Generating function for nucleus-nucleus scattering amplitudes in Glauber theory

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A new approach to deal with the scattering amplitudes in Glauber theory is proposed. It relies on the use of generating function, that has been explicitly found. The method is applied to the analytical calculation of the nucleus-nucleus elastic scattering amplitudes in the all interaction orders of the Glauber theory.

The amplitude of the elastic scattering of the incident nucleus A on the fixed target nucleus B:

$$F^{el}_{AB}(q) = \frac{ik}{2\pi} \int d^2b\, e^{iqb} \big[1 - S_{AB}(b)\big],$$

q is the transferred momentum, k is the mean momentum carrying by a nucleon in nucleus A.
Glauber theory:

$$S_{AB}(b) = \langle A, B \,|\, \left\{ \prod_{ij} \big[1 - \Gamma_{NN}(b + x_i - y_j)\big] \right\} |\, A, B \rangle,$$

$$\Gamma_{NN}(b) = 1 - e^{i\chi_{NN}(b)} = \frac{1}{2\pi ik} \int d^2q\, e^{-iqb} f^{el}_{NN}(q),$$

$f^{el}_{NN}(q)$ is the nucleon-nucleon scattering

$$
\begin{aligned}
S_{AB}(b) &= \int \prod_{i=1}^{A} d^2x_i \int \prod_{j=1}^{B} d^2y_j \rho_A^{\perp}(x_1 - b, \ldots, x_A - b)\, \rho_B^{\perp}(y_1, \ldots, y_B) \\
&\times \left\{ \prod_{ij} \left[1 - \Gamma_{NN}(x_i - y_j)\right] \right\}.
\end{aligned}
$$

Nuclear densities

$$\rho_N^{\perp}(x_1, \ldots, x_N) = \int \prod_{i=1}^{N} dz_i\, \rho_N(z_1, x_1, \ldots, z_N, x_N).$$

$$\rho_N(r_1, \ldots, r_N) = \prod_{i=1}^{N} \rho_N(r_i), \qquad \rho_N^{\perp}(x_1, \ldots, x_N) = \prod_{i=1}^{N} \rho_N^{\perp}(x_i)$$

$$\int d^3r \rho_N(r) = 1, \qquad \int d^2x \rho_N^{\perp}(x) = 1.$$

$$C_0 \int D\Phi D\Phi^* \exp\Big\{ - \int d^2x d^2y\, \Phi(x)\Delta^{-1}(x-y)\Phi^*(y) + \sum_i \Phi(x_i) + \sum_j \Phi^*(y_j) \Big\} = \exp\Big\{ \sum_{i,j} \Delta(x_i - y_j) \Big\} = \prod_{i,j} e^{\Delta(x_i - y_j)},$$

$$e^{\Delta(x-y)} - 1 = -\Gamma_{NN}(x-y).$$

$$S_{AB}(b) = C_0 \int D\Phi D\Phi^* \exp\Big\{ - \int d^2x d^2y\, \Phi(x)\Delta^{-1}(x-y)\Phi^*(y) \Big\} \times \Big[\int d^2x\, \rho_A^{\perp}(x-b) e^{\Phi(x)} \Big]^A \Big[\int d^2y\, \rho_B^{\perp}(y) e^{\Phi^*(y)} \Big]^B.$$

Generating function:

$$Z(u,v) = \int D\Phi D\Phi^* \exp\Big\{ - \int d^2x d^2y\, \Phi(x)\Delta^{-1}(x-y)\Phi^*(y) + u \int d^2x\, \rho_{\perp}(x-b) e^{\Phi(x)} + v \int d^2x\, \rho_{\perp}(x) e^{\Phi^*(x)} \Big\},$$

$$S_{AB}(b) = \frac{1}{Z(0,0)} \frac{\partial^A}{\partial u^A} \frac{\partial^B}{\partial v^B} Z(u,v) \Big|_{u=v=0}.$$

$$f^{el}_{NN}(q) \;=\; ik\frac{\sigma^{tot}_{NN}}{4\pi}e^{-\frac{1}{2}\beta q^2},\;\Rightarrow\;\Gamma_{NN}(x) \;=\; \frac{\sigma^{tot}_{NN}}{4\pi\beta}e^{-\frac{x^2}{2\beta}},$$

σ^{tot}_{NN} is the total nucleon-nucleon cross section.

$$\Gamma_{NN}(x) \;\simeq\; \frac{1}{2}\,\sigma^{tot}_{NN}\,\delta^{(2)}(x).$$

$a = \sqrt{2\pi\beta} \simeq$ interaction radius.

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$$\Gamma_{NN}(x) \;\simeq\; \frac{1}{2}\,\sigma^{tot}_{NN}\,\delta^{(2)}(x).$$

$a = \sqrt{2\pi\beta} \simeq$ interaction radius. The discrete version:

$$C_0 \prod_{x_n} \int \frac{d\Phi(x_n) d\Phi^*(x_n)}{2\pi} \exp\left\{ -\sum_n \frac{1}{y}\Phi(x_n)\Phi^*(x_n) + \sum_i \Phi(x_i) + \sum_j \Phi^*(y_j) \right\}$$

$$= \exp\{y \sum_{i,j} \delta_{x_i,y_j}\}, \qquad e^{\,y\delta_{x_i,y_j}} \;=\; 1 \;+\; (e^{\,y} \;-\; 1)\delta_{x_i,y_j}$$

$$z_y \;=\; e^{\,y} \;-\; 1 \;=\; -\frac{1}{2}\,\frac{\sigma^{tot}_{NN}}{a^2}.$$

$$Z(u,v) = C\, e^{W_y(u,v)}, \qquad z_y = 1 - \frac{1}{2}\frac{\sigma_{NN}^{tot}}{a^2}, \qquad \begin{array}{l} a = \sqrt{2\pi\beta} \\ \simeq \text{interaction radius} \end{array},$$

$$W_y(u,v) = \frac{1}{a^2}\int d^2x\, \ln\Big(\sum_{M\leq A, N\leq B} \frac{z_y^{M\cdot N}}{M!N!}\left[a^2 u \rho_A^{\perp}(x-b)\right]^M \left[a^2 v \rho_B^{\perp}(x)\right]^N\Big).$$

$$t_{m,n}(b) = \frac{1}{a^2}\int d^2x\, \left[a^2 \rho_A^{\perp}(x-b)\right]^m \left[a^2 \rho_B^{\perp}(x)\right]^n.$$

$$t_{0,1}(b) = t_{1,0}(b) = 1 \Rightarrow W_y(u,v) = u + v + F(u,v).$$

$$S_{AB}(b) = \sum_{k,j\leq A,B} \frac{A!B!}{(A-k)!(B-j)!}\frac{1}{k!}\frac{\partial^k}{\partial u^k}\frac{1}{j!}\frac{\partial^j}{\partial v^j}\, e^{F(u,v)}\Big|_{u=v=0}$$

$$\underset{i,j\ll A,B}{\approx} e^{F(A,B)}.$$

Optical approximation:

$$t_{11}(b) \neq 0,$$

$$S_{AB} = e^{-\frac{1}{2}\,\sigma_{NN}^{tot}\,T_{AB}(b)},$$

$$T_{AB}(b) = A \cdot B \cdot t_{1,1}(b).$$

Rigid target approximation:

$$t_{1n}(b) \neq 0, \quad n \geq 1$$

$$S_{AB}(b) = \left[T_{rg}(b)\right]^A,$$

$$T_{rg}(b) = \int d^2x\,\rho_A^{\perp}(x-b) \times e^{-\frac{1}{2}\sigma_{NN}^{tot}\rho_B^{\perp}(x)}.$$

The total interaction cross section:

$$\sigma_{AB}^{tot} = \frac{4\pi}{k} \mathrm{Im} F_{AB}^{el}(q=0) = 2 \int d^2 b \left[1 - S_{AB}(b)\right].$$

The integrated elastic cross section:

$$\sigma_{AB}^{el} = \int d^2 b \left[1 - S_{AB}(b)\right]^2.$$

The total inelastic, or reaction, cross section:

$$\sigma_{AB}^{r} = \sigma_{AB}^{tot} - \sigma_{AB}^{el} = \int d^2 b \left[1 - |S_{AB}(b)|^2\right].$$

$$\rho_A(r) = \rho_0\left[1+\frac{1}{6}(A-4)\frac{r^2}{\lambda^2}\right]e^{-\frac{r^2}{\lambda^2}},$$

The reaction and the total cross sections of $^{12}C - {}^{12}C$ collision at the energy 950 MeV per nucleon and $R_{\rm rms} = 2.49$ fm.

	optical approximation	rigid target approximation	assuming $A \gg 1$	exact differentiating
σ^r, mb	952	911	857	867
σ^{tot}, mb	1572	1470	1371	1363

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