LXXI International conference «NUCLEUS-2021. Nuclear physics and elementary particle physics. Nuclear physics technology»
Saint-Petersburg
20-25 September 2021

Binding energies of light hypernuclei

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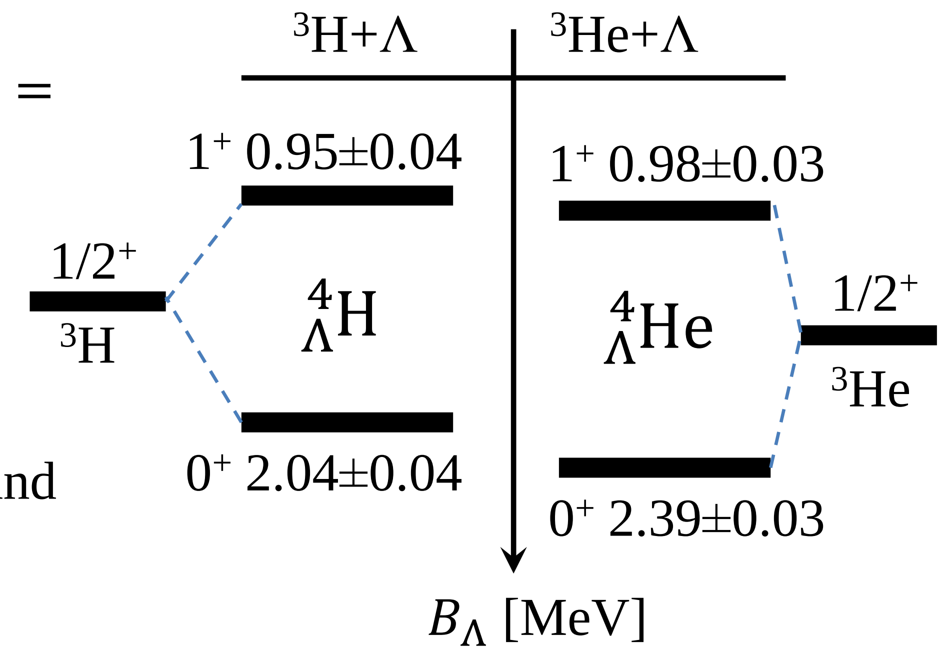
Light hypernuclei are natural laboratory for testing NN & YN interactions, and conversion $YN \to Y'N'$

- Binding energies E_b
- Hyperon separation energy

$B_{\mathrm{Y}}(A) = M(A) - M(\mathrm{Y}) - M(A - 1_Y) =$
$= |E_b(A - 1_Y)| - |E_b(A)|$

- Charge symmetry breaking (CSB)

$B_\Lambda({}^4_\Lambda\mathrm{H}) - B_\Lambda({}^4_\Lambda\mathrm{He}) = -0.35\ (0^+)$



The simplest hypernucleus ${}^3_\Lambda\mathrm{H}$ is bound and has no excited states.

	DATA	PRC48(1993)2576	PRC51(1995)2905	SC89+Bonn B (PRL88(2002)172501)	SC89+Nijm'93(PRL88(2002)172501)	Jülich'04(NPA914(2013)140)	PRC65(2001)11301	PRC65(2001)01301 w/o $(NNN)_{3/2}\Sigma$
$B_\Lambda({}^3_\Lambda\mathrm{H})$	0.13±0.05	unbound	0.13	0.155	0.143	0.13	-	-
$B_\Lambda({}^4_\Lambda\mathrm{H})$	2.04±0.04	-	-	-	1.8	1.8	2.33	2.19

Solving the 3-body problem using Faddeev's integral equations

Local potential $V(\vec{p}',\vec{p})$

$$V(\vec{p}',\vec{p})=\sum_{JM,L_fL_iM_{L_f}M_{L_i}} \mathrm{y}^{*JM}_{L_fS_f}(\hat{p}')V_{L_fL_i}(p',p)\mathrm{y}^{JM}_{L_iS_i}(\hat{p})$$

Separable potential

$$V_{L_fL_i}(p',p) = \sum_{i,j=1,\mathrm{range}} \chi_i^{L_f}(p')\lambda_{ij}\chi_j^{L_i}(p$$

We are looking for a solution of homogeneous Faddeev's equation for transition amplitudes

$X^L_{\beta\alpha}(p',p,E) = \langle p',\beta,\chi|R_0U^LR_0|p,\alpha,\chi\rangle$

U^L – component of transition operator

$$U = PR_0^{-1} + PT$$

R_0 – free resolvent in 3-body phase space

P – cyclic permutation operator

$$P = P_{12}P_{23} + P_{13}P_{23}$$

After projection 2d-equation is transformed into an infinite system of coupled 1d-equations.

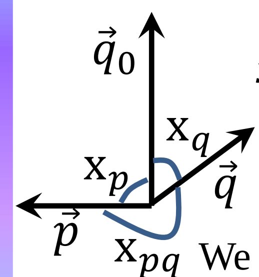
Reasonable simplification

The action of the form factors χ only in limited number of partial waves

Direct integration of 3d Faddeev equation

$$T(\vec{p},\vec{q};E,q_0) \equiv T\left(p,x_p,q,x_q,x_{pq};E,q_0\right)$$

5 – independent variables



$$x_{pq} = \sqrt{1-x_q^2}\sqrt{1-x_p^2}\cos(\phi_q-\phi_p)$$

Integration over azimuth angle in integral kernels

We are looking for a solution of homogeneous Faddeev's equation for T –matrix $T = tP + tPR_0T$;

$$\langle\vec{p},\vec{q}|tP|\Psi\rangle = \int d^3q'd^3p'd^3q''d^3p''\langle\vec{p},\vec{q}|t|\vec{p}',\vec{q}'\rangle\cdot$$

$$\cdot\langle\vec{p}',\vec{q}'|P|\vec{p}'',\vec{q}''\rangle\langle\vec{p}''\vec{q}''|\Psi\rangle;$$

$$\langle\vec{p},\vec{q}|t|\vec{p}',\vec{q}'\rangle = \delta^3(\vec{q}-\vec{q}')t(\vec{p},\vec{p}',E)$$

$$\langle\vec{p}',\vec{q}'|P|\vec{p}'',\vec{q}''\rangle = \langle\vec{p}',\vec{q}'|\vec{p}'',\vec{q}''\rangle + \langle\vec{p}',\vec{q}'|\vec{p}'',\vec{q}''\rangle;$$

$$\vec{q}_1',\vec{p}_1' = \begin{cases} f(\vec{q}_2,\vec{p}_2); \\ f(\vec{q}_3,\vec{p}_3). \end{cases} \vec{q}_2',\vec{p}_2' = \begin{cases} f(\vec{q}_3,\vec{p}_3); \\ f(\vec{q}_1,\vec{p}_1). \end{cases} \vec{q}_3',\vec{p}_3' = \begin{cases} f(\vec{q}_1,\vec{p}_1); \\ f(\vec{q}_2,\vec{p}_2). \end{cases}$$

More about permutation operator is in FBSY27(1999)83.

Setting the kinetic energy E in the cycle, we find the positions of the matrix equation roots

$$\det(1-PtR_0)\Big|_{E=E_b} = 0$$

2-Body ingredients

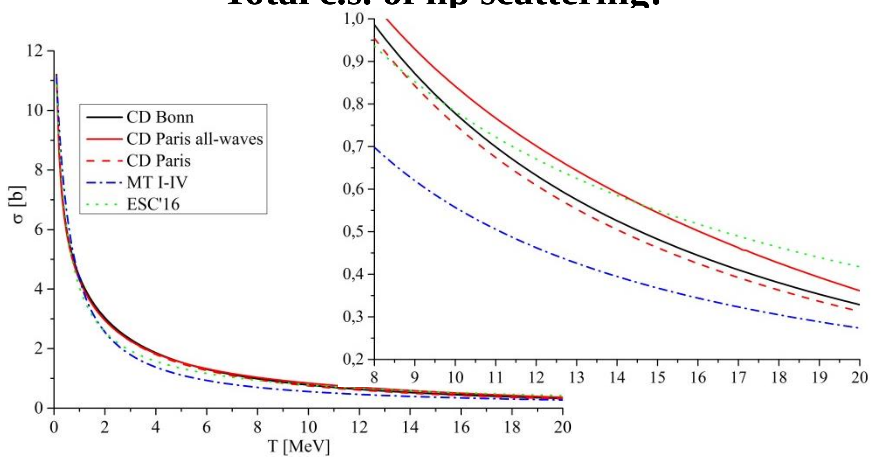
$$V_{NN} = \begin{cases} \text{CD Bonn}, {}^1S_0, {}^3S_1 - {}^3D_1, \\ \text{CD Paris}, {}^1S_0, {}^3S_1 - {}^3D_1, \\ \text{CD Paris}, \quad \text{up to } {}^3F_2, \\ \text{MTI} - \text{IV} \\ \text{ESC'16 (www.nn} - \text{online.org)} \end{cases}$$

$$V_{YN} = \begin{cases} \text{separable phase equivalent}, & \text{I} \\ \text{ESC'16 (www.nn} - \text{online.org)}, & \text{II} \end{cases}$$

Necessary formulas for potentials are in nucl-th:1409.5593v1

Only central and spin-spin parts are used in calc.

Total c.s. of np scattering:



$$\begin{cases} t_{\Lambda\Lambda} = V_{\Lambda\Lambda} + V_{\Lambda\Lambda} G_\Lambda t_{\Lambda\Lambda} + 2V_{\Lambda\Sigma} G_\Sigma t_{\Sigma\Lambda}; \\ t_{\Sigma\Lambda} = V_{\Sigma\Lambda} + V_{\Sigma\Lambda} G_\Lambda t_{\Lambda\Lambda} + 2V_{\Sigma\Sigma} G_\Sigma t_{\Sigma\Lambda}; \\ t_{\Sigma\Sigma} = V_{\Sigma\Sigma} + V_{\Sigma\Lambda} G_\Lambda t_{\Lambda\Sigma} + 2V_{\Sigma\Sigma} G_\Sigma t_{\Sigma\Sigma}. \end{cases}$$

$$V_{L_f L_i}(p', p) = \sum_{i,j=1,\text{range}} \chi_i^{L_f}(p') \lambda_{ij} \chi_j^{L_i}(p)$$

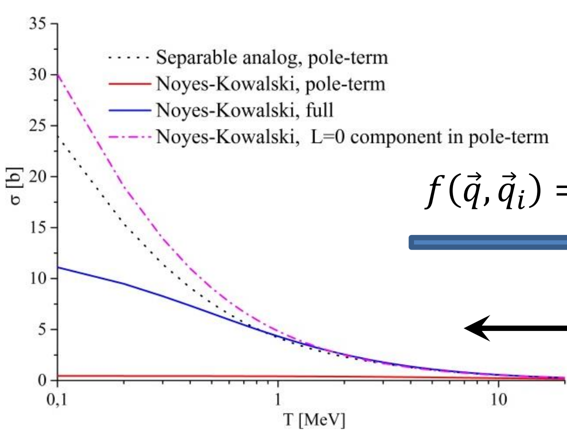
Exact solution

$$V(\vec{p}', \vec{p})$$

Generalization of the Noyes--Kowalski procedure

$$T(\vec{q}_f, \vec{q}_i) = \frac{f(\vec{q}_f, \vec{q}_i) V(\vec{q}_i, \vec{q}_i)}{1 - \frac{1}{(2\pi)^3} \int_0^\infty q''^2 dq'' \int_{-1}^1 dx_{q''} \int_0^{2\pi} d\phi_{q''} \frac{V(\vec{q}_f, \vec{q}'') f(\vec{q}'', \vec{q}_i)}{E(q_i) - E(q'') + i\varepsilon}}$$

$$f(\vec{q}, \vec{q}_i) = \frac{V(\vec{q}, \vec{q}_i)}{V(\vec{q}_i, \vec{q}_i)} + \frac{1}{(2\pi)^3} \int_0^\infty q'^2 dq' \int_{-1}^1 dx_{q'} \int_0^{2\pi} d\phi_{q'} \left[V(\vec{q}, \vec{q}') - \frac{V(\vec{q}, \vec{q}_i)}{V(\vec{q}_i, \vec{q}_i)} V(\vec{q}_i, \vec{q}') \right] \frac{f(\vec{q}', \vec{q}_i)}{E(q_i) - E(q') + i\varepsilon}$$



Developed a method for 2-body t-matrix with local potentials

For local MT I-IV potential.

For separable potentials procedure gives an exact solution.

3-Body T-matrix

$$\begin{cases}\langle\vec{p},\vec{q}|T_1|\phi_{23}\phi_1\rangle = \langle\vec{p},\vec{q}|t_1|\phi_{31}\phi_2\rangle + \langle\vec{p},\vec{q}|t_1\ |\phi_{12}\phi_3\rangle + \langle\vec{p},\vec{q}|t_1R_0T_2|\phi_{31}\phi_2\rangle + \langle\vec{p},\vec{q}|t_1R_0T_3|\phi_{12}\phi_3\rangle \\ \langle\vec{p},\vec{q}|T_2|\phi_{31}\phi_2\rangle = \langle\vec{p},\vec{q}|t_2|\phi_{23}\phi_1\rangle + \langle\vec{p},\vec{q}|t_2\ |\phi_{12}\phi_3\rangle + \langle\vec{p},\vec{q}|t_2R_0T_1|\phi_{23}\phi_1\rangle + \langle\vec{p},\vec{q}|t_2R_0T_3|\phi_{12}\phi_3\rangle \\ \langle\vec{p},\vec{q}|T_3|\phi_{12}\phi_3\rangle = \langle\vec{p},\vec{q}|t_3|\phi_{23}\phi_1\rangle + \langle\vec{p},\vec{q}|t_3\ |\phi_{31}\phi_2\rangle + \langle\vec{p},\vec{q}|t_3R_0T_1|\phi_{23}\phi_1\rangle + \langle\vec{p},\vec{q}|t_3R_0T_2|\phi_{31}\phi_2\rangle\end{cases}$$

Homogeneous equation:

$$\det\left[\begin{pmatrix}1_{N\times N} & 0 & 0\\ 0 & 1_{N\times N} & 0\\ 0 & 0 & 1_{N\times N}\end{pmatrix} - \begin{pmatrix}0 & a_{12} & a_{13}\\ a_{21} & 0 & a_{23}\\ a_{31} & a_{32} & 0\end{pmatrix}\right]_{E=|E_b|} = 0$$

$$a_{12} = \int d^3q''\left\langle\vec{p},\vec{q}\middle|t_1\middle|-\vec{q}\frac{m_1}{m_1+m_3}-\vec{q}'',\vec{q}\right\rangle\left(-E-\frac{\vec{q}^2}{2\mu_{23}}-\frac{\vec{q}''^2}{2\mu_{13}}-\frac{qq''x_{q''}}{m_3}\right)^{-1}$$

$$a_{13} = \int d^3q''\left\langle\vec{p},\vec{q}\middle|t_1\middle|\vec{q}\frac{m_1}{m_1+m_2}+\vec{q}'',\vec{q}\right\rangle\left(-E-\frac{\vec{q}^2}{2\mu_{23}}-\frac{\vec{q}''^2}{2\mu_{12}}-\frac{qq''x_{q''}}{m_2}\right)^{-1}$$

$$a_{21} = \int d^3q''\left\langle\vec{p},\vec{q}\middle|t_2\middle|\vec{q}\frac{m_2}{m_3+m_2}+\vec{q}'',\vec{q}\right\rangle\left(-E-\frac{\vec{q}^2}{2\mu_{31}}-\frac{\vec{q}''^2}{2\mu_{23}}-\frac{qq''x_{q''}}{m_3}\right)^{-1}$$

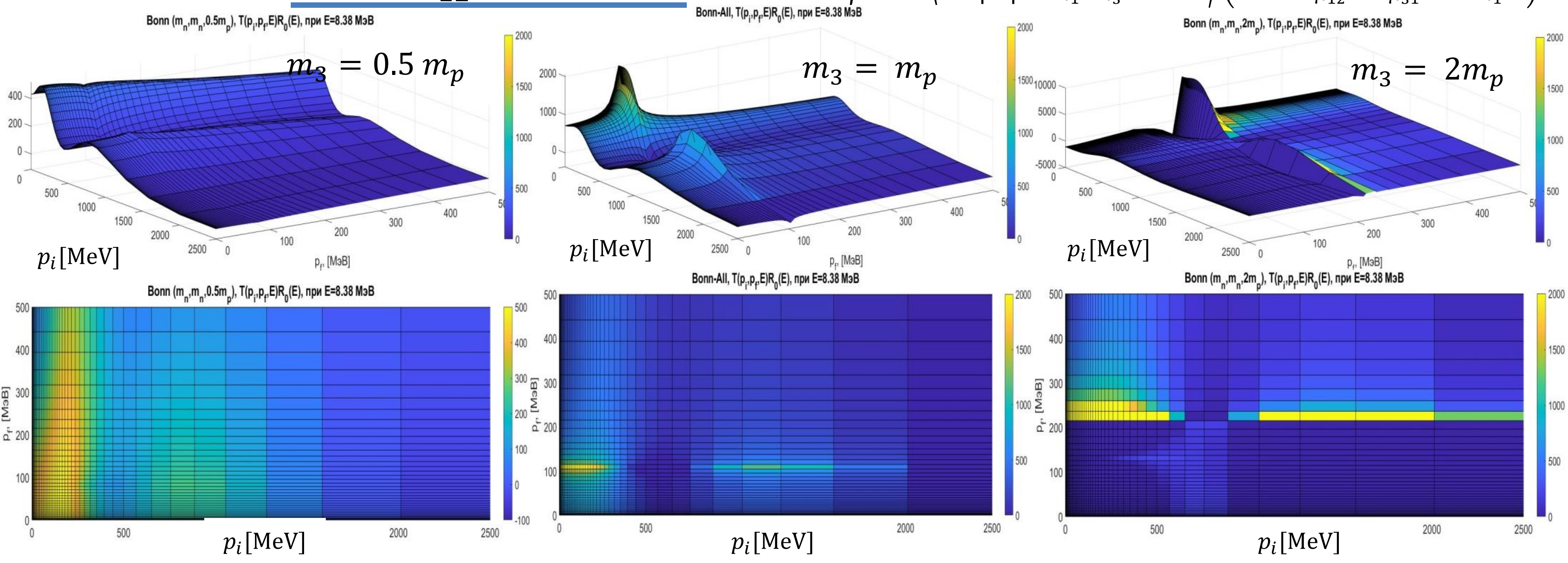
$$a_{23} = \int d^3q''\left\langle\vec{p},\vec{q}\middle|t_2\middle|\vec{q}\frac{m_1}{m_1+m_2}+\vec{q}'',\vec{q}\right\rangle\left(-E-\frac{\vec{q}^2}{2\mu_{31}}-\frac{\vec{q}''^2}{2\mu_{12}}-\frac{qq''x_{q''}}{m_1}\right)^{-1}$$

$$a_{31} = \int d^3q''\left\langle\vec{p},\vec{q}\middle|t_3\middle|\vec{q}\frac{m_2}{m_3+m_2}+\vec{q}'',\vec{q}\right\rangle\left(-E-\frac{\vec{q}^2}{2\mu_{12}}-\frac{\vec{q}''^2}{2\mu_{23}}-\frac{qq''x_{q''}}{m_2}\right)^{-1}$$

$$a_{32} = \int d^3q''\left\langle\vec{p},\vec{q}\middle|t_3\middle|-\vec{q}\frac{m_1}{m_1+m_3}-\vec{q}'',\vec{q}\right\rangle\left(-E-\frac{\vec{q}^2}{2\mu_{12}}-\frac{\vec{q}''^2}{2\mu_{31}}-\frac{qq''x_{q''}}{m_1}\right)^{-1}$$

✓ In correspondence with the solution of one homogeneous equation [PRC72(2005)054003]
At $V_{NN} = V_{np}$, and $m_1 = m_2 = m_3 \equiv m_N$.
For V_{np} (CD Bonn), calculation gives $E_b \approx -8.38$ МэВ.

Quantities a_{12} at E=8.38 MeV:



Binding energies: ^{3}H, ^{3}He, $^3_\Lambda$H, $^4_\Lambda$H

NN potentials + YN – I, ⧗ – in process..

See details NPA1009(2021)122172:

Nucleus	CD Bonn ($^1S_0$, $^3S_1$-$^3D_1$)	CD Paris (all waves)	ESC'16	MTI-IV	DATA
^{3}H	8.38	8.37	8.21	7.9	8.481
^{3}He	~7.7	7.66	7.78	7.6	7.718
$^3_\Lambda$H,(V_{YN}-I)	2.1	2.1	⧗	⧗	2.094
$^4_\Lambda$H,(V_{YN}-I)	6.3	6.3	-	-	6.441

Only separable calculation

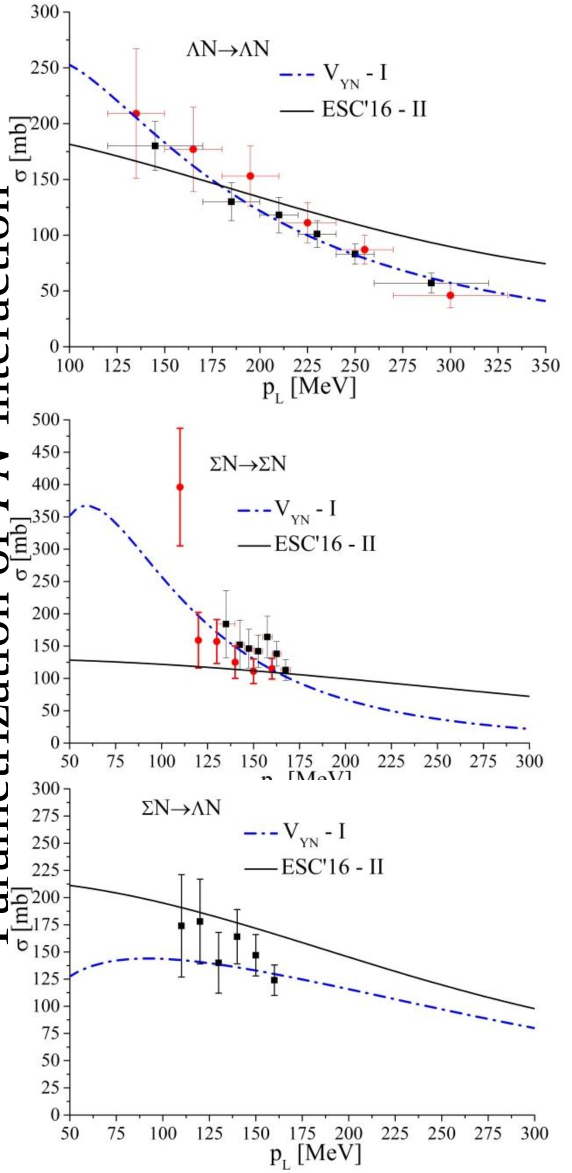
$$B_\Lambda(A) = M(A) - M(\Lambda) - M(A-1_\Lambda) = |E_b(A-1_\Lambda)| - |E_b(A)|$$

1. Calculations with partial-wave expansion with separable potentials agree well with calculations without that expansion
2. One can neglects the influence of higher partial waves beginning form $^3P_0$-uncoupled on the binding energy.

Variation of YN models at V_{NN} (CD Bonn) for separation energy B_Λ:

	Present work	ESC'16	DATA
$B_\Lambda(^3_\Lambda\text{H})$	0.13	0.09	0.13±0.05
$B_\Lambda(^4_\Lambda\text{H})$	2.08	⧗	2.04±0.04

Parametrization of YN interaction



Discussion and plans

- Work with local potentials in phase space is more promising, but time consuming.

3b-phase space $(3 \times 45, 3 \times 45)$

We are looking for determinant of this matrix

2-body t-matrices (45,7)

Radial mesh points, angular mesh points

1 det. ⇒ few days

3b-phase space $(3 \times 25, 3 \times 25)$

2-body t-matrices (45,7)

4d spline interpolation (15,7)→(45,7)

FBSY22(1997)107

1 det. ⇒ ~1 hour!!

- It has been presented and tested a method for calculating the binding energies (& separation energies) of light hypernuclei with different potentials.
- Work continues on calculating the binding energies of hyperons in light hypernuclei.
- Work has begun on calculating the binding energies of kaons in light kaonic nuclei.

Is it possible to transform the ESC potentials into separable form?

It is also interesting to calculate the kaon-hyperon-nucleon 3-body interaction.

Thank you!

The reported study was funded by RFBR, Project No. 20-02-00004A.