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Direct calculation of quasi-stationary states of the neutron plus nonspherical nucleus system

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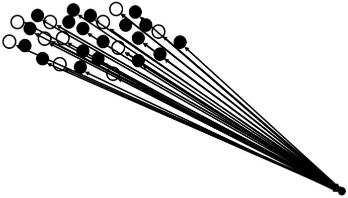
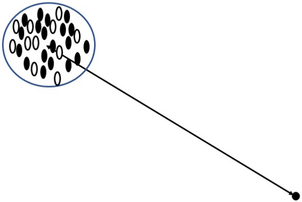
The today's statement

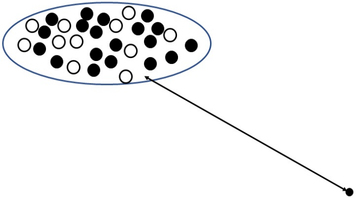




It's real situation It's usual calculation*

For calculation of scattering at non-spherical nuclei some folding models have been used.

This work is way to include some aspect of different forms of nuclei



* *Some authors use different methods which include non-spherical form (A.S. Umar, Sekizawa, A.V. Karpov).*

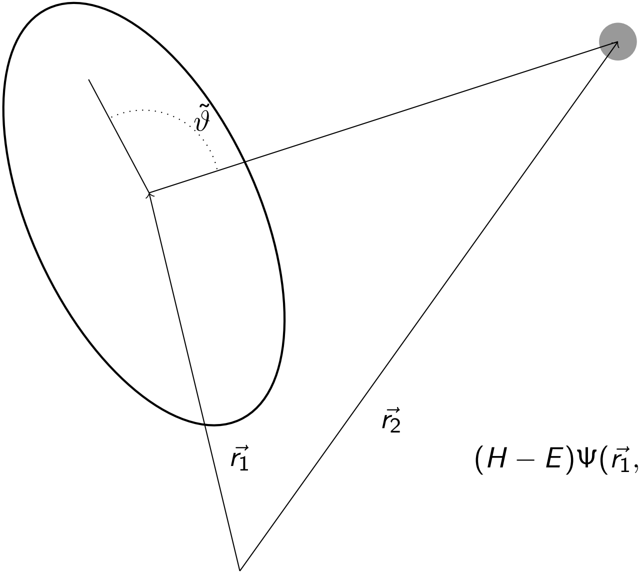
In the next works **the particle-rotor model of the nucleus** has been presented

- V. P. Bugrov and S. G. Kadmensky, Sov. J. Nucl. Phys. 49, 967 (1989);
- S. G. Kadmensky and V. P. Bugrov, Phys. At. Nucl. 59, 399 (1996);

This work is alternative way for description the property of nuclei.

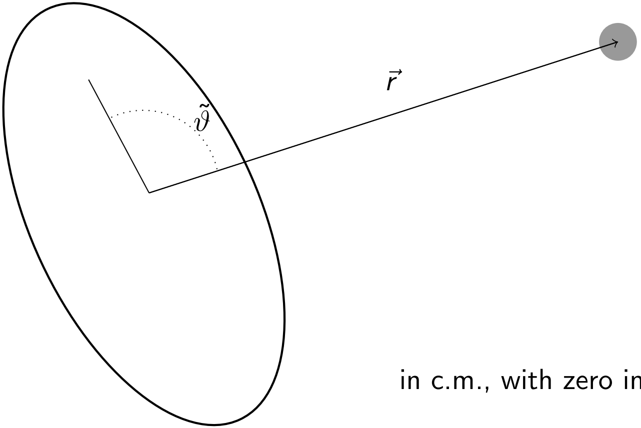
Interaction

of the point particle with axial symmetrical object



$$(H - E)\Psi(\vec{r_1}, \vec{r_2}, \tilde{\vartheta}) = 0$$

Center-of-momentum frame



in c.m., with zero impact parameter

$$(H - E)\Psi(\vec{r}; \tilde{\vartheta}) = 0$$

Theory description

The Schrödinger equation

$$\Delta\Psi(r,\vartheta,\varphi) - V(r,\vartheta)\Psi(r,\vartheta,\varphi) = -k^2\Psi(r,\vartheta,\varphi),$$
$$\Delta = \frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{1}{\sin(\vartheta)}\frac{\partial}{\partial\vartheta}\sin(\vartheta)\frac{\partial}{\partial\vartheta} + \frac{1}{r^2\sin^2(\vartheta)}\frac{\partial^2}{\partial\varphi^2}$$

Theory description

The Schrödinger equation for axial-symmetrical field

$$\Delta\Psi(r,\vartheta,\varphi) - V(r,\vartheta)\Psi(r,\vartheta,\varphi) = -k^2\Psi(r,\vartheta,\varphi),$$

$$\Delta = \frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{1}{\sin(\vartheta)}\frac{\partial}{\partial\vartheta}\sin(\vartheta)\frac{\partial}{\partial\vartheta} + \frac{1}{r^2\sin^2(\vartheta)}\frac{\partial^2}{\partial\varphi^2}$$

can be written in form

$$\Delta_m\Psi_m(r,\vartheta) - V(r,\vartheta)\Psi_m(r,\vartheta) = -k^2\Psi_m(r,\vartheta),$$

$$m = 0, \pm 1, \pm 2, \ldots$$

$$\Delta_m = \frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{1}{\sin(\vartheta)}\frac{\partial}{\partial\vartheta}\sin(\vartheta)\frac{\partial}{\partial\vartheta} - \frac{m^2}{r^2\sin^2(\vartheta)},$$

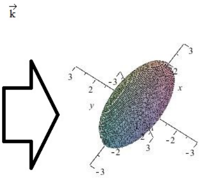
where wave function $\Psi(r,\vartheta,\varphi) = \sum_m \Psi_m(r,\vartheta)e^{im\varphi}$. Here, $k^2 = 2mE$ is the wave number, E is the energy of the system. The potential is restricted within a certain domain: $V = 0$ at $r > r_V$.

Theory description

For scattering problem

with incoming wave haves random wive number $\vec{k}$

$$\Psi(r,\vartheta,\varphi) = e^{i\vec{k}\vec{r}} + \Phi(r,\vartheta,\varphi)$$
$$= e^{ikr\cos\tilde{\vartheta}} + \Phi(r,\vartheta,\varphi),$$



and the equations for components of $\Phi(r,\vartheta,\varphi) = \sum_m \Phi(r,\vartheta)e^{im\varphi}$ is

$$\Delta_m \chi_m(r,\vartheta) - V(r,\vartheta)\Phi_m(r,\vartheta) =$$
$$-k^2\Phi_m(r,\vartheta) + V(r,\vartheta)F_m(\vec{k},\vec{r}),$$

where

$$F_m(\vec{k},\vec{r}) = \int_0^{2\pi} e^{ikr\cos\tilde{\vartheta}} e^{im\varphi} d\varphi.$$

F_m-function

The axial symmetry can be used here.
When $\vec{k}$ is in plane XOZ and $\vec{k}$ has angel ϑ' with direction OZ, so scalar product $(\vec{k}\vec{e_z}) = \cos\vartheta'$, it can be showed that

$$F_m = i^m e^{ikr\cos\vartheta\cos\vartheta'} J_m(kr\sin\vartheta\sin\vartheta'),$$

where $J_m(x)$ is the Bessel function.

It is analog of free wave's decomposition into the spherical functions for the spherical symmetrical problem.

The boundary conditions

For $r = 0$

the standard substitution has been used.

$$\Psi(r,\vartheta,\varphi) = \frac{\psi(r,\vartheta,\varphi)}{r}, \psi(0,\vartheta,\varphi) = 0 \Rightarrow$$
$$\Rightarrow \Phi_m(r,\vartheta) = \chi_m(r,\vartheta)/r, \chi_m(0,\vartheta) = 0.$$

For $\vartheta = 0, \pi$

the property of (un)parity has been used

$$F_m(\vartheta,\ldots) = (-1)^m F_m(-\vartheta,\ldots), \chi_m(r,\vartheta) = (-1)^m \chi_m(r,-\vartheta)$$

And for $r >> r_V$

where solution has asymptotical form ($r \sim 100 \div 200 r_V$)

$$\Psi(r,\vartheta,\varphi) = e^{ikr\cos\tilde{\vartheta}} + f(\vartheta,\varphi)\frac{e^{ikr}}{r} \Rightarrow \chi_m(r,\vartheta) = f_m(\vartheta)e^{ikr}$$

Result could be seem as

From last condition for every m the $f_m(\vartheta)$ can be found.

symmetrical reference frame

$f_m \neq f_m(\varphi)$

$\frac{d\sigma}{d\Omega} = |f|^2 \neq$

Frame where $\vec{k} \upuparrows \vec{e_z}$

$f_m(\tilde{\vartheta}, \tilde{\varphi}) = f_m(\vartheta) e^{im\varphi}$

$\neq \frac{d\sigma}{d\tilde{\Omega}}$

but, exactly

$\sigma_m = \int \frac{d\sigma}{d\Omega} d\Omega$ and $\sigma = \sum_m \sigma_m$ is identical.

The method of solution

The algorithm of using the finite difference method has been approached. The two-dimension differential equation have been represented as two-dimension matrix equation.

$$\Delta_m \chi_m(r,\vartheta) = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \dots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix} \begin{pmatrix} \chi_m(r-\Delta r, \Delta\vartheta) \\ \chi_m(r-\Delta r, 2\Delta\vartheta) \\ \vdots \\ \chi_m(r-\Delta r, n\Delta\vartheta) \end{pmatrix} +$$

$$\begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & \ddots & \dots \\ b_{n1} & b_{n2} & \cdots & b_{nn} \end{pmatrix} \begin{pmatrix} \chi_m(r, \Delta\vartheta) \\ \chi_m(r, 2\Delta\vartheta) \\ \vdots \\ \chi_m(r, n\Delta\vartheta) \end{pmatrix} + \mathbf{C} \begin{pmatrix} \chi_m(r+\Delta r, \Delta\vartheta) \\ \chi_m(r+\Delta r, 2\Delta\vartheta) \\ \vdots \\ \chi_m(r+\Delta r, n\Delta\vartheta) \end{pmatrix}$$

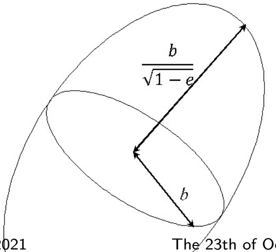
The sweep matrix schema has been used. The sets of programs KANTBP have been used before for checking of precision. Another method of control is using the optical theorem.

The elastic scattering of a neutron at the uranium isotopes has been considered.

General ideas of calculation:

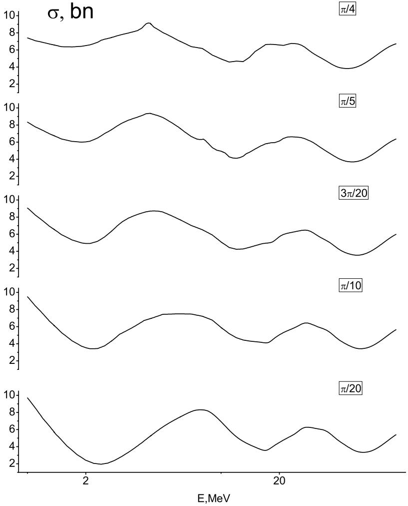
- The optical potential (in Wood-Saxon form)[1] without the *ls*−interaction has been used.
- Elliptical potential's form has been received from transformation of the spherical form. The deformation parameter β and the equal of volume of the sphere with the radius R and the ellipsoid with the two semi-minor axes b and eccentricity e have been taken into account.

[1] Yinlu Han, et al., Phys. Rew. C 81, 0246

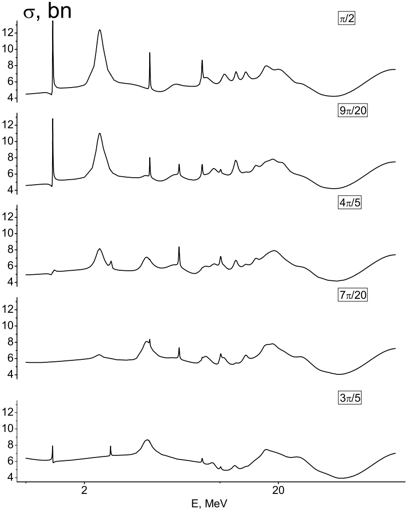


Results of test

$0 < \vartheta' \leq \pi/4$

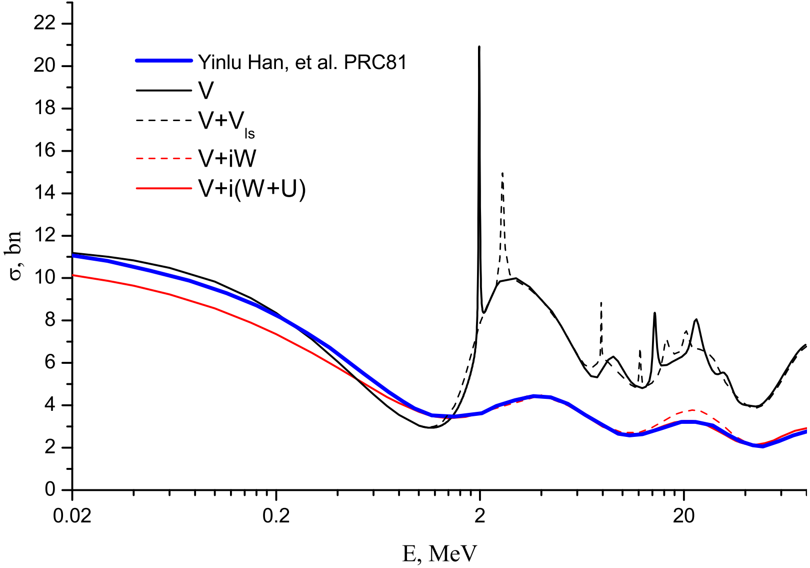


$\pi/4 < \vartheta' \leq \pi/2$



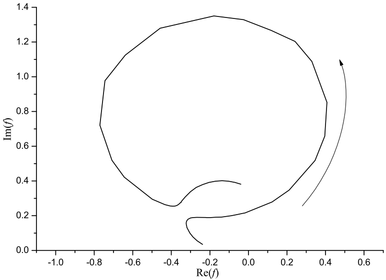
Results of test

The result of data's averaging.

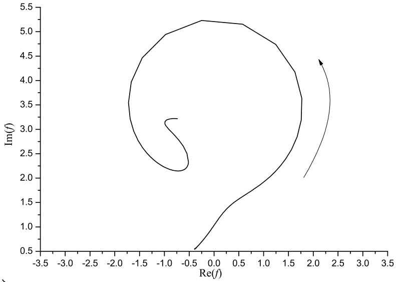


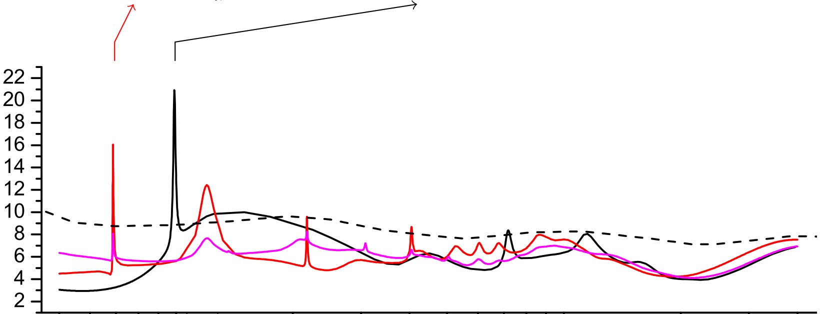
Resonance?

$\vartheta = \pi/2, m = 2$

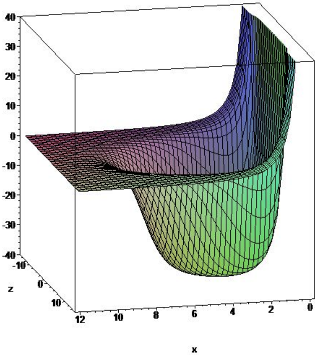


spherical





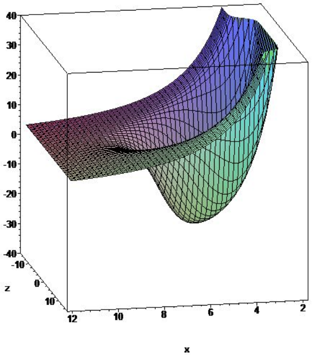
$$V(r,\vartheta)+\frac{m^2}{2\mu r^2\sin^2(\vartheta)}\frac{\hbar^2}{2\mu}$$



The reason of resonance's exist is the step which is produced by the "a centrifugal" potential and the potential of nuclei. For elliptical potential this effect may be slightly differ from spherical symmetrical potential.

The short description of resonance's reason

$$V(r,\vartheta) + \frac{\hbar^2 m^2}{2\mu r^2 \sin^2(\vartheta)}$$



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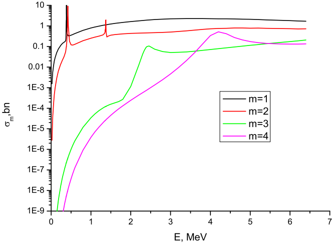
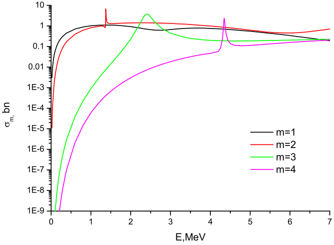
Results of test

$\vartheta' = \pi/4$



$\vartheta' = \pi/2$



Preliminary calculation for Be^{1}1

Journal of Physics G: Nuclear and Particle Physics

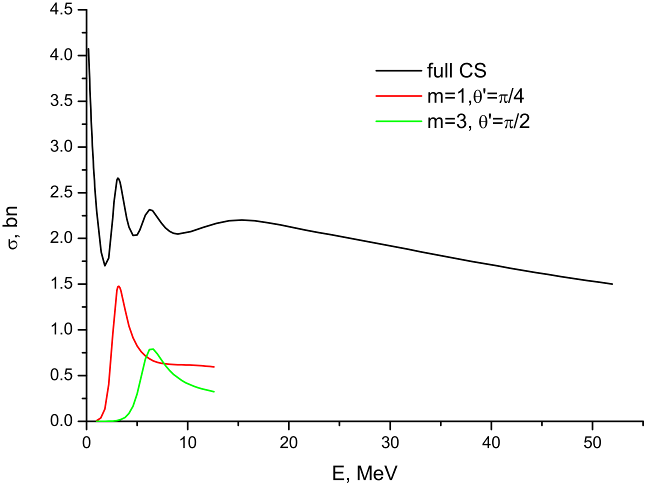
Properties of ^{12}Be and ^{11}Be in terms of single-particle motion in deformed potential

To cite this article: I Hamamoto and S Shimoura 2007 *J. Phys. G: Nucl. Part. Phys.* **34** 2715

The WoodsSaxon potential from this work has been used.

Preliminary calculation for Be[1]1

The ls interaction has't included.



CONCLUSION

- The schema of solution for non-spherical potential has been developed;
- The possibility of calculation for property of states are showed;
- The strong development (ls-including) is needed for correct result.

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Thank you for attention