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Z(4)-DDM : A γ-rigid solution of the Bohr Hamiltonian with Davidson potential for β and $\gamma = 30°$

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- The energy spectra
- B(E2) Transition rates

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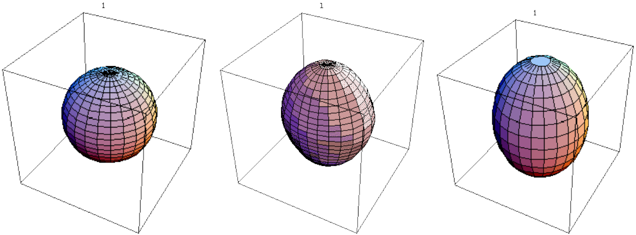
Introduction : Geometrical Collective Model

Geometrical collective (Bohr-Mottelson) Hamiltonian has 5 degrees of freedom, namely the two shape variables β and γ and the three Euler angles :

A. Bohr, and B. R. Mottelson, Mat. Fys. Medd. K. Dan. Vidensk. Selsk. 27 (1953) No. 16.

$$H\Psi(\beta,\gamma,\theta_1,\theta_2,\theta_3) = E\Psi(\beta,\gamma,\theta_1,\theta_2,\theta_3)$$

$$H = -\frac{\hbar^2}{2B}\left[\frac{1}{\beta^4}\frac{\partial}{\partial\beta}\beta^4\frac{\partial}{\partial\beta} + \frac{1}{\beta^2 \sin 3\gamma}\frac{1}{\partial\gamma}\sin 3\gamma\frac{\partial}{\partial\gamma} - \frac{1}{4\beta^2}\sum_{k=1,2,3}\frac{Q_k^2}{\sin^2(\gamma - \frac{2}{3}\pi k)}\right] + V(\beta,\gamma)$$



Introduction : Geometrical Collective Model

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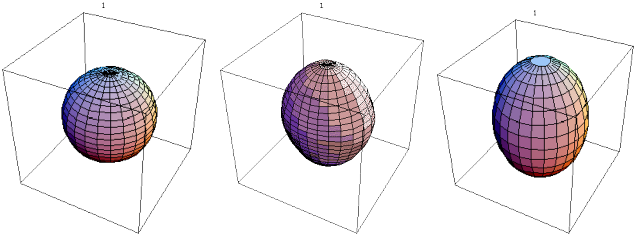

$$H_B = \beta\text{vibration} \quad + \quad \gamma\text{vibration} \quad + \quad \text{rotation} \quad + \quad \text{potential}$$

- Imposing a certain value for the γ shape variable, one reaches the γ-rigid version of the collective model which is interesting by itself due to its description of the basic rotation-vibration coupling.

Davydaov-Chaban model

The γ-rigid Hamiltonian for $\gamma = \pi/6$:

$$H\Psi(\beta, \theta_1, \theta_2, \theta_3) = E\Psi(\beta, \theta_1, \theta_2, \theta_3)$$

$$H = -\frac{\hbar^2}{2B}\left[\frac{1}{\beta^3}\frac{\partial}{\partial\beta}\beta^3\frac{\partial}{\partial\beta} - \frac{1}{4\beta^2}\sum_{k=1,2,3}\frac{Q_k^2}{\sin^2(\gamma - \frac{2}{3}\pi k)}\right] + V(\beta)$$

- **Z(4) :** **A. S. Davydov, and A. A. Chaban, Nucl. Phys. 20 (1960) 499.**
  ⇝ Generally appropriate for non-axial even-even nuclei, which are soft with respect to β vibrations of nuclear surface.

Motivations

- The Davydov-Chaban model with a constant mass is insufficient :
  - Detailed comparisons to experimental data have pointed out that the mass tensor of the collective Hamiltonian cannot be considered as a constant and should be taken as a function of the collective coordinates.
    - ⇝ [R. V. Jolos and P. von Brentano PRC 78, 064309 (2008)/PRC 79, 044310 (2009)]
  - An exactly solvable Hamiltonian emerges for the Davidson and the Kratzer potentials by applying the formalism of DDM, works well for deformed nuclei.
    - ⇝ [D. Bonatsos, P. E. Georgoudis, D. Lenis, N. Minkov, and C. Quesne, PRC 83, 044321 (2011)]
    - ⇝ [D. Bonatsos, P. E. Georgoudis, N. Minkov, D. Petrellis, C. Quesne, PRC 88 034316 (2013)]
    - ⇝ [M. Chabab, A. Lahbas, M. Oulne, PRC 91 064307 (2015)]
- Purpose :
  - Construct a Davydov-Chaban Hamiltonian by allowing the nuclear mass to depend on the deformation.

Plan

Davydov-Chaban Hamiltonian with deformation-dependent effective mass

- Allowing the nuclear mass to depend on the deformation : $B_0 \rightarrow B = \frac{B_0}{f(\beta)^2}$

  $\rightsquigarrow$ **in accordance with the formalism of position dependent effective mass : O. von Roos, Phys. Rev. B 27, 7547 (1983).**
- DC Hamiltonian with DDM $\Leftrightarrow$ to a modified DC hamiltonian with different metric and different effective potentials.

$$\left[-\frac{1}{2}\frac{\sqrt{f}}{\beta^3}\frac{\partial}{\partial\beta}\beta^3 f\frac{\partial}{\partial\beta}\sqrt{f}+\frac{f^2}{4\beta^2}\sum_{k=1,2,3}\frac{Q_k^2}{\sin^2(\gamma-\frac{2}{3}\pi k)}+v_{eff}\right]\psi=\epsilon\psi$$

with $v_{eff}=v(\beta)+\frac{1}{4}(1-\delta-\lambda)f\,\nabla^2 f+\frac{1}{2}\left(\frac{1}{2}-\delta\right)\left(\frac{1}{2}-\lambda\right)(\nabla f)^2$,

$\epsilon=2B_0E/\hbar^2, \quad v=2B_0V/\hbar^2$.

- Considering the total wave function of the form $\psi(\beta,\Omega)=\chi(\beta)\phi(\Omega)$, the associated Schrödinger equation is separated in two parts :

$$\left[-\frac{1}{2}\frac{\sqrt{f}}{\beta^3}\frac{\partial}{\partial\beta}\beta^3 f\frac{\partial}{\partial\beta}\sqrt{f}+\frac{f^2}{2\beta^2}\Lambda+v_{eff}\right]\chi(\beta)=\epsilon\chi(\beta),$$

$$\left[\frac{1}{4}\sum_{k=1,2,3}\frac{Q_k^2}{\sin^2(\gamma-\frac{2}{3}\pi k)}-\Lambda\right]\phi(\Omega)=0.$$

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Solution of angular part

In the case of $\gamma = \pi/6$, the angular momentum term can be written as

$$\sum_{k=1,2,3} \frac{Q_k^2}{\sin^2(\gamma - \frac{2}{3}\pi k)} = 4(Q_1^2 + Q_2^2 + Q_3^2) - 3Q_1^2.$$

angular equation has been solved by Meyer-ter-Vehn [Nuclear Physics A 249 (1975) 111-140], with the results

$$\Lambda = L(L+1) - \frac{3}{4}\alpha^2,$$

$$\phi(\Omega) = \phi^L_{\mu,\alpha}(\Omega) = \sqrt{\frac{2L+1}{16\pi^2(1+\delta_{\alpha,0})}} \left[\mathcal{D}^{(L)}_{\mu,\alpha}(\Omega) + (-1)^L \mathcal{D}^{(L)}_{\mu,-\alpha}(\Omega)\right],$$

we introduce the wobbling quantum number $n_w = L - \alpha$, the eigenvalues of the angular part are written as

$$\Lambda = L(L+1) - \frac{3}{4}(L - n_w)^2,$$

Solution of β part

- We consider the special case of the Davidson potential
  - $v(\beta) = \beta^2 + \frac{\beta_0^4}{\beta^2}$,
- We choose the deformation function in the following special form
  - $f(\beta) = 1 + a\beta^2, \qquad a << 1.$

⇝ Solving the radial equation through Asymptotic Iteration Method (AIM), we obtain :

- the generalized formula of the energy eigenvalues
  $\epsilon_{n_\beta n_w L} = \frac{1}{2}\left[k_0 + \frac{a}{2}(3 + 2p + 2q + pq) + 2a(2 + p + q)n_\beta + 4an_\beta^2\right]$,
- The radial wave function :
  $R_{n_\beta n_w L}(t) =$
  $N_{n_\beta n_w L}\, 2^{-1/2-(q+p)/4}\, a^{-(1+p)/4}\, (1-t)^{(1+q)/4}\, (1+t)^{(1+p)/4}\, P_{n_\beta}^{(p/2,q/2)}(t)$,
- The parametrization used :

$k_2 = 2 + a^2\left[(1-\delta-\lambda) + (1-2\delta)(1-2\lambda) + \frac{7}{4} + \Lambda\right]$	$q = \sqrt{1 + 4\frac{k_2}{a^2}}$
$k_0 = a\left[(1-\delta-\lambda) + \frac{7}{2} + 2\Lambda\right]$	$p = \sqrt{1 + 4k_{-2}}$
$k_{-2} = \Lambda + \frac{3}{4} + 2\beta_0^4$	$t = \frac{-1+a\beta^2}{1+a\beta^2}$
$N_{n_\beta L} = \left(a^{p/2+1} n_\beta!\, q\right)^{\frac{1}{2}} \left[\frac{\Gamma(n_\beta + \frac{q+p}{2} + 1)}{\Gamma(n_\beta + \frac{q}{2} + 1)\Gamma(n_\beta + \frac{p}{2} + 1)}\right]^{\frac{1}{2}}$	

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Numerical results :
The energy spectra

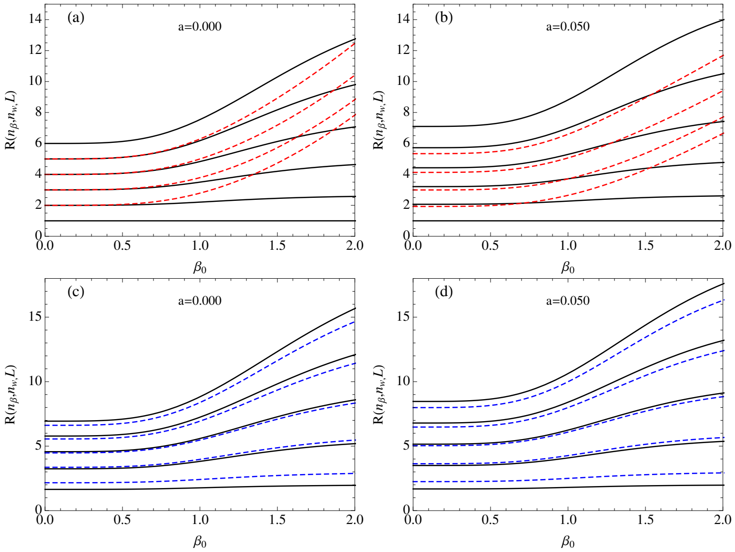
- The energy ratios $R_{n_\beta,n_w,L}$ are defined by : $R_{n_\beta,n_w,L} = \frac{\epsilon_{n_\beta,n_w,L} - \epsilon_{0,0,0}}{\epsilon_{0,0,2} - \epsilon_{0,0,0}}$,
- The bands in the present model are classified by the following quantum numbers :
  - For gsb : $n_\beta = 0$ and $n_w = 0$;
  - For β band : $n_\beta = 1$ and $n_w = 0$;
  - For γ band : $n_\beta = 0$ and $n_w = 2$ for even L levels and $n_\beta = 0$ and $n_w = 1$ for odd L levels.
- We determine the optimal values of the free model's parameters by making use of the quality measure :

$$\sigma = \sqrt{\frac{\sum_{i=1}^{N}(E_i(Exp) - E_i(th))^2}{(N-1)E(2_g^+)}}.$$

- The values of free parameters fitted to the experimental data

nucleus	β_0	a
^{192}Pt	1.32	0.002
^{194}Pt	1.31	0.058
^{196}Pt	1.14	0.122

Numerical results :
The energy spectra



Numerical results : The energy spectra of the $^{192,194,196}Pt$ isotopes

	^{192}Pt				^{194}Pt				^{196}Pt			
	Exp	D	Ref 1	Ref 2	Exp	D	Ref 1	Ref 2	Exp	D	Ref 1	Ref 2
$R_{0,0,4}$	2.479	2.374	2.439	2.396	2.470	2.445	2.415	2.406	2.465	2.362	2.513	2.481
$R_{0,0,6}$	4.314	3.960	3.787	3.834	4.298	4.202	3.835	3.902	4.290	3.968	3.709	3.701
$R_{0,0,8}$	6.377	5.674	5.773	5.761	6.392	6.201	5.880	5.896	6.333	5.770	5.579	5.559
$R_{0,0,10}$	8.624	7.473	7.350	7.484	8.672	8.408	7.573	7.713	8.558	7.752	6.914	6.932
$R_{1,0,0}$	3.776	3.714	3.397	3.537	3.858	3.666	3.706	3.809	3.192	2.970	2.954	2.977
$R_{1,0,2}$	4.547	4.726	4.995	5.162	4.603	4.730	5.409	5.493	3.828	4.047	4.308	4.364
$R_{1,0,4}$		6.118	7.002	7.113		7.511	6.265	7.490		5.511	6.238	6.280
$R_{0,2,2}$	1.935	1.857	1.653	1.664	1.894	1.892	1.661	1.676	1.936	1.848	1.646	1.643
$R_{0,1,3}$	2.910	2.620	2.302	2.345	2.809	2.711	2.332	2.378	2.852	2.608	2.249	2.252
$R_{0,2,4}$	3.795	4.366	4.229	4.200	3.743	4.667	4.268	4.273	3.636	4.388	4.179	4.150
$R_{0,1,5}$	4.682	4.563	4.342	4.360	4.563	4.894	4.402	4.446	4.526	4.593	4.243	4.227
$R_{0,2,6}$	5.905	6.686	6.358	6.466		7.430	6.524	6.645	5.644	6.874	6.041	6.049
$R_{0,1,7}$	6.677	6.523	6.065	6.215		7.230	6.235	6.392		6.694	5.737	5.754
$R_{0,2,8}$	8.186	8.925	9.163	9.203		10.269	-	9.508	7.730	9.424	8.564	8.573
rms		0.526	0.614	0.593		0.338	0.543	0.515		0.550	0.682	0.683

- Ref1 : P. Buganu, R. Budaca, PRC 91 (2015) 014306 ⇝ Z(4)-Sextic for k=5 .
- Ref2 : R. Budaca, P. Buganu, M. Chabab, A. Lahbas, M. Oulne, Ann Phys 375 (2016) 65. ⇝ Z(4)-Sextic for k=20

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- Ref2 : R. Budaca, P. Buganu, M. Chabab, A. Lahbas, M. Oulne, Ann Phys 375 (2016) 65. ⇝ Z(4)-Sextic for k=20

B(E2) Transition rates

- The $B(E2)$ transition rates are given by

$$B(E2; L_i\alpha_i \to L_f\alpha_f) = \frac{5}{16\pi} \frac{\left| \left\langle L_f\alpha_f \mid\mid T^{(E2)} \mid\mid L_i\alpha_i \right\rangle \right|^2}{(2L_i+1)}$$

- The quadrupole operator for triaxial nuclei around $\gamma = \pi/6$ is given by

$$T_M^{(E2)} = t\beta \left[\mathcal{D}^{(2)}_{M,0}(\theta_i)\cos(\gamma - \frac{2\pi}{3}) + \frac{1}{\sqrt{2}}(\mathcal{D}^{(2)}_{M,2}(\theta_i) + \mathcal{D}^{(-2)}_{M,0}(\theta_i))\sin(\gamma - \frac{2\pi}{3}) \right]$$

- The general expression for E2 transition probabilities is

$$B(E2; L_i\alpha_i \to L_f\alpha_f) = \frac{5}{16\pi}\frac{t^2}{2}\frac{1}{(1+\delta_{\alpha_i,0})(1+\delta_{\alpha_f,0})}[(L_i2L_f|\alpha_i2\alpha_f)$$

$$+(L_i2L_f|\alpha_i - 2\alpha_f) + (-1)^{L_f}(L_i2L_f|\alpha_i - 2 - \alpha_f)]^2 \times [I_\beta(n_i, L_i, \alpha_i, n_f, L_f, \alpha_f)]^2$$

$$I_\beta(n_i, L_i, \alpha_i, n_f, L_f, \alpha_f) = \int_0^\infty \beta\xi_{n_i,L_i,\alpha_i}(\beta)\xi_{n_f,L_f,\alpha_f}(\beta)\beta^3 d\beta$$

Numerical results : B(E2) transition rates of the ^{192,194,196}Pt isotopes

nuleus	$\frac{4g\to 2g}{2g\to 0g}$	$\frac{6g\to 4g}{2g\to 0g}$	$\frac{8g\to 6g}{2g\to 0g}$	$\frac{10g\to 8g}{2g\to 0g}$	$\frac{2\gamma\to 2g}{2_1\to 0g}$	$\frac{2\gamma\to 0g}{2g\to 0g}$ $\times 10^3$	$\frac{0\beta\to 2g}{2g\to 0g}$	$\frac{2\beta\to 0g}{2g\to 0g}$ $\times 10^3$	rms
^{192}Pt	1.56(12)	1.23(55)			1.91(16)	9.5(9)			
	1.60	2.34	3.02	3.69	1.63	0.0	0.80	15.12	**0.2851**
^{194}Pt	1.73(13)	1.36(45)	1.02(30)	0.69	1.81(25)	5.9(9)	0.01		
	1.60	2.34	3.01	3.65	1.62	0.0	1.15	52.33	**0.5034**
^{196}Pt	1.48(3)	1.80(23)	1.92(23)			0.4	0.07(4)	0.06(6)	
	1.68	2.54	3.33	4.10	1.70	0.0	1.46	45.70	**0.3531**

Plan

Conclusion

- A new solution for the Davydov-Chaban Hamiltonian within the DDM for the Davidson potential is proposed, called Z(4)-DDM Davidson.
- This work is achieved through the use of Iteration Asymptotic Method IAM, exact analytical expressions for the spectra and wave functions have been obtained ;
- The numerical realization of this model consisted of calculating energy spectra and transition probabilities of ^{192,194,196}Pt isotopes using Davidson as collective potential compared to experimental data and some models calculations.
- The predicted energy spectra are in good agreement with the experimental data for the studied nuclei.

Thank You!

Open discussion