

HIGH-SPIN STATES OF YRAST-BANDS EVEN ISOTOPES Pu, Cm, Fm, No

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Аннотация

In some isotopes of Pu, Cm, Fm, No the yrast-band state energies are experimentally known up to the spin 32^+ , like this holds in ^{248}Cm . What is unique is that all states of these super long bands remain purely collective. Collectivity of the bands is manifested in the fact that the band state energies are reproduced with a high degree of accuracy within the framework of the phenomenology of the IBM1. The uncertainty of the model parameters is reduced under condition that the depth of the deformation energy obtained on the basis of realistic interactions was equal to the deformation energy depth in the internal state, calculated with the found parameters of the IBM1 Hamiltonian.

1 RELATIONSHIP OF THE MODEL OF INTERACTING BOSONS WITH VARIABLE GEOMETRIC MODELS

The assumption that the algebra of phonon operators is closed leads to a restriction on the maximum number of bosons, and this is the main difference between the MBE1 and the geometric models of the nucleus. The way of correlating generalized coordinates in geometric models and boson parameters was obtained through the use of an internal state [J. N. Ginocchio and M. W. Kirson, Nucl. Phys. A **350**, 31 (1980). A. Bohr and B. R. Mottelson, Phys. Scr. **22**, 468 (1980).]

$$|\Phi\rangle = \left\{ s^+ + \tilde{\beta} \left[d_0^+ \cos \tilde{\gamma} + \frac{1}{\sqrt{2}} (d_2^+ + d_{-2}^+) \sin \tilde{\gamma} \right] \right\}^\Omega; \quad \langle \Phi | \Phi \rangle = \Omega (1 + \tilde{\beta}^2)^2. \quad (1)$$

Hamiltonian IBM1

$$H_{\text{IBM}} = \varepsilon_d \hat{n}_d + k_1 (d^+ \cdot d^+ s s + \text{H.c.}) + k_2 \left((d^+ d^+)^{(2)} \cdot d s + \text{H.c.} \right) + \frac{1}{2} \sum_L C_L (d^+ d^+)^{(L)} \cdot (d d)^{(L)}, \quad (2)$$

The internal state function has the form

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$$\begin{aligned} \langle H_{\text{IBM}} \rangle = & \langle \Phi | H_{\text{IBM}} | \Phi \rangle (\langle \Phi | \Phi \rangle)^{-1} = \frac{\Omega \tilde{\beta}^2}{(1 + \tilde{\beta}^2)^2} \left\{ \varepsilon_d + 2k_1(\Omega - 1) \right. \\ & \left. - \sqrt{\frac{8}{7}} k_2(\Omega - 1) \tilde{\beta} \cos 3\tilde{\gamma} + \left[\left(\frac{C_0}{10} + \frac{C_2}{7} + \frac{9C_4}{35} \right) (\Omega - 1) + \varepsilon_d \right] \tilde{\beta}^2 \right\}. \end{aligned} \quad (3)$$

The quantity $\langle H_{\text{IBM}} \rangle$ as a function of deformation characteristics $\tilde{\beta}$ and $\tilde{\gamma}$ as PES can be called a deformation surface energy.

$$\hat{T}(E2) = e^* \left(d^+ s + s^+ d + \chi_{E2} d^+ d \right)^{(2)}, \quad (4)$$

$$\begin{aligned} \langle Q_{\text{IBM}}^{(0)} \rangle = & \langle \Phi | 4\sqrt{\frac{\pi}{5}} \hat{T}(E2) | \Phi \rangle (\langle \Phi | \Phi \rangle)^{-1} \\ = & 4\sqrt{\frac{\pi}{5}} \frac{2\Omega \tilde{\beta} \cos \tilde{\gamma}}{1 + \tilde{\beta}^2} e^* \left(1 - \sqrt{\frac{1}{14}} \chi_{E2} \tilde{\beta} \frac{\cos 2\tilde{\gamma}}{\cos \tilde{\gamma}} \right), \end{aligned} \quad (5)$$

$R = 1.2A^{1/3}$ fm and $\beta^2 \ll 1$

$$Q^{(0)} = \frac{3}{\sqrt{5\pi}} Z e R^2 \beta \cos \gamma. \quad (6)$$

So

$$\beta \cos \gamma = \frac{4\pi}{3} \frac{\tilde{\beta} \cos \tilde{\gamma}}{1 + \tilde{\beta}^2} \frac{2e^* \Omega}{Z e R^2} \left(1 - \sqrt{\frac{1}{14}} \chi_{E2} \tilde{\beta} \frac{\cos 2\tilde{\gamma}}{\cos \tilde{\gamma}} \right). \quad (7)$$

The same with $1/2(Q_2 + Q_{-2})$, and so

$$\text{tg } \gamma = \text{tg } \tilde{\gamma} \frac{1 + \sqrt{2/7} \chi_{E2} \tilde{\beta} \cos \tilde{\gamma}}{1 - \sqrt{1/14} \chi_{E2} \tilde{\beta} (\cos 2\tilde{\gamma} / \cos \tilde{\gamma})}. \quad (8)$$

$\gamma = \tilde{\gamma} = 0$ и $\gamma = \tilde{\gamma} = \pi/3$,

$$\beta = 1.4179 \frac{\tilde{\beta}}{1 + \tilde{\beta}^2} \frac{e_{\text{W}}^* \Omega}{Z} \left(1 - 0.2673 \chi_{E2} \tilde{\beta} \right). \quad (9)$$

e_{W}^* corresponds to the case when the values of $B(E2)$ are considered in single-particle units W.u. $= 0.0594A^{4/3}e^2 \text{ fm}^4$. $e^* = 0.2437A^{2/3}e_{\text{W}}^*$.

The energies of the yrast bands and a number of states of the γ bands of the designated nuclei are analyzed within the framework of the phenomenology of the IBM1. An additional condition is the reproduction of the depth of the potential energy of deformation obtained on the basis of the Gonya cite map interaction.

2 ANALYSIS OF A NUMBER OF NUCLEI IN THE TRANSURANE REGION IN THE PHENOMENOLOGY OF IBM1

In some cases, the total number of bosons is used $\Omega = 36$. In this case, the size of the matrices reaches 551×551 for the spin $I = 16^+$. The search for parameters was carried out in the range of their values, which gives the value of the maximum potential energy of deformation, close to the estimate obtained in the work

[S. Hilaire and M. Girod, *Eur. Phys. J. A* **33**, 237 (2007); http://www-phynu.cea.fr/science_en_ligne/carte_potentiels_microscopiques/choix/choixisotopes.html].

Microscopic version of IBM1:

A. D. Efimov, *Phys. Atom. Nucl.* **83**, 380 (2020)

Таблица 1: The value of boson parameters in MeV for Pu isotopes

Параметр	$^{236}_{94}\text{Pu}$	$^{238}_{94}\text{Pu}$	$^{240}_{94}\text{Pu}$		$^{242}_{94}\text{Pu}$	$^{244}_{94}\text{Pu}$
ε_d	-0.599648	-0.616820	-0.617153	-0.647031	-0.603417	-0.604948
k_1	-0.036678	-0.035646	-0.037240	-0.030975	-0.038054	-0.038016
k_2	0.016832	0.018168	0.021340	0.011139	0.019278	0.017834
C_0	0.240945	0.197461	0.241177	0.554837	0.212750	0.229193
C_2	0.021790	-0.007801	-0.005771	0.080469	-0.008387	-0.011953
C_4	0.030488	0.033498	0.032335	0.048073	0.027480	0.030619
Ω	24	24	24	36	24	24
V_{\min}	-15	-16.2	-16.6	-16.6	-17	-16.5
$\langle H_{\text{IBM}} \rangle_{\min}$	-14.97	-16.27	-16.50	-16.40	-16.95	-16.40
$E(0_1^+)$	-15.49	-16.84	-17.06	-16.95	-17.52	-17.00

Таблица 2: The value of boson parameters in MeV for Cm isotopes

Параметр	$^{242}_{96}\text{Cm}$	$^{246}_{96}\text{Cm}$			$^{248}_{96}\text{Cm}$		
ε_d	-0.760776	-0.767505	-0.313958	-0.495547	-0.775108	-0.664738	-0.595
k_1	-0.047578	-0.047365	-0.012914	-0.021580	-0.045880	-0.038703	-0.033
k_2	0.030710	0.027934	0.012363	0.013514	0.024586	0.013733	0.0105
C_0	0.517326	0.515156	-0.110052	-0.022396	0.469563	0.525903	0.5864
C_2	-0.024683	-0.022940	0.059870	0.052000	-0.019301	0.027832	0.0473
C_4	0.047627	0.048715	0.027771	0.030190	0.048427	0.039679	0.0386
Ω	24	24	24, $SU(3)$ -1	24, $SU(3)$ -2	24	30	36
V_{\min}	-18.25	-18.45	-18.45	-18.45	-17.8	-17.8	-17.7
$\langle H_{\text{IBM}} \rangle_{\min}$	-18.51	-18.12	-9.34	-12.64	-17.81	-17.39	-17.7
$E(0_1^+)$	-19.11	-18.70	-10.21	-13.36	-18.40	-17.89	-18.2

Таблица 3: Exp. and Th. energy for Pu isotopes (^{240}Pu th. 1 — $\Omega = 24$, th. 2 — с $\Omega = 36$)

I^π	$^{236}_{96}\text{Pu}$		$^{238}_{96}\text{Pu}$		$^{240}_{96}\text{Pu}$			$^{242}_{96}\text{Pu}$		$^{244}_{96}\text{Pu}$	
	экс.	теор.	экс.	теор.	экс.	теор. 1	теор. 2	экс.	теор.	экс.	теор.
2 ⁺	44.6	44.5	44.1	43.8	42.8	42.4	42.9	44.5	44.3	44.2	45.1
4 ⁺	147.5	147.2	145.9	145.2	141.7	140.6	141.7	147.3	146.9	149.9	149.4
6 ⁺	305.8	305.4	303.4	302.6	294.3	293	293.6	306.4	305.7	313	310.8
8 ⁺	515.7	515.5	512.6	513.4	497.4	497	495.3	518.1	517.8	530.2	526.3
10 ⁺	773.5	773.3	771.9	774.6	747.4	749.8	743.7	778.6	779.7	797.8	792.4
12 ⁺	1074.3	1074.3	1077.7	1082.8	1041.1	1048	1036	1084.4	1087	1111.4	1105
14 ⁺	1413.6	1414	1426.4	1434	1374.8	1387	1368	1431.7	1437	1466.7	1459
16 ⁺	1786	1787.9	1815.5	1825	1745.7	1765	1740	1816.7	1823	1859.2	1852
18 ⁺		2191.5	2241.7	2251	2151.6	2176	2148	2236	2243	2284.5	2277
20 ⁺			2702.3	2709	2590.2	2616	2591	2686	2690	2737.9	2730
22 ⁺			3195.4	3194	3059.8	3083	3068	3163	3161	3211	3208
24 ⁺			3717.1	3702	3559	3572	3577	3662	3651	3686.3	3704
26 ⁺			4263.7	4229	4086.3	4079	4117	4172	4155	4145.2	4214
28 ⁺			4833.3	4771.5	4639.4	4600	4688			4606.1	4733
30 ⁺			5426.5	5324	5220.3	5133	5287			5085.7	5256
32 ⁺			1028.5	1029	5819.3	5675	5916			5589.6	5776
34 ⁺			1069.9	1072						6119.7	6289
2 ₂ ⁺		1099	1125.8	1128	1137	1.137	1125	1102	1102	1015	1015.5
3 ₁ ⁺		1145.3		1781	1177.6	1.180	1178		1145		1059
4 ₂ ⁺		1205.7			1232.5	1.234	1245		1202		1117
0 ₂ ⁺		1922				1.961	1997		1907		1273

Таблица 4: Exp. and th. energy for Cm isotopes (^{248}Cm th. 1 — $\Omega = 24$, th. 2 — $\Omega = 30$, th. 3 — $\Omega = 36$)

I^π	$^{242}_{96}\text{Cm}$		$^{246}_{96}\text{Cm}$				$^{248}_{96}\text{Cm}$			
	экс.	теор.	экс.	теор.	$SU(3)$ -1	$SU(3)$ -2	экс.	теор. 1	теор. 2	теор. 3
2^+	42.1	41.7	42.9	42.6	42.6	42.8	43.4	43.2	43.7	43.9
4^+	138.1	138.2	141	141.2	141.1	141.6	143.8	143.4	144.5	144.8
6^+	288.3	287.8	294.1	294	293.1	294.4	298.9	298.5	300	300
8^+	489.1	488.2	498.7	498.4	495.7	498	506.4	506.1	506.4	505.7
10^+	735.9	736.1	751.5	751.1	745.5	749.2	762.8	762.6	760.3	758.2
12^+	1026.2	1027.9	1045.3	1048	1039	1044	1064.1	1064	1058	1054
14^+	1355.2	1360	1385.3	1386	1373	1380	1406.1	1407	1395	1389
16^+	1720.8	1728	1758.4	1760	1744	1752	1783.9	1787	1769	1762
18^+	2119.5	2128	2163.3	2165	2149	2158	2192.6	2198	2175	2168
20^+	2549.3	2556	2596.3	2598	2586	2594	2627	2638	2612	2606
22^+	3008.8	3007	3054.2	3054	3050	3057	3083.4	3100	3075	3074
24^+	3497.4	3477	3533.3	3528	3541	3544	3559.5	3581	3563	3570
26^+			4031.4	4015	4054	4051	4055.3	4074	4074	4091
28^+				4512	4589	4576	4572.3	4575	4605	4637
30^+				5016	5142	5115	5113.9	5076	5154	5206
32^+				5565	5709	5665	5680.7	5605	5720	5798
2^+_{2}		1200	1124.3	1125	1120	1126	1050	1054	1048	1043
3^+_{1}		1237	1165.5	1163	1169	1174		1093	1095	1093
4^+_{2}		1286	1220	1214	1232	1236	1144	1145	1156	1158
0^+_{2}		1787		1631	1170	1820		1730	1843	1846

Таблица 5: Parameters for Fm and No

Параметр	$^{248}_{100}\text{Fm}$	$^{250}_{100}\text{Fm}$	$^{252}_{102}\text{No}$	$^{254}_{102}\text{No}$
ε_d	-0.884069	-0.900130	-0.836100	-0.857295
k_1	-0.045806	-0.045133	-0.040628	-0.035551
k_2	0.025660	0.026923	0.019023	0.019891
C_0	0.384586	0.363519	0.167861	0.124074
C_2	-0.024916	-0.025427	-0.010254	-0.025006
C_4	0.053072	0.051547	0.035493	0.043086
Ω	24	24	24	24
V_{\min}	-20.5	-21	-21.5	-21.5
$\langle H_{\text{IBM}} \rangle_{\min}$	-20.34	-20.89	-21.25	-21.32
$E(0^+_1)$	-20.98	-21.57	-21.92	-22.02

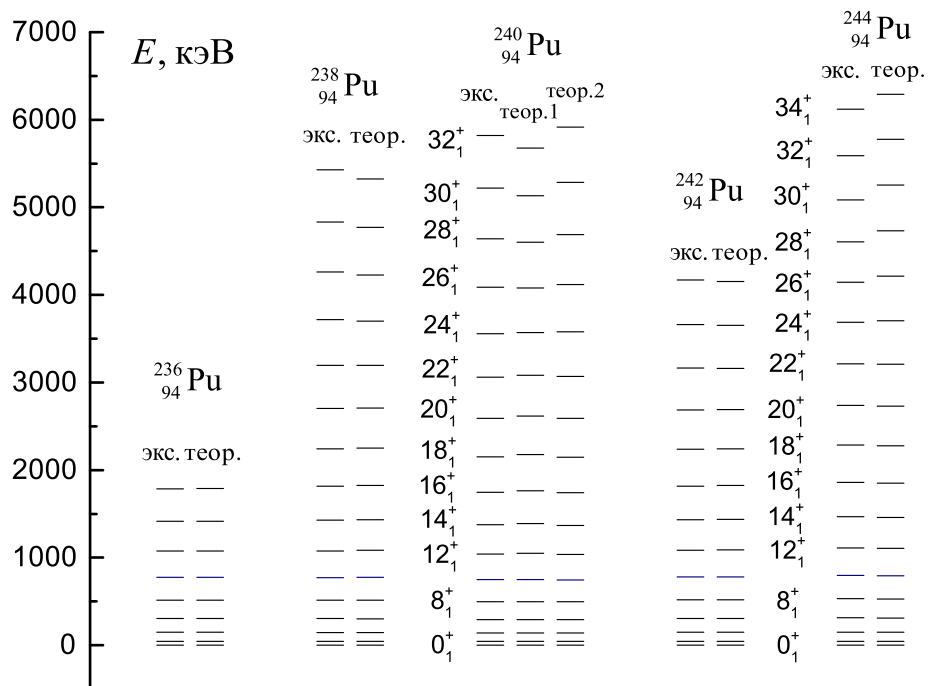


Рис. 1: Эксп. и теор. значения энергий в изотопах Pu. Вариант расчета, обозначенный как теор. 2, соответствует полному числу бозонов $\Omega = 36$, в остальных случаях $\Omega = 24$. Experimental and calculated values energies in Pu isotopes. The variant of the calculation, designated as theor. 2 corresponds to the total number of bosons $\Omega = 36$, in other cases $\Omega = 24$.

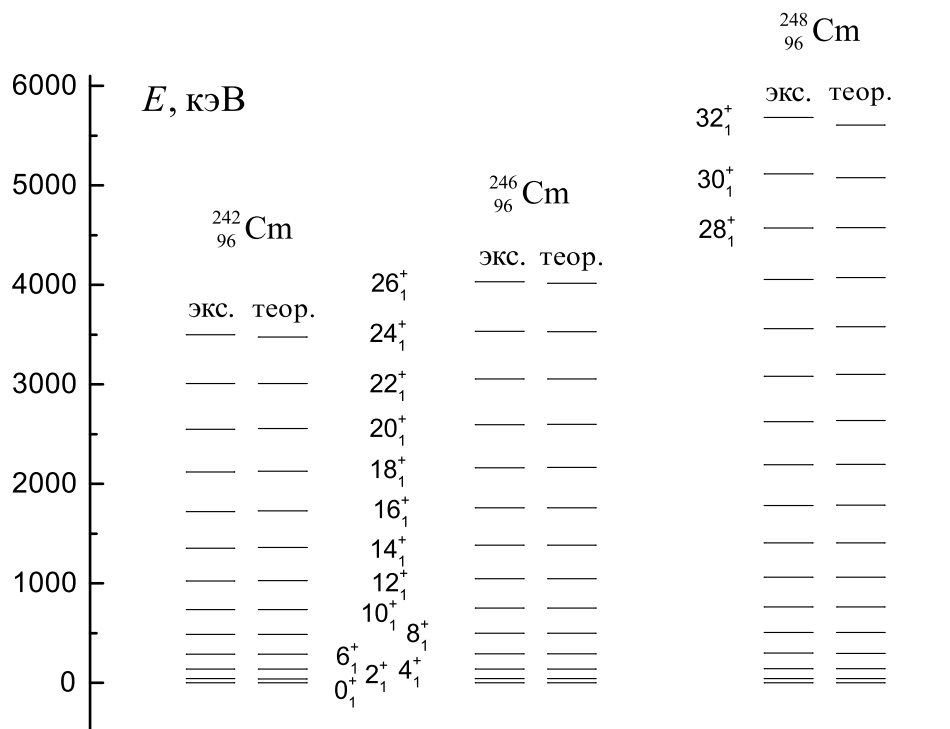


Рис. 2: Экспериментальные и расчетные значения энергий в изотопах Cm с $\Omega = 24$.
Experimental and calculated values of energies in isotopes Cm with $\Omega = 24$

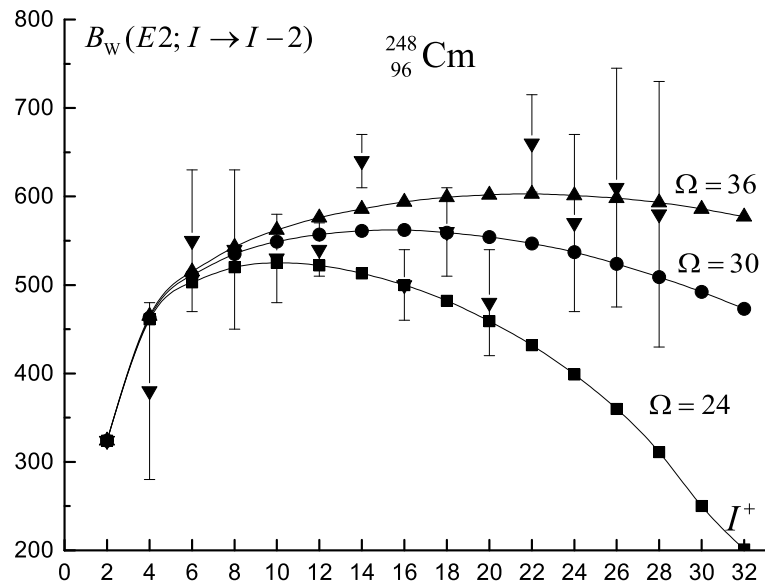


Рис. 3: Значения $B_W(E2; I \rightarrow I-2)$ для ^{248}Cm , $W.u. = 92.55 e^2 \text{fm}^4$. Расчеты представлены последовательно для трех полных значений бозонов $\Omega = 24, 30$ и 36 . Values $B_W(E2; I \rightarrow I-2)$ for ^{248}Cm , $W.u. = 92.55 e^2 \text{fm}^4$, experimental values. The calculations are presented sequentially for three total values of the bosons $\Omega = 24, 30$ and 36 .

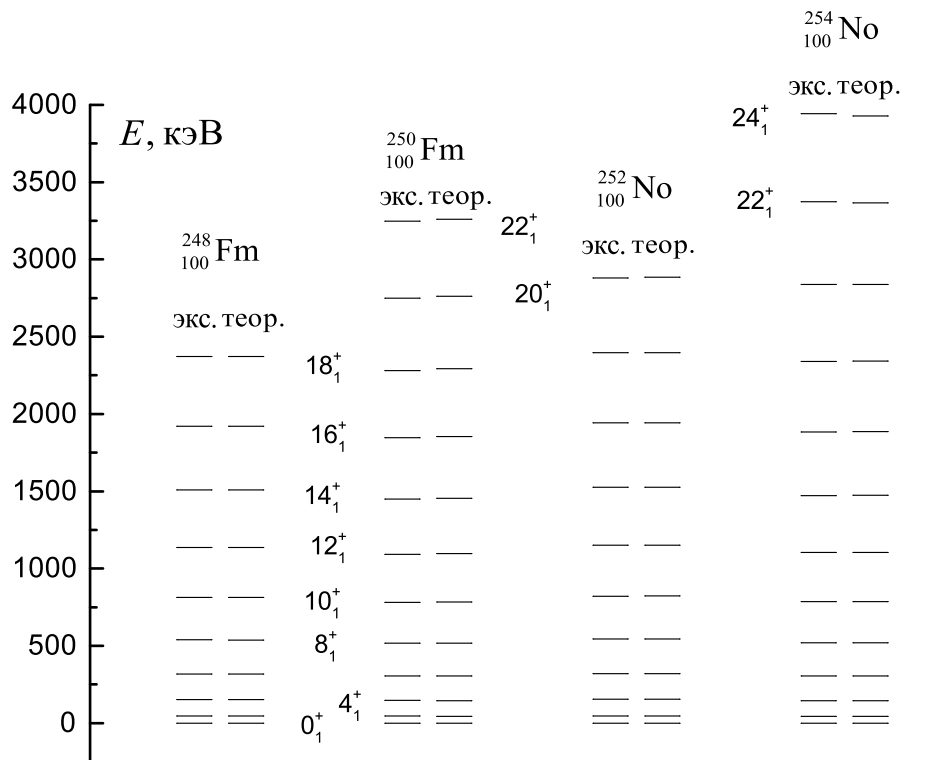


Рис. 4: Экспериментальные и расчетные значения энергий в изотопах Fm и No.
 Experimental and calculated the values of energies in the isotopes Fm and No.

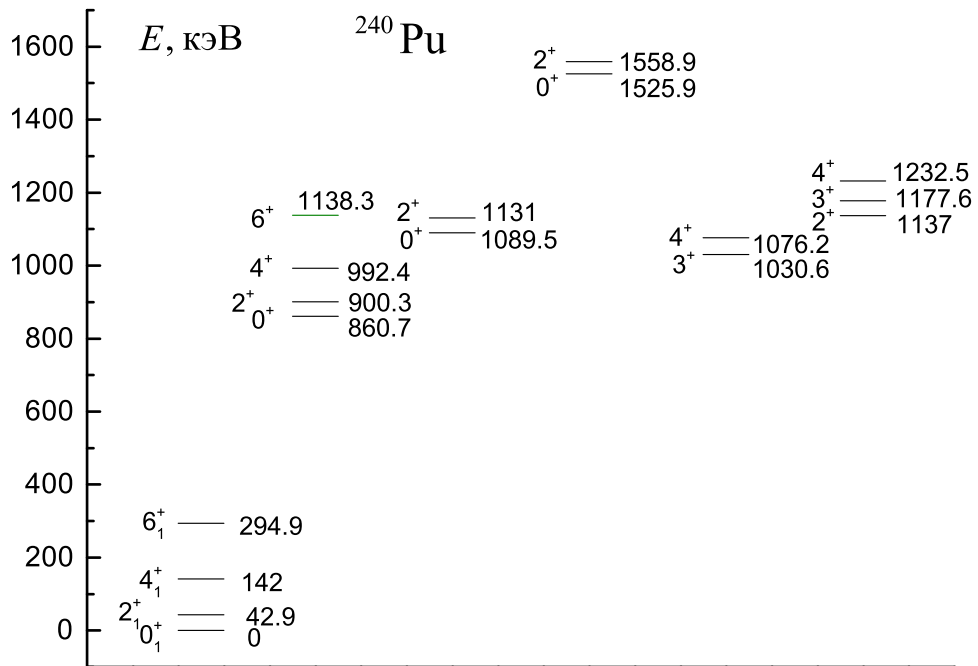


Рис. 5: Фрагмент схемы ядерных уровней ^{240}Pu . Fragment of nuclear level diagram ^{240}Pu .

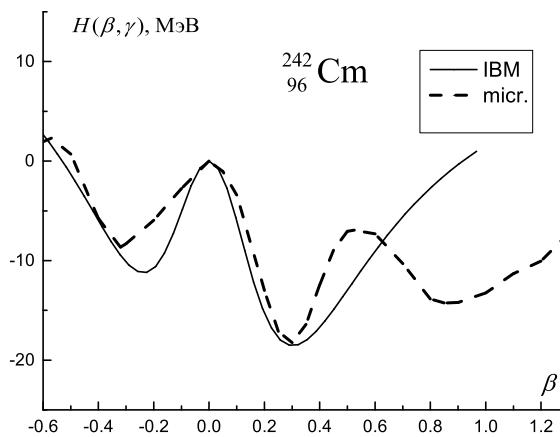


Рис. 6: Потенциальная энергия деформации ^{242}Cm , а также вычисленная в соответствии с ("IBM") с параметрами из табл. 2; Potential strain energy ^{242}Cm , and denoted as "micr.", and also computed according to (3) ("IBM") with parameters from table 2.

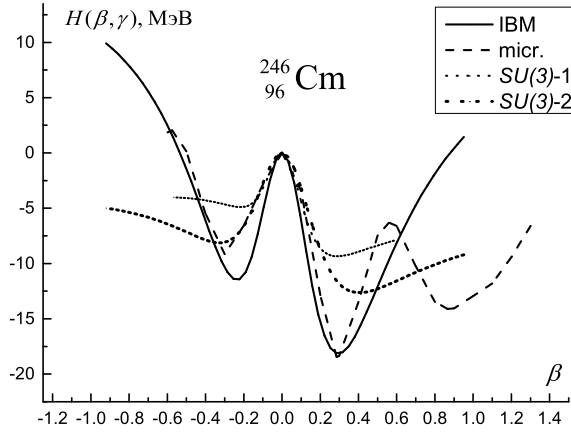


Рис. 7: Потенциальная энергия деформации $^{246}_{96}\text{Cm}$, полученная в [?] и обозначенная как "micr.", ("IBM") с параметрами из табл. 2, $\Omega' = 39$. Обозначение $SU(3)$ соответствует кривой, полученной с параметрами, найденными вблизи соответствующего предела МВБ1. Эти параметры дают соответствие с экспериментальными энергиями состояний ирраст-полосы и 2_2^+ -состояния, но различающиеся значения для 0_2^+ -состояния. Для $SU(3)$ -1-варианта эта энергия меньше, чем для варианта $SU(3)$ -2. Potential strain energy ^{246}Cm obtained as "micr.", And also calculated in accordance with ("IBM ") with parameters from table 2, $\Omega' = 39$. Designation $SU(3)$ corresponds to the curve obtained with the parameters found near the corresponding limit of IBM1. These parameters agree with the experimental energies of the yrast band states and the 2_2^+ state, but differing values for the 0_2^+ state. For the $SU(3)$ -1-variant, this energy is less than for the $SU(3)$ - 2 variant.

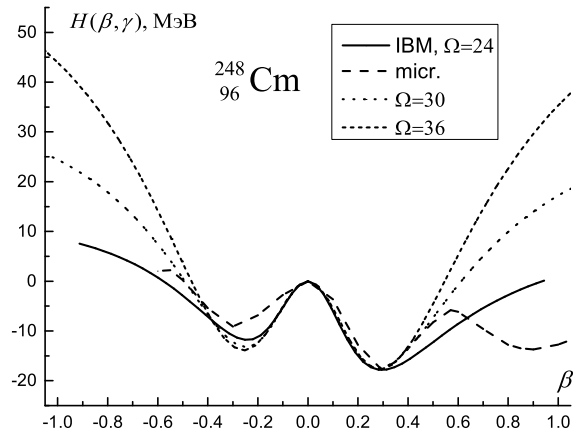


Рис. 8: Потенциальная энергия деформации ^{248}Cm с разными числами бозонов $\Omega = 24, 30, 36$. Соответствующие им значения $\Omega' = 39, 45, 52$. Potential strain energy ^{248}Cm "micr.", And also calculated in accordance with ("IBM ") with parameters from Table 2 with different numbers of bosons $\Omega = 24, 30, 36$. The corresponding values are $\Omega' = 39, 45, 52$.

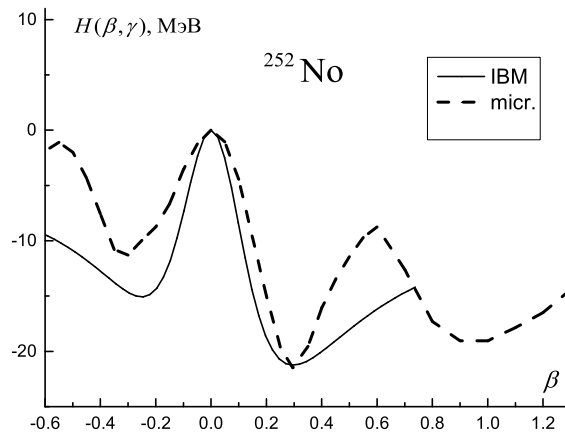


Рис. 9: Потенциальная энергия деформации ^{252}No , обозначенная как "micr.", а также вычисленная в соответствии ("IBM") с параметрами из табл. 5, $\Omega' = 31$. Potential strain energy ^{252}No , denoted as "micr.", And also calculated ("IBM ") with parameters from table 5, $\Omega' = 31$.

3 CONCLUSION

In this work, within the framework of the phenomenology of the IBM1, we analyzed the yrast bands up to the extremely high observed spins in the even isotopes Pu, Cm, Fm, No. In all of them, a very satisfactory description was obtained, and the effect of quasiparticle powers was not found. The only exception is ^{244}Pu , but even for it a good description of the energies was obtained up to spin $I = 22^+$. At the same time, it was possible to reproduce several states of the γ -band at the same time. The ambiguity of the parameters of the Hamiltonian MVB1 was partially minimized due to the fact that the minimum in the potential energy of deformation, determined in the IBM1 using the internal state function, approximately corresponded to the microscopic calculation in the approximation Hartree–Fock–Bogolyubov using the forces of Gonya. In this case, the depth of the well in the potential energy of deformation in these nuclei is very impressive, about 20 MeV. The work uses a phenomenological approach and, as a rule, it is associated with the description of the known energies of collective states.

4 BEHAVIOR OF THE MOMENT OF INERTIA

Discrepancy between experimental and calculated energies is available and it is convenient to identify it through dependency graphs effective moment of inertia from the square of the frequency, namely

$$\frac{2J}{\hbar^2} = \frac{4I - 2}{E(I \rightarrow I - 2)}; \quad (\hbar\omega)^2 = \frac{\left(E(I \rightarrow I - 2)\right)^2}{\left(\left(I(I + 1)\right)^{1/2} - \left((I - 2)(I - 1)\right)^{1/2}\right)^2}. \quad (10)$$

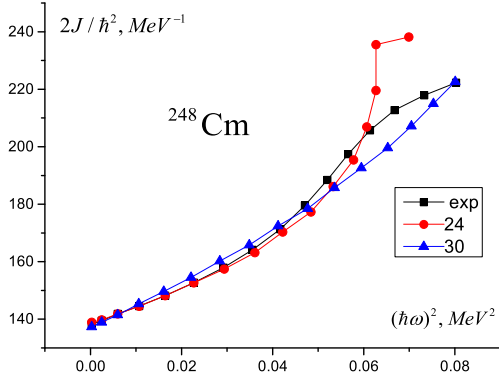


Рис. 10: ^{248}Cm , $I_{\max} = 32^+$.

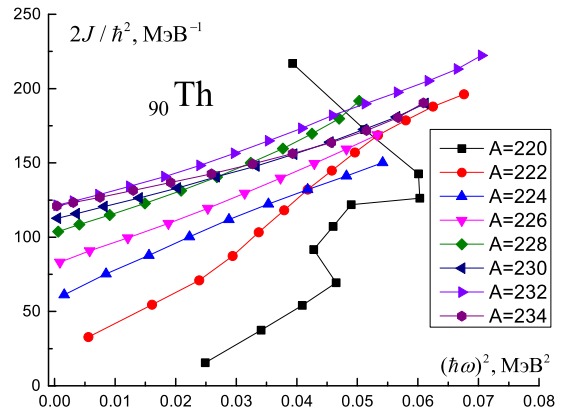


Рис. 11: Th.

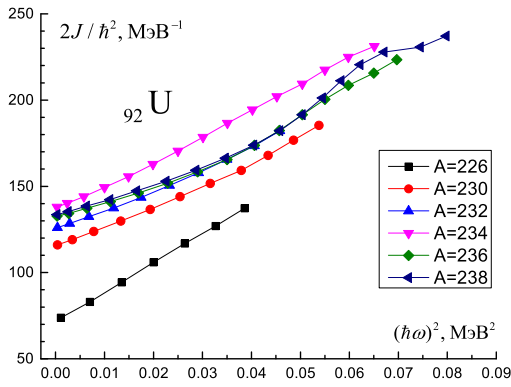


Рис. 12: U .

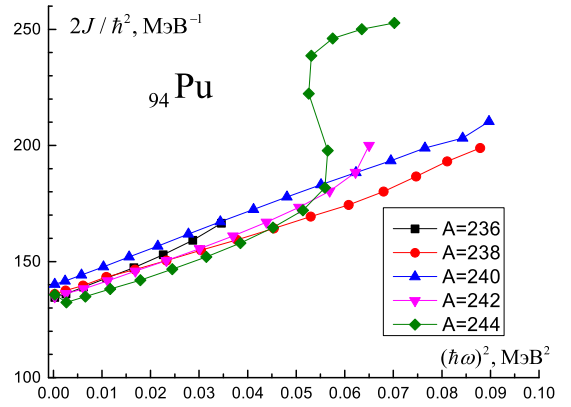


Рис. 13: Pu.

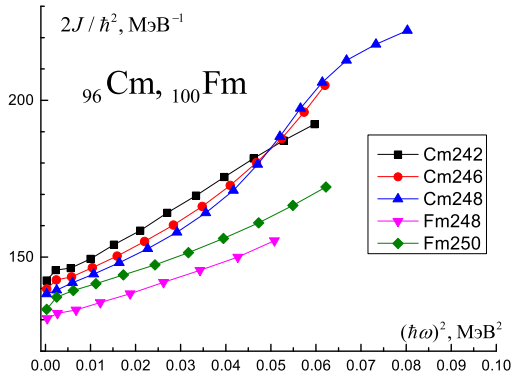


Рис. 14: The same for $^{242,246,248}\text{Cm}$ и $^{248,250}\text{Fm}$.

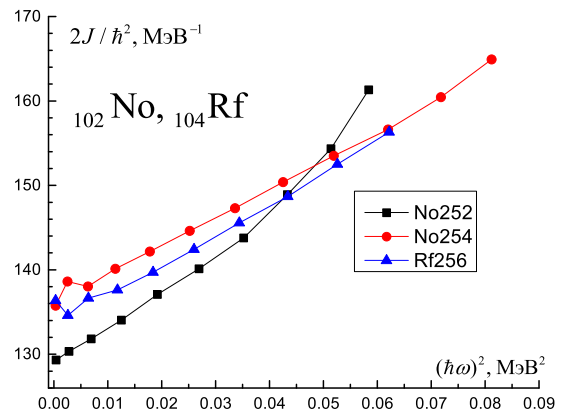


Рис. 15: The same for $^{252,254}\text{No}$ и ^{256}Rf .

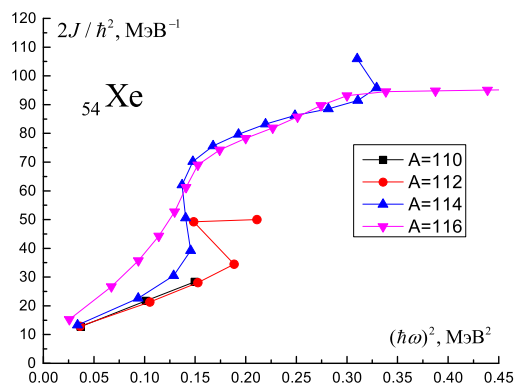


Рис. 16: The same for light even isotopes Xe.

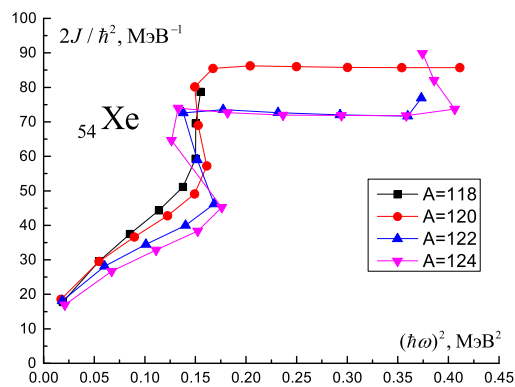


Рис. 17: And for heavy isotopes Xe.

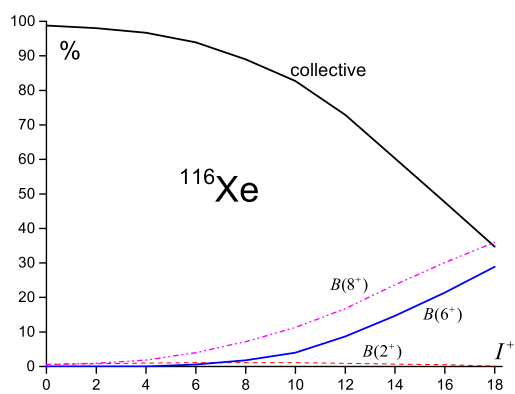


Рис. 18: Boson composition of the yrast band wave functions ^{116}Xe .

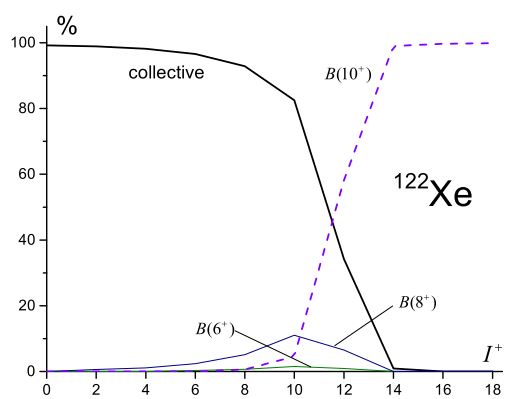


Рис. 19: The same for ^{122}Xe .

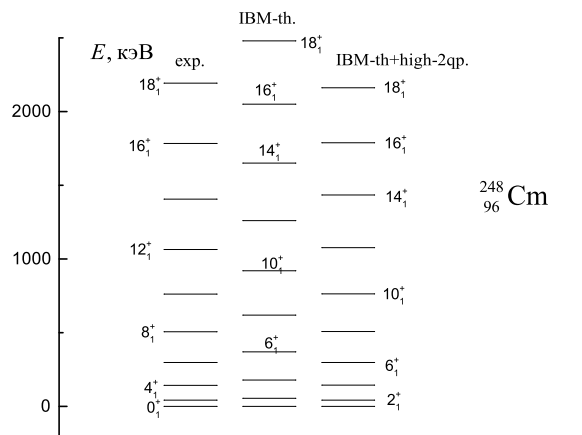


Рис. 20: ^{248}Cm .

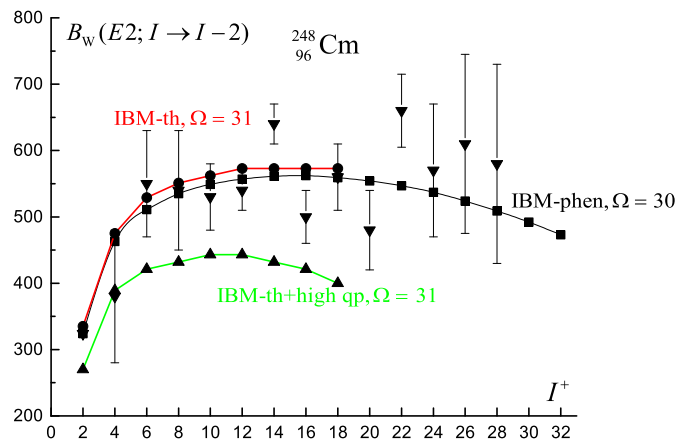


Рис. 21: ^{248}Cm .

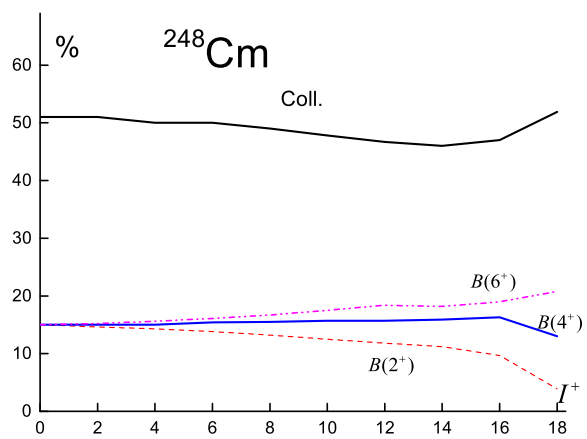


Рис. 22: Phonon compositions for ^{248}Cm .