Study of ground states of ^{13,14}C, ^{13,14}N, ^{14}O nuclei by Feynman's continual integrals

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The ground states of ^{13,14}C, ^{13,14}N, ^{14}O nuclei were studied in two complementary few-body models.

In first model the studied isotopes were considered as cluster nuclei with the following configurations: ^{13}C (3α + *n*), ^{14}C (3α + 2*n*), ^{13}N (3α + *p*), ^{14}N (3α + *n* + p), ^{14}O (3α + 2*p*). In more simple model the studied isotopes were considered as systems consisting from nuclear core {^{12}C} and one or two valence nucleons. The wave functions and energies of these few-body systems were calculated by Feynman's continual integrals method in imaginary (Euclidean) time τ [1–5].

[1] Feynman R.P. and Hibbs A.R. Quantum Mechanics and Path Integrals (McGraw-Hill, New York, 1965).

[2] Shuryak E.V.and Zhirov O.V., Nucl. Phys. B. 1984. V.242. P.393.

[3] Samarin V.V. and Naumenko M.A., Phys. Atom. Nucl. 2017. V.80. P.877.

[4] Samarin, V.V. and Naumenko, M.A., Bull. Russ. Acad. Sci.: Phys. 2019.V. 83. P. 411.

[5] Samarin V.V., Bull. Russ. Acad.Sci.: Phys. 2020. V. 84. P. 981.

$$\hbar \ln K_{\rm E}\left(q,\tau;q,0\right) \to \hbar\left|\Psi_0(q)\right|^2 - E_0\tau,\ \tau\to\infty \quad (1)$$

The ground state energy E_0 was calculated as the slope of the linear part of the graph of propagator $K_{\rm E}\left(q,\tau;q,0\right)$ (fig. 1,2). The probability density $\left|\Psi_0(q)\right|^2$ was calculated as the value of $K_{\rm E}\left(q,\tau;q,0\right)$.

The values of the propagator were calculated based on Jacobi coordinates(fig. 3), the Monte-Carlo method, and averaging over random trajectories. The algorithm of parallel calculations was implemented in C++ programming language using NVIDIA CUDA technology [6]. Experimental data are taken from [7].

[6] Naumenko M.A. and Samarin V.V.Supercomp. Front. Innov. 2016. V.3. P.80.

[7] Zagrebaev V. I. *et al.*, NRV Web Knowledge Base on Low-Energy Nuclear Physics, http://nrv.jinr.ru/.

The graphs of $\ln K_{\rm E}\left(q,\tau;q,0\right)$ having linear parts are shown in fig. 1, 2 ($t_0 = m_0 x_0^2/\hbar \approx 1.57\cdot 10^{-23}$, $x_0 = 1$ fm, m_0 is the neutron mass, as in [3–5]).

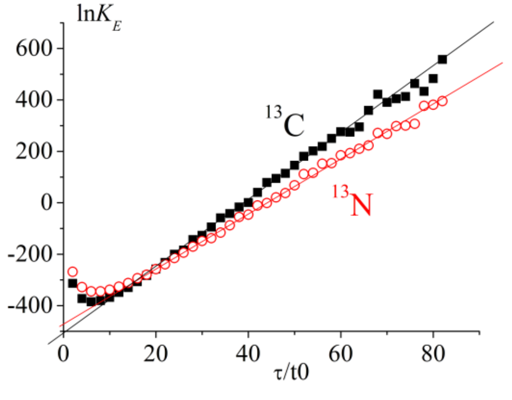


Fig. 1. The graphs of propagator $K_{\rm E}\left(q,\tau;q,0\right)$ for : ^{13}C (3α + *n*) and ^{13}N (3α + *p*)

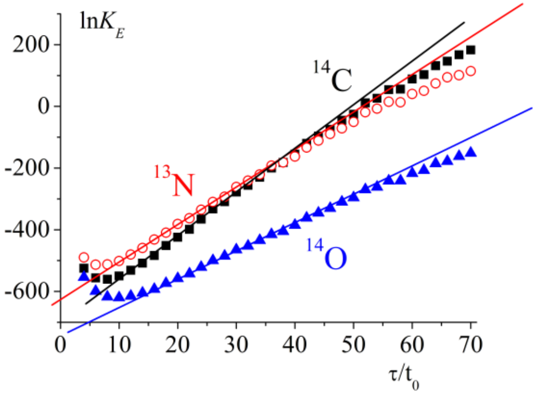


Fig. 2. The graphs of propagator $K_{\rm E}\left(q,\tau;q,0\right)$ for : ^{14}C (3α + 2*n*), ^{14}N (3α + *n* + p), ^{14}O (3α + 2*p*).

Nucleus	Separation energy (MeV), $(-E_0)$	
	Experiment [7]	Theory
^{13}C (3α + *n*)	12.220	13.4±0.1
^{13}N (3α + *p*)	9.217	11.1±0.1
^{14}C (3α + 2*n*)	20.397	14.6±0.2
^{14}N (3α + *n* + p)	19.771	11.9±0.2
^{14}O (3α + 2*p*)	13.844	9.6±0.4

For ^{13}C, ^{13}N the shift is $\Delta E_0 \approx 1\div 2$.

For ^{14}C, ^{14}N, ^{14}O the shift is $\Delta E_0 \approx -5$.

The α-α, α-*n*, α-*p*, *n*-*n* and *p*-*n* potentials may be changed to improving the agreement with experimental data.

The probability density $|\Psi_0(q)|^2$ was calculated as the value of $K_E(q,\tau;q,0) \approx \text{const} \cdot |\Psi_0(q)|^2$. The graphs of propagator $K_E(q,\tau;q,0)$ for ^{13}C are shown in fig. 4.

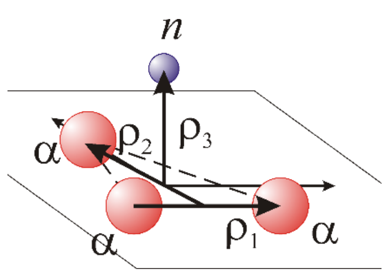
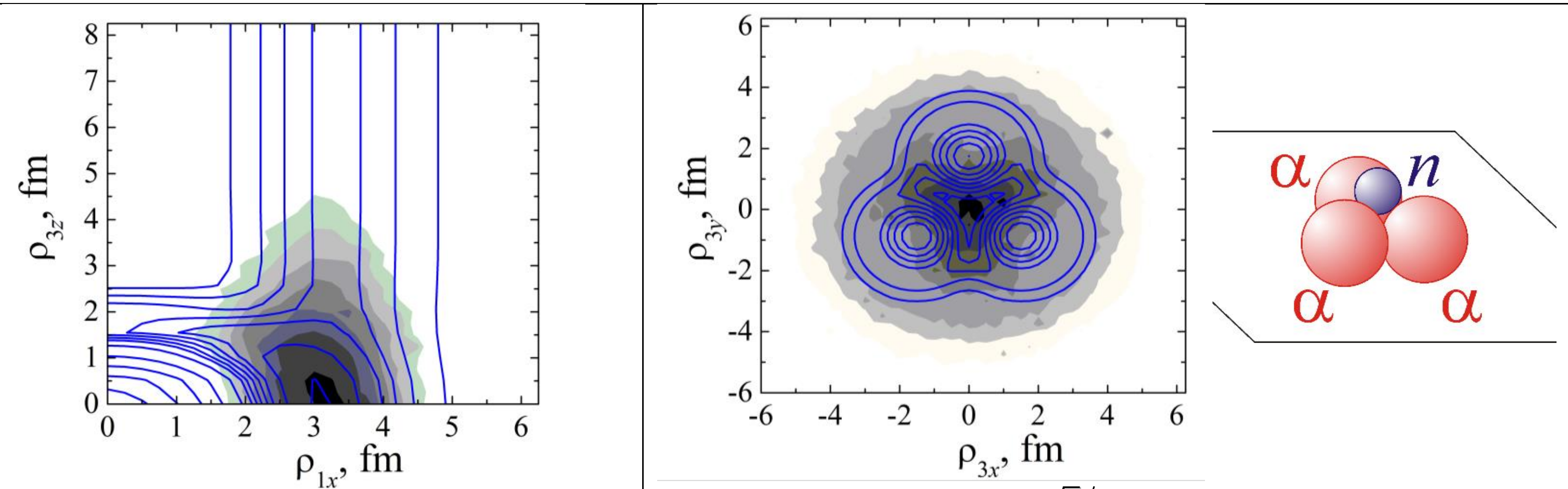


Fig. 3. Jacobi coordinates for ^{13}C ($3\alpha + n$)



$\vec{\rho}_1 \perp \vec{\rho}_2 \perp \vec{\rho}_3,\ \rho_2 = \rho_1\sqrt{3}/2$

$\vec{\rho}_1 \perp \vec{\rho}_2,\ \rho_2 = \rho_1\sqrt{3}/2,\ \ \rho_1 = 3$ fm, $\ \rho_{3z} = 1$ fm

Fig. 4. Topography (shades of grey; logarithmic scale) of the propagator K_E equal to the probability density $|\Psi_0|^2$ for the ground state of the ^{13}C ($3\alpha + n$) along with the potential landscape (curves) for the regular triangle configuration [7] of the 3α subsystem.

[7] Samarin V.V. J. Phys.: Conf. Ser. 2020. V. 1555. 012030.

Results of the few-body model correspond to results of the shell model of deformed nuclei [8, 9].

[8] Samarin V.V. Phys. Atom. Nucl. 2010. V.73. P. 1416.

[9] Samarin V.V. Phys. Atom. Nucl. 2015. V.78. P. 128.

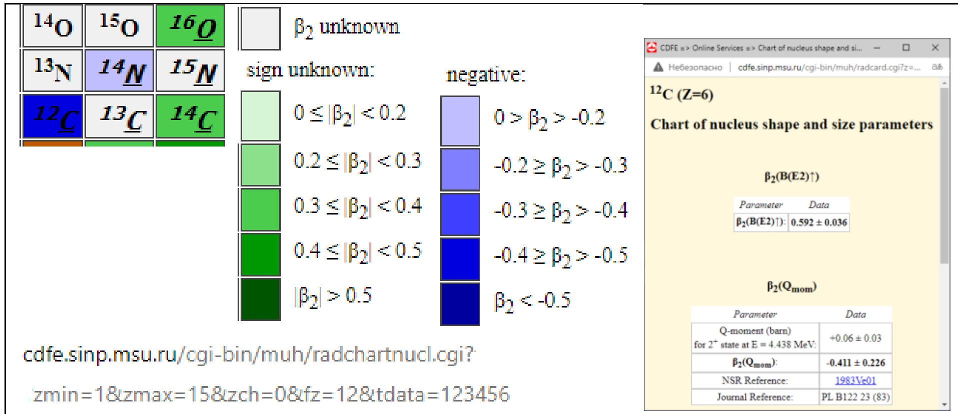


Fig. 5. Deformations of the nuclei near the ^{13}C nucleus.

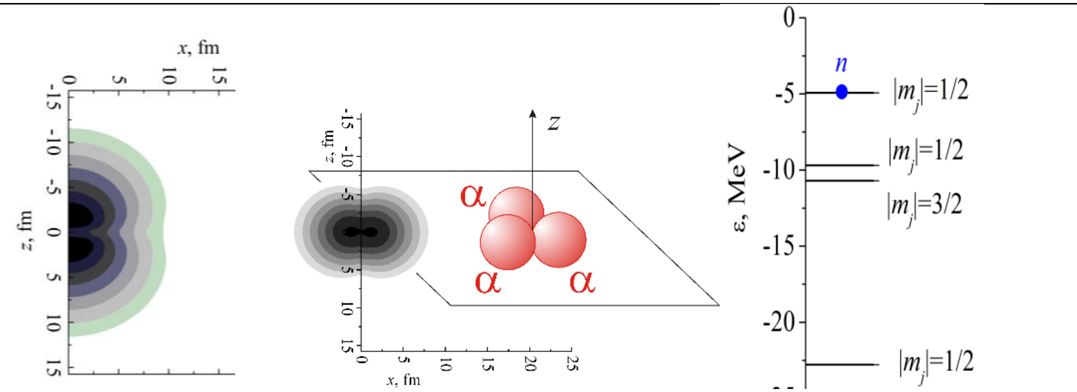


Fig. 6. Results of the application of the shell model of deformed nuclei to the ^{13}C nucleus.