National Research Centre "Kurchatov Institute"

PETERSBURG NUCLEAR PHYSICS INSTITUTE

Analysis of neutron-induced fission cross-sections of Pb isotopes near the closed neutron shell

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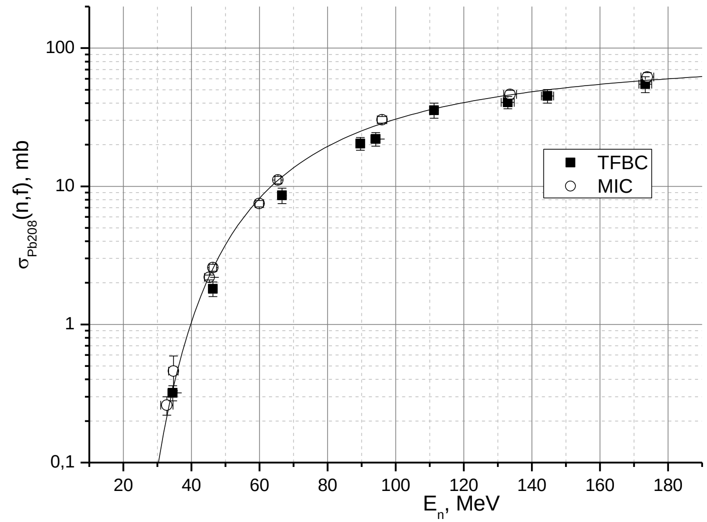
Contents

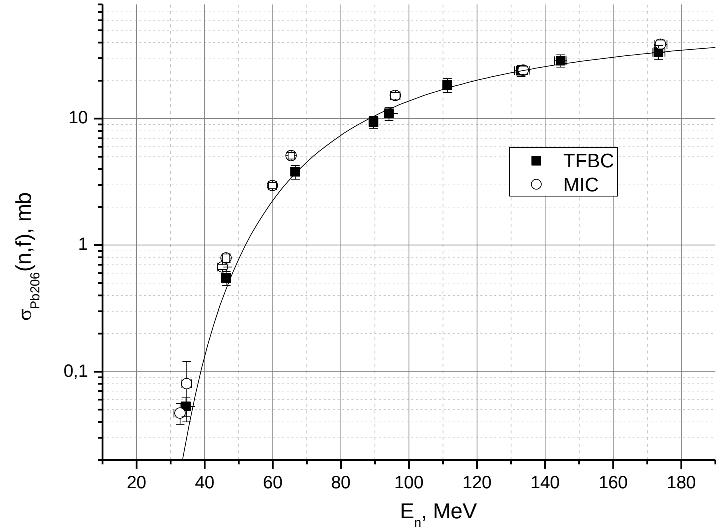
- Experimental fission cross-section of (n,f) reaction for ^{204}Pb, $^{206-208}Pb$ targets
- Review of the earlier analysis of (p,f) and (e,f) reactions with Pb isotopes

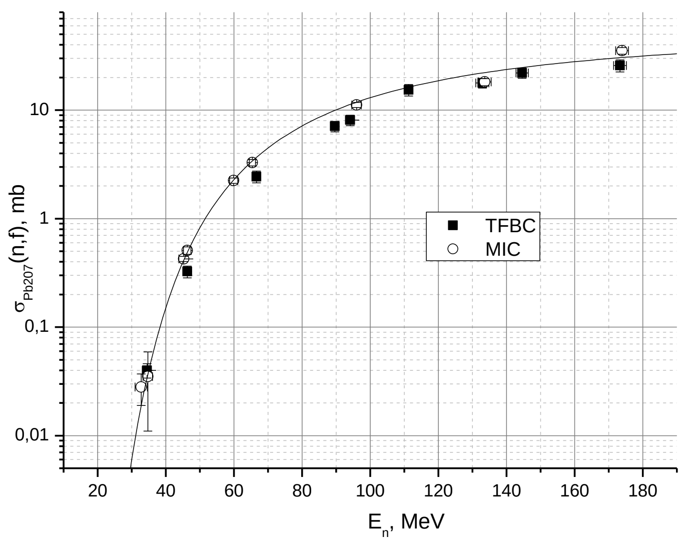
fissioning. Fission barriers results.

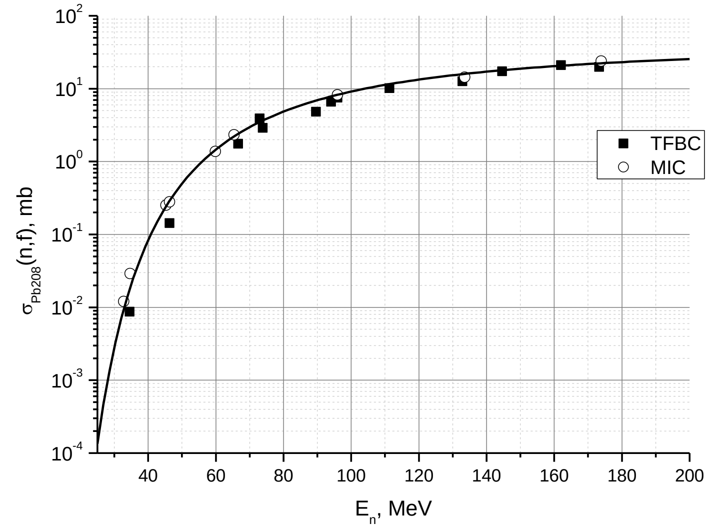
- Using of Talys code for description of (n,f) reaction
- Hauser-Feshbah compound model
- Level densities
- Fission barriers
- Fitting of fission barriers
- Results and comparing with the previous works

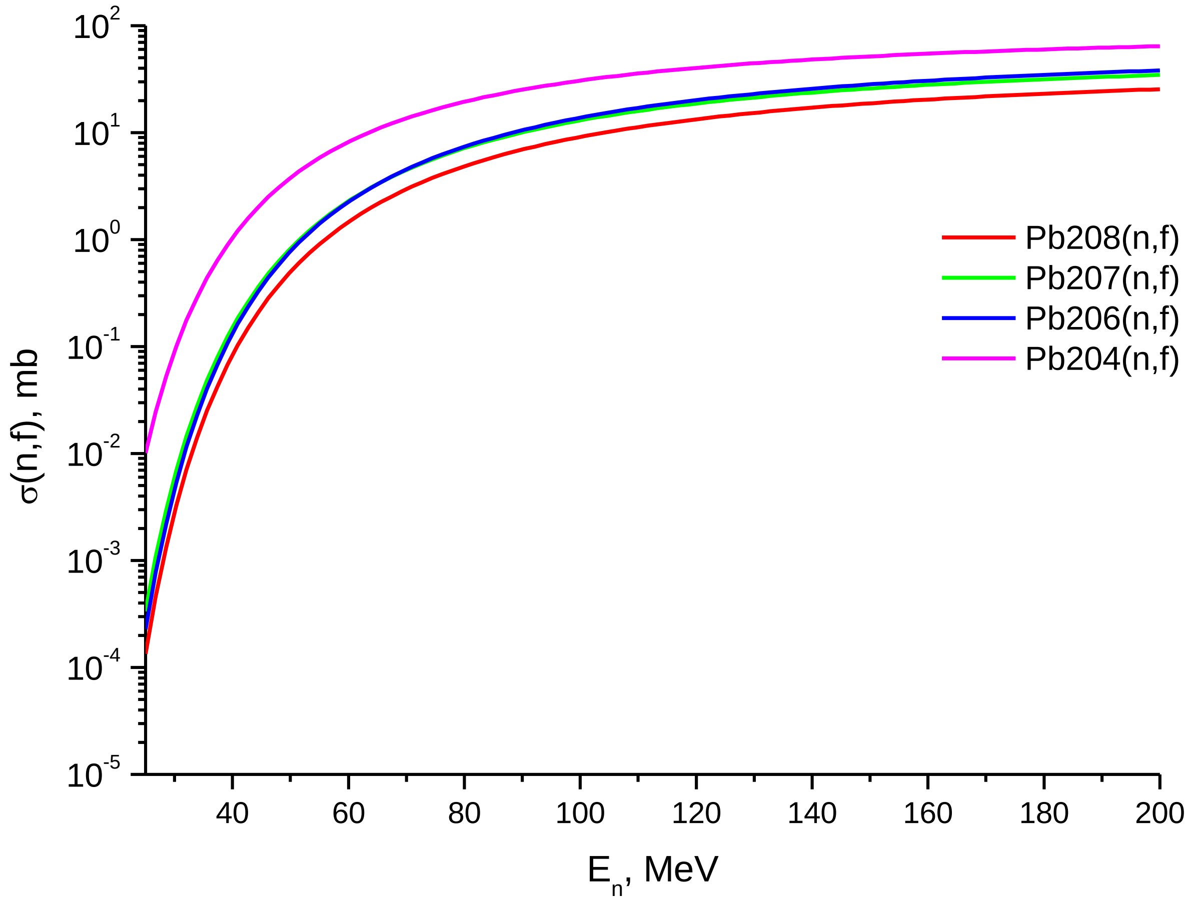
I.V.Ryzhov *et al*. NIM A **562** (2006) 439-448











$$\sigma_f(E_n) = p_1 \exp\left(-\left(p_2 / E_n\right)^{3/2}\right)$$

Fission and compound cross-sections

I. Ignatyuk A.V., Smirenkin G.N., Itkis M.G. et al approach **(Physics of Particles and Nuclei, v.16, #4 (1985))**

$$\sigma_f(E_n) = \frac{\pi}{k^2}\sum_J g(J)T_J(E_n)\frac{\Gamma_f^J}{\Gamma_f^J + \Gamma_n^J}$$

$$g(J)=\frac{2J+1}{(2s+1)(2I_0+1)}$$

Fissility

$$P_f(\mathrm{J})=\frac{\Gamma_f^J}{\Gamma_f^J + \Gamma_n^J}; \qquad P_f(E_n) = \frac{\sigma_f(E_n)}{\sigma_c(E_n)} = \sum_J P_f(J)\sigma_c(J)/\sigma_c$$

$$\sigma_c(E_n) \cong \sigma_R(E_n)$$
$$\sigma_R(E_n) = \sigma_{non-el}(E_n) - \sigma_{comp-el}(E_n)$$

Fission width

According to Bohr and Wheeler theory of fission:

$$\Gamma_f^J = \frac{1}{2\pi\rho_c(U_c, J)} \int_0^{U_f - B_f} \rho_f(U_f - B_f - E, J)\,dE$$

Here

$U_c = E^* - \Delta_c; \quad \Delta_c = 12\chi / \sqrt{A_c},$

$\chi = 0, 1, 2$ for odd-odd,odd-even or even-even nuclei

$U_f = E^* - \Delta_f; \quad \Delta_f = 14\chi / \sqrt{A_c},$

$\chi = 0, 1, 2$ for odd-odd,odd-even or even-even nuclei

$\boldsymbol{B_f} - fission\ barrier\ height$ (**first main parameter**)

Decay width

According to Weisskopf statistical theory of particle emission:

$$\Gamma_n^J = \frac{(2s+1)m_n}{(\pi\hbar)^2\,\rho_c(U_c,J)} \int_0^{U_n-B_n} \sigma_{inv}(E)\rho_n(U_n-B_n-E,J)EdE$$

$U_n = E^* - \Delta_n;\quad \Delta_c = 12\chi/\sqrt{A_n},$

$\chi = 0,1,2$ for odd-odd,odd-even or even-even nuclei

I approximation:

$$\sigma_{inv}(E) = \pi\left(r_0 A^{1/3}\right)^2$$

$$\Gamma_n^J = \frac{A_n^{2/3}}{\pi\omega\rho_c(U_c,J)} \int_0^{U_n-B_n} \rho_n(U_n-B_n-E)EdE$$

where

$$\omega = \hbar^2/2m_n r_0^2 \approx 10\text{ MeV}$$

Level densities

$$\rho_f(U,J)=\frac{\rho_f(U)}{2\sqrt{2\pi}\sigma_{||}}\sum_{K=-J}^{J}\exp\left\{-\frac{J(J+1)}{2\sigma_{\perp}^2}-\frac{K^2}{2\sigma_{eff}^2}\right\}$$

$$\sigma_i^2=\Im_i t/\hbar^2;\ \Im_{eff}=\Im_{\perp}\Im_{||}/\left(\Im_{\perp}-\Im_{||}\right)$$

I approximation:

$$\frac{\Gamma_f^J}{\Gamma_n^J}=\frac{\omega}{2A^{2/3}}\gamma(J)\frac{\int_0^{U_f-B_f}\rho_f(U_f-B_f-E,0)dE}{\int_0^{U_n-B_n}\rho_n(U_n-B_n-E,0)EdE}$$

where

$$\gamma(J)=\frac{\sqrt{2\pi}K_0}{2J+1}\exp\left[\beta\left(J+1/2\right)^2\right]erf\left(\frac{J+1/2}{\sqrt{2}K_0}\right)$$

$$\beta=\frac{1}{2}\left(\sigma_{\perp n}^{-2}-\sigma_{\perp f}^{-2}\right);\quad K_0=\sigma_{eff}^{(f)}$$

$$P_f(E)=\frac{\Gamma_f^0}{\Gamma_n^0}\bar{\gamma}(J_{\max});$$

Fissility **I** approximation:

$$\bar{\gamma}(J_{\max})=J_{\max}^{-2}\int_0^{J_{\max}}(2J+1)\gamma(J)dJ$$

$$\rho_F(U,J)=\frac{(2J+1)}{\sqrt{2}\sigma^3}\frac{\sqrt{\pi}}{24}a^{-1/4}\left(U-\delta\right)^{-5/4}\exp[2\sqrt{a\left(U-\delta\right)}-\frac{\left(J+1/2\right)^2}{2\sigma^2}]$$

$$\sigma^2=\frac{6\overline{m}^2}{\pi^2}\sqrt{a\left(U-\delta\right)}$$

a – level density parameter

δ – odd – even correction

$$\overline{m}^2\simeq\frac{\pi^2\Im_0}{6a}\simeq 0.24A^{2/3}$$

$$a_{f,n}(U,Z,N)=\tilde{a}_{f,n}(A)\left[1+\delta W_{f,g}(Z,N)f\left(U\right)/U\right]$$

$$f\left(U\right)=1-\exp(-\gamma U) \qquad \delta W_g=M_{exp}-M_{LDM}$$

$\boldsymbol{a_{f,n}}$ **– three additional parameters**:

$$\tilde{a}_n;\ \ \tilde{a}_f/\tilde{a}_n;\ \ \ \delta W_f$$

Ignatyuk A.V. et al *Journal of Nuclear Physics v.40, 1984, p.1404*

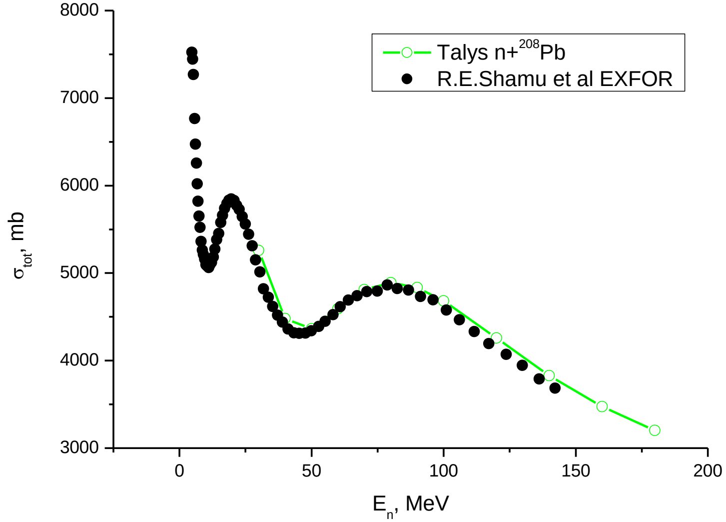
$$\tilde{a}_n = 0.094A \ \text{MeV}^{-1}; \quad \delta W_f = 0; \quad \tilde{a}_f = 1.03\tilde{a}_n$$

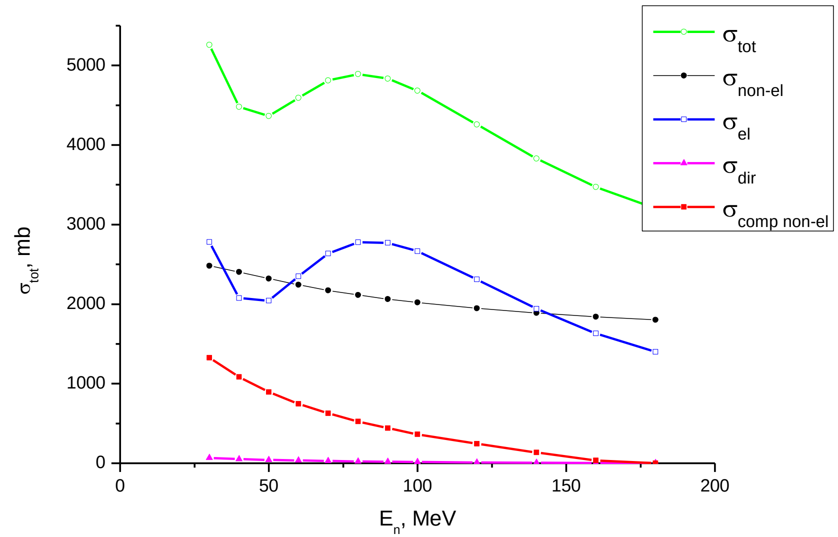
Nuclei	B_f, MeV	a_f, 1/MeV	a_n, 1/MeV
204	23.2	19.6	14.5 – 15.4
206	25.3	19.7	12.5 – 14.9
207	27.0	19.8	12.6 – 14.7
208	27.4	19.9	12.6 – 14.3

RIPL-2,3 fission barriers sistematic (G.N. Smirenkin compilation in RIPL-2 – Nucl.Data Sheets, **110** (2009) p.3107)

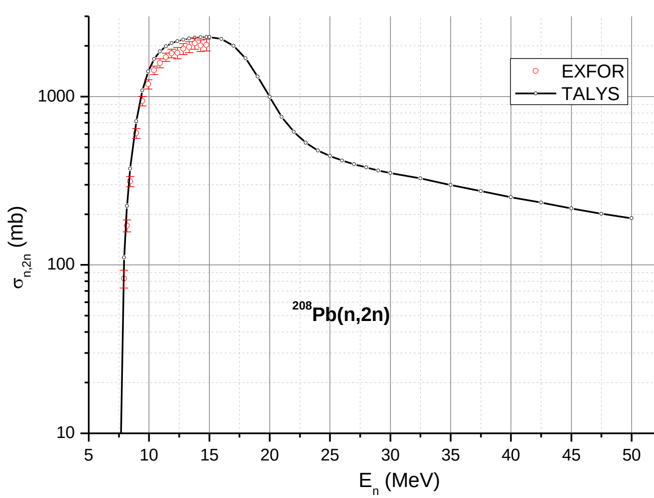
Z	A	Element	B_f
82	204	Pb	23.5
82	205	Pb	24.6
82	206	Pb	25.3
82	207	Pb	27.0
82	208	Pb	27.4

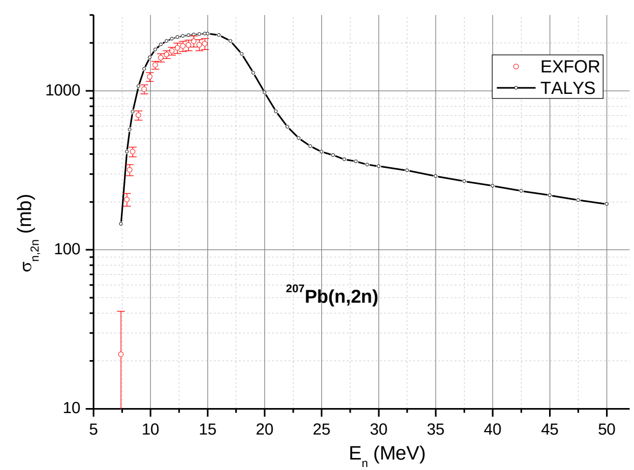
n+^{208}Pb





n+^{207}Pb

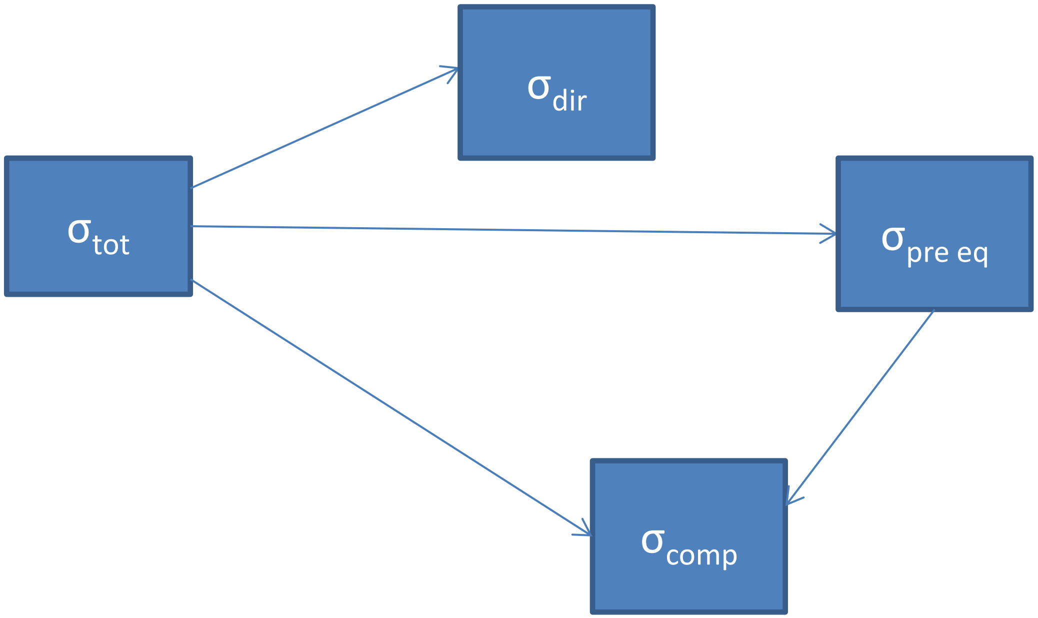




Code TALYS

Hierarchy of reaction steps are considered:

- Direct step – ECIS (coupled channel code)
- Pre-equilibrium step (Exciton model)
- Compound multi-step (**Hauser-Feshbach model)**



Fission and compound cross-sections: Hauser-Feshbach model

$$\sigma_f(J\Pi, E_n) = \sigma^C_{J\Pi}(E_{tot}) \frac{\Gamma_f(J\Pi, E_{tot})}{\Gamma(J\Pi, E_{tot})}$$

$$\Gamma(J\Pi, E_{tot}) = \Gamma_f(J\Pi, E_{tot}) + \Gamma_n(J\Pi, E_{tot}) + \Gamma_\gamma(J\Pi, E_{tot})$$

$$\sigma_f(E_n) = \sum_{J=\mathrm{mod}(I+s,1)}^{I+s+l_{\max}} \sum_{\Pi=-1}^{1} \sigma_f(J\Pi, E_{tot})$$

Level densities

$$\rho(E^*, J, \Pi) = K_{vib} K_{rot} \rho_{\text{int}}(E^*, J, \Pi)$$

For non-collective internal nuclear excitations it is used Fermi-gas expression:

$$\rho_{\text{int}}^{tot}(E^*) = \frac{1}{\sqrt{2\pi}\sigma} \frac{\sqrt{\pi}}{12} a^{-1/4} U^{-5/4} \exp[2\sqrt{aU}] \qquad R_F(E^*, J) = \frac{2J+1}{2\sigma^2} \exp\left[-\frac{(J+1/2)^2}{2\sigma^2} \right]$$

a – level density parameter

$$a(U, Z, N) = \tilde{a}(A)\left[1 + \delta W_g(Z, N) f(U)/U\right]$$

$$f(U) = 1 - \exp(-\gamma U) \qquad \delta W_g = M_{exp} - M_{LDM}$$

M_{LDM} – Myers and Swiatecki approximation

Back-shifted FG model

$$U = E^* - \Delta^{BFM}$$

$$\Delta^{BFM} = \chi \frac{12}{\sqrt{A}} + \delta$$

$\chi =$ -1 for odd-odd
0 for odd-even
+1 for even-even

Nuclei	δW_g, MeV	S_n, MeV	δ
Pb-204	-6.702	8.395	-0.36001
Pb-205	-7.563	6.732	0.06938
Pb-206	-8.393	8.087	-0.25303
Pb-207	-9.553	6.738	1.07741
Pb-208	**-9.961**	7.368	**1.43561**
Pb-209	-8.607	3.937	0.79324

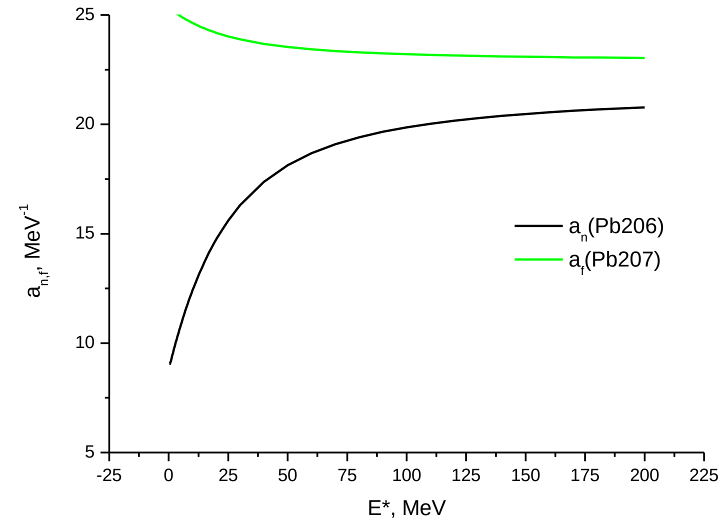
Nuclei	$D_{0\ exp}$, keV	$D_{0\ th}$, keV
Pb-205	2 ± 0.5	2.0
Pb-206	-	0.27
Pb-207	32 ± 6	24.0
Pb-208	38 ± 8	11.7
Pb-209	**400 ± 80**	675

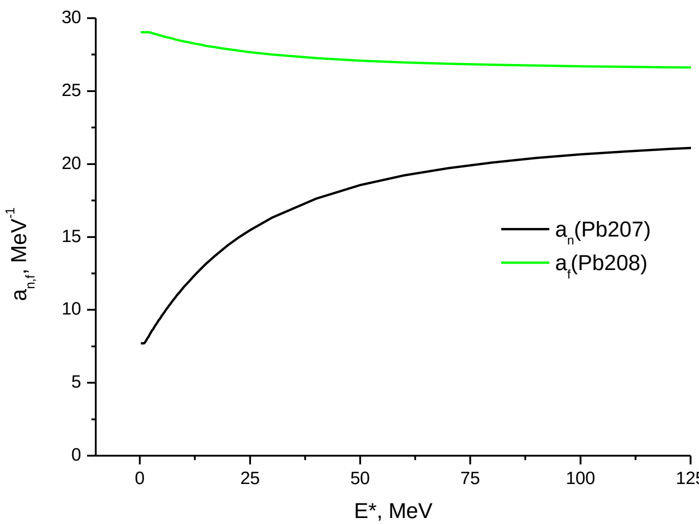
Ground state channel level densities
Back-Shifted Fermi Gas model

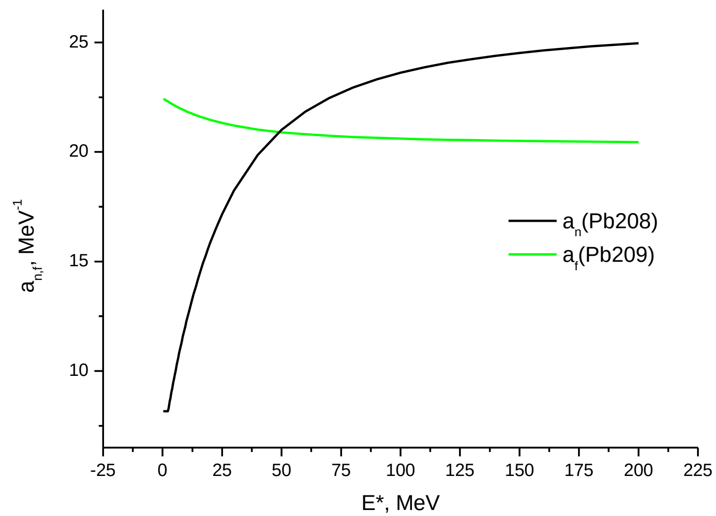
Nuclei	$a(S_n)$, MeV $^{-1}$	$\tilde{a}$, MeV^{-1}
Pb-205	12.65	21.86
Pb-206	11.84	21.69
Pb-207	10.33	22.86
Pb-208	11.01	**26.30**
Pb-209	9.44	20.30

Fission channel

Nuclei	δW_f, MeV	$\tilde{a}$, MeV^{-1}
Pb-205	1.5	
Pb-206	1.5	21.69
Pb-207	1.5	22.86
Pb-208	**1.5**	26.30
Pb-209	1.5	20.30







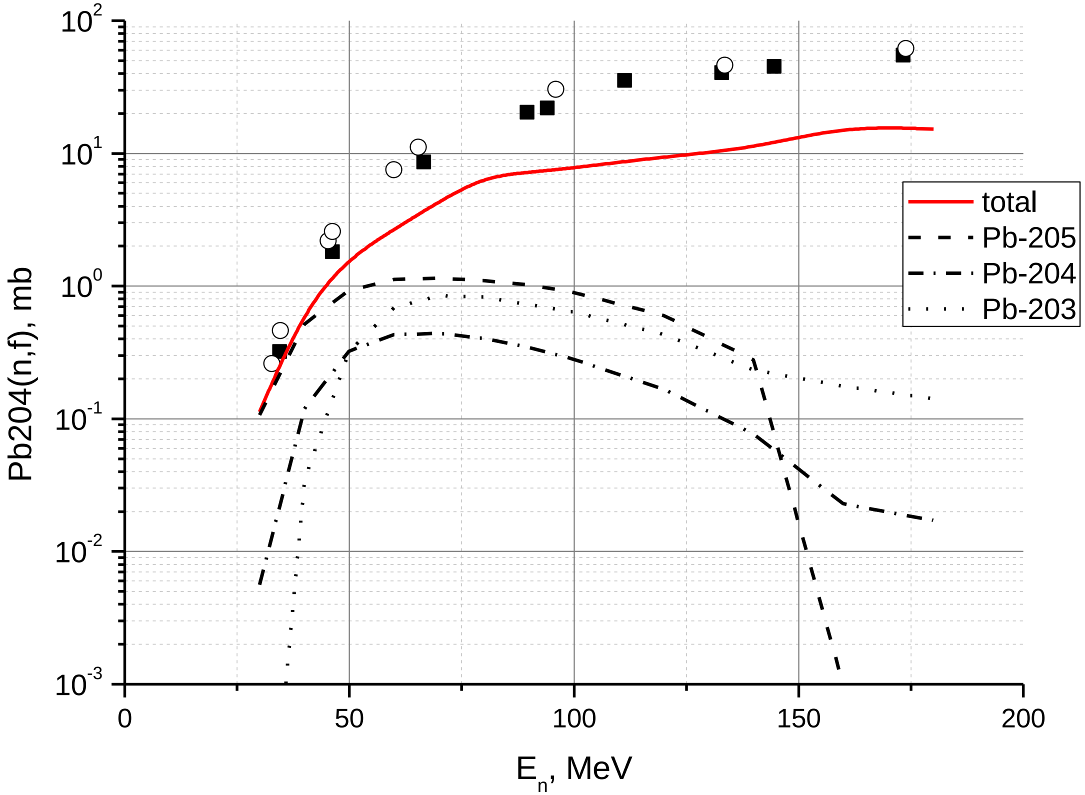
Fission barriers

Rotating-Finite-Range Model (RFRM) by Sierk: single-humped fission barrier heights are determined within a rotating liquid drop model, extended with finite-range effects in the nuclear surface energy and finite surface-diffuseness effects in the Coulomb energy. (Phys.Rev. C **33** (1986) p.2039)

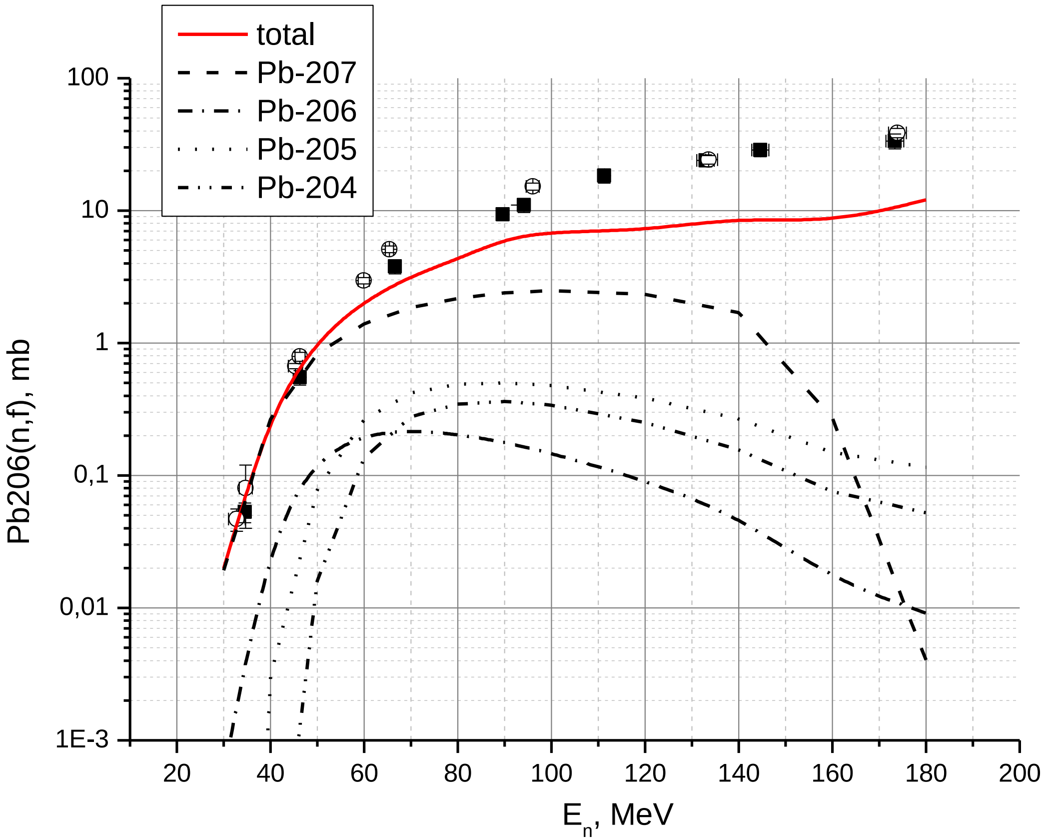
$$B_f = B_{RFRM} - (\delta W_{gs} - \delta W_f)$$

Nuclei	B_{RFRM} MeV	B_f, MeV	B_{f_ep}, MeV
Pb-205	12.85	21.9	**24.6**
Pb-207	12.98	24.0	**27.0**
Pb-208	13.15	24.6	**27.4**
Pb-209	13.22	23.3	

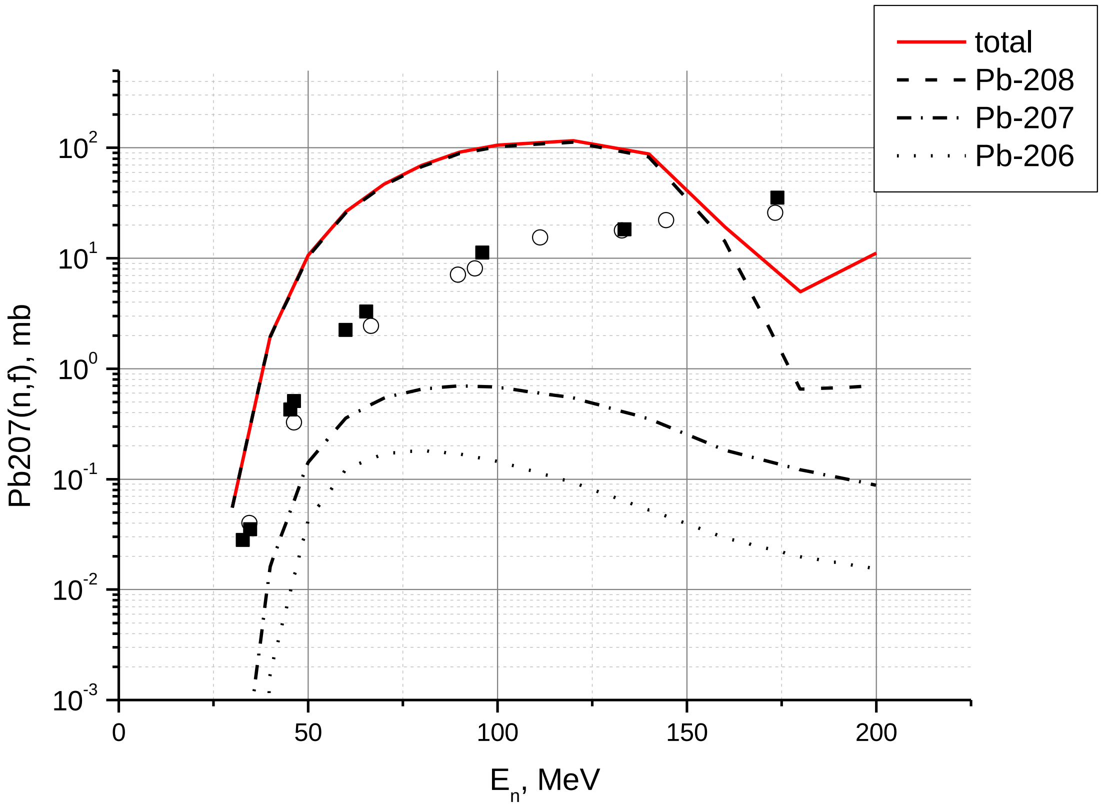
^{204}Pb(n,f) – reaction cross-section



^{206}Pb(n,f) – reaction cross-section

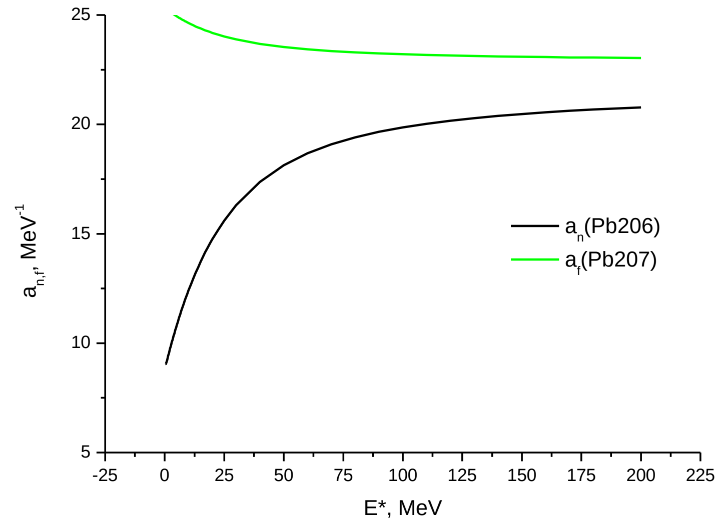


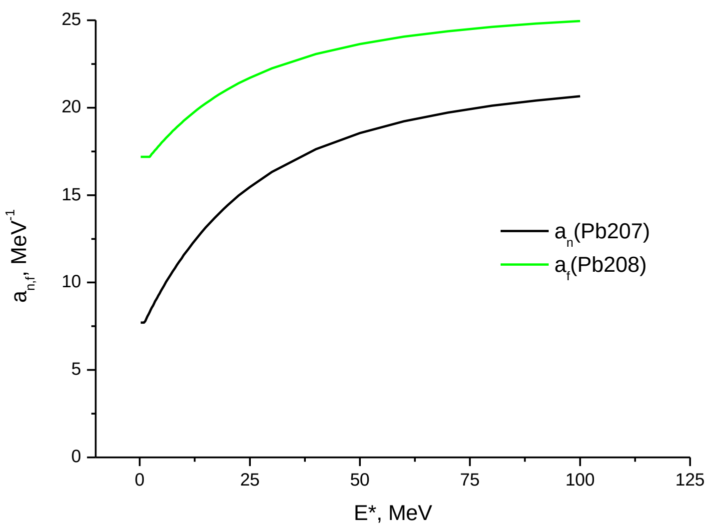
^{207}Pb(n,f) – reaction cross-section

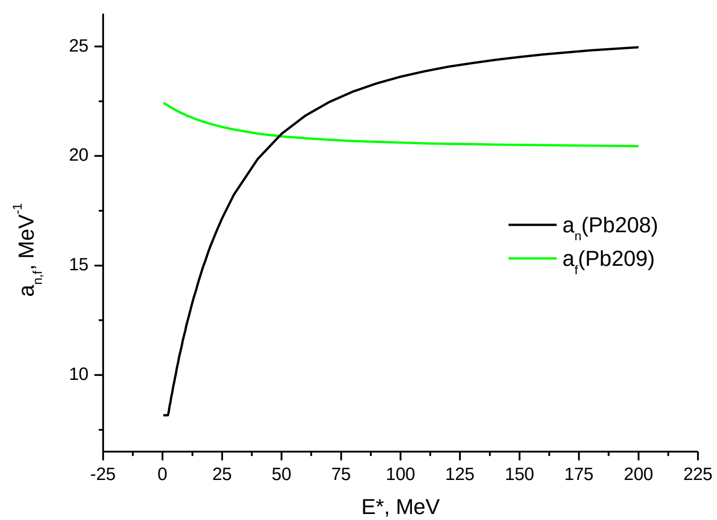


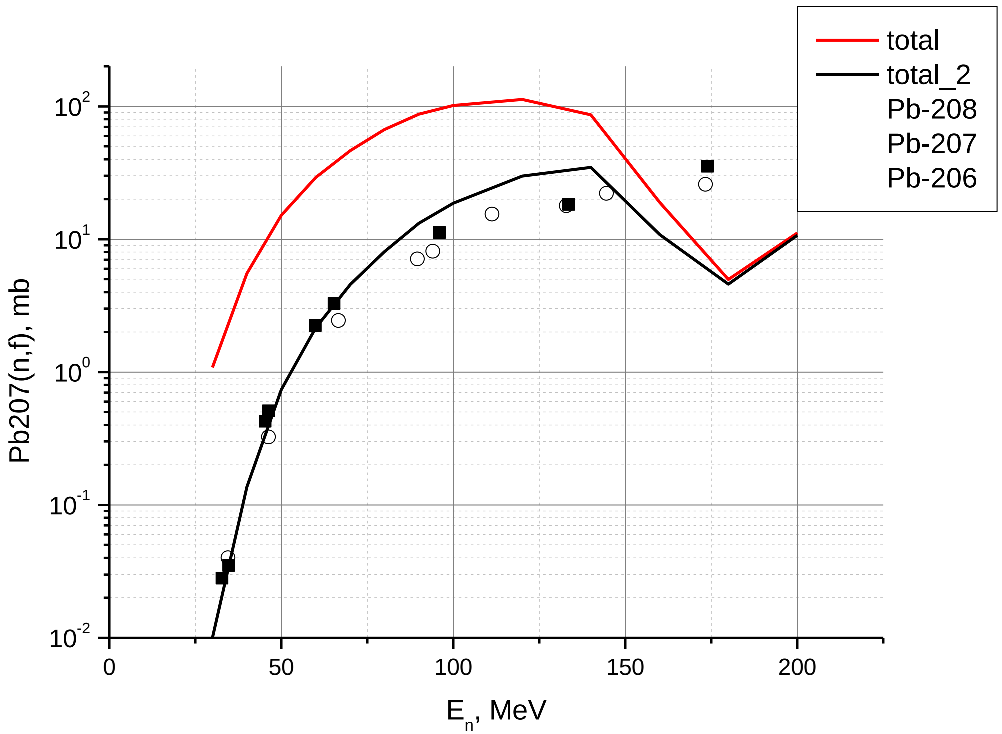
Fission channel

Nuclei	δW_f, MeV	$\tilde{a}$, MeV^{-1}
Pb-205	1.5	
Pb-206	1.5	21.69
Pb-207	1.5	22.86
Pb-208	**-5.**	26.30
Pb-209	1.5	20.30





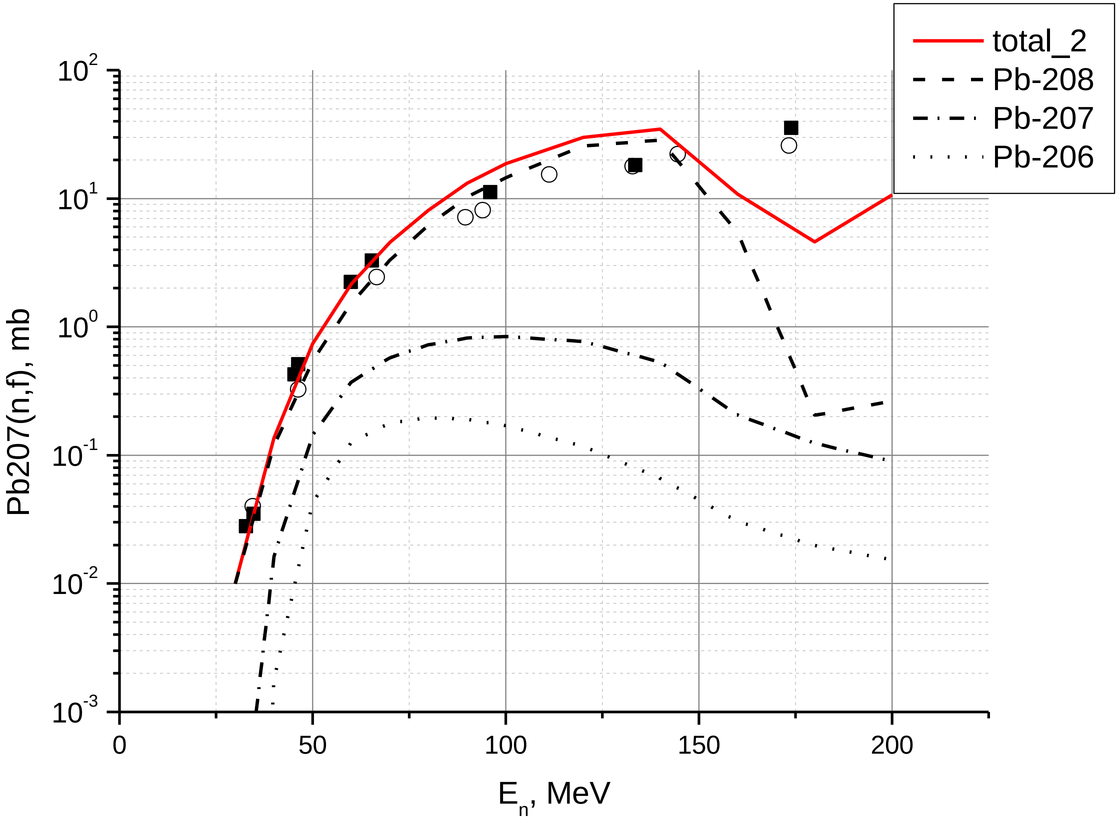




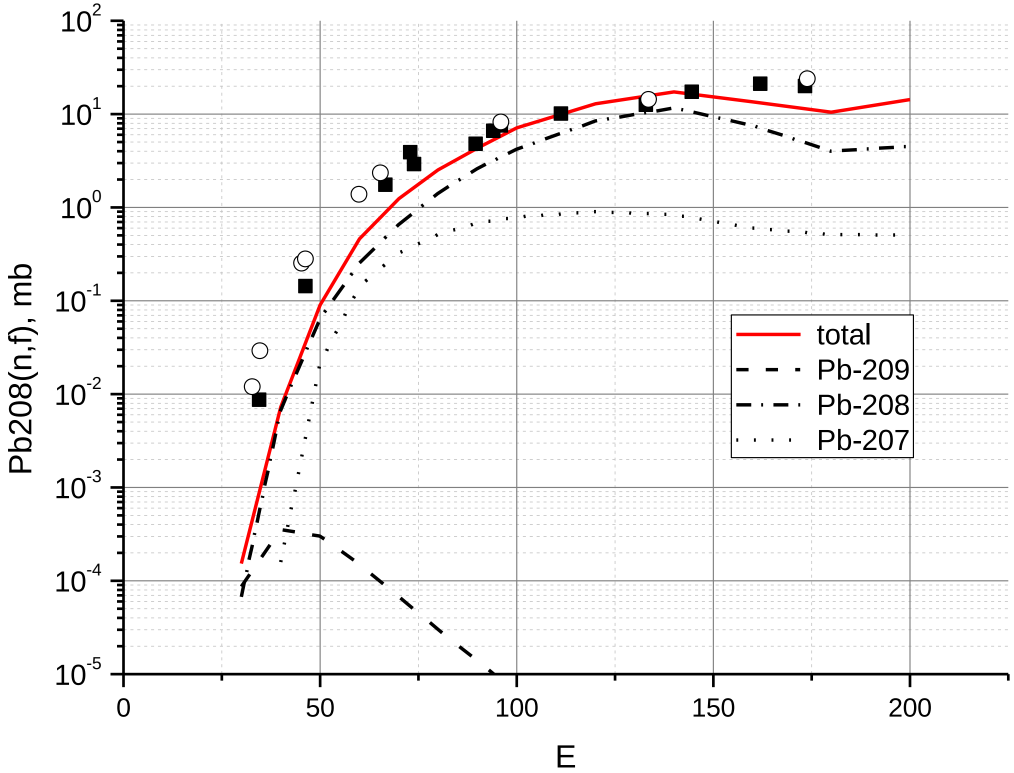
$$B_f = B_{RFRM} - (\delta W_{gs} - \delta W_f)$$

Nuclei	B_{RFRM} MeV	B_f, MeV	$B_{f\ mod}$, MeV
Pb-205	12.85	21.9	
Pb-207	12.98	24.0	
Pb-208	13.15	*18.1*	**22.0**
Pb-209	13.22	23.3	

^{207}Pb(n,f) – reaction cross-section



^{208}Pb(n,f) – reaction cross-section



Results

Z	A	Element	B_f, MeV RIPL-III	B_f, MeV This work
82	205	Pb	24.6	*21.9*
82	207	Pb	27.0	*24.0*
82	208	Pb	27.4	***22.0***
82	209	Pb		*23.3*

Main assumptions:

- TALYS code analysis
- Back-Shifted FG model of level densities

Thank you for the attention!