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Thermal shift of atomic levels in hydrogen: influence on the determination of the proton radius

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Background

- Progress in theoretical/experimental studies of the hydrogen atom has paved the way for the determination of fundamental physical constants: Rydberg constant and proton charge radius.

  The results of theoretical and experimental work are collected in CODATA:

  E. Tiesinga, P.J. Mohr, D.B. Newell, B.N. Taylor, Rev. Mod. Phys. **93**, 025010 (2021)
- Most accurate measurements of transition frequencies in hydrogen:

  $1s - 2s$ - with fractional uncertainty 4.2×10^{-15}

  C.G. Parthey et al., Phys. Rev. Lett. **107**, 203001 (2011); A. Matveev et al., Phys. Rev. Lett. **110**, 230801 (2013)

  $1s - 3s$ - with fractional uncertainty 2.5×10^{-13}

  A. Grinin et al., Science **370**, 1061 (2020)
- To find the physical constants one should write a set of equations with

$$E_{nlj} = R_\infty \left(-\frac{1}{n^2} + f_{nlj}\left(\alpha, \frac{m_e}{m_p}, r_p \dots \right) \right) \qquad (1)$$

$$E_{nlj} - E_{n'l'j'} = \Delta E^{\mathrm{exp}}_{nlj-n'l'j'} \qquad (2)$$

Since the question is about finding two variables (R_∞ and r_p), it is necessary to solve two equations for independent transitions (one of them is $1s - 2s$ or $1s - 3s$).

Proton radius puzzle

Until recently (CODATA - 2012, 2016):

$$R_\infty = 10973731.568508(68)\ \mathrm{m}^{-1} \text{ and } r_p = 0.8759(77)\ \mathrm{fm}$$

Hower, the muonic hydrogen value (the Lamb shift measurement in μH)

$$r_p = 0.84169(66)\ \mathrm{fm}$$

R. Pohl et al., Nature **466**, 213 (2010)

$$r_p = 0.84087(39)\ \mathrm{fm}$$

A. Antognini et al., Science **339**, 417 (2013) (mixing of hyperfine sub-levels)

The problem was called "the proton radius puzzle": eH vs μH - 5.6σ

The problem could not be solved for about 10 years:

in μH very clear experiment ⇒ theory test

in eH clear theory ⇒ experiment test

Modern status: nonresonant corrections

A. Beyer et al., Science **358**, 79 (2017): $2s - 4p$ transition frequency measurements

$$R_\infty = 10973731.568076(96)\ \mathrm{m}^{-1} \text{ and } r_p = 0.8335(95)\ \mathrm{fm}$$

have partially solved the proton radius puzzle (coincides with μH experiment within the uncertainty region) by taking into account nonresonant effects (NR).

Theory: F. Low, Phys. Rev. **88**, 53 (1952)

Prof. L.N. Labzovsky for HCI: J. Phys. B **27**, L439 (1994); Physica Scripta **56**, 271 (1997)

eH: Phys. Rev. Lett. **87**, 140003 (2001)

U. D. Jentschura, P. J. Mohr, Can. J. Phys. **80**, 633 (2002) (quantum interference effect - QIE)

Interfering transitions between the nearest atomic levels lead to an asymmetry of the observed line shape at a level of several tens of kilohertz.

The Fano-Voight profile model was applied to compensate for the QIE, Science **358**, 79 (2017)

The theory is not so unambiguous and strongly depends on the experimental measurement technique

D. Solovyev et al., J. Phys. B: **53**, 125002 (2020) and references therein.

μH experiments are unsensitive to NR corrections

The $r_p = 0.8335(95)$ fm value is the most important step in solving the proton radius puzzle.

Modern status: nonresonant corrections

Up to the Science **358**, 79 (2017) experiment, the QIE was considered theoretically for the one-photon scattering process in eH and μH atoms.

CODATA includes two-photon transitions $2s - nS/nD$ in the eH atom.

The process-dependent NR corrections (QIE as the dominant part of them) were investigated in A. Anikin, T. Zalialiutdinov, D. Solovyev, Phys. Rev. A **103**, 022833 (2021); JETP Letters **114**, 212 (2021), see also J. Phys. B (2021).

The results for the $2s - nD$ two-photon transitions can be presented as

$$\delta_{\rm NR} = -\frac{C_{ab}\Gamma_{nd}^2}{4C_a\Delta_{fs}} + \mathcal{O}\left(\frac{\Gamma_{nd}^4}{\Delta_{fs}^3}\right), \tag{3}$$

where Γ_{nd} is the level width, Δ_{fs} is the fine splitting energy, and the coefficients C_{ab}, C_a are calculated numerically based on thetheory of angular momentum. Numerical results

	Δ_{fs}, Hz	Γ_{nd}, Hz	$\delta_{\rm NR}$, Hz	Exp. unc., Hz
$4d$	4.557026×10^8	4.40503×10^6	-8691.82	$24. \times 10^3$
$6d$	1.350231×10^8	1.33682×10^6	-2701.67	$10. \times 10^3$
$8d$	5.69628×10^7	5.72382×10^5	-1174.02	6.4×10^3
$12d$	1.68779×10^7	1.72261×10^5	-358.88	7.0×10^3

Thermal shift: QED theory

The influence of blackbody radiation has been well known since the early 1980s: T.F. Gallagher, W.E. Cooke, Phys. Rev. Lett. **42**, 835 (1979); J.W. Farley, W.H. Wing, Phys. Rev. A **23**, 2397 (1981) - the QM description for the Stark effect is still used.

QED theory of BBR-induced effects:

D. Solovyev et al., Phys. Rev. A **92**, 022508 (2015); D. Solovyev, Ann. Phys. **415**, 168128 (2020) The Stark shift is the same:

$$\Delta E_a^{\mathrm{Stark}} = \frac{4e^2}{3\pi}\sum_n |\langle a|\vec{r}|n\rangle|^2 \int\limits_0^\infty n_\beta(\omega)\frac{E_{an}\omega^3}{E_{na}^2-\omega^2}d\omega, \qquad (4)$$

$E_{na} = E_n - E_a$, n_β - the Planck distribution function.

There is an additional thermal effect:

$$\Delta E_a^\beta = -\frac{2Ze^2}{3\pi}\int\limits_0^\infty n_\beta(\omega)\omega^2\langle a|r^2|a\rangle \overset{eH}{=} -\frac{4Ze^2\zeta(3)}{3\pi\beta^3}\frac{n_a^2}{2}\left[5n_a^2+1-3l_a(l_a+1)\right]a_0^2. \qquad (5)$$

Stark shift $\sim T^4$ (defomation polarizability), thermal shift $\sim T^3$ (orientation polarizability).

The magnitude reaches several kHz for highly excited states at room temperature.

Thermal and NR corrections: proton radius

Thermal shifts combined with NR (QIE) corrections should be taken into account to determine the proton charge radius.

M. Horbatsch, E.A. Hessels, Phys. Rev. A **93**, 022513 (2016), CODATA - 2021:

$$R_\infty = 10973731.568160(21)\ \mathrm{m}^{-1} \text{ and } r_p = 0.8414(19)\ \mathrm{fm}.$$

The hyperfine structure has not been resolved for two-photon $2s - nS/nD$ transitions.

	r_p^{HH}, fm	R_∞^{HH} m^{-1}	$r_p^{\mathrm{HH}+\beta}$, fm	$R_\infty^{\mathrm{HH}+\beta}$ m^{-1}
$1s-2s, 2s-8d_{3/2}$	0.8412	10973731.568152	0.8403	10973731.568142
$1s-3s,\ 2s-8d_{3/2}$	0.8407	10973731.568149	0.8396	10973731.568139
$1s-2s,\ 2s-8d_{5/2}$	0.8413	10973731.568153	0.8404	10973731.568144
$1s-3s,\ 2s-8d_{5/2}$	0.8408	10973731.568151	0.8398	10973731.568141
$1s-2s, 2s-12d_{3/2}$	0.8413	10973731.568152	0.8343	10973731.568082
$1s-3s,\ 2s-12d_{3/2}$	0.8407	10973731.568150	0.8328	10973731.568075
$1s-2s, 2s-12d_{5/2}$	0.8412	10973731.568151	0.8342	10973731.568081
$1s-3s,\ 2s-12d_{5/2}$	0.8406	10973731.568149	0.8327	10973731.568074
$1s-2s, 2s-4d_{5/2}-\frac{1}{4}(1s-2s)$	0.8398	10973731.568138	0.8403	10973731.568143
$1s-3s, 2s-4d_{5/2}-\frac{1}{4}(1s-2s)$,	0.8398	10973731.568141	0.8403	10973731.568146
$1s-2s, 2s-6d_{5/2}-\frac{1}{4}(1s-3s)$	0.8413	10973731.568153	0.8394	10973731.568134
$1s-3s, 2s-6d_{5/2}-\frac{1}{4}(1s-3s)$	0.8413	10973731.568201	0.8394	10973731.568183
rms$(1s-2s)$	0.8410	10973731.568150	0.8383	10973731.568121
rms$(1s-3s)$	0.8406	10973731.568157	0.8374	10973731.568126
rms	0.8408	10973731.568153	0.8379	10973731.568123

Thermal and NR corrections: proton radius

Proton charge radius, r_p, and rydberg constant, R_∞, using data of M. Horbatsch, E.A. Hessels, Phys. Rev. A **93**, 022513 (2016) and A. Grinin et al., Science **370**, 1061 (2020), thermal effects and NR corrections.

	$r_p^{\mathrm{HH}+\beta}$, fm	$R_\infty^{\mathrm{HH}+\beta}$ m^{-1}	$r_p^{\mathrm{HH}+\beta+\mathrm{NR}}$, fm	$R_\infty^{\mathrm{HH}+\beta+\mathrm{NR}}$ m^{-1}
$1s-2s, 2s-8d_{3/2}$	0.8403	10973731.568142	0.8410	10973731.568150
$1s-3s, 2s-8d_{3/2}$	0.8396	10973731.568139	0.8405	10973731.568147
$1s-2s, 2s-8d_{5/2}$	0.8404	10973731.568144	0.8409	10973731.568149
$1s-3s, 2s-8d_{5/2}$	0.8398	10973731.568141	0.8403	10973731.568146
$1s-2s, 2s-12d_{3/2}$	0.8343	10973731.568082	0.8345	10973731.568084
$1s-3s, 2s-12d_{3/2}$	0.8328	10973731.568075	0.8330	10973731.568077
$1s-2s, 2s-12d_{5/2}$	0.8342	10973731.568081	0.8344	10973731.568082
$1s-3s, 2s-12d_{5/2}$	0.8327	10973731.568074	0.8328	10973731.568075
$1s-2s, 2s-4d_{5/2}-\frac{1}{4}(1s-2s)$	0.8403	10973731.568143	0.8481	10973731.568223
$1s-3s, 2s-4d_{5/2}-\frac{1}{4}(1s-2s)$,	0.8403	10973731.568146	0.8481	10973731.568220
$1s-2s, 2s-6d_{5/2}-\frac{1}{4}(1s-3s)$	0.8394	10973731.568134	0.8414	10973731.568154
$1s-3s, 2s-6d_{5/2}-\frac{1}{4}(1s-3s)$	0.8394	10973731.568183	0.8414	10973731.568202
rms$(1s-2s)$	0.8383	10973731.568121	0.8401	10973731.568140
rms$(1s-3s)$	0.8374	10973731.568126	0.8394	10973731.568145
rms	0.8379	10973731.568123	0.8397	10973731.568142

Conclusions

Proton radius puzzle solved according to CODATA-2021
$r_p = 0.8414(19)$ fm

- μH experiment leads to stable value of r_p
- Most significant contribution arises from the hyperfine splitting in the eH atom
    - $r_p = 0.8759(77)$ fm $\rightarrow r_p = 0.8414(19)$ fm
    - The latter is the same as μH results: $r_p = 0.84169(66), 0.84087(39)$ fm
- Nonresonant corrections in the eH atom: QIE as the most significant part
- Further experimental improvements intersect with the need for an accurate analysis of NR corrections for two-photon transitions
- Thermal environment:
    - The thermal Stark shift is well known and included into the analysis
    - There are other thermal-induced energy shifts as well
- Further experimental improvements intersect with the need for an accurate analysis of the thermal shifts affecting the last digits of the proton charge radius

Thank you for the attention

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