauge
- no $\gamma-Z$ mixing at tree level or at one loop on shell
- no $\gamma-h$ mixing


## The old $\gamma_{5}$ problem

- use anticommuting $\gamma_{5}$ whenever possible
$\rightarrow$ Trueman (1996), Jegerlehner (2001)
- $\gamma_{5}$ in closed fermion loops problematic
- $\epsilon^{\mu \nu \lambda \sigma}$ in SMEFT vertices
- in general, 't Hooft-Veltman scheme generates spurious anomalies $\Rightarrow$ requires symmetry-restoring counterterms
- use chiral vertices and check Ward identities in critical cases


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(4) Non-perturbative effects in $\mu \rightarrow e \gamma$

## Matching to Chiral Perturbation Theory (ChPT)

$\rightarrow$ Dekens, Jenkins, Manohar, Stoffer, JHEP 01 (2019) 088


- at the hadronic scale, QCD is non-perturbative
- ChPT can be formulated including LEFT effects
- non-perturbative matching required, e.g. using lattice QCD simulations


## Lepton-flavor violation: $\mu \rightarrow e \gamma$

$\rightarrow$ Dekens, Jenkins, Manohar, Stoffer, JHEP 01 (2019) 088

- hadronic effects can show up in a purely leptonic process
- LF violation due to many operators, e.g.

$$
\begin{aligned}
& \mathcal{O}_{e q}^{S, R R}=\left(\bar{e}_{L p} e_{R r}\right)\left(\bar{q}_{L s} q_{R t}\right) \\
& \mathcal{O}_{e q}^{V, L L}=\left(\bar{e}_{L p} \gamma^{\mu} e_{L r}\right)\left(\bar{q}_{L s} \gamma_{\mu} q_{L t}\right) \\
& \mathcal{O}_{e q}^{T, R R}=\left(\bar{e}_{L p} \sigma^{\mu \nu} e_{R r}\right)\left(\bar{q}_{L s} \sigma_{\mu \nu} q_{R t}\right)
\end{aligned}
$$

## Lepton-flavor violation: $\mu \rightarrow e \gamma$

- matching of semileptonic operators to ChPT is standard: external scalar, vector, and tensor sources
$\rightarrow$ Gasser, Leutwyler (1984), Catà, Mateu (2007)
- at lowest order in $\alpha_{\text {QED }}:\langle\gamma(p, \epsilon)| S|0\rangle$ and $\langle\gamma(p, \epsilon)| V^{\mu}|0\rangle$ vanish due to Lorentz and gauge invariance
- semileptonic tensor operators contribute to $\mu \rightarrow e \gamma$ :

- non-perturbative effects not suppressed by light quark masses

Lepton-flavor violation: $\mu \rightarrow e \gamma$

- constraints on SMEFT operators at the weak scale through matching SMEFT $\Rightarrow$ LEFT
- two competing effects:
- perturbative RGE mixing of tensor operators into dipoles when running from $\mu=M_{W}$ to $\mu=2 \mathrm{GeV}$
- non-perturbative matching effect
- $\mu \rightarrow e \gamma$ (MEG) gives best limit for strange-quark operator at the electroweak scale $\left(c_{T}=\mathcal{O}(1)\right)$ :

$$
\left(c_{T}-3.1\right) L_{\substack{e d \\ e \mu s s}}^{T, R R}<2.8 \times 10^{-5} \mathrm{TeV}^{-2}
$$

- $\mu \rightarrow e$ conversion in nuclei (SINDRUM II) gives stronger limits for up- and down-quark operators


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## LEFT

- we constructed the full LEFT operator basis up to dimension six
- complete one-loop RGE, including (dim-5) ${ }^{2}$ effects
- one-loop matching to SMEFT at the weak scale
- unified framework to compute leading-log effects from the scale of New Physics down to low energies
- first step towards next-to-leading-log accuracy
- many interesting applications: LF violation, $C P$-violation (EDMs), flavor physics, connection with collider physics
$\mu \rightarrow e \gamma$
- contributions of semileptonic tensor operators: RGE effect and non-perturbative matching effect
- non-perturbative effects can lead to interesting enhancements and new constraints
- non-perturbative effect dominates for up- and down-quarks
- $\mu \rightarrow e \gamma$ gives strongest constraints on strange-quark operator (better than $\mu \rightarrow e$ conversion):
$\Lambda_{\text {BSM }} \gtrsim 450 \mathrm{TeV}$
- analogous effect also present in $(g-2)_{\mu}$
$\rightarrow$ Aebischer, Dekens, Jenkins, Manohar, Sengupta, Stoffer: 2102.08954 [hep-ph]



## Backup

## LEFT basis

## (6) Backup

## LEFT basis

| $(\bar{L} L)(\bar{L} L)$ |  | $(\bar{L} L)(\bar{R} R)$ |  | $(\bar{L} R)(\bar{L} R)+$ h.c. |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathcal{O}_{\nu \nu}^{V, L L}$ | $\left(\bar{\nu}_{L p} \gamma^{\mu} \nu_{L r}\right)\left(\bar{\nu}_{L s} \gamma_{\mu} \nu_{L t}\right)$ | $\mathcal{O}_{\nu e}^{V, L R}$ | $\left(\bar{\nu}_{L p} \gamma^{\mu} \nu_{L r}\right)\left(\bar{e}_{R s} \gamma_{\mu}\right.$ | $\mathcal{O}_{e e}^{S, R R}$ | $\left(\bar{e}_{L p} e_{R r}\right)\left(\bar{e}_{L s} e_{R t}\right)$ |
| $\mathcal{O}_{e e}^{V, L L}$ | $\left(\bar{e}_{L p} \gamma^{\mu} e_{L r}\right)\left(\bar{e}_{L s} \gamma_{\mu} e_{L t}\right.$ | $\mathcal{O}_{e e}^{V, L R}$ | $\left(\bar{e}_{L p} \gamma^{\mu} e_{L r}\right)\left(\bar{e}_{R s} \gamma_{\mu}\right.$ | $\mathcal{O}_{e u}^{S, R R}$ | $\left.\bar{e}_{L p} e_{R r}\right)\left(\bar{u}_{L s} u_{R t}\right)$ |
| $\mathcal{O}_{\nu e}^{V, L L}$ | $\left(\bar{\nu}_{L p} \gamma^{\mu} \nu_{L r}\right)\left(\bar{e}_{L s} \gamma_{\mu} \epsilon\right.$ | $\mathcal{O}_{\nu u}^{V, L R}$ | $\left(\bar{\nu}_{L p} \gamma^{\mu} \nu_{L r}\right)\left(\bar{u}_{R}\right.$ | $\mathcal{O}_{e u}^{T, R R}$ | $\left(\bar{e}_{L p} \sigma^{\mu \nu} e_{R r}\right)\left(\bar{u}_{L s} \sigma_{\mu \nu} u_{R t}\right)$ |
| $\mathcal{O}_{\nu u}^{V, L L}$ | $\left(\bar{\nu}_{L p}\right.$ | $\mathcal{O}_{\nu d}^{V, L R}$ | $\left(\bar{\nu}_{L P}\right.$ | $\mathcal{O}_{\text {ed }}^{S, R R}$ | $\left(\bar{e}_{L p} e_{R r}\right)\left(\bar{d}_{L s} d_{R t}\right)$ |
| $\mathcal{O}_{\nu d}^{V, L L}$ | $\left(\bar{\nu}_{L p}\right.$ | $\mathcal{O}_{e u}^{V, L R}$ | $\left.\left(e_{L p}\right) e_{L r}\right)\left(\bar{u}_{R s} \gamma_{\mu} u_{R t}\right.$ | $\mathcal{O}_{e d}^{T, R R}$ | $\left(\bar{e}_{L p} \sigma^{\mu \nu} e_{R r}\right)\left(\bar{d}_{L s} \sigma_{\mu \nu} d_{R t}\right)$ |
| $\mathcal{O}_{e u}^{V, L L}$ | ( $\bar{e}^{\prime}$ | $\mathcal{O}_{e d}^{V, L R}$ | $\left(\bar{e}_{L}\right.$ | $\mathcal{O}_{\text {vedu }}^{S, R R}$ | $\left.\bar{\nu}_{L p} e_{R r}\right)\left(\bar{d}_{L s} u_{R t}\right)$ |
| $\mathcal{O}_{e d}^{V, L L}$ | ( $\bar{e}_{L}$ | $\mathcal{O}_{u e}^{V, L R}$ | $\left(\bar{u}_{L p} \gamma^{\mu} u_{L r}\right)\left(\bar{e}_{R s} \gamma_{\mu} e_{R}\right)$ | $\mathcal{O}_{\nu e d u}^{T, R R}$ | $\left(\bar{\nu}_{L p} \sigma^{\mu \nu} e_{R r}\right)\left(\bar{d}_{L s} \sigma_{\mu \nu} u_{R t}\right)$ |
| $\mathcal{O}_{\text {vedu }}^{V, L L}$ | $\left(\bar{\nu}_{L p} \gamma\right.$ | $\mathcal{O}_{\text {de }}^{V, L R}$ | $\left(\bar{d}_{L p} \gamma^{\mu} d_{L r}\right)\left(\bar{e}_{R s} \gamma_{\mu} e_{R t}\right)$ | $\mathcal{O}_{u u}^{S 1, R R}$ | t) |
| $\mathcal{O}_{u L}^{V, L L}$ | ( | $\mathcal{O}_{\nu e d u}^{V, L R}$ | $\left(\bar{\nu}_{L p}\right.$ | $\mathcal{O}_{u u}^{S 8, R R}$ | $\left(\bar{u}_{L p} T^{A} u_{R r}\right)\left(\bar{u}_{L s} T^{A} u_{R t}\right)$ |
| $\mathcal{O}_{d d}^{V, L L}$ | $\bar{d}_{L p}$ | $\mathcal{O}_{u u}^{V 1, L R}$ | $\left(\bar{u}_{L p} \gamma^{\mu} u_{L r}\right)\left(\bar{u}_{R s} \gamma_{\mu} u_{R t}\right)$ | $\mathcal{O}_{u d}^{S 1, R R}$ | $\left(\bar{u}_{L p} u_{R r}\right)\left(\bar{d}_{L s} d_{R t}\right)$ |
| $\mathcal{O}_{u d}^{V 1, L L}$ | $\left(\bar{u}_{L p}\right.$ | $\mathcal{O}_{u u}^{V 8, L R}$ | $\left(\bar{u}_{L p} \gamma^{\mu} T^{A} u_{L r}\right)\left(\bar{u}_{R s} \gamma_{\mu} T^{A} u_{R t}\right.$ | $\mathcal{O}_{u d}^{S 8, R R}$ | $\left(\bar{u}_{L p} T^{A} u_{R r}\right)\left(\bar{d}_{L s} T^{A} d_{R t}\right)$ |
| $\mathcal{O}_{u d}^{V 8, L L}$ | $\left(\bar{u}_{L p}\right.$ | $\mathcal{O}_{u d}^{V 1, L R}$ | $(\bar{u}$ | $\mathcal{O}_{d d}^{S 1, R R}$ | $\left(\bar{d}_{L p} d_{R r}\right)\left(\bar{d}_{L s} d_{R t}\right)$ |
| $(\bar{R} R)(\bar{R} R)$ |  | $\mathcal{O}_{u d}^{V 8, L R}$ | $\left(\bar{u}_{L p} \gamma\right.$ | $\mathcal{O}_{d d}^{S 8, R R}$ | $\left(\bar{d}_{L p} T^{A} d_{R r}\right)\left(\bar{d}_{L s} T^{A} d_{R t}\right)$ |
| $\mathcal{O}_{e e}^{V, R R}$ | $\left(\bar{e}_{R p} \gamma^{\mu} e_{R r}\right)\left(\bar{e}_{R s} \gamma_{\mu} e_{R t}\right)$ | $\mathcal{O}_{d u}^{V 1, L R}$ | $\left(\bar{d}_{L p} \gamma^{\mu} d_{L r}\right)\left(\bar{u}_{R s} \gamma_{\mu} u_{R t}\right)$ | $\begin{aligned} & \mathcal{O}_{u d d u}^{S 1, R R} \\ & \mathcal{O}^{S 8, R R} \end{aligned}$ | $\left(\bar{u}_{L p} d_{R r}\right)\left(\bar{d}_{L s} u_{R t}\right)$ |
| $\begin{aligned} & \mathcal{O}_{e u}^{V, R R} \\ & \mathcal{O}_{e d}^{V, R R} \end{aligned}$ | $\begin{aligned} & \left(\bar{e}_{R p} \gamma^{\mu} e_{R r}\right)\left(\bar{u}_{R s} \gamma_{\mu} u_{R t}\right) \\ & \left(\bar{e}_{R p} \gamma^{\mu} e_{R r}\right)\left(\bar{d}_{R s} \gamma_{\mu} d_{R t}\right) \end{aligned}$ | $\mathcal{O}_{d d}^{V 1, L R}$ | $\left(\bar{d}_{L p} \gamma^{\mu} d_{L r}\right)\left(\bar{d}_{R s} \gamma_{\mu} d_{R t}\right)$ | $(\bar{L} R)(\bar{R} L)+$ h.c. |  |
| $\mathcal{O}_{u L}^{V, R R}$ | $\left(\bar{u}_{R p} \gamma^{\mu} u_{R r}\right)\left(\bar{u}_{R s} \gamma_{\mu} u_{R t}\right)$ | $\mathcal{O}_{d d}^{V 8, L R}$ | $\left(\bar{d}_{L p} \gamma^{\mu} T^{A} d_{L r}\right)\left(\bar{d}_{R s} \gamma_{\mu} T^{A} d_{R t}\right)$ | $\overline{\mathcal{O}_{e u}^{S, R L}}$ | $\left(\bar{e}_{L p} e_{R r}\right)\left(\bar{u}_{R s} u_{L t}\right)$ |
| $\mathcal{O}_{d d}^{V, R R}$ | $\left(\bar{d}_{R p} \gamma^{\mu} d_{R r}\right)\left(\bar{d}_{R s} \gamma_{\mu} d_{R t}\right)$ | $\mathcal{O}_{u d d u}^{V 1, L R}$ | $\left(\bar{u}_{L p} \gamma^{\mu} d_{L r}\right)\left(\bar{d}_{R s} \gamma_{\mu} u_{R t}\right)+$ h.c. | $\mathcal{O}_{e d}^{S, R L}$ | $\left(\bar{e}_{L p} e_{R r}\right)\left(\bar{d}_{R s} d_{L t}\right)$ |
| $\mathcal{O}_{u d}^{V 1, R R}$ | $\left(\bar{u}_{R p} \gamma^{\mu} u_{R r}\right)\left(\bar{d}_{R s} \gamma_{\mu} d_{R t}\right)$ | $\mathcal{O}_{u d d u}^{V 8, L R}$ | $\left(\bar{u}_{L p} \gamma^{\mu} T^{A} d_{L r}\right)\left(\bar{d}_{R s} \gamma_{\mu} T^{A} u_{R t}\right)+$ h.c. | $\mathcal{O}_{\text {vedu }}^{S, R L}$ | $\left(\bar{\nu}_{L p} e_{R r}\right)\left(\bar{d}_{R s} u_{L t}\right)$ |
| $\mathcal{O}_{u d}^{V 8, R R}$ | $\left(\bar{u}_{R p} \gamma^{\mu} T^{A} u_{R r}\right)\left(\bar{d}_{R s} \gamma_{\mu} T^{A} d_{R t}\right)$ |  |  |  |  |

## (6) Backup

## LEFT basis

$$
\begin{array}{c|c}
\boldsymbol{\Delta}=\mathbf{4}+\mathbf{h . c .} \\
\hline \mathcal{O}_{\nu \nu}^{S, L L} & \left(\nu_{L p}^{T} C \nu_{L r}\right)\left(\nu_{L s}^{T} C \nu_{L t}\right)
\end{array}
$$

| $\Delta L=2+$ h.c. |  | $\Delta B=\Delta L=1+$ h.c. |  | $\Delta B=-\Delta L=1+$ h.c. |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathcal{O}_{\nu e}^{S, L L}$ | $\left(\nu_{L p}^{T} C \nu_{L r}\right)\left(\bar{e}_{R s} e_{L t}\right)$ | $\mathcal{O}_{u}^{\text {u,LL }}$ | $\epsilon_{\alpha \beta \gamma}\left(u_{L p}^{\alpha T} C d_{L r}^{\beta}\right)\left(d_{L s}^{\gamma T} C \nu_{L t}\right)$ | $\overline{\mathcal{O}_{d d d}^{S, L L}}$ | $\epsilon_{\alpha \beta \gamma}\left(d_{L p}^{\alpha T} C d_{L r}^{\beta}\right)\left(\bar{e}_{R s} d_{L t}^{\gamma}\right)$ |
| $\mathcal{O}_{\nu \text { 仡 }}^{T, L L}$ | $\left(\nu_{L p}^{T} C \sigma^{\mu \nu} \nu_{L r}\right)\left(\bar{e}_{R s} \sigma_{\mu \nu} e_{L t}\right)$ | $\mathcal{O}_{\text {duu }}^{S, L L}$ | $\epsilon_{\alpha \beta \gamma}\left(d_{L p}^{\alpha T} C u_{L r}^{\beta}\right)\left(u_{L s}^{\gamma T} C e_{L t}\right)$ | $\mathcal{O}_{u d d}^{S, L R}$ | $\epsilon_{\alpha \beta \gamma}\left(u_{L p}^{\alpha T} C d_{L r}^{\beta}\right)\left(\bar{\nu}_{L s} d_{R t}^{\gamma}\right)$ |
| $\mathcal{O}_{\nu \nu e}^{S, L R}$ | $\left(\nu_{L p}^{T} C \nu_{L r}\right)\left(\bar{e}_{L s} e_{R t}\right)$ | $\mathcal{O}_{u u d}^{S, L R}$ | $\epsilon_{\alpha \beta \gamma}\left(u_{L p}^{\alpha T} C u_{L r}^{\beta}\right)\left(d_{R s}^{\gamma T} C e_{R t}\right)$ | $\mathcal{O}_{\text {ddu }}^{S, L R}$ | $\epsilon_{\alpha \beta \gamma}\left(d_{L p}^{\alpha T} C d_{L r}^{\beta}\right)\left(\bar{\nu}_{L s} u_{R t}^{\gamma}\right)$ |
| $\mathcal{O}_{\nu u}^{S, L L}$ | $\left(\nu_{L p}^{T} C \nu_{L r}\right)\left(\bar{u}_{R s} u_{L t}\right)$ | $\mathcal{O}_{\text {duu }}^{S, L R}$ | $\epsilon_{\alpha \beta \gamma}\left(d_{L p}^{\alpha T} C u_{L r}^{\beta}\right)\left(u_{R s}^{\gamma T} C e_{R t}\right)$ | $\mathcal{O}_{\text {ddd }}^{S, L R}$ | $\epsilon_{\alpha \beta \gamma}\left(d_{L p}^{\alpha T} C d_{L r}^{\beta}\right)\left(\bar{e}_{L s} d_{R t}^{\gamma}\right)$ |
| $\mathcal{O}_{\nu u}^{T, L L}$ | $\left(\nu_{L p}^{T} C \sigma^{\mu \nu} \nu_{L r}\right)\left(\bar{u}_{R s} \sigma_{\mu \nu} u_{L t}\right)$ | $\mathcal{O}_{u u d}^{S, R L}$ | $\epsilon_{\alpha \beta \gamma}\left(u_{R p}^{\alpha T} C u_{R r}^{\beta}\right)\left(d_{L s}^{\gamma T} C e_{L t}\right)$ | $\mathcal{O}_{d d d}^{S, R L}$ | $\epsilon_{\alpha \beta \gamma}\left(d_{R p}^{\alpha T} C d_{R r}^{\beta}\right)\left(\bar{e}_{R s} d_{L t}^{\gamma}\right)$ |
| $\mathcal{O}_{\nu \nu u}^{S, L R}$ | $\left(\nu_{L p}^{T} C \nu_{L r}\right)\left(\bar{u}_{L s} u_{R t}\right)$ | $\mathcal{O}_{\text {duu }}^{S, R L}$ | $\epsilon_{\alpha \beta \gamma}\left(d_{R p}^{\alpha T} C u_{R r}^{\beta}\right)\left(u_{L s}^{\gamma T} C e_{L t}\right)$ | $\mathcal{O}_{u d d}^{S, R R}$ | $\epsilon_{\alpha \beta \gamma}\left(u_{R p}^{\alpha T} C d_{R r}^{\beta}\right)\left(\bar{\nu}_{L s} d_{R t}^{\gamma}\right)$ |
| $\mathcal{O}_{\nu d}^{S, L L}$ | $\left(\nu_{L p}^{T} C \nu_{L r}\right)\left(\bar{d}_{R s} d_{L t}\right)$ | $\mathcal{O}_{\text {dud }}^{S, R L}$ | $\epsilon_{\alpha \beta \gamma}\left(d_{R p}^{\alpha T} C u_{R r}^{\beta}\right)\left(d_{L s}^{\gamma T} C \nu_{L t}\right)$ | $\mathcal{O}_{d d d}^{S, R R}$ | $\epsilon_{\alpha \beta \gamma}\left(d_{R p}^{\alpha T} C d_{R r}^{\beta}\right)\left(\bar{e}_{L s} d_{R t}^{\gamma}\right)$ |
| $\mathcal{O}_{\nu d}^{T, L L}$ | $\left(\nu_{L p}^{T} C \sigma^{\mu \nu} \nu_{L r}\right)\left(\bar{d}_{R s} \sigma_{\mu \nu} d_{L t}\right)$ | $\mathcal{O}_{\text {ddu }}^{S, R L}$ | $\epsilon_{\alpha \beta \gamma}\left(d_{R p}^{\alpha T} C d_{R r}^{\beta}\right)\left(u_{L s}^{\gamma T} C \nu_{L t}\right)$ |  |  |
| $\mathcal{O}_{\nu d}^{S, L R}$ | $\left(\nu_{L p}^{T} C \nu_{L r}\right)\left(\bar{d}_{L s} d_{R t}\right)$ | $\mathcal{O}_{\text {duu }}^{S, R R}$ | $\epsilon_{\alpha \beta \gamma}\left(d_{R p}^{\alpha T} C u_{R r}^{\beta}\right)\left(u_{R s}^{\gamma T} C e_{R t}\right)$ |  |  |
| $\mathcal{O}_{\text {vedu }}^{\text {S,LL }}$ | $\left(\nu_{L p}^{T} C e_{L r}\right)\left(\bar{d}_{R s} u_{L t}\right)$ |  |  |  |  |
| $\mathcal{O}_{\nu \text { vedu }}^{\text {P,LL }}$ | $\left(\nu_{L p}^{T} C \sigma^{\mu \nu} e_{L r}\right)\left(\bar{d}_{R s} \sigma_{\mu \nu} u_{L t}\right)$ |  |  |  |  |
| $\mathcal{O}_{\nu \text { vedu }}^{\text {S,LR }}$ | $\left(\nu_{L p}^{T} C e_{L r}\right)\left(\bar{d}_{L s} u_{R t}\right)$ |  |  |  |  |
| $\mathcal{O}_{\nu e d u}^{V, R L}$ | $\left(\nu_{L p}^{T} C \gamma^{\mu} e_{R r}\right)\left(\bar{d}_{L s} \gamma_{\mu} u_{L t}\right)$ |  |  |  |  |
| $\mathcal{O}_{\text {vedu }}^{\text {V,RR }}$ | $\left(\nu_{L p}^{T} C \gamma^{\mu} e_{R r}\right)\left(\bar{d}_{R s} \gamma_{\mu} u_{R t}\right)$ |  |  |  |  |

