Enhanced production cross sections of ^{123,124,126,128}I via incomplete fusion reactions:

Scope in Nuclear Medicine

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Plan of the talk

Why study long-lived Iodine isotopes?

Enhanced production through incomplete fusion reaction

Scope in Nuclear Medicine

- Cross sections

  Complete fusion (CF) channels: Cs isotopes (A = 127, 128, 129, 130)

  Incomplete fusion (ICF) channels: Iodine isotopes (A = 123, 124, 126, 128)

- Case study of ^{11}B + ^{122}Sn

  Off-line γ-ray spectrometry

- Sum-rule model (original-SRM)

  Modified SRM

- Summary

Cross sections (CF vs. ICF)

- Nuclei produced through online irradiation of Sn isotopes (foils) using Boron beams (55-78 MeV) using the Heavy-ion accelerator.

Examples:

CF: ^{124}Sn(^{11}B, xn)$^{135-x}$Cs, ^{124}Sn(^{10}B, xn)$^{134-x}$Cs, ^{122}Sn(^{11}B, xn)$^{133-x}$Cs

ICF: ^{124}Sn(^{11}B, αxn)$^{135-x-4}$I, ^{124}Sn(^{10}B, αxn)$^{134-x-4}$I, ^{122}Sn(^{11}B, αxn)$^{133-x-4}$I

- Cross sections measured by the intensities of characteristic γ-rays emitted by the produced nuclei with long life (min-days).

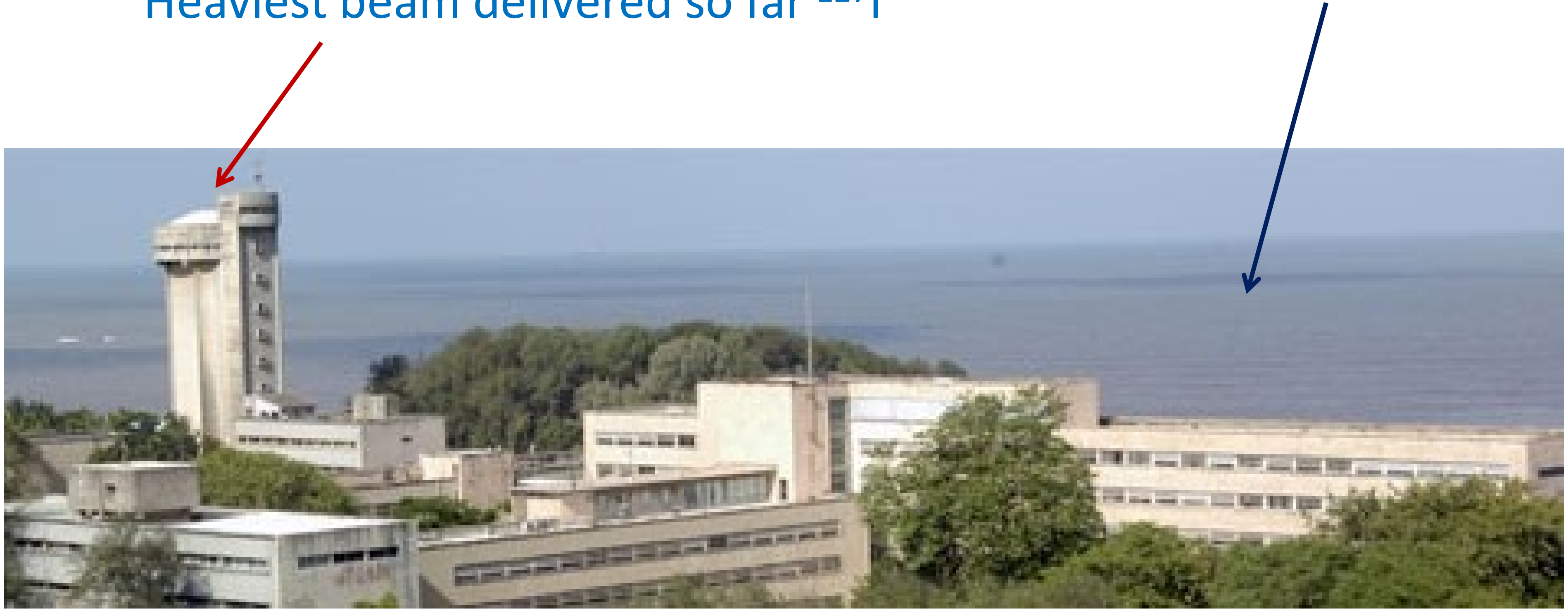
⟶ Off-line γ-ray spectrometry

- High-purity coaxial germanium detector utilized for the measurement.

B. Bhujang, Pragya Das *et al.*, Journal of Phys.: Conference Series **420,** 012128 (2013)

Tata Institute of Fundamental Research (TIFR)
Mumbai-400005, India

Experimental facility



14 MV Heavy ion accelerator (Pelletron)

Heaviest beam delivered so far ^{127}I

Arabian Sea

CF and ICF

Statistical model code (PACE) utilized for CF cross sections

A. Gavron, Phys. Rev. **C 21,** 230 (1980)

Enhanced cross sections for ICF (much above PACE estimates)

→ Different mechanism

ICF cross sections higher than Sum rule model estimates!

- Original SRM applied for ICF reactions at beam energies > 10 MeV/A

J. Wilczyński et al., Phys. Rev. Lett. **45,** 606 (1980)

- Modified SRM (MSRM1) for beam energies < 8 MeV/A

R. Tripathi et al., Phys. Rev. C 79 (2009) 064604

- Our study: (MSRM2)

Energy dependence in defining critical angular momentum

Production of iodine isotopes: Scope in Nuclear Medicine

Common Radiotracers in nuclear medicine

^{131}I ($\tau_{1/2}$ = 8.04 d, E_γ = 364 keV) produced using reactor

^{123}I ($\tau_{1/2}$ = 13.2 h, E_γ = 159 keV) cyclotron

- Gamma energies high enough to be penetrating through human body
- Half-lives high enough for practical utilization

Nuclei produced in our experiments (using heavy-ion accelerator)

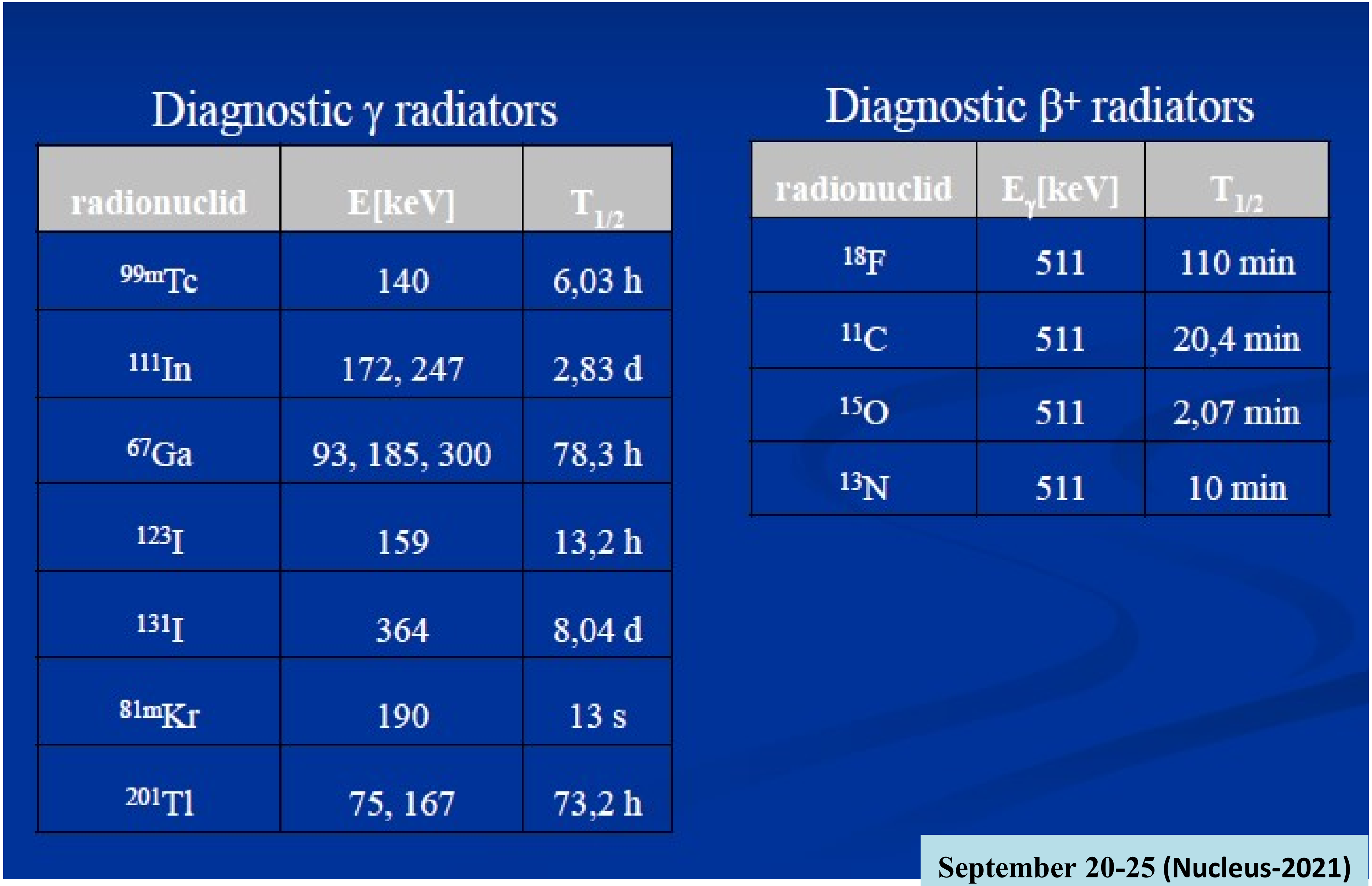
^{123}I ($\tau_{1/2}$ = 13.2 h, E_γ = 159 keV)

^{124}I ($\tau_{1/2}$ = 4.2 d, E_γ = 603 keV)

^{126}I ($\tau_{1/2}$ = 12.9 d, E_γ = 389 keV)

^{128}I ($\tau_{1/2}$ = 25 m, E_γ = 443 keV)

Radionuclides used in diagnostics



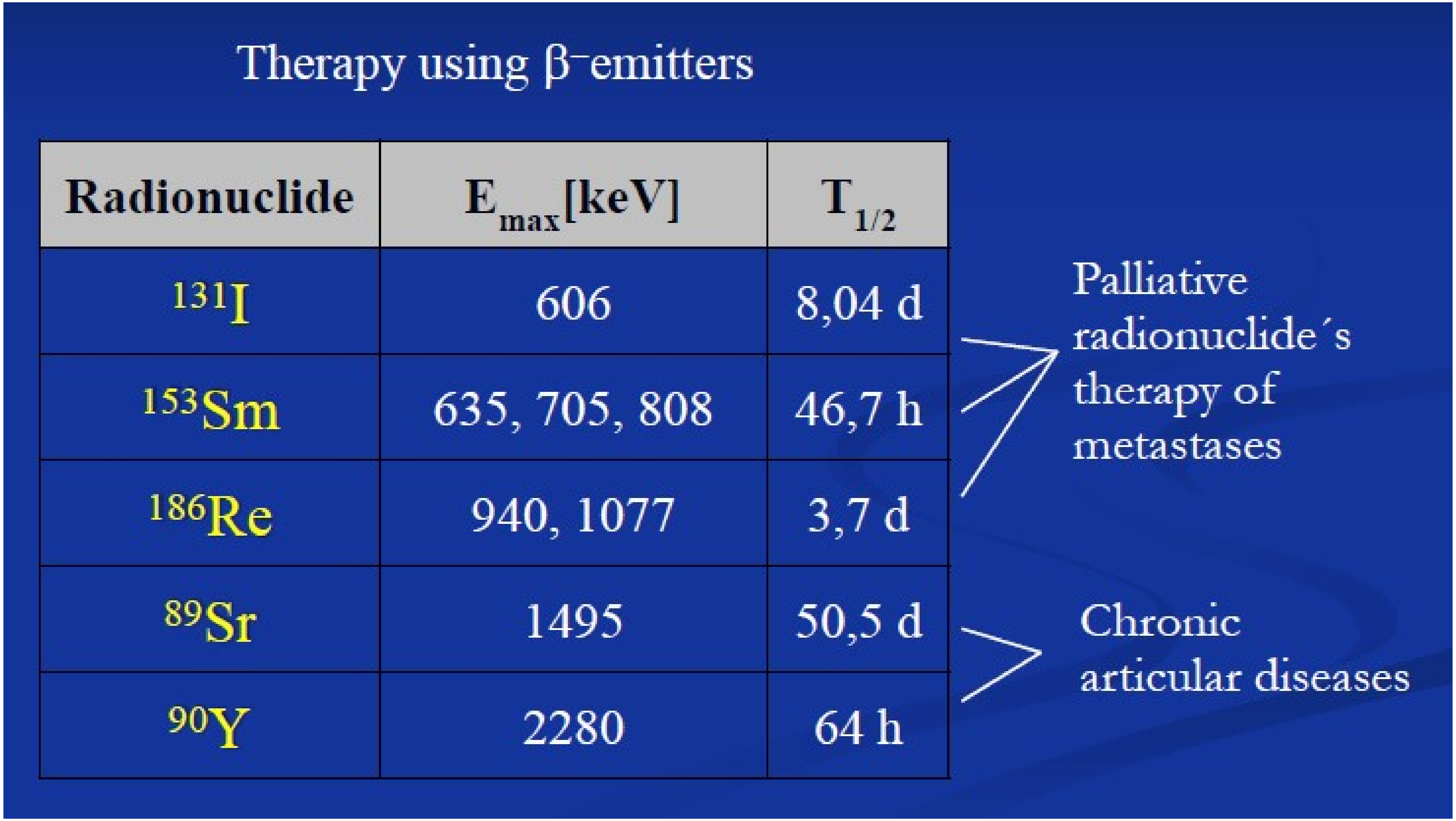
Diagnostic γ radiators

radionuclid	E[keV]	$T_{1/2}$
^{99m}Tc	140	6,03 h
^{111}In	172, 247	2,83 d
^{67}Ga	93, 185, 300	78,3 h
^{123}I	159	13,2 h
^{131}I	364	8,04 d
^{81m}Kr	190	13 s
^{201}Tl	75, 167	73,2 h

Diagnostic β⁺ radiators

radionuclid	E_γ[keV]	$T_{1/2}$
^{18}F	511	110 min
^{11}C	511	20,4 min
^{15}O	511	2,07 min
^{13}N	511	10 min

Radionuclides in Therapy



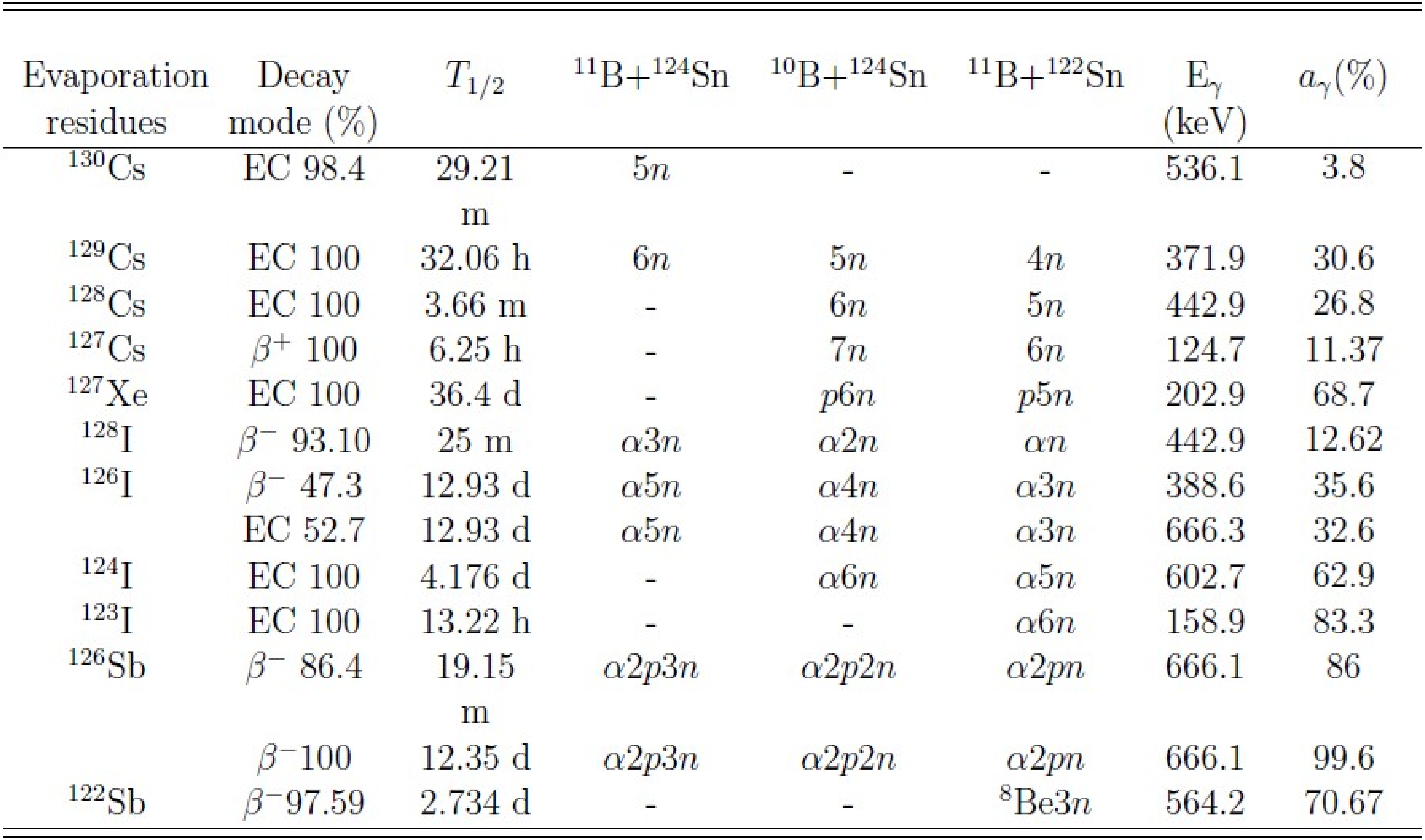
Therapy using β^-emitters

Radionuclide	E_{max}[keV]	$T_{1/2}$
^{131}I	606	8,04 d
^{153}Sm	635, 705, 808	46,7 h
^{186}Re	940, 1077	3,7 d
^{89}Sr	1495	50,5 d
^{90}Y	2280	64 h

Palliative radionuclide´s therapy of metastases

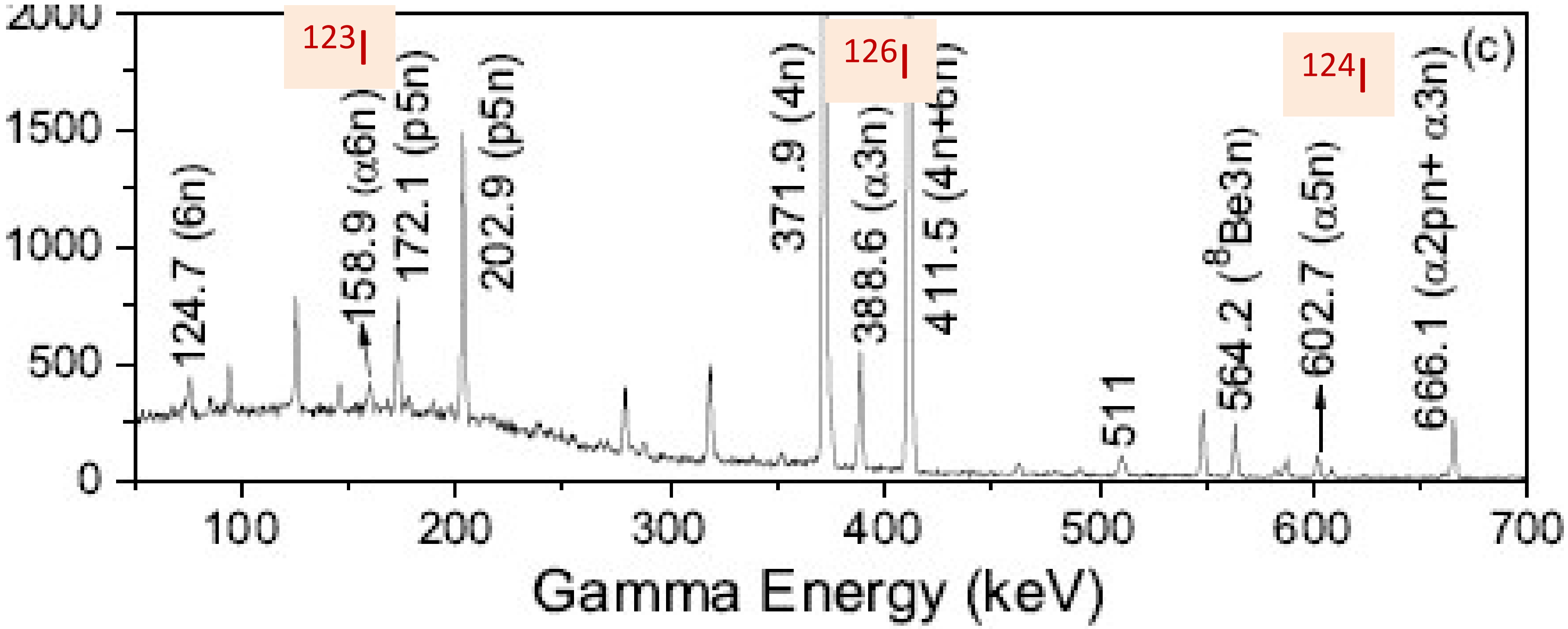
Chronic articular diseases

Residual nuclei formed in our studied reactions



Evaporation residues	Decay mode (%)	$T_{1/2}$	^{11}B+^{124}Sn	^{10}B+^{124}Sn	^{11}B+^{122}Sn	E$_\gamma$ (keV)	a_γ(%)
^{130}Cs	EC 98.4	29.21 m	$5n$	-	-	536.1	3.8
^{129}Cs	EC 100	32.06 h	$6n$	$5n$	$4n$	371.9	30.6
^{128}Cs	EC 100	3.66 m	-	$6n$	$5n$	442.9	26.8
^{127}Cs	β^+ 100	6.25 h	-	$7n$	$6n$	124.7	11.37
^{127}Xe	EC 100	36.4 d	-	$p6n$	$p5n$	202.9	68.7
^{128}I	β^- 93.10	25 m	$\alpha 3n$	$\alpha 2n$	αn	442.9	12.62
^{126}I	β^- 47.3	12.93 d	$\alpha 5n$	$\alpha 4n$	$\alpha 3n$	388.6	35.6
	EC 52.7	12.93 d	$\alpha 5n$	$\alpha 4n$	$\alpha 3n$	666.3	32.6
^{124}I	EC 100	4.176 d	-	$\alpha 6n$	$\alpha 5n$	602.7	62.9
^{123}I	EC 100	13.22 h	-	-	$\alpha 6n$	158.9	83.3
^{126}Sb	β^- 86.4	19.15 m	$\alpha 2p3n$	$\alpha 2p2n$	$\alpha 2pn$	666.1	86
	β^-100	12.35 d	$\alpha 2p3n$	$\alpha 2p2n$	$\alpha 2pn$	666.1	99.6
^{122}Sb	β^-97.59	2.734 d	-	-	^{8}Be$3n$	564.2	70.67

Case study of ^{11}B + ^{122}Sn



- Irradiation of ^{122}Sn foil of thickness ~ 2 mg/cm^2 for a few hours.
- Immediately after, singles data collection repeatedly for few days.
- Absolute detector efficiency determination using ^{152}Eu radioactive source.
- Finding cross-sections at different beam energies from the intensity of γ-rays.

Data Analysis

Nuclei studied: $\tau_{1/2} > 3$ min

^{128}I (25 m), ^{126}I (12.9 d), ^{128}Cs (3.6 m), ^{123}I (13.2 hr), ^{124}I (4.2 d), ^{127}Cs (6.3 h), ^{122}Sb (2.7 d), ^{129}Cs (32.1 h), ^{130}Cs (29.2 m), ^{126}Sb (19.2 m)

Single decay:

$$\sigma = \frac{\lambda I}{N_T\, a_\gamma\, \varepsilon_\gamma\, I_b [1 - exp(1 - \lambda T)][exp(-\lambda t_1) - exp(-\lambda t_2)]}$$

λ - decay constant,

I - intensity of gamma transition per unit time,

N_T – number of target nuclei per unit volume,

a_γ - branching intensity,

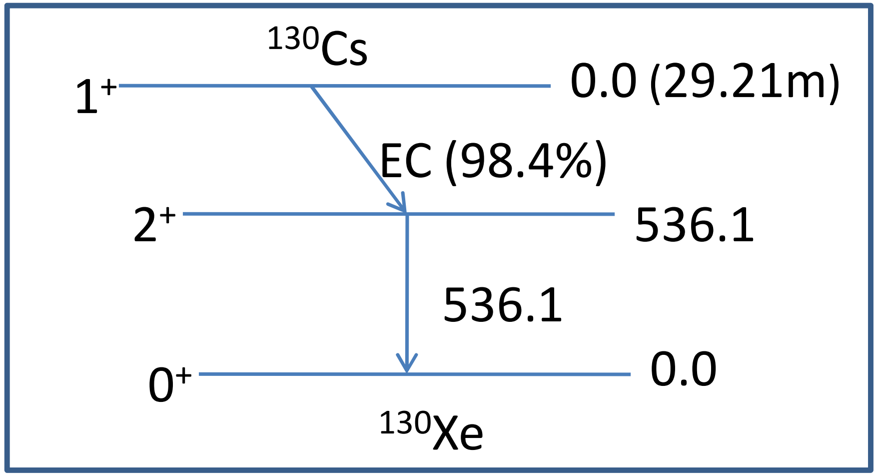
ε_γ - absolute efficiency,

I_b - beam intensity,

T – irradiation time,

t_1 – start time of data acquisition,

t_2 – start time of data acquisition



^{130}Cs

1^+ 0.0 (29.21m)

EC (98.4%)

2^+ 536.1

536.1

0^+ 0.0

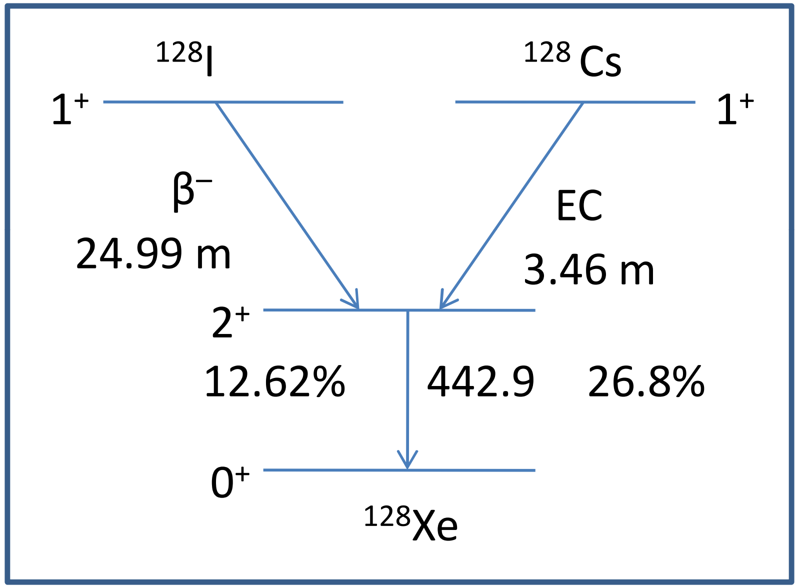
^{130}Xe

Double decay

Peak area (γ-ray) = $P_1\sigma_1 + P_2\sigma_2$

$$P_1 \equiv \frac{N_T}{\lambda_1} a_{\gamma_1} \varepsilon_{\gamma_1} I_b \left[1 - exp(-\lambda_1 T)\right]\left[exp(-\lambda_1 t_1) - exp(-\lambda_1 t_2)\right]$$

$$P_2 \equiv \frac{N_T}{\lambda_2} a_{\gamma_2} \varepsilon_{\gamma_2} I_b \left[1 - exp(-\lambda_2 T)\right]\left[exp(-\lambda_2 t_1) - exp(-\lambda_2 t_2)\right]$$



^{128}I

^{128}Cs

1^+

1^+

β^-

24.99 m

EC

3.46 m

2^+

12.62%

442.9

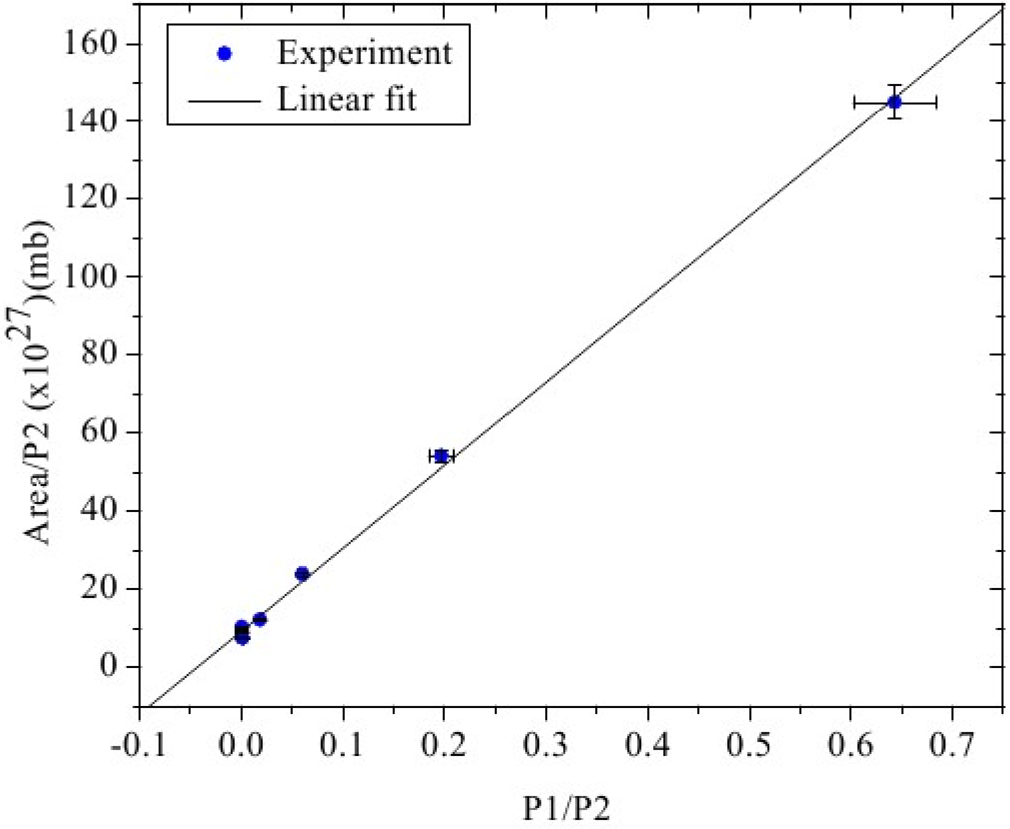
26.8%

0^+

^{128}Xe

Straight line plot of Area/P2 Vs P1/P2

[Data points for different t_1 , t_2]

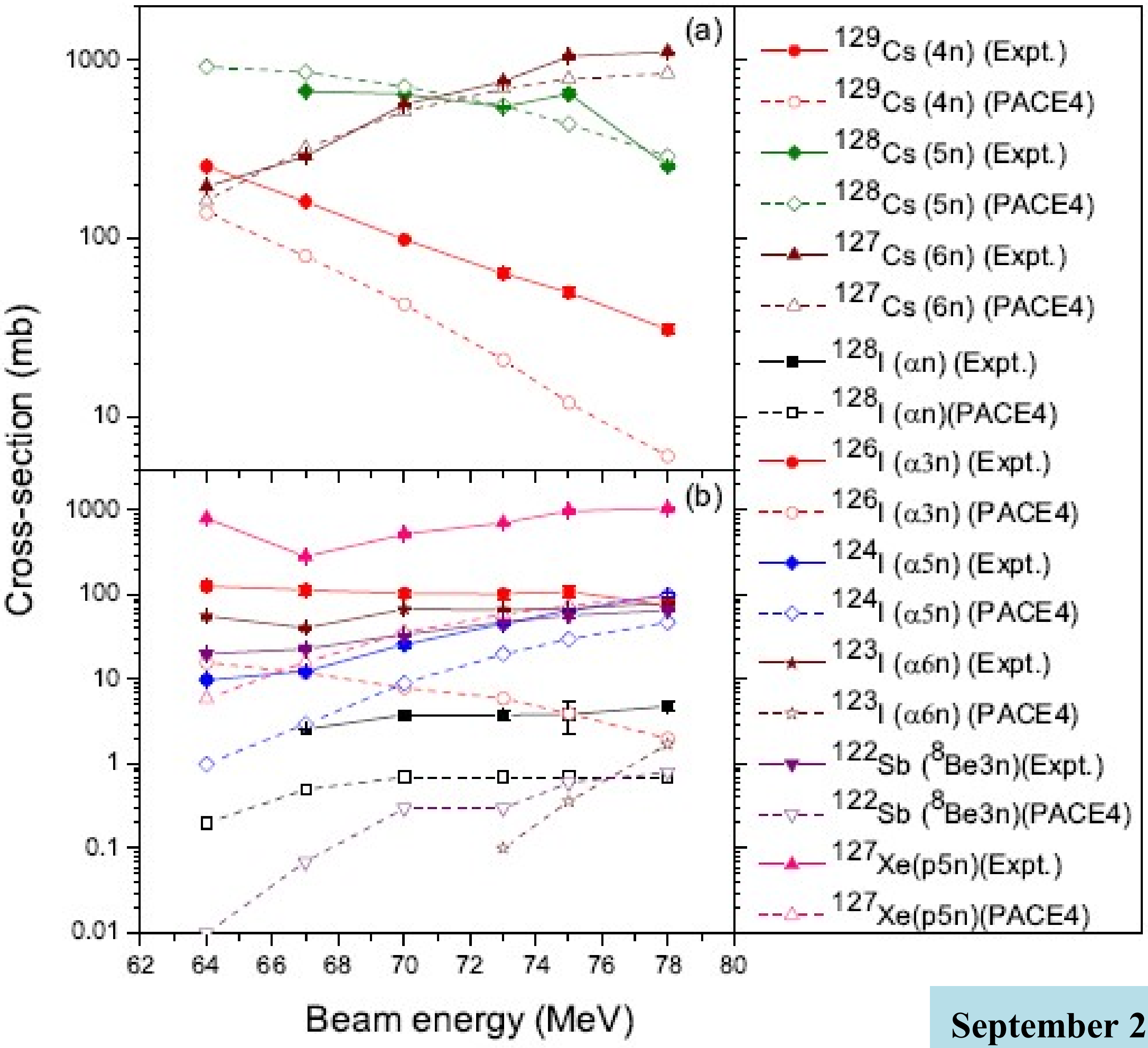


From the straight line fit

σ_1 = (213 ± 3) mb for ^{128}Cs

σ_2 = (9.4 ± 0.8) mb for ^{128}I

^{11}B + ^{122}Sn (Experiment vs. PACE)



September 20-25 (Nucleus-2021)

Sum Rule Model

K. Siwek-Wilczyńska *et al.* Phys. Rev. Lett. 42, 1599 (1979)
J. Wilczyński *et al.*, Phys. Rev. Lett. 45, 606 (1980)

Identical treatment for

Complete fusion (CF) and Incomplete fusion (ICF) reactions

- Concept of generalized angular momentum

→ Successive "ℓ-windows" with overlaps

- Higher average value of ℓ for the higher mass of the emitted projectile like fragment (PLF)

Examples of PLF: ^{4}He, ^{7}Li, ^{8}Be . . . .

- Complete capture of the projectile (CF)

No emission of PLF

Cont…

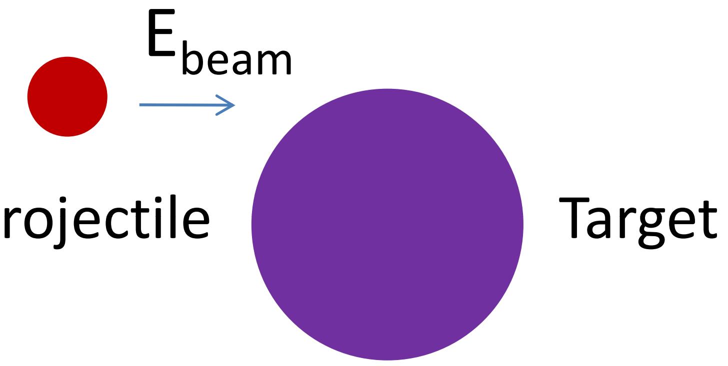
Probability for the i^{th} reaction channel $p(i) = \exp\left[\left\{\frac{Q_{gg}(i) - Q_c(i)}{T}\right\}\right]$

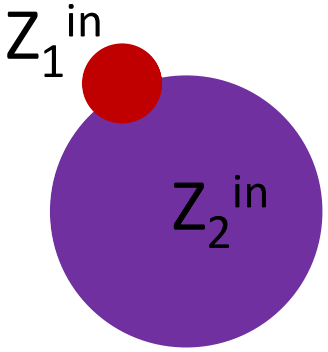
Ground state *Q*-value

Effective temperature (parameter)

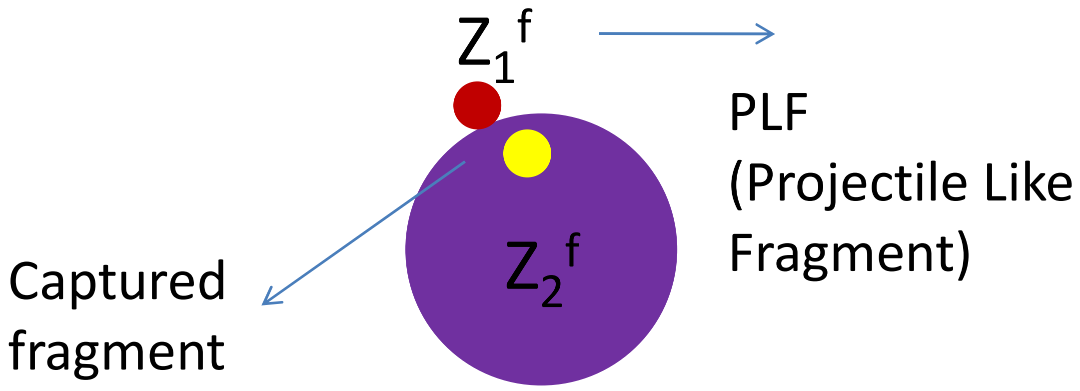
Change in Coulomb energy due to transfer of charge

$$Q_c = q_c\left(Z_1^f Z_2^f - Z_1^{in} Z_2^{in}\right) e^2$$





Initial Dinuclear system



Final Dinuclear system

Cont…

Limiting angular momentum

$$\ell_{lim}(i) = \frac{mass\ of\ projectile}{mass\ of\ captured\ fragment} \times \ell_{cr}(target + captured\ fragment)$$

Critical angular momentum

$$\left(\ell_{cr} + \frac{1}{2}\right)^2 = \frac{\mu (C_1 + C_2)^3}{\hbar^2}\left[4\pi\gamma\frac{C_1 C_2}{C_1 + C_2} - \frac{Z_1 Z_2 e^2}{(C_1 + C_2)^2}\right]$$

C_1 , C_2 → half - density radii

γ → surface tension coefficient

$$C_i = f_c r_o A_i^{\frac{1}{3}}.$$

MSRM2

Add a term inside square bracket

$$\frac{E}{(C_1 + C_2)}$$

Concept of "critical angular momentum"

Classical turning point

Cont...

Transmission coefficient

$$T_\ell\left(i\right) = \left[1 + \exp\left(\frac{\ell - \ell_{lim}\left(i\right)}{\Delta_\ell}\right)\right]^{-1}$$

Normalization of probability

$$N_\ell \sum_i T_\ell(i)\, p(i) = 1$$

Summation runs for All the reaction channels including CF

MSRM1

Multiply $T_\ell(i)$ by a factor

$$[\exp(n\ell f_t)]^{-1}$$

No. of nucleons transferred to the target

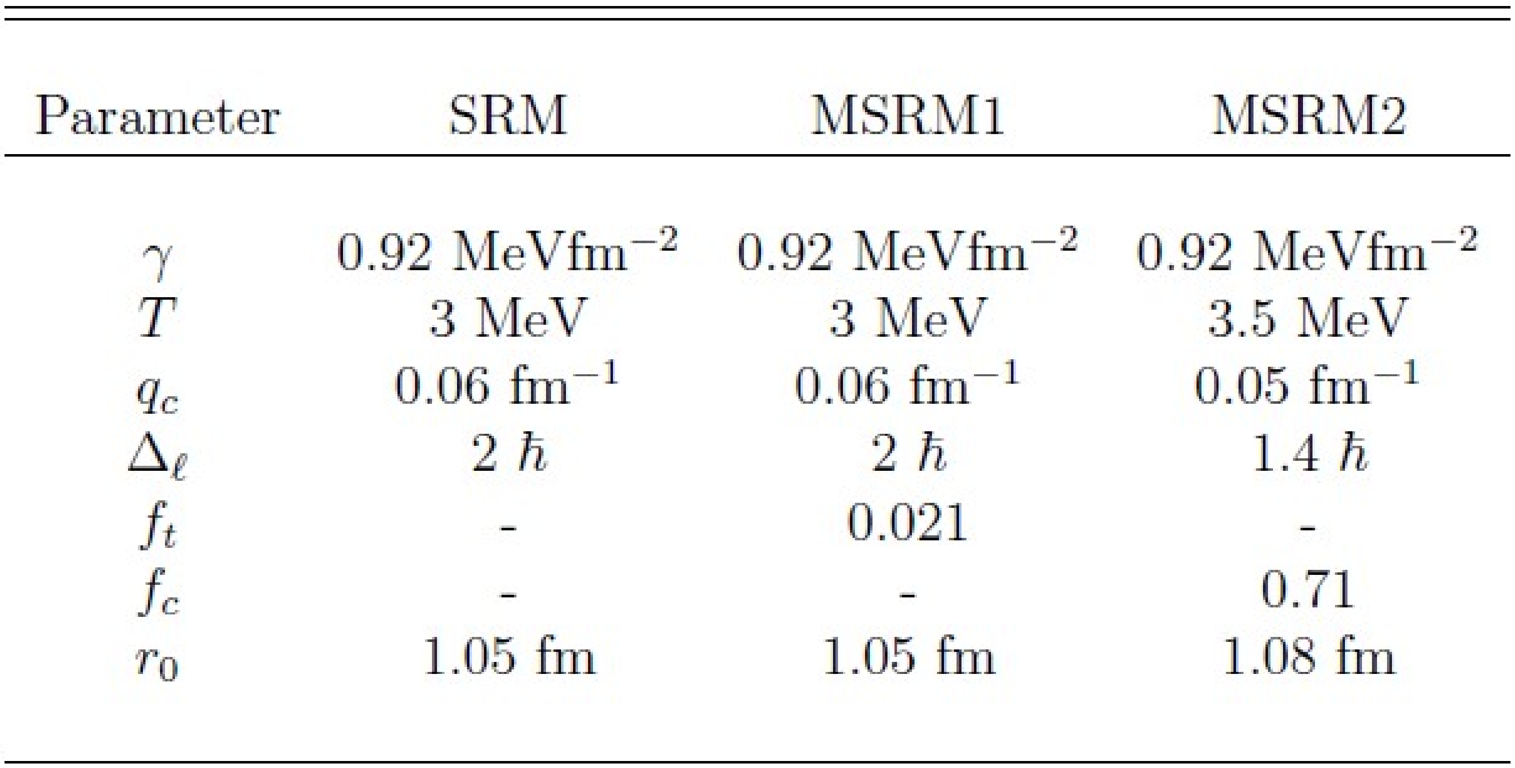
A parameter

Reaction cross-section

$$\sigma(i) = \frac{\lambda^2}{4\pi} \sum_{\ell=0}^{\ell_{max}} (2\ell + 1) N_\ell T_\ell(i) p(i)$$

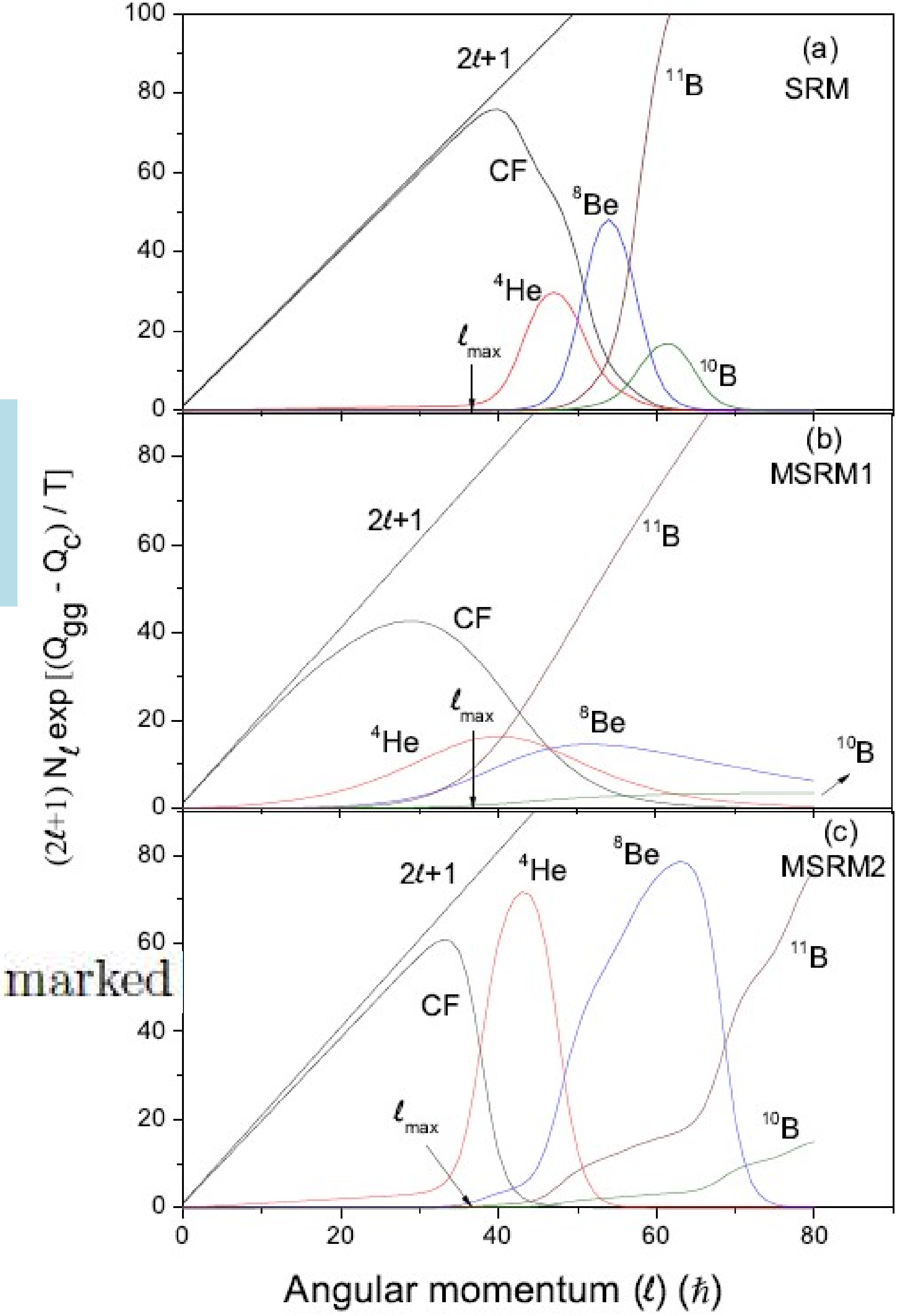
Summation runs over the range of partial waves that confines CF and ICF

Choice of parameters



Parameter	SRM	MSRM1	MSRM2
γ	$0.92\ \mathrm{MeVfm}^{-2}$	$0.92\ \mathrm{MeVfm}^{-2}$	$0.92\ \mathrm{MeVfm}^{-2}$
T	3 MeV	3 MeV	3.5 MeV
q_c	$0.06\ \mathrm{fm}^{-1}$	$0.06\ \mathrm{fm}^{-1}$	$0.05\ \mathrm{fm}^{-1}$
Δ_ℓ	$2\ \hbar$	$2\ \hbar$	$1.4\ \hbar$
f_t	-	0.021	-
f_c	-	-	0.71
r_0	1.05 fm	1.05 fm	1.08 fm

Theoretical Probability distribution function

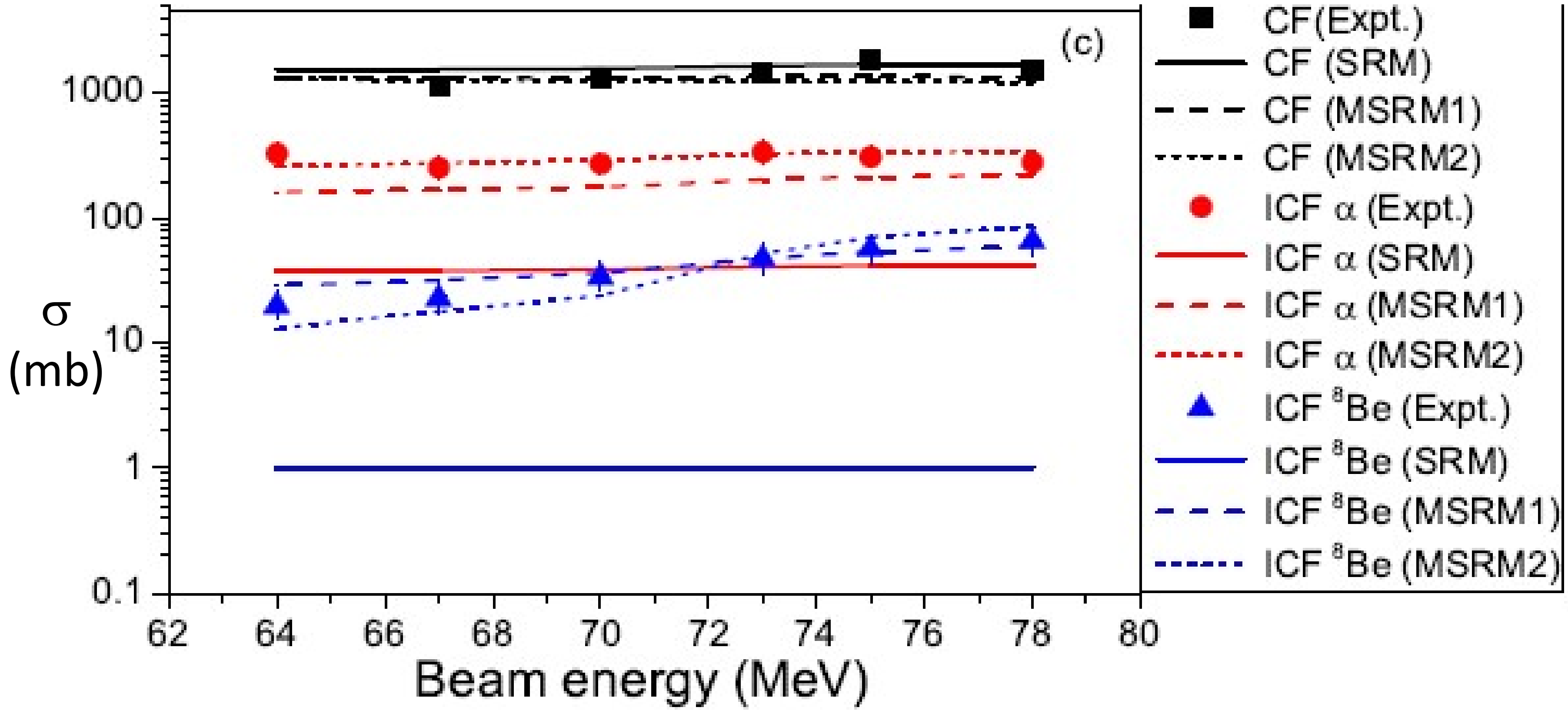


ℓ_{max} at 64 MeV beam energy is marked

Cross sections (^{11}B + ^{122}Sn)

σ_{Cf}(exp) = σ (^{129}Cs) + σ (^{128}Cs) + σ (^{127}Cs)

$\sigma_{ICF(\alpha)}$(exp) = σ (^{128}I) + σ (^{126}I) + σ (^{124}I) + σ (^{123}I)



Much better match for ICF using MSRM1 and MSRM2 compared to original SRM

Summary

- Production of Long-lived iodine isotopes ($^{123,\,124,\,126,\,128}$I) through ICF:

  Enhanced cross sections measured for beam energies < 8 MeV/A

  Above coulomb barrier

- Original sum rule model (SRM) worked earlier for beam energies >10 MeV/A

  SRM underestimated cross section for our studied ICF reactions.

- Two independently suggested modifications (MSRM1 and MSRM2) worked well for our studied reactions.

- ICF reactions provide an alternative way of producing long lived Iodine isotopes to be used as radiotracers for diagnostic and therapeutic purposes.

- ICF can also be utilized for studying nuclear structure properties for nuclei close to β-stability (not discussed here).


Thank you