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"NUCLEUS – 2021. Nuclear physics and elementary particle physics.  
Nuclear physics technologies"



## NUCLEON DENSITY PROFILES AND NUCLEUS-NUCLEUS INTERACTION POTENTIAL

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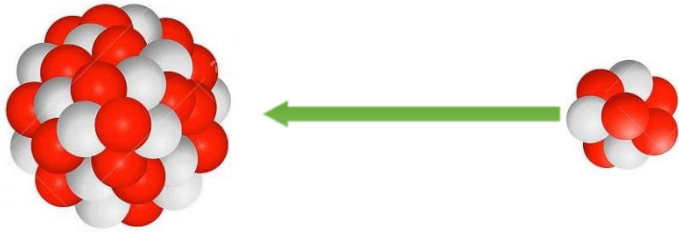
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Saint Petersburg  
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## Heavy ions collision: diabatic potential

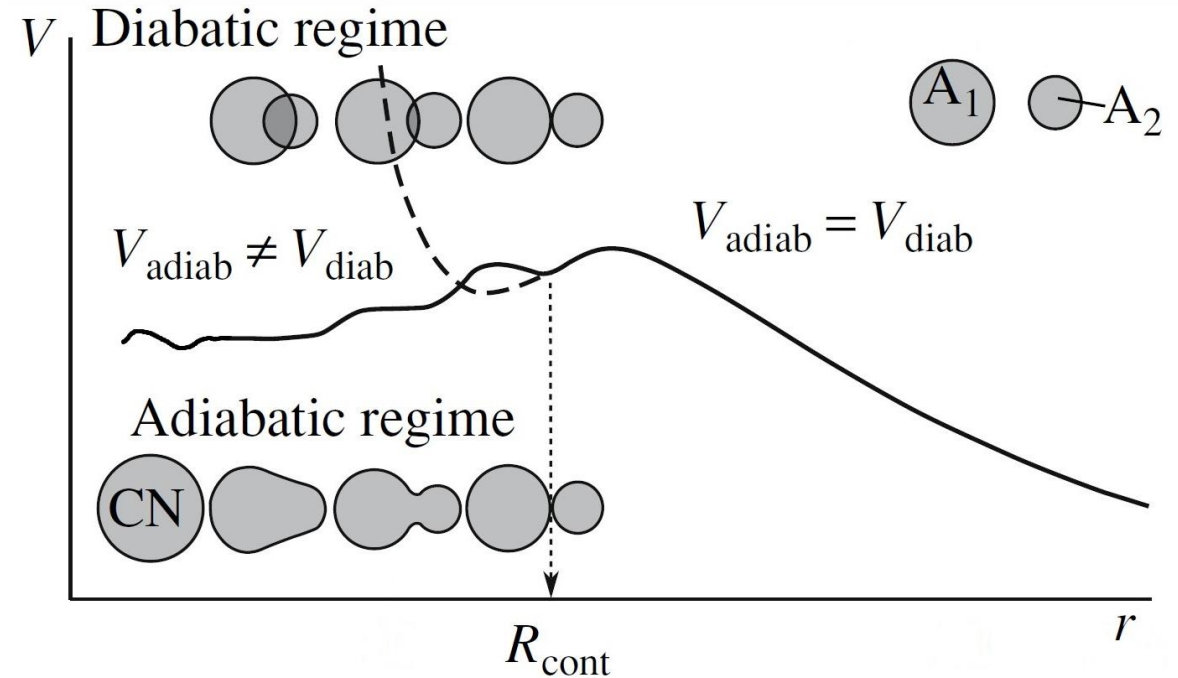
Reactions with heavy ions:

1. Two collision regime: diabatic and adiabatic
2. Fast collisions: no dynamic deformations => double density and repulsion



### Objectives:

1. Investigate how *nucleon density* parameters impact on profile of diabatic potential
2. Select density parameters and calculate *diabatic potential* for spherical nuclei with  $Z \geq 8$ ,  $N \geq 8$



Adopted from: Zagrebaev V. et al.// Phys. Part. Nucl., 2007. 38, 4, 469–491

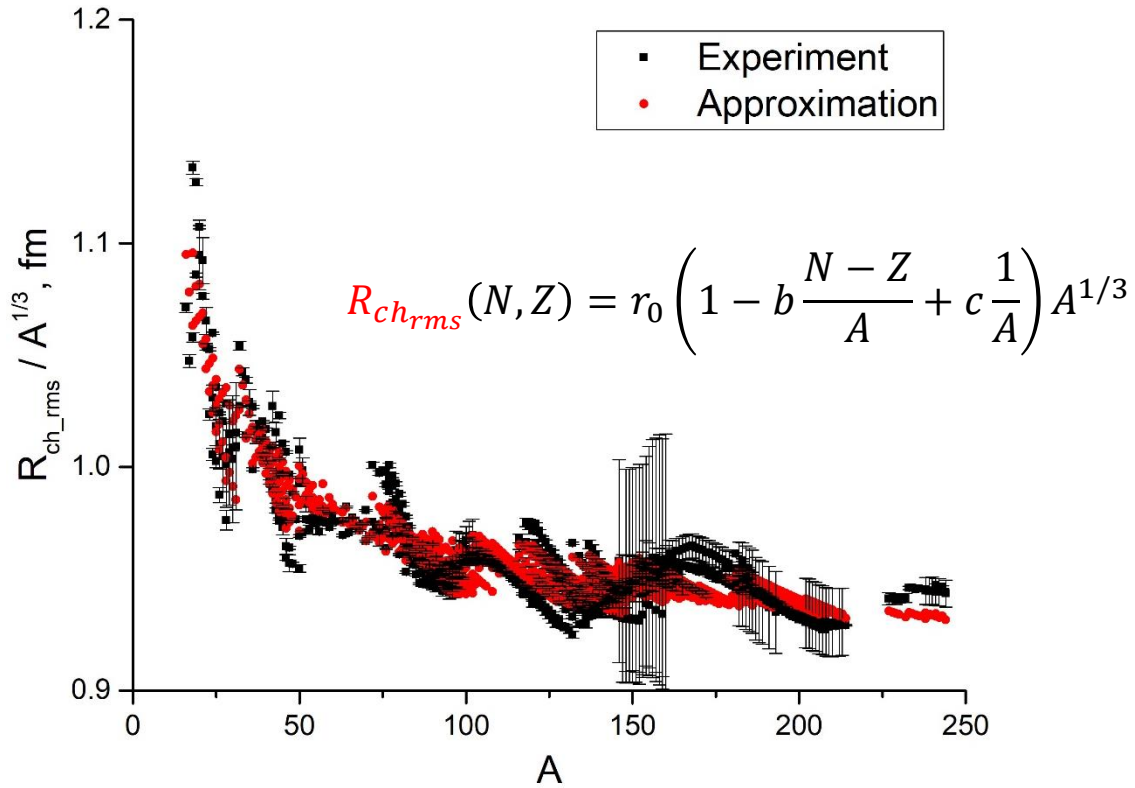
$$V_{diab}(r) = V_{NN}(r) + V_{coulomb}(r)$$

$$V_{NN}(r) = \int_{V_1} \rho_1(\mathbf{r}_1) \int_{V_2} \rho_2(\mathbf{r}_2) v_{NN}^{Migdal}(\mathbf{r} + \mathbf{r}_2 - \mathbf{r}_1) d^3r_2 d^3r_1$$

### Charge and proton density

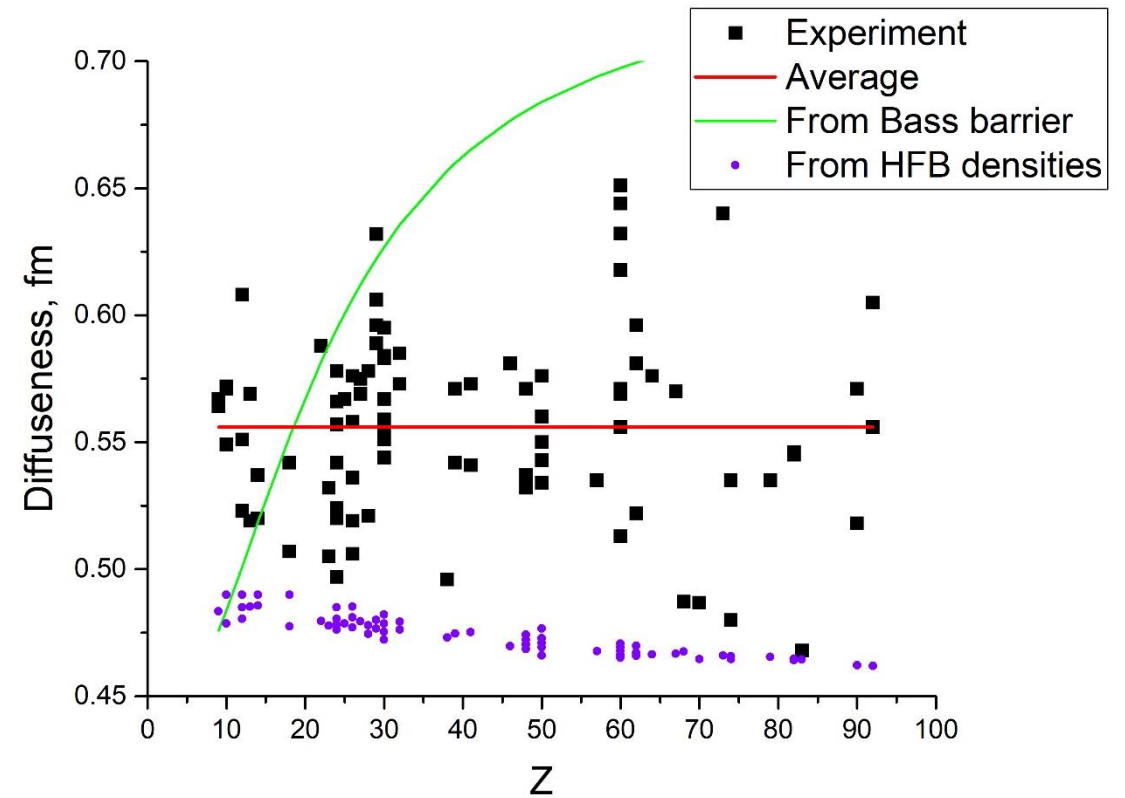
$$\text{Charge density: } \rho_{ch}(r) = \frac{\rho_{och}}{1 + \exp\left(\frac{r - R_{ch}}{a_{ch}}\right)}$$

$$R_{ch_{rms}}(\text{approximated}) \rightarrow R_{ch} \rightarrow R_p$$



Exp. data: I. Angeli et al. *At. Data Nucl. Data Tables* 99, 69 (2013).

$$a_{ch} = 0.556(4) \text{ fm (const)}$$



Exp. data: H. de Vries et al. *At. Data Nucl. Data Tables* 36, 495–536 (1987).  
 Data from **HFB** calculations: G. G. Adamian et al. *Phys. Rev. C* 94, 1 (2016).  
 Data from **Bass barrier**: A. V. Karpov et al. *AIP Conf. Proc.*, 912, 286 (2007).

## Neutron density

$$\text{Neutron density: } \rho_n(r) = \frac{\rho_{0n}}{1 + \exp\left(\frac{r - R_n}{a_n}\right)}$$

Neutron skin thickness:

$$\Delta R_{np} = R_{n_{rms}} - R_{p_{rms}}$$

Antiprotons-nuclei scattering

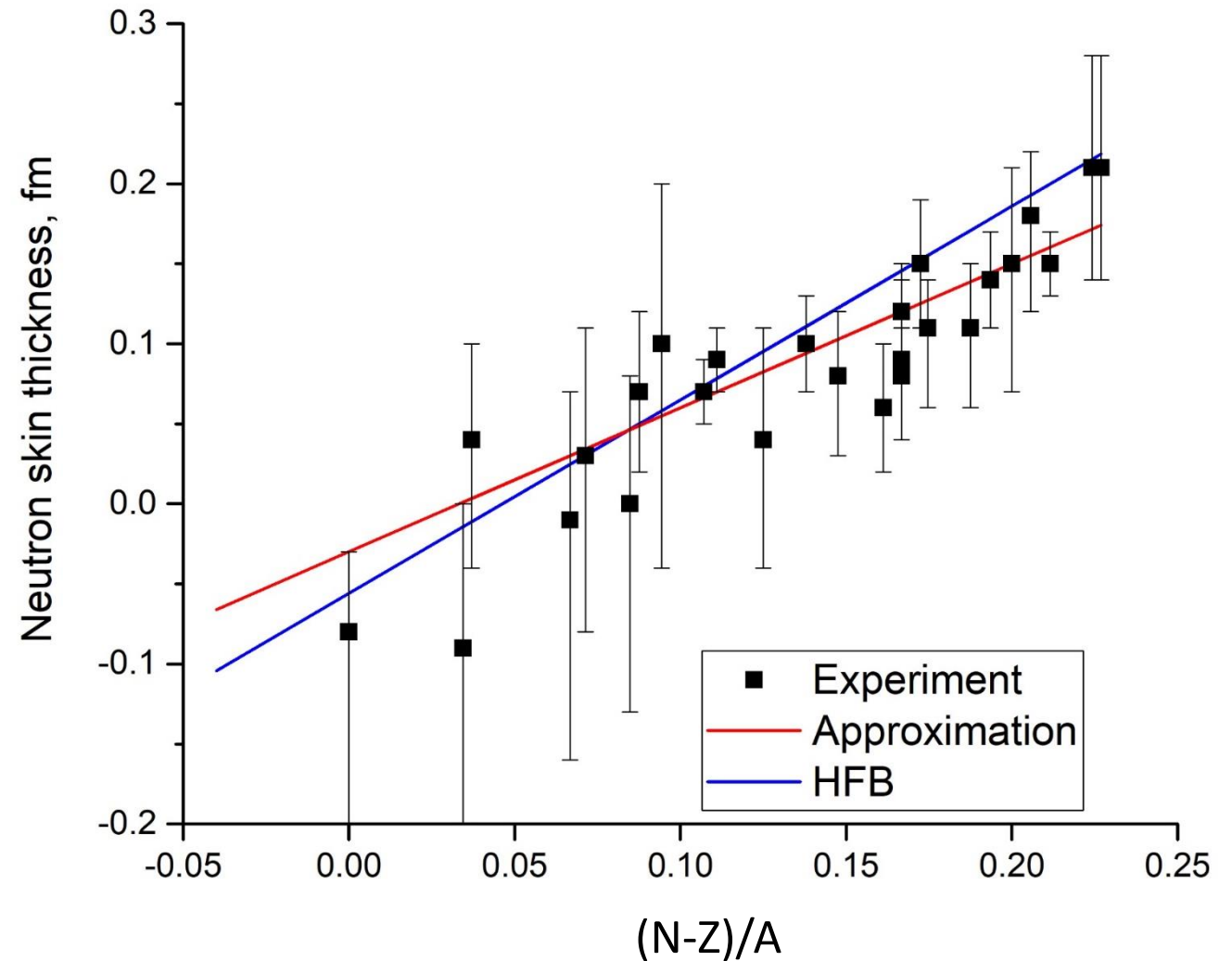
(approximation of experimental data):

$$\Delta R_{np} = a \frac{N - Z}{A} - b$$

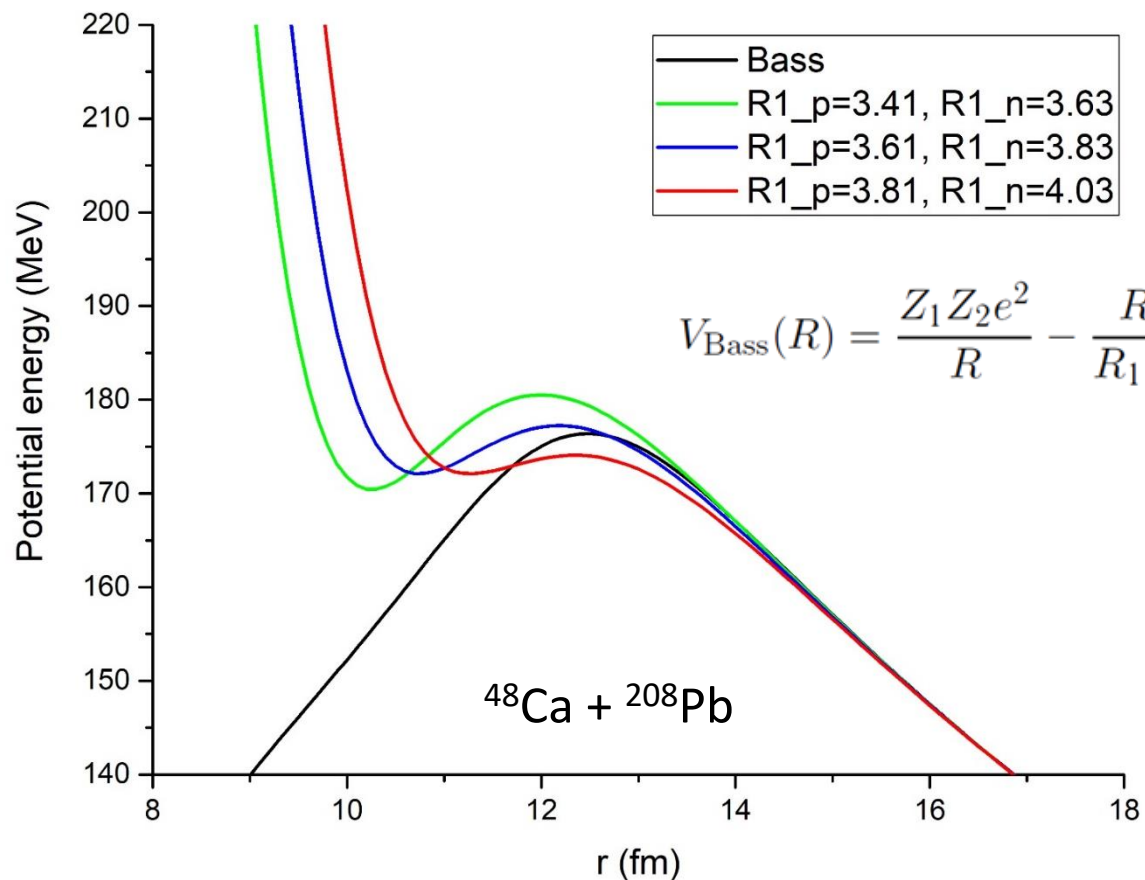
*J. Jastrzebski et al. IJMP E, vol. 13. (2004)*

$$R_{ch_{rms}} \rightarrow R_{p_{rms}} \rightarrow \Delta R_{np} \rightarrow R_{n_{rms}} \rightarrow R_n$$

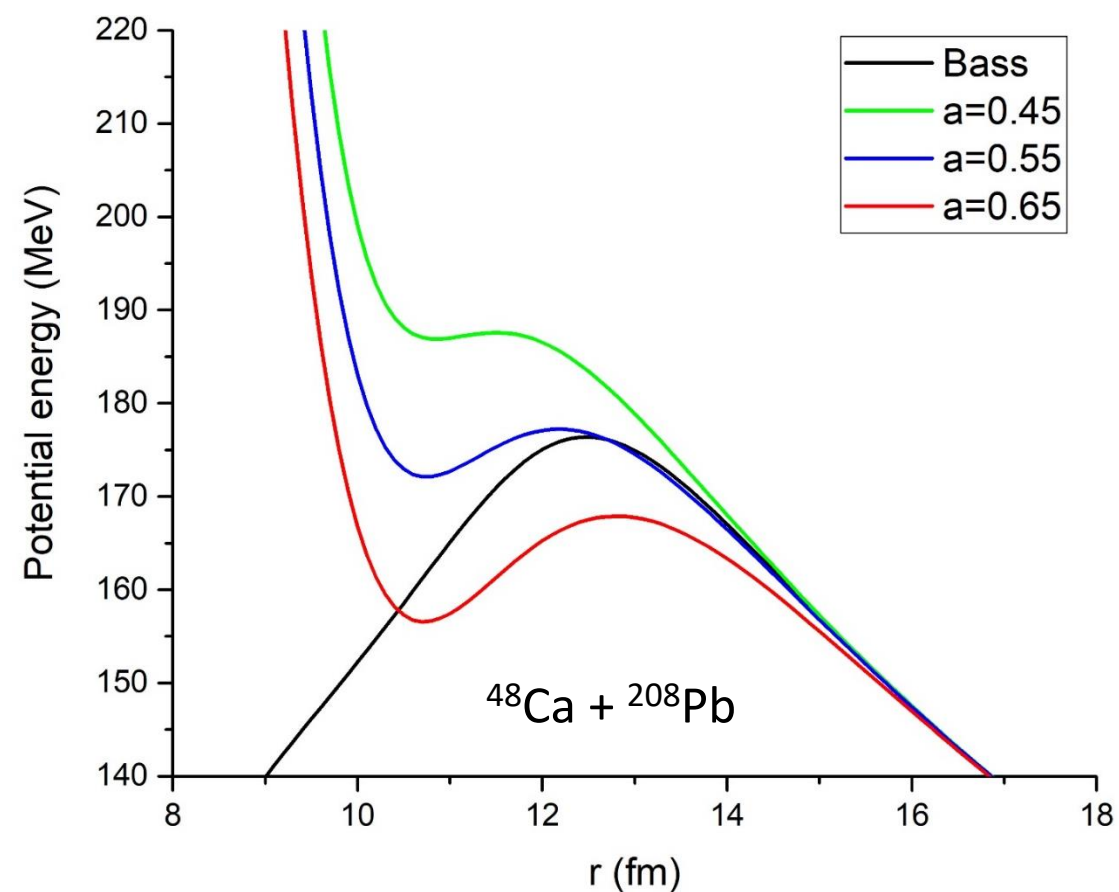
$$\text{Assumption: } a_{ch} \approx a_n \approx a_p$$



Variation of parameters

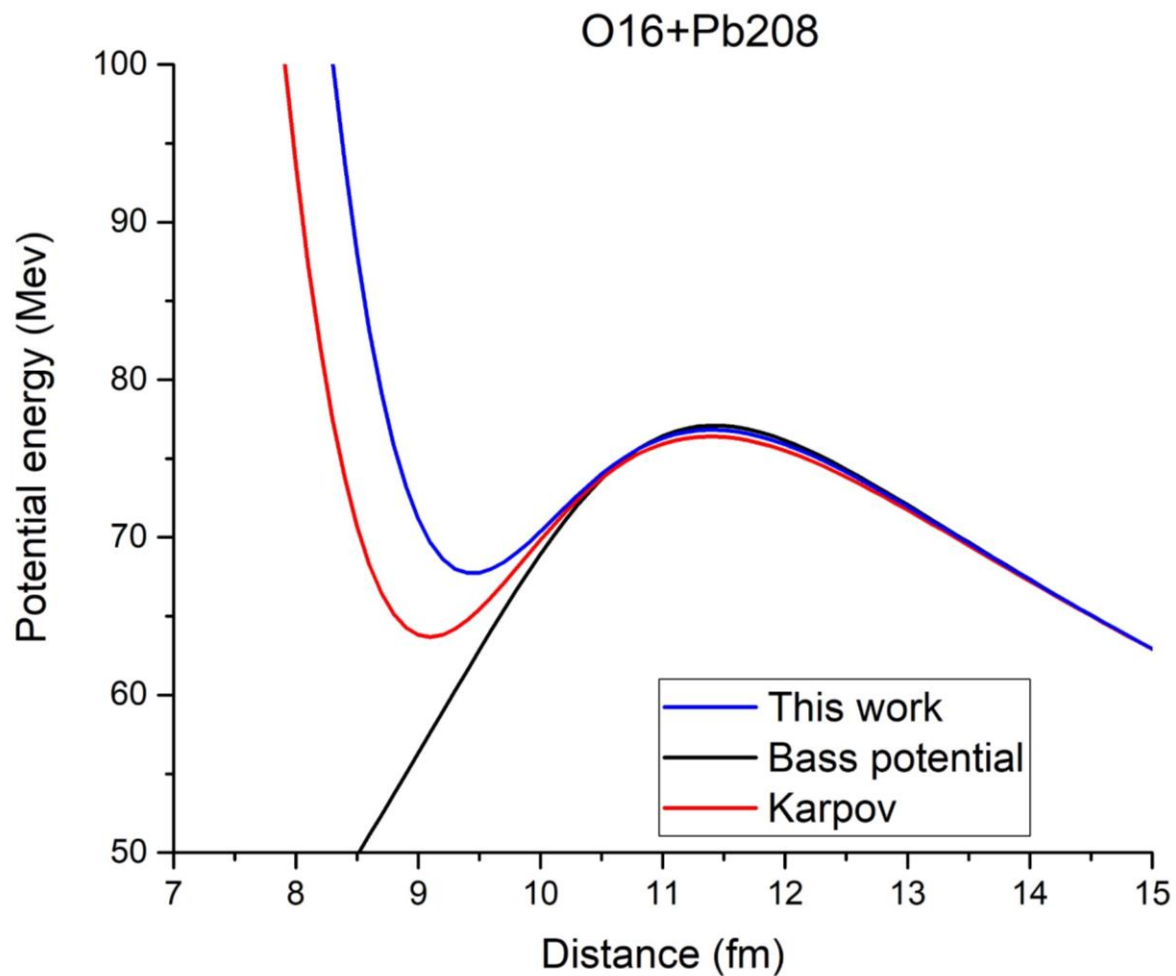


Half-density radii variation:  $R_{1p} = (3.61 \pm 0.2) \text{ fm}$

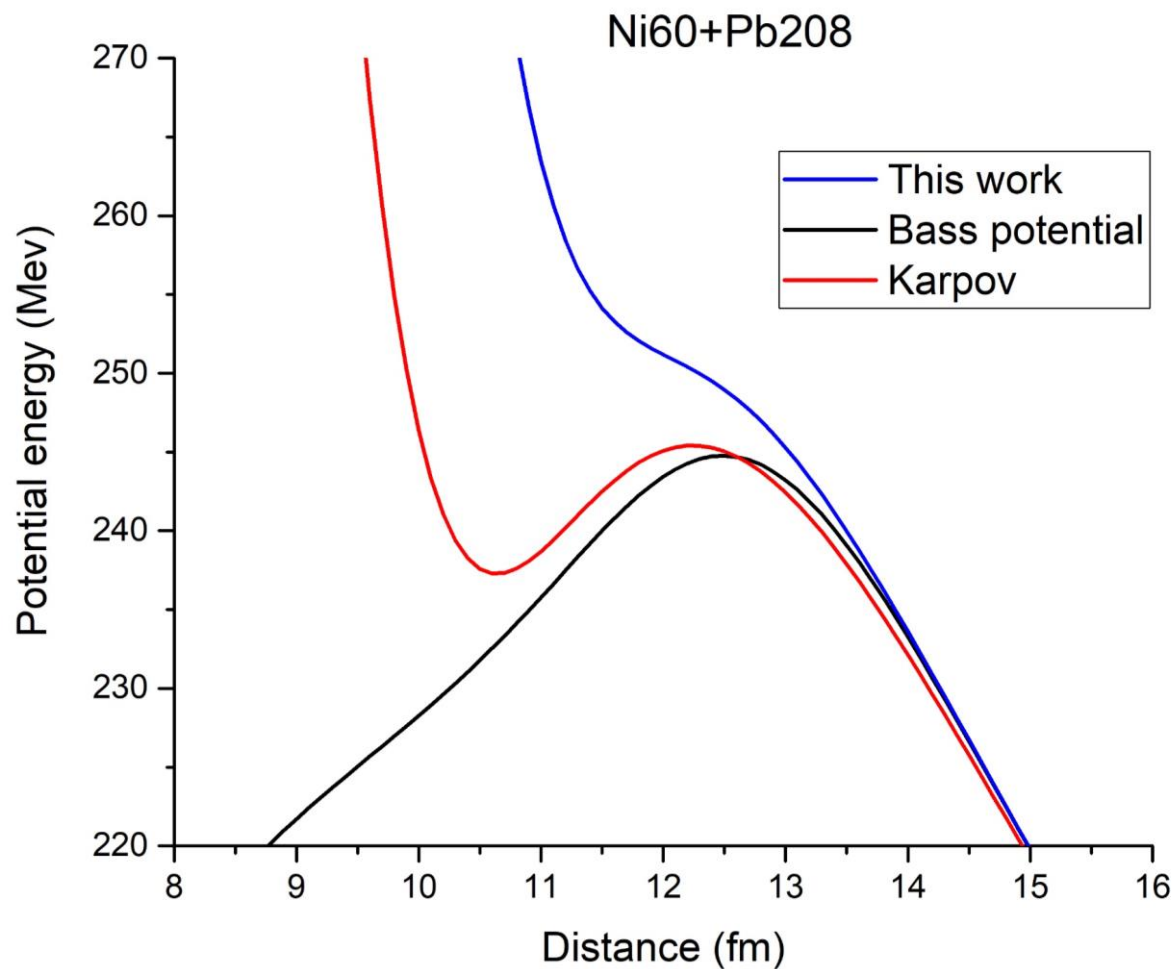


Diffuseness variation:  $a = (0.55 \pm 0.1) \text{ fm}$

Results



System of light+heavy nuclei



System of middle+heavy nuclei

## Conclusion

1. Approximation coefficients are obtained on the base of experimental data for:
  - rms. charge radius
  - diffuseness
2. Neutron radii are determined from neutron skin thickness and proton radii.
3. Sensitivity of diabatic potential to parameters of nucleonic densities is shown.
4. Nucleus-nucleus interaction potential is calculated for spherical nuclei with  $Z \geq 8$ ,  $N \geq 8$ . Comparison with Bass potential for nuclei system of different mass is carried out.

**Thank you for your attention!**



## Appendix: Model

1. Folding potential: 
$$V_{NN}(r) = \int_{V_1} \rho_1(\mathbf{r}_1) \int_{V_2} \rho_2(\mathbf{r}_2) v_{NN}(\mathbf{r} + \mathbf{r}_2 - \mathbf{r}_1) d^3r_2 d^3r_1$$

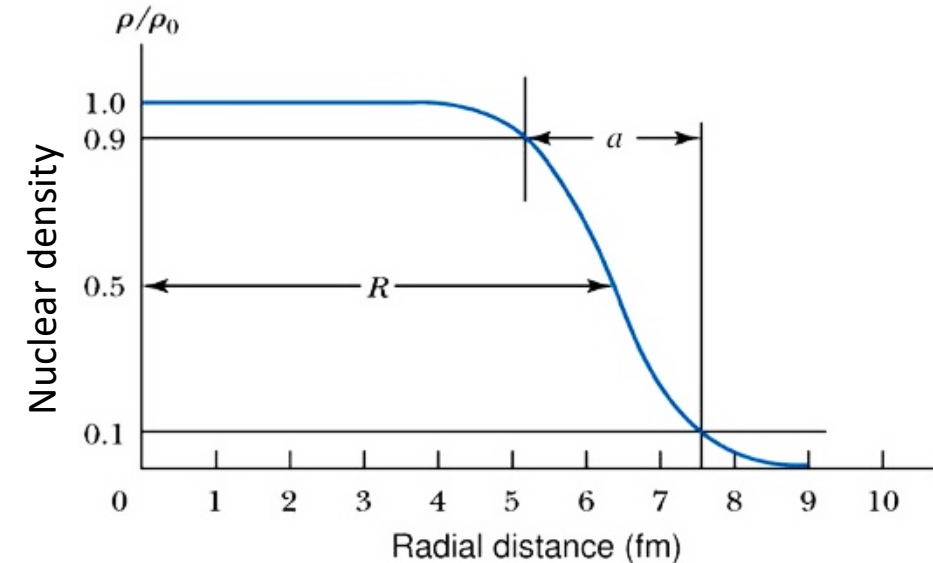
2. Effective Migdal forces: 
$$v_{NN}(\mathbf{r}_{12}) = C \left[ F_{\text{ex}} + (F_{\text{in}} - F_{\text{ex}}) \frac{\rho_1(\mathbf{r}_1) + \rho_2(\mathbf{r}_2)}{\rho_2(0) + \rho_2(0)} \right] \delta(\mathbf{r}_{12})$$

3. Total nucleonic density:

$$\rho_{1,2}(r) = \rho_{1,2}^p(r) + \rho_{1,2}^n(r),$$

$$\rho(r) = \frac{\rho_0}{1 + \exp\left(\frac{r - R}{a}\right)} \text{ for } p, n$$

4. Spherical nuclei  $Z \geq 8, N \geq 8$



## Appendix: Data and relations for density parameters

### 1. Radii data:

- Charge rms. radius (exp)  
*I. Angeli et al. At. Data Nucl. Data Tables 99, 69 (2013).*
- Proton rms. radius
- Rms. neutron radius
- Rms. and half-density radius
- Neutron skin thickness from antiprotons-nuclei scattering data (exp)  
*J. Jastrzebski et al. IJMP E, vol. 13. (2004)*

$$R_{ch_{rms}} = 0.956 \left( 1 - 0.153 \frac{N - Z}{A} + 2.326 \frac{1}{A} \right) A^{1/3}$$

$$R_{p_{rms}} = \sqrt{R_{ch_{rms}}^2 - r_{proton}^2}$$

$$R_i = R_{i_{rms}} \sqrt{\frac{5}{3} - \frac{7}{3} \left[ \frac{\pi a_i}{R_{i_{rms}}} \right]^2}$$

$$R_{n_{rms}} = R_{p_{rms}} + \Delta R_{np}$$

$$\Delta R_{np} = (0.90 \pm 0.15) \frac{N - Z}{A} - (0.03 \pm 0.02)$$

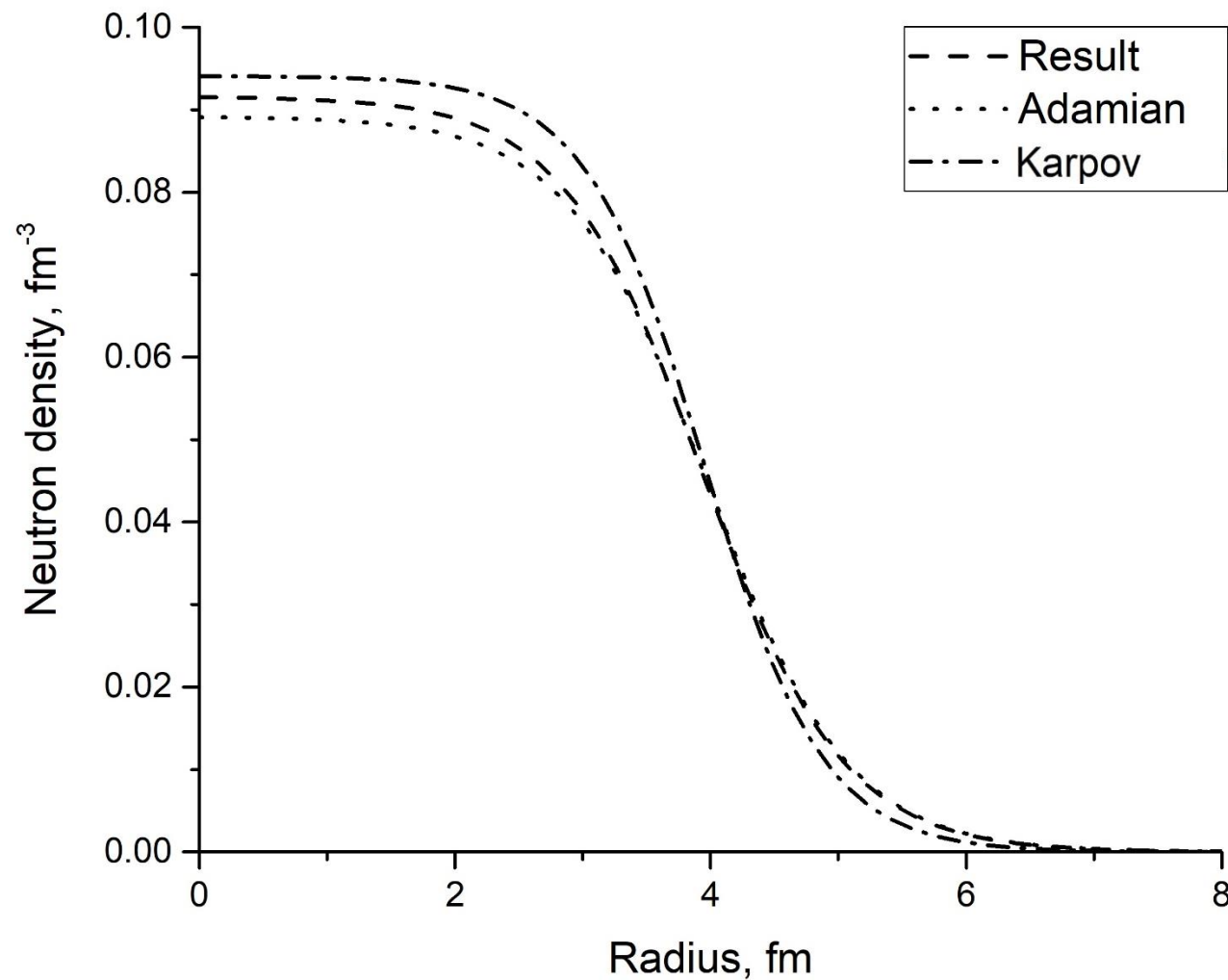
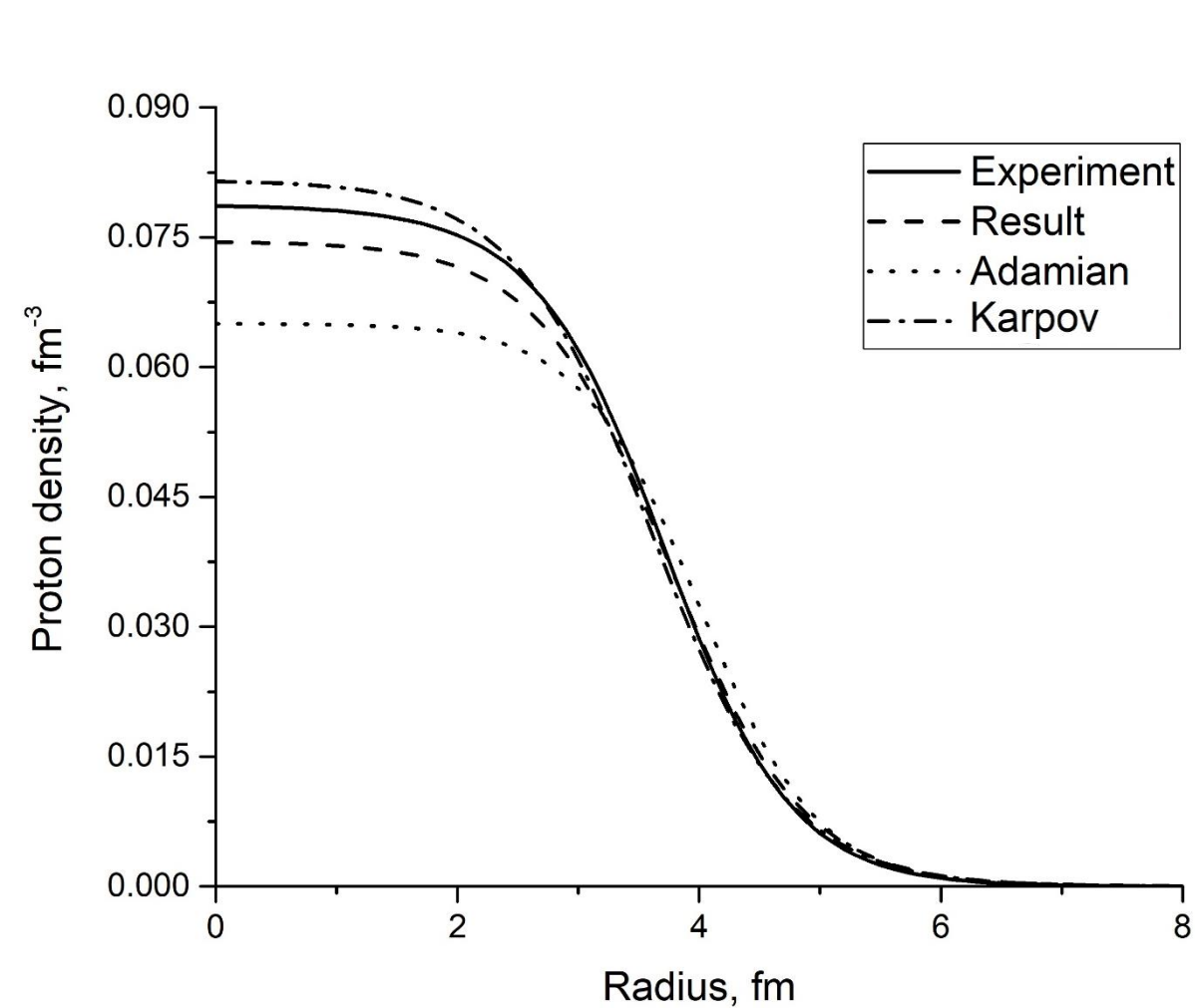
### 2. Diffuseness data:

- From HFB-densities  
*G. G. Adamian et al. Phys. Rev. C 94, 1 (2016).*
- From comparison with Bass barrier  
*A. V. Karpov et al. AIP Conf. Proc., 912, 286 (2007).*

$$a(N, Z) = 0.4899 - 0.1236 \frac{N - Z}{A}$$

$$a(Z) = 0.734 - \frac{150}{Z^2 + 500}$$

An example: densities for  $^{48}\text{Ca}$



Experimental data:  
*J.B. Bellicard et al. Phys. Rev. Lett. 19, 527 (1967)*

*A. V. Karpov et al. AIP Conf. Proc. (EXON06), 912 (2007) 286.*  
*G. G. Adamian et al. Phys. Rev. C 94, 1 (2016).*

## Appendix: Potentials

Effective nucleon-nucleon Migdal potential:

$$v_{NN}(\mathbf{r}_{12}) = C \left[ F_{\text{ex}} + (F_{\text{in}} - F_{\text{ex}}) \frac{\rho_1(\mathbf{r}_1) + \rho_2(\mathbf{r}_2)}{\rho_2(0) + \rho_2(0)} \right] \delta(\mathbf{r}_{12}), \quad F_{\text{ex(in)}} = f_{\text{ex(in)}} \pm f'_{\text{ex(in)}}$$

$C, \text{MeV}\cdot\text{fm}^{-3}$	$f_{\text{in}}$	$f_{\text{ex}}$	$f'_{\text{in}}$	$f'_{\text{ex}}$
300	0.09	-2.59	0.42	0.54

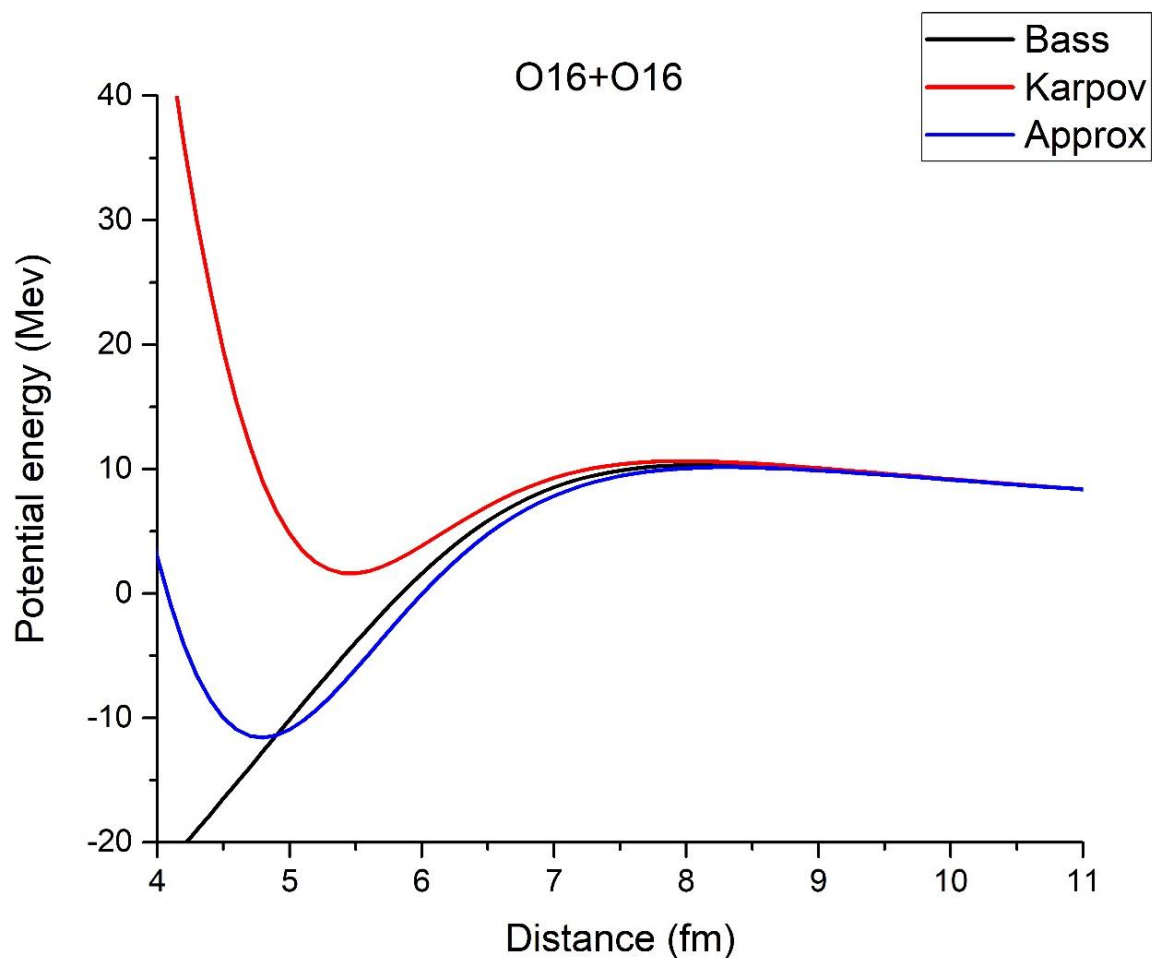
Bass potential:

$$V_{\text{Bass}}(R) = \frac{Z_1 Z_2 e^2}{R} - \frac{R_1 R_2}{R_1 + R_2} g(\xi) \quad g(\xi) = \left[ A \exp\left(\frac{\xi}{d_1}\right) + B \exp\left(\frac{\xi}{d_2}\right) \right]^{-1}$$

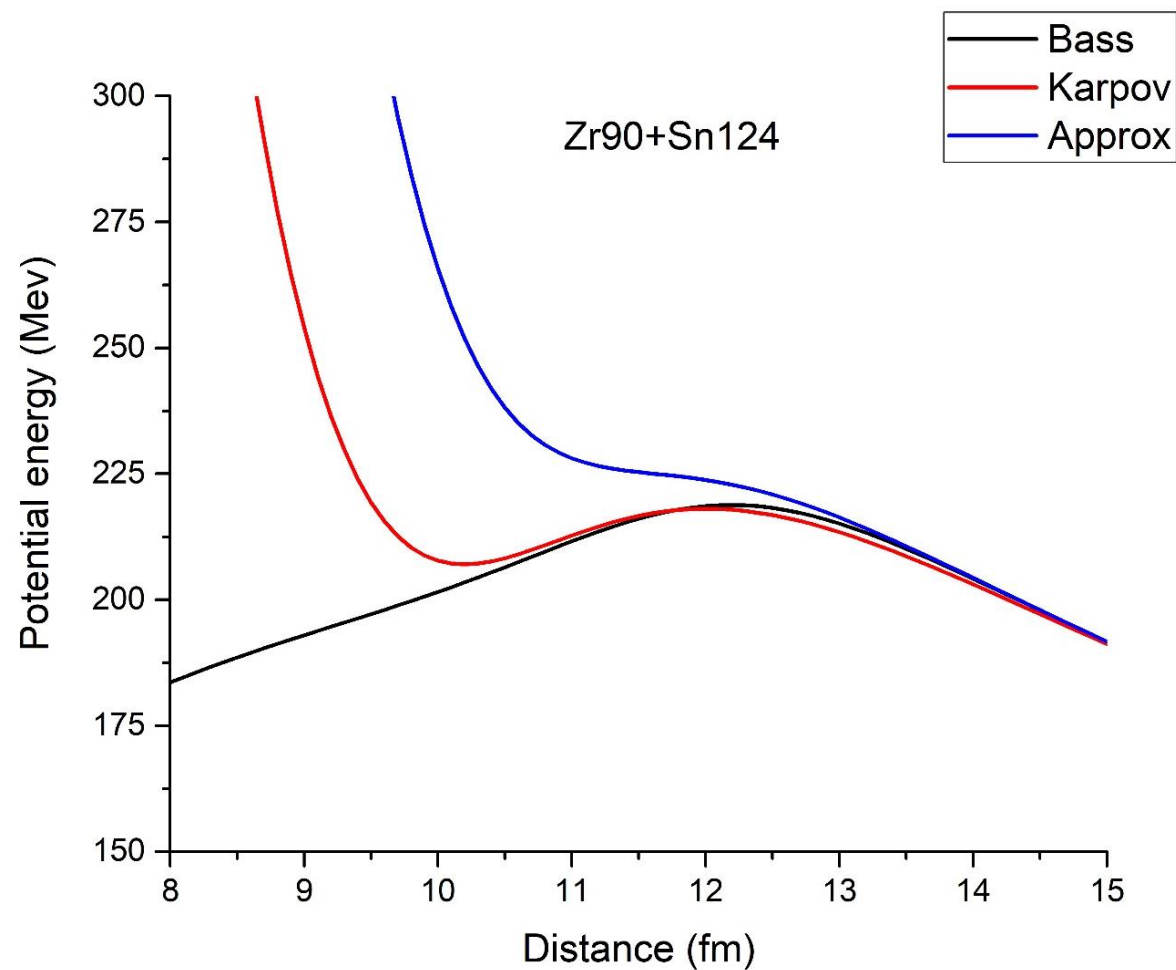
$$\xi = R - (R_1 + R_2)$$

$A, \text{MeV}^{-1}\cdot\text{fm}$	$B, \text{MeV}^{-1}\cdot\text{fm}$	$d_1, \text{fm}$	$d_2, \text{fm}$
0.03	0.0061	3.3	0.65

Appendix: Results



System light + light nuclei



System middle+middle nuclei