The role of spin-spin forces in calculations of transition probabilities between the first one-phonon states.

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Plan

- General relations
- Second order effects within quantum many body theory
- Phonons
- Transition probabilities between one phonon states
- Preliminary calculation results for even Sn isotopes
- Conclusions

Self-consistency

In our approach we used only one small set of parameters by using the energy density functional method with the well known Fayans functional parameters DF3-a.

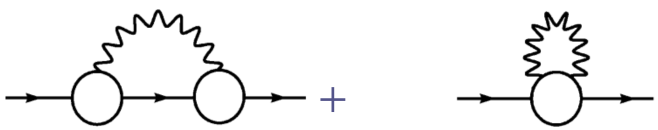
- Mean field (ground state) is determined by the first derivative of the functional
- Effective pp- and ph-interactions are the second derivative of the same functional

➢ This provides a high predictive power of our approach

General relations

- Small parameter

$$\alpha = \frac{|\langle 1||g_s||2\rangle|^2}{(2j_1+1)\omega_s^2} < 1$$

- It is necessary to take into account all the g^2 terms in the mass operator, it can be realized by phonon tadpole effect:
  - $\delta M =$ 
- Phonons and mean field are described within the Theory of Finite Fermi Systems. In symbolic form:
  - Equation for effective field

$$\hat{V}(\omega) = e_q\hat{V}_0(\omega) + \hat{F}\hat{A}(\omega)\hat{V}(\omega)$$

  - Photon creation amplitude

$$\hat{g}_s(\omega) = \hat{F}\hat{A}(\omega)\hat{g}_s(\omega)$$

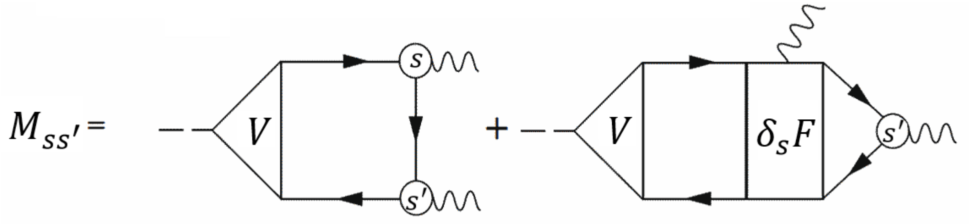
V.A. Khodel, Sov. J. Nucl. Phys. 24, 704 (1976)

Second order effects within quantum many body theory

- Following the method of Khodel, we have for transition amplitude with phonons s and s`:

$$M_{ss'} = VGg_sGg_{s'}G + VGG\delta_sFGGg_{s'}$$

- In Feynman diagram language:



- To take into account pairing effects, we must to add diagrams with anomalous Green function F:

$$G_1(\varepsilon) = G_1^h(-\varepsilon) = \frac{u_1^2}{\varepsilon - E_1 + \imath\delta} + \frac{v_1^2}{\varepsilon + E_1 - \imath\delta}$$

$$F_1^{(1)}(\varepsilon) = F_1^{(2)}(\varepsilon) = -\frac{\Delta_1}{2E_1}\left[\frac{1}{\varepsilon - E_1 + \imath\delta} + \frac{1}{\varepsilon + E_1 - \imath\delta}\right],$$

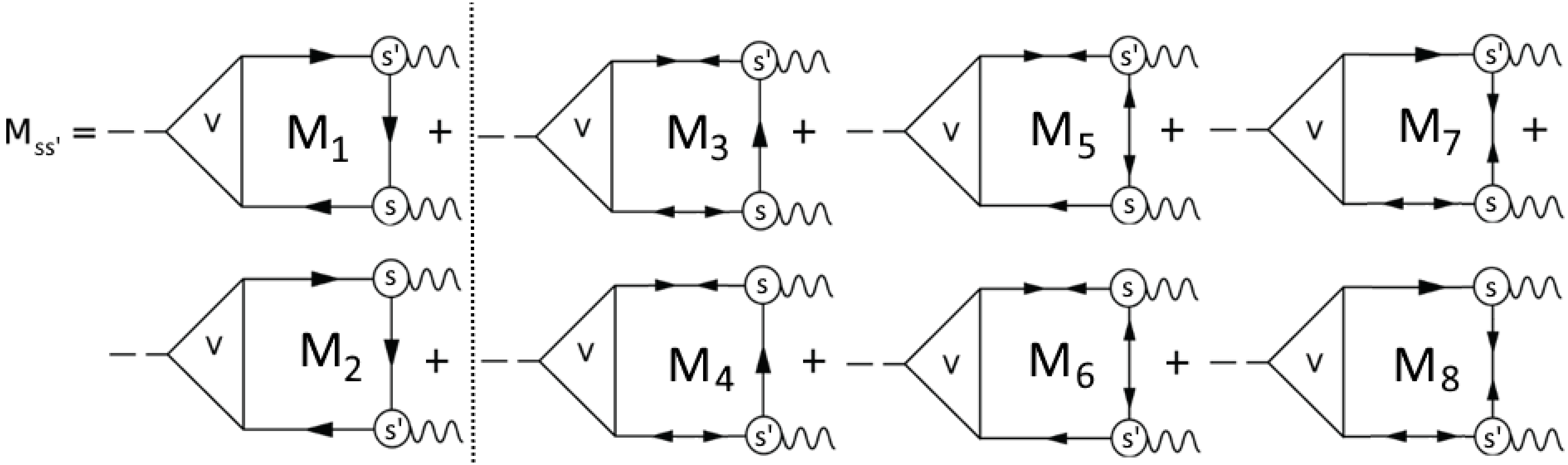
where

$$E_1 = \sqrt{(\varepsilon_1 - \mu)^2 + \Delta_1^2}, u_1^2 = (E_1 + \varepsilon_1 - \mu)/\,2E_1 = 1 - v_1^2$$

❖low indices mean the set of single-particle quantum numbers 1=(n_1 ; j_1 ; l_1 ; m_1)

As it was shown for for a close task the calculation of the quadruple moments*, graphs with δF give a small input.

Thus, we obtain 8 Feynman diagrams, describing the transition amplitude.



- For transition amplitude with phonons I_s and $I_{s'}$ with energy ω_s and $\omega_{s'}$:

$$M_{ss'} = M_{12} + M_{34} + M_{76} + M_{58}$$

Where

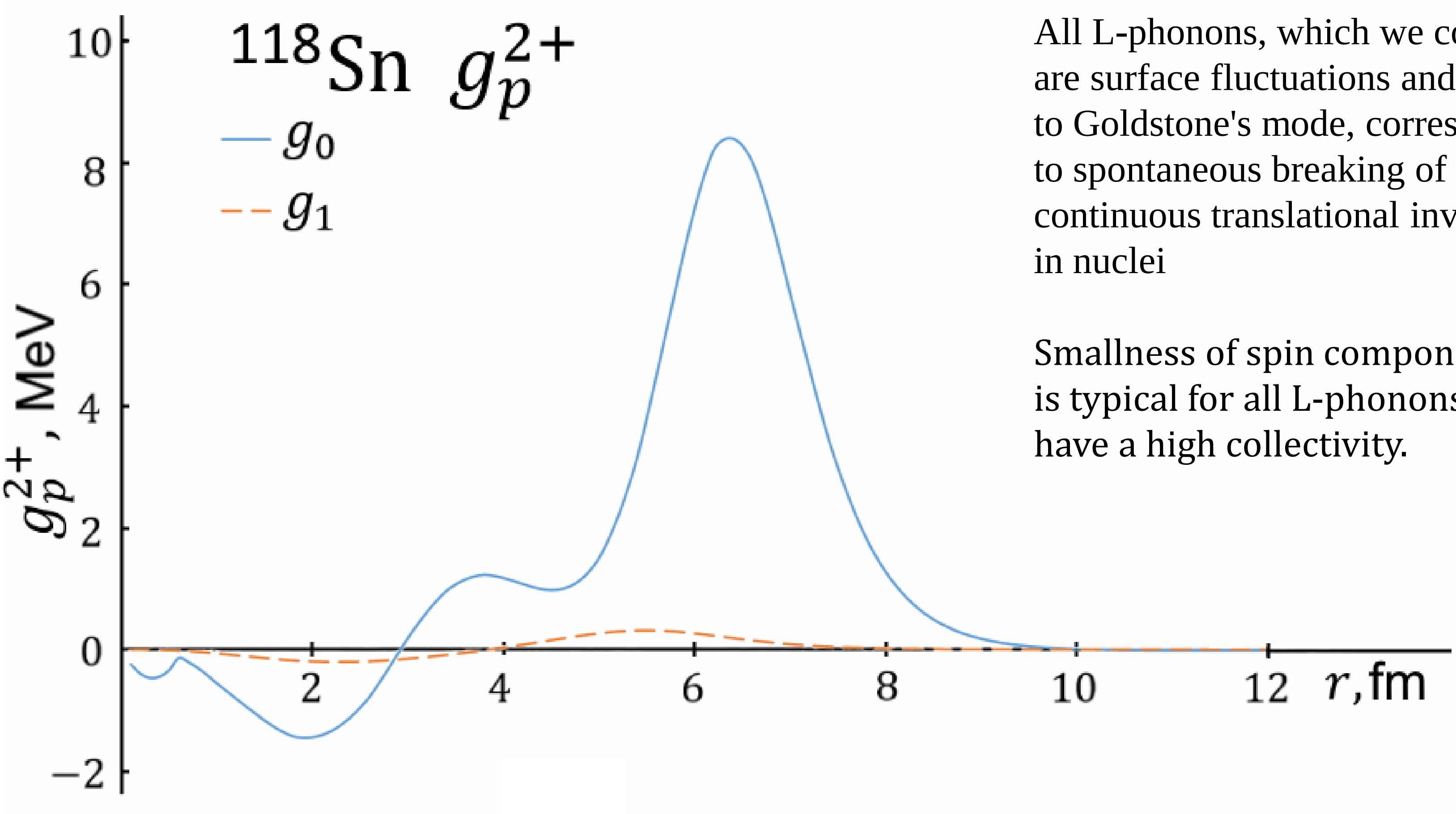
$$M_1 = \sum_{123} \langle 1|V|2\rangle\langle 2|g^{s'}|3\rangle\langle 3|g^{s}|1\rangle \int G_1(\varepsilon)G_2(\varepsilon+\omega)G_3(\varepsilon+\omega_s)d\varepsilon$$

*D. Voitenkov, S. Kamerdzhiev, S. Krewald, E. E. Saperstein, and S. V. Tolokonnikov, Phys. Rev. C 85, 054319 (2012).

Phonons

The vertex g_L is the sum of two components with spins S = 0 and S = 1:

- $g_s = g^s_{I_s 0} Y_{I_s M} + g^s_{I_s 1} [\boldsymbol{Y}_{\boldsymbol{I_s}} \otimes \boldsymbol{\sigma}]_{I_s M}$



All L-phonons, which we consider are surface fluctuations and belong to Goldstone's mode, corresponding to spontaneous breaking of continuous translational invariance in nuclei

Smallness of spin component g_1^s is typical for all L-phonons, which have a high collectivity.

2_1^+ Phonons

- Characteristics of the low-lying 2_1^+-phonons in even Sn isotopes, ω_2(MeV) and B(E2)↑ e²b²

A	ω_2^{th}	ω_2^{exp}	$B(E2)^{th}$	$B(E2)^{exp}$
102	1.453	1.472	0.065	-
104	1.388	1.260	0.107	-
106	1.316	1.207	0.142	0.195 (0.039)
108	1.231	1.206	0.155	0.222 (0.019)
110	1.162	1.212	0.188	0.220 (0.022)
112	1.130	1.257	0.197	0.240 (0.014)
114	1.156	1.300	0.193	0.24 (0.05)
116	1.186	1.294	0.182	0.209 (0.006)
118	1.217	1.230	0.172	0.209 (0.008)
120	1.240	1.171	0.152	0.202 (0.004)
122	1.290	1.141	0.158	0.192 (0.004)
124	1.350	1.132	0.147	0.166 (0.004)

3_1^- Phonons

- Characteristics of the low-lying 3_1^--phonons in even Sn isotopes, ω_3(MeV) and B(E3)↑ e^2b^3

- What is especially important for the g^2 problem, we have good agreement with experiment for the probabilities B(E2)↑ and B(E3)↑

A	ω_3	ω_3^{exp}	$B(E3)$	$B(E3)^{exp}$
100	5.621	–	0.109	–
102	3.959	–	0.0565	–
104	3.643	–	0.0760	–
106	3.457	–	0.0901	–
108	3.350	–	0.0959	–
110	3.282	2.459	0.0996	–
112	3.221	2.355	0.102	0.087(12)
114	3.157	2.275	0.106	0.100(12)
116	3.100	2.266	0.106	0.127(17)
118	3.072	2.325	0.106	0.115(10)
120	3.069	2.401	0.112	0.115(15)
122	3.112	2.493	0.107	0.092(10)
124	3.208	2.614	0.103	0.073(10)

Transition probabilities between one phonon states:

For the reduced $I_s \rightarrow I_{s'}$ transition probability with energy $\omega = \omega_{s'} - \omega_s$ we obtain:

$$\mathrm{B(EL)} = \frac{1}{2\mathrm{I_s}+1} |\langle \mathrm{I_s}||\mathrm{M_L}||\mathrm{I_{s'}}\rangle|^2$$

- In the simple case: $g_s = g^s_{I_s 0} Y_{I_s M}$

And we have for transition amplitude to the states with two phonons $M_{ss'}$:

$$\langle I_s||M_L||I_{s'}\rangle = \sum_{123} \begin{Bmatrix} I_s & I_{s'} & L \\ j_2 & j_1 & j_3 \end{Bmatrix} V_{12} g^s_{31} g^{s'}_{23} \left[A^{(12)}_{123} + A^{(34)}_{123} + (-1)^{L+Is} A^{(76)}_{123} + (-1)^{L+Is'} A^{(58)}_{123}\right]$$

- In the case of full phonon amplitude: $g_s = g^s_{I_s 0} Y_{I_s M} + g^s_{I_s 1} [\boldsymbol{Y_{I_s}} \otimes \boldsymbol{\sigma}]_{I_s M}$

We have slightly more complex expression for $M_{ss'}$:

$$\langle I_s||M_L||I_{s'}\rangle = \sum_{123} \begin{Bmatrix} I_s & I_{s'} & L \\ j_2 & j_1 & j_3 \end{Bmatrix} V_{12} \cdot$$

$$\cdot \left(g0^s_{31} g0^{s'}_{23} - g1^s_{31} g1^{s'}_{23} - g0^s_{31} g1^{s'}_{23} - g1^s_{31} g0^{s'}_{23}\right) \left[A^{(12)}_{123} + A^{(34)}_{123} + (-1)^{L+Is} A^{(76)}_{123} + (-1)^{L+Is'} A^{(58)}_{123}\right]$$

Ground-state correlation

$$A_{123}^{(12)} = A_{123}^{(1)} + A_{123}^{(2)} = \frac{2(E_{13}E_{23} + \omega_s\omega_{s'})}{(E_{13}^2 - \omega_s^2)(E_{23}^2 - \omega_{s'}^2)}(u_1^2u_2^2v_3^2 - v_1^2v_2^2u_3^2) +$$

$$\frac{2(E_{32}E_{12} + \omega\omega_{s'})}{(E_{32}^2 - \omega_{s'}^2)(E_{12}^2 - \omega^2)}(u_1^2v_2^2u_3^2 - v_1^2u_2^2v_3^2) + \frac{2(E_{31}E_{21} - \omega\omega_s)}{(E_{31}^2 - \omega_s^2)(E_{21}^2 - \omega^2)}(v_1^2u_2^2u_3^2 - u_1^2v_2^2v_3^2),$$

GSC

$$A_{123}^{(34)} = A_{123}^{(3)} + A_{123}^{(4)} =$$

$$\frac{\Delta_1\Delta_2}{4E_1E_2}(u_3^2 - v_3^2)\cdot\left(\frac{2(E_{13}E_{23} + \omega_s\omega_{s'})}{(E_{13}^2 - \omega_s^2)(E_{23}^2 - \omega_{s'}^2)} + \frac{2(E_{32}E_{12} + \omega\omega_{s'})}{(E_{32}^2 - \omega_{s'}^2)(E_{12}^2 - \omega^2)} + \frac{2(E_{31}E_{21} - \omega\omega_s)}{(E_{31}^2 - \omega_s^2)(E_{21}^2 - \omega^2)}\right),$$

$$A_{123}^{(76)} = A_{123}^{(7)} + A_{123}^{(6)} =$$

$$\frac{\Delta_1\Delta_3}{4E_1E_3}(u_2^2 - v_2^2)\cdot\left(\frac{2(E_{13}E_{23} + \omega_s\omega_{s'})}{(E_{13}^2 - \omega_s^2)(E_{23}^2 - \omega_{s'}^2)} + \frac{2(E_{32}E_{12} - \omega\omega_{s'})}{(E_{32}^2 - \omega_{s'}^2)(E_{12}^2 - \omega^2)} + \frac{2(E_{31}E_{21} + \omega\omega_s)}{(E_{31}^2 - \omega_s^2)(E_{21}^2 - \omega^2)}\right),$$

$$A_{123}^{(58)} = A_{123}^{(5)} + A_{123}^{(8)} =$$

$$\frac{\Delta_2\Delta_3}{4E_2E_3}(u_1^2 - v_1^2)\cdot\left(\frac{2(E_{13}E_{23} + \omega_s\omega_{s'})}{(E_{13}^2 - \omega_s^2)(E_{23}^2 - \omega_{s'}^2)} + \frac{2(E_{32}E_{12} - \omega\omega_{s'})}{(E_{32}^2 - \omega_{s'}^2)(E_{12}^2 - \omega^2)} + \frac{2(E_{31}E_{21} + \omega\omega_s)}{(E_{31}^2 - \omega_s^2)(E_{21}^2 - \omega^2)}\right).$$

Preliminary calculation results for even Sn isotopes:

- Transition probabilities B(E1) $(3_1^- \rightarrow 2_1^+)e^2fm^2$ g_0

A	B(E1) g_0	B(E1) $g_0 + g_1$	B(E1)^{Exp}
124	0,0018	0,0015	0,0020±0,0002
122	0,0020	0,0017	0,0018±0,0002
120	0,0020	0,0016	0,0020±0,0001
118	0,0020	0,0016	0,0017±0,0004
116	0,0015	0,0012	0,0014±0,0002
114	0,0036	0,0027	0,0003±0,0002
112	0,0029	0,0022	0,0014±0,0001
110	0,0023	0,0017	-
108	0,0013	0,0010	-
106	0,0006	0,0004	-
104	0,0003	0,0003	-
102	0,0011	0,0014	-
100	0,0060	0,0056	-

Experimental data for $^{124-116}$Sn: L.I. Govor, A.M. Demidov, O.K. Zhuravlev, I.V. Michailov and E.Yu. Shkuratova, Soy. J. Nucl. Phys. 54 (1991) 196.

Discussion

- As usual in our works, for E1 we have good agreement with experiment due to difference of two big effects: while GCS increase transition magnitude by an order, polarizability effects decrease it, but this decrease (2-3 times) is not so significant, as for magic nuclei (by an order of magnitude).

$$B(E1)(3^- \to 2^+),\ e^2 fm^2$$

1	2	3	4	5	6
Nucl.	$V = e_q V^{(0)}$ GSC $= 0$	$V = V_{\text{pol.}}$ GSC $= 0$	$V = e_q V^{(0)}$ GSC $\neq 0$	$V = V_{\text{pol.}}$ GSC $\neq 0$	Exp.
^{118}Sn	0.00044	0.00011	0.00939	0.00202	0.0017 ± 0.0004
^{120}Sn	0.00044	0.00012	0.00901	0.00199	0.0020 ± 0.0001
^{122}Sn	0.00047	0.00014	0.00899	0.00199	0.0018 ± 0.0002
^{124}Sn	0.00041	0.00012	0.00785	0.00180	0.0020 ± 0.0002

- Pairing effect give significant contribution to the size of transition probabilities, and M58(the amount of contributions of the respective diagrams) differs in magnitude by an order relative to other pairing terms
  - Which is apparently due to the proximity of the transition energy to the energy of 2+ phonon

$$\langle I_s||M_L||I_{s'}\rangle = \sum_{123} \begin{Bmatrix} I_s & I_{s'} & L \\ j_2 & j_1 & j_3 \end{Bmatrix} V_{12} g^{s}_{31} g^{s'}_{23} \times$$

$$\times \left[A^{(12)}_{123} + A^{(34)}_{123} + (-1)^{L+Is} A^{(76)}_{123} + (-1)^{L+Is'} A^{(58)}_{123} \right]$$

$$B(EL) = \frac{1}{2I_s + 1}\left(M^{p}_{ss'} + M^{n}_{ss'}\right)^2$$

$B(E1)(3^- \rightarrow 2^+),\ e^2 fm^2$

Sn118	M12	M34	M76	M58	**$M_{ss'}$**	B(E1) Theor	B(E1) Exp
n	-0,18489	-0,00800	-0,00660	0,07308	-0,12640	0,00202	0,0017±0,0004
p	0,24546	0,00000	0,00000	0,00000	0,24546		
Sn120							
n	-0,17416	-0,00563	-0,00630	0,06736	-0,11874	0,00199	0,0020±0,0001
p	0,23684	0,00000	0,00000	0,00000	0,23684		
Sn122							
n	-0,17043	-0,00393	-0,00620	0,06431	-0,11625	0,00199	0,0018±0,0002
p	0,23433	0,00000	0,00000	0,00000	0,23433		
Sn124							
n	-0,14941	-0,00103	-0,00423	0,04776	-0,10690	0,00180	0,0020±0,0002
p	0,21917	0,00000	0,00000	0,00000	0,21917		

Conclusions

- Using parameters of Fayans functional DF3-a we achieve reasonable description of transitions between the first one-phonon 3^{-}- and 2^{+}-states in most of the calculated Sn isotopes. But did not explain the outstanding ^{114}Sn.
- Good agreement with experiment achieved **only** if we take into account **two** significant effects: nuclear polarizability and GSC-effects.
- For the considered problem, it was obtained a large role of the paring terms.
- Despite the small value, spin component g_1^s can affect the final value of the transition probability B(E1), but in their current form, they don't improve the description of the experimental data.

Prospects

- Our next goal is to calculate M2 transitions, where spin effects can be more significant.

- There is a work by Fayans, in which for 4 magic isotopes(^{16}O, ^{40,48}Ca, ^{208}Pb) was shown, that spin-orbital interaction influences nuclear low-lying collective states quite noticeably and accounting for it leads to a better description of transition probabilities for natural-parity states. So, our challenge is to include LS-interaction in our scheme to improve the description of experimental data.

S.A. Fayans, E.L. Trykov and Dietrich Zawischa, Nuclear Physics A, 568, 523-543, (1994)

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Thank you for your attention!

Self-consistency

In our approach we used only one small set of parameters by using the energy density functional method with the known Fayans functional parameters DF3-a.

1. Mean field (ground state) is determined by the first derivative of the functional
2. Effective pp- and ph-interactions are the second derivative of the same functional

$$\mathcal{F} = \frac{\delta^2 \mathcal{E}}{\delta \rho^2} \qquad \mathcal{F}^{\xi} = \frac{\delta^2 \mathcal{E}}{\delta \nu^2}$$

Таблица 1. Параметры нормальной части ЭФП Фаянса

Параметр	DF3 [29]	DF3-a [30]
μ_0 [МэВ]	-16.05	-16.05
r_0 [фм]	1.147	1.145
K_0 [МэВ]	200	200
β [МэВ]	28.7	28.7
a_+^v	-6.598	-6.575
h_{1+}^v	0.163	0.163
h_{2+}^v	0.724	0.725
a_-^v	5.565	5.523
h_{1-}^v	0	0
h_{2-}^v	3.0	3.0
a_+^s	-11.4	-11.1
h_+^s	0.31	0.31
a_-^s	-4.11	-4.10
h_-^s	0	0
r_c [фм]	0.35	0.35
κ	0.216	0.190
κ'	0.077	0.077
g_1	0	0
g_1'	-0.123	-0.308

$$\hat{V} = \begin{pmatrix} V \\ d_1 \\ d_2 \end{pmatrix}, \quad \hat{V}_0 = \begin{pmatrix} V_0 \\ 0 \\ 0 \end{pmatrix},$$

$$\hat{\mathcal{F}} = \begin{pmatrix} \mathcal{F} & \mathcal{F}^{\omega\xi} & \mathcal{F}^{\omega\xi} \\ \mathcal{F}^{\xi\omega} & \mathcal{F}^{\xi} & \mathcal{F}^{\xi\omega} \\ \mathcal{F}^{\xi\omega} & \mathcal{F}^{\xi\omega} & \mathcal{F}^{\xi} \end{pmatrix},$$

$$\hat{A}(\omega) = \begin{pmatrix} \mathcal{L}(\omega) & \mathcal{M}_1(\omega) & \mathcal{M}_2(\omega) \\ \mathcal{O}(\omega) & -\mathcal{N}_1(\omega) & \mathcal{N}_2(\omega) \\ \mathcal{O}(-\omega) & -\mathcal{N}_1(-\omega) & \mathcal{N}_2(-\omega) \end{pmatrix}$$

$$\mathcal{F}_1^s = C_0 r_0^2 (g_1 + g_1' \boldsymbol{\tau}_1 \boldsymbol{\tau}_2) \times \\ \times \delta(\mathbf{r}_1 - \mathbf{r}_2)(\boldsymbol{\sigma}_1 \boldsymbol{\sigma}_2)(\mathbf{p}_1 \mathbf{p}_2).$$

$$\mathcal{F}^{\xi} = C_0 f^{\xi} =$$
$$= C_0 \left(f_{\text{ex}}^{\xi} + h^{\xi} x^{2/3} + f_{\nabla}^{\xi} r_0^2 (\nabla x)^2 \right)$$

$$\delta_L \mathcal{F} = \frac{\delta \mathcal{F}(\rho)}{\delta \rho} \delta \rho_L, \quad \delta \rho_L = A g_L$$