



# Time-Dependent Description of Reactions with Weakly Bound Nuclei $^{11}\text{Li}$ , $^{11}\text{Be}$

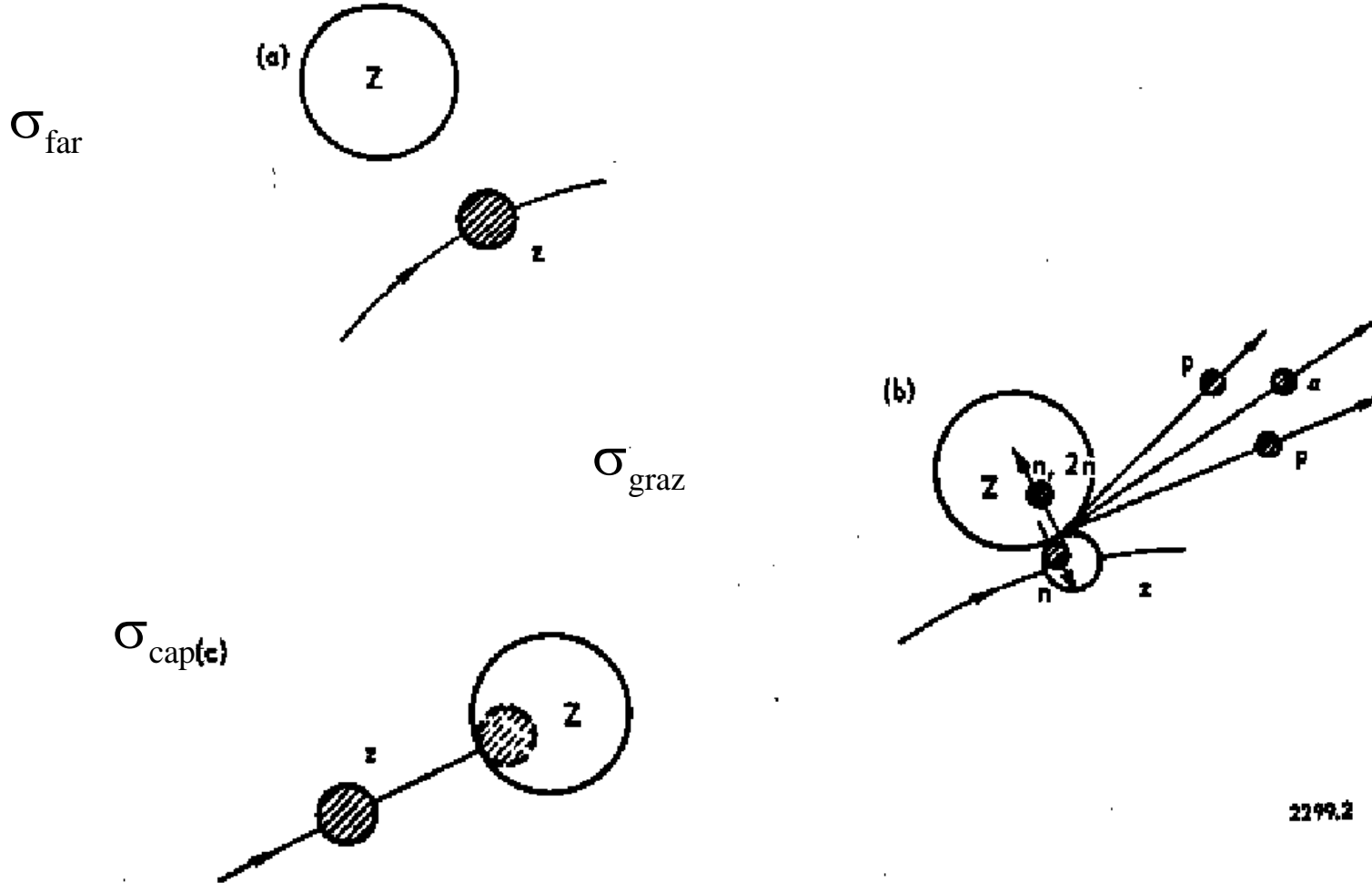
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Joint Institute for Nuclear Research, Dubna, Russia

# Basic processes at nucleus-nucleus collisions

The far interaction (*a*), grazing interaction (*b*) and fusion (capture) (*c*).

Total reaction cross section:  $\sigma_R = \sigma_{\text{capt}} + \sigma_{\text{graz}} + \sigma_{\text{far}}$



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# Theoretical model basics

The microscopic approach based on the numeric solution of the time-dependent Schrödinger equation [1-4] for the outer and inner neutrons and protons of colliding nuclei.

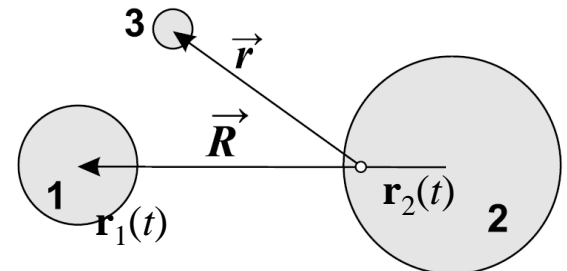
- Classical motion of cores.
- Time-dependent Schrödinger equation (**TDSE**) to describe neutron rearrangement and modification of the barrier of the nucleus-nucleus potential

$$i\hbar \frac{\partial}{\partial t} \Psi(\mathbf{r}, t) = \left\{ -\frac{\hbar^2}{2m} \Delta + V_1(|\mathbf{r} - \mathbf{r}_1(t)|) + V_2(|\mathbf{r} - \mathbf{r}_2(t)|) + \hat{V}_{LS}^{(1)}(\mathbf{r} - \mathbf{r}_1(t)) + \hat{V}_{LS}^{(2)}(\mathbf{r} - \mathbf{r}_2(t)) \right\} \Psi(\mathbf{r}, t), \quad \Psi(\mathbf{r}, t) = \begin{pmatrix} \psi(\mathbf{r}, t) \\ \phi(\mathbf{r}, t) \end{pmatrix}$$

- The initial wave functions were determined from the spherical shell model with parameters providing reasonable values of charge radius and separation energies of outer neutrons and protons.
- The initial conditions for the proton wave functions included the long-range character of the Coulomb interaction with the other nucleus. For instance, the proton wave function in the isolated projectile nucleus at a finite distance from the target nucleus was preliminarily subjected to slow (adiabatic) switching of the Coulomb interaction with the target nucleus. Thus, the polarization effects of the proton cloud were already taken into account in the initial condition.

- [1] V. Samarin. Phys. Atom. Nucl. **78**, 128 (2015).
- [2] V. Samarin, EPJ Web Conf. **66**, 03075 (2014).
- [3] V. Samarin, EPJ Web Conf. **86**, 00040 (2015).
- [4] V. Samarin. Phys. Atom. Nucl. **81**, 486 (2018).

Light quantum particle (nucleon) 3



Two heavy classical particles (cores) 1 and 2

# Halo Nucleus $^{11}\text{Li}$

## Chart of nucleus shape and size parameters

### Nucleus quadrupole deformation parameter $\beta_2$ :

- $\beta_2(B(E2)\uparrow)$  obtained using reduced transition probabilities.  
 [\[S.Raman, C.W.Nestor, P.Tikkanen, At.Data Nucl.Data Tables 78, 1 \(2001\)\]](#)
- $\beta_2(Q_{mom})$  obtained from the electric quadrupole moment values.  
 [\[N.J.Stone, At.Data Nucl.Data Tables, 90 75 \(2005\)\]](#)
- Calculated  $\beta_2$ -calc.  
 [\[B.S.Ishkhanov, V.N.Orlin, Yad.Fiz. 68, 1407 \(2005\)\]](#)



Z=5	$^6\text{B}$	$^7\text{B}$	$^8\text{B}$	$^9\text{B}$	$^{10}\text{B}$	$^{11}\text{B}$	$^{12}\text{B}$	$^{13}\text{B}$
Z=4	$^5\text{Be}$	$^6\text{Be}$	$^7\text{Be}$	$^8\text{Be}$	$^9\text{Be}$	$^{10}\text{Be}$	$^{11}\text{Be}$	$^{12}\text{Be}$
Z=3	$^3\text{Li}$	$^4\text{Li}$	$^5\text{Li}$	$^6\text{Li}$	$^7\text{Li}$	$^8\text{Li}$	$^9\text{Li}$	$^{10}\text{Li}$
Z=2	$^3\text{He}$	$^4\text{He}$	$^5\text{He}$	$^6\text{He}$	$^7\text{He}$	$^8\text{He}$	$^9\text{He}$	$^{10}\text{He}$

sign unknown:

	$0 \leq  \beta_2  < 0.2$
	$0.2 \leq  \beta_2  < 0.3$
	$0.3 \leq  \beta_2  < 0.4$
	$0.4 \leq  \beta_2  < 0.5$
	$ \beta_2  > 0.5$

positive:

	$0 < \beta_2 < +0.2$
	$+0.2 \leq \beta_2 < +0.3$
	$+0.3 \leq \beta_2 < +0.4$
	$+0.4 \leq \beta_2 < +0.5$
	$\beta_2 > +0.5$

negative:

	$0 > \beta_2 > -0.2$
	$-0.2 \geq \beta_2 > -0.3$
	$-0.3 \geq \beta_2 > -0.4$
	$-0.4 \geq \beta_2 > -0.5$
	$\beta_2 < -0.5$

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### $^{11}\text{Li}$ (Z=3)

## Chart of nucleus shape and size parameters

$\beta_2(Q_{mom})$

Parameter	Data
Q-moment (barn) for $3/2^-$ state at E = 0.000 MeV:	$-0.031 \pm 0.005$
$\beta_2(Q_{mom})$ :	$-0.636 \pm 0.121$
NSR Reference:	<a href="#">1992Ma12</a>
Journal Reference:	PL B281 16 (92)

# Halo Nucleus $^{11}\text{Li}$

## Chart of nucleus shape and size parameters

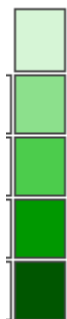
### Nucleus quadrupole deformation parameter $\beta_2$ :

- 1.  $\beta_2(B(E2)\uparrow)$  obtained using reduced transition probabilities.  [\[S.Raman, C.W.Nestor, P.Tikkanen, At.Data Nucl.Data Tables 78, 1 \(2001\)\]](#)
- 2.  $\beta_2(Q_{mom})$  obtained from the electric quadrupole moment values.  [\[N.J.Stone, At.Data Nucl.Data Tables, 90 75 \(2005\)\]](#)
- 3. Calculated  $\beta_2$ -calc.  [\[B.S.Ishkhanov, V.N.Orlin, Yad.Fiz. 68, 1407 \(2005\)\]](#)



	Z=5	$^6\text{B}$	$^7\text{B}$	$^8\text{B}$	$^9\text{B}$	$^{10}\text{B}$	$^{11}\text{B}$	$^{12}\text{B}$	$^{13}\text{B}$	
	Z=4	$^5\text{Be}$	$^6\text{Be}$	$^7\text{Be}$	$^8\text{Be}$	$^9\text{Be}$	$^{10}\text{Be}$	$^{11}\text{Be}$	$^{12}\text{Be}$	
	Z=3	$^3\text{Li}$	$^4\text{Li}$	$^5\text{Li}$	$^6\text{Li}$	$^7\text{Li}$	$^8\text{Li}$	$^9\text{Li}$	$^{10}\text{Li}$	$^{11}\text{Li}$
	Z=2	$^3\text{He}$	$^4\text{He}$	$^5\text{He}$	$^6\text{He}$	$^7\text{He}$	$^8\text{He}$	$^9\text{He}$	$^{10}\text{He}$	

sign unknown:



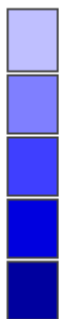
- $0 \leq |\beta_2| < 0.2$
- $0.2 \leq |\beta_2| < 0.3$
- $0.3 \leq |\beta_2| < 0.4$
- $0.4 \leq |\beta_2| < 0.5$
- $|\beta_2| > 0.5$

positive:



- $0 < \beta_2 < +0.2$
- $+0.2 \leq \beta_2 < +0.3$
- $+0.3 \leq \beta_2 < +0.4$
- $+0.4 \leq \beta_2 < +0.5$
- $\beta_2 > +0.5$

negative:



- $0 > \beta_2 > -0.2$
- $-0.2 \geq \beta_2 > -0.3$
- $-0.3 \geq \beta_2 > -0.4$
- $-0.4 \geq \beta_2 > -0.5$
- $\beta_2 < -0.5$

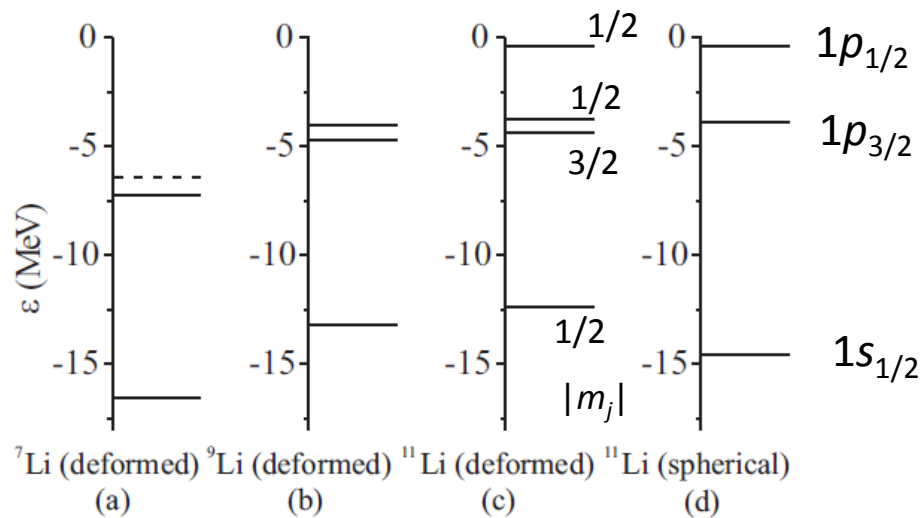


FIG. 7. Schemes of neutron levels for the nuclei  $^7\text{Li}$  (a),  $^9\text{Li}$  (b), and  $^{11}\text{Li}$  (c) in the shell model of the deformed nucleus and in the shell model of the spherical nucleus  $^{11}\text{Li}$  (d).

## Halo Nucleus $^{11}\text{Li}$

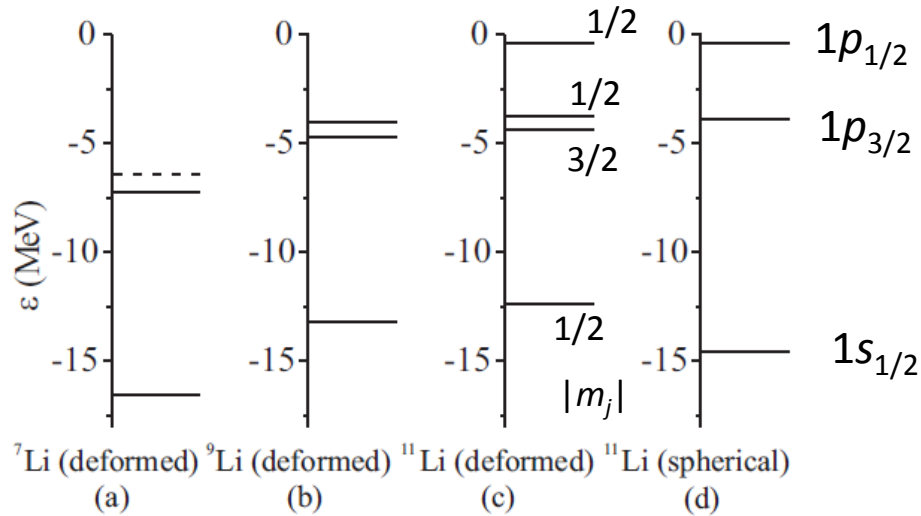
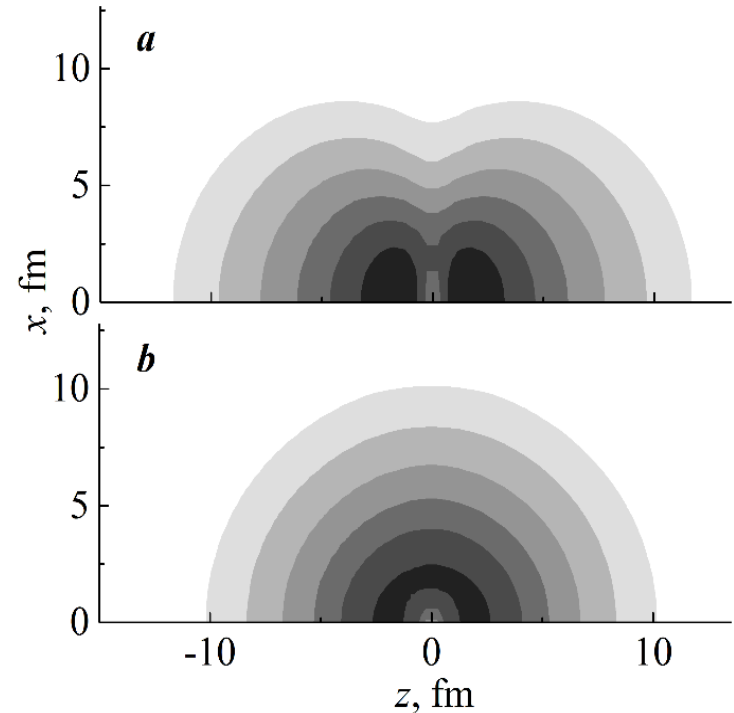


FIG. 7. Schemes of neutron levels for the nuclei  $^7\text{Li}$  (a),  $^9\text{Li}$  (b), and  $^{11}\text{Li}$  (c) in the shell model of the deformed nucleus and in the shell model of the spherical nucleus  $^{11}\text{Li}$  (d).

Results of the shell model of deformed nuclei [1, 2].



Probability density for the outer neutron level of  $^{11}\text{Li}$  nucleus with quantum number  $|m_j|=1/2$ :  
 (a) in the shell model of deformed nucleus ( $\beta_2=-0.6$ );  
 (b) in the shell model of spherical nucleus.

[1] Samarin V.V. Phys. Atom. Nucl. 2010. V.73. P. 1416.

[2] Samarin V.V. Phys. Atom. Nucl. 2015. V.78. P. 128.

# Halo Nucleus $^{11}\text{Be}$

## Chart of nucleus shape and size parameters

### Nucleus quadrupole deformation parameter $\beta_2$ :

- 1.  $\beta_2(B(E2)\uparrow)$  obtained using reduced transition probabilities. [\[S.Raman, C.W.Nestor, P.Tikkanen, At.Data Nucl.Data Tables 78, 1 \(2001\)\]](#)
- 2.  $\beta_2(Q_{\text{mom}})$  obtained from the electric quadrupole moment values. [\[N.J.Stone, At.Data Nucl.Data Tables, 90 75 \(2005\)\]](#)
- 3. Calculated  $\beta_2$ -calc. [\[B.S.Ishkhanov, V.N.Orlin, Yad.Fiz. 68, 1407 \(2005\)\]](#)



Z=5	$^6\text{B}$	$^7\text{B}$	$^8\text{B}$	$^9\text{B}$	$^{10}\text{B}$	$^{11}\text{B}$	$^{12}\text{B}$	$^{13}\text{B}$
Z=4	$^5\text{Be}$	$^6\text{Be}$	$^7\text{Be}$	$^8\text{Be}$	$^9\text{Be}$	$^{10}\text{Be}$	$^{11}\text{Be}$	$^{12}\text{Be}$
Z=3	$^3\text{Li}$	$^4\text{Li}$	$^5\text{Li}$	$^6\text{Li}$	$^7\text{Li}$	$^8\text{Li}$	$^9\text{Li}$	$^{10}\text{Li}$
Z=2	$^3\text{He}$	$^4\text{He}$	$^5\text{He}$	$^6\text{He}$	$^7\text{He}$	$^8\text{He}$	$^9\text{He}$	$^{10}\text{He}$

sign unknown:

	$0 \leq  \beta_2  < 0.2$
	$0.2 \leq  \beta_2  < 0.3$
	$0.3 \leq  \beta_2  < 0.4$
	$0.4 \leq  \beta_2  < 0.5$
	$ \beta_2  > 0.5$

positive:

	$0 < \beta_2 < +0.2$
	$+0.2 \leq \beta_2 < +0.3$
	$+0.3 \leq \beta_2 < +0.4$
	$+0.4 \leq \beta_2 < +0.5$
	$\beta_2 > +0.5$

negative:

	$0 > \beta_2 > -0.2$
	$-0.2 \geq \beta_2 > -0.3$
	$-0.3 \geq \beta_2 > -0.4$
	$-0.4 \geq \beta_2 > -0.5$
	$\beta_2 < -0.5$

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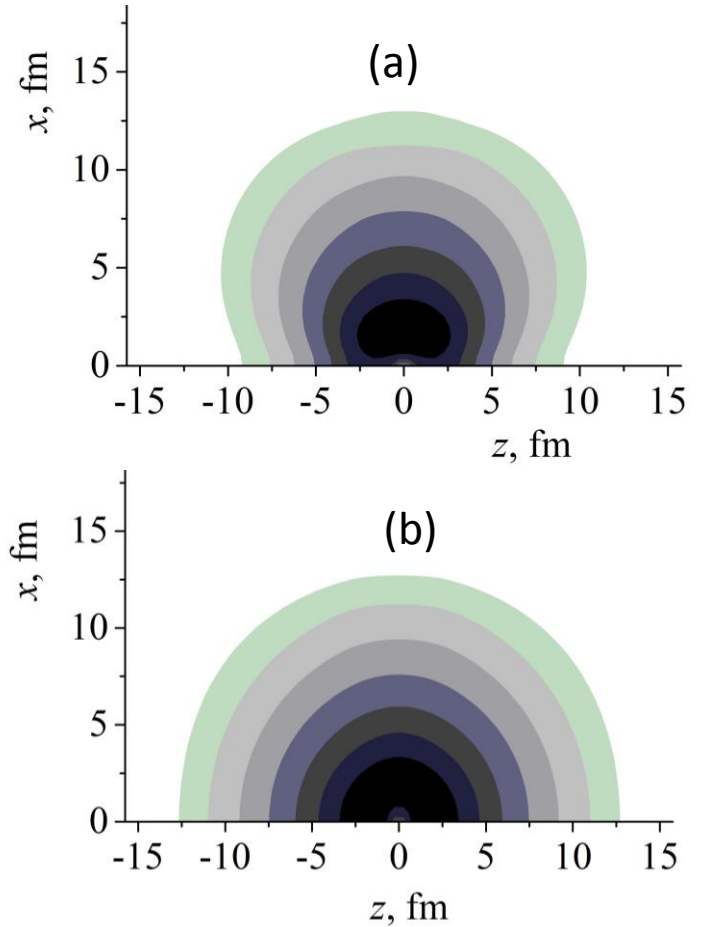
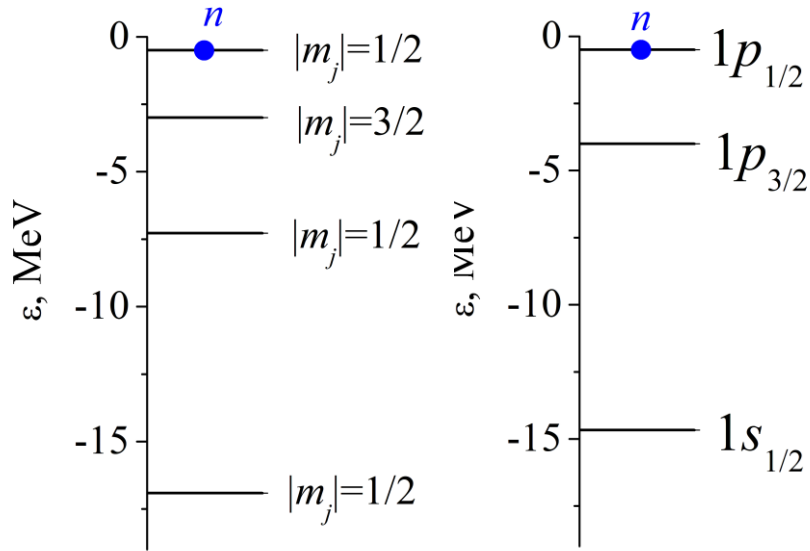
## $^{11}\text{B}$ (Z=5)

### Chart of nucleus shape and size parameters

$\beta_2(Q_{\text{mom}})$

Parameter	Data
Q-moment (barn) for $3/2^-$ state at $E = 0.000$ MeV:	$+0.0407 \pm 0.0003$
$\beta_2(Q_{\text{mom}})$ :	$+0.498 \pm 0.029$
NSR Reference:	<a href="#">1970Ne21</a>
Journal Reference:	PR A2 1208 (70)

# Halo Nucleus $^{11}\text{Be}$



Neutron level of  $^{11}\text{Be}$  nucleus:  
 (a) in the shell model of deformed nucleus ( $\beta_2=0.5$ );  
 (b) in the shell model of spherical nucleus.

Results of the shell model of deformed nuclei [1, 2].

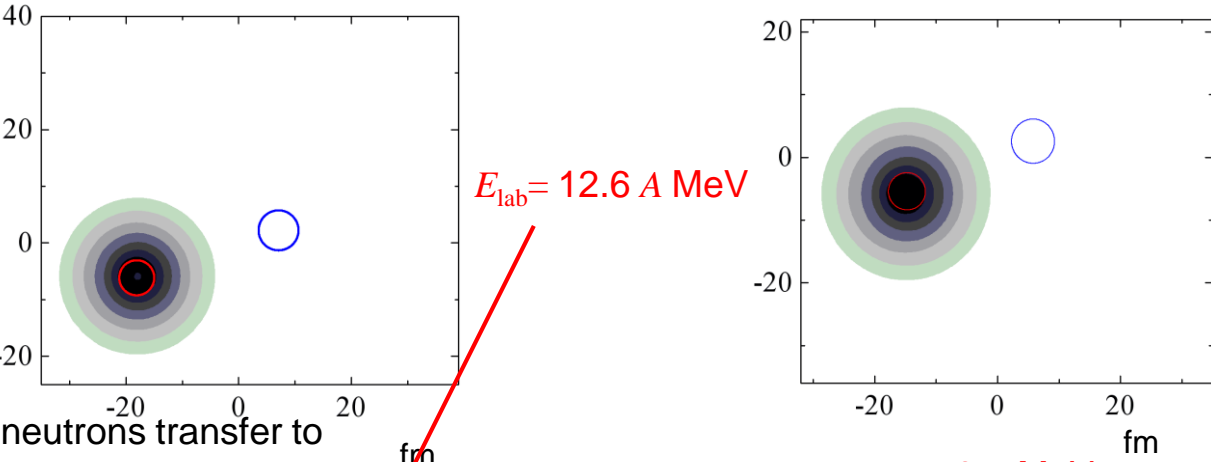
[1] Samarin V.V. Phys. Atom. Nucl. 2010. V.73. P. 1416.  
 [2] Samarin V.V. Phys. Atom. Nucl. 2015. V.78. P. 128.

Probability density for the outer neutron level of  $^{11}\text{Be}$  nucleus with quantum number  $|m_j|=1/2$ :  
 (a) in the shell model of deformed nucleus ( $\beta_2=0.5$ );  
 (b) in the shell model of spherical nucleus.

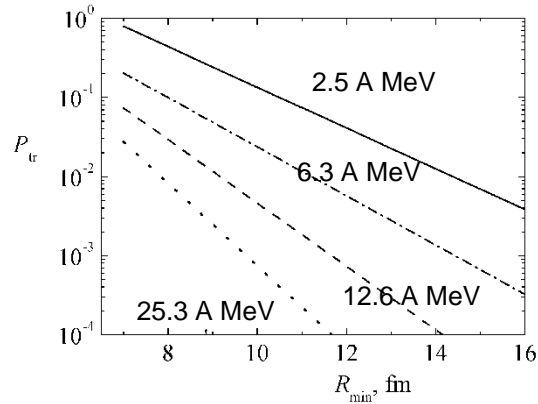


# Dynamics of outer halo neutrons of $^{11}\text{Li}$ during collision $^{11}\text{Li} + ^{28}\text{Si}$ in the center of mass

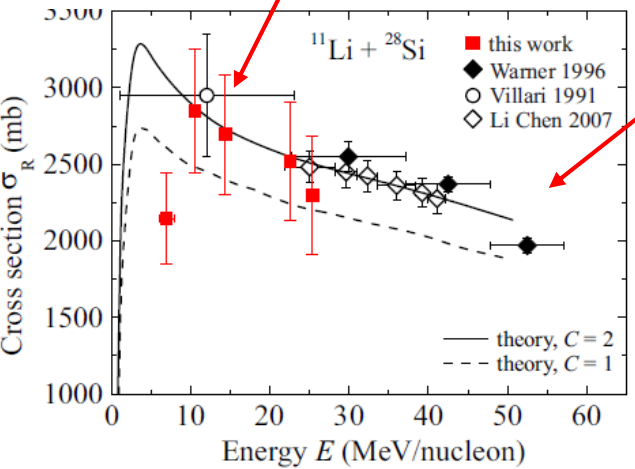
$^{11}\text{Li}$  was represented as core  $^9\text{Li} + n + n$



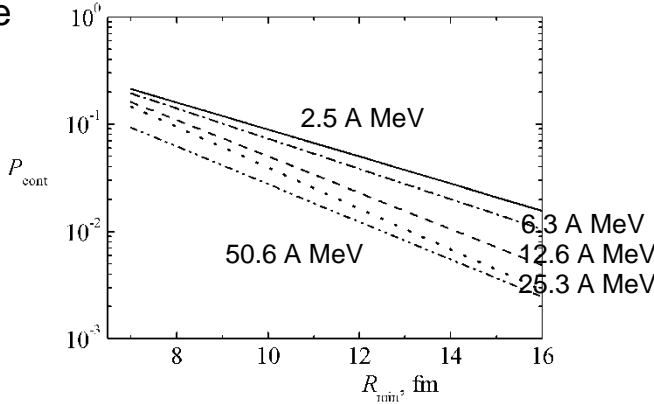
neutrons transfer to bound and unbound states



The probabilities of neutron transfer to unoccupied bound states of discrete spectrum of the  $^{28}\text{Si}$



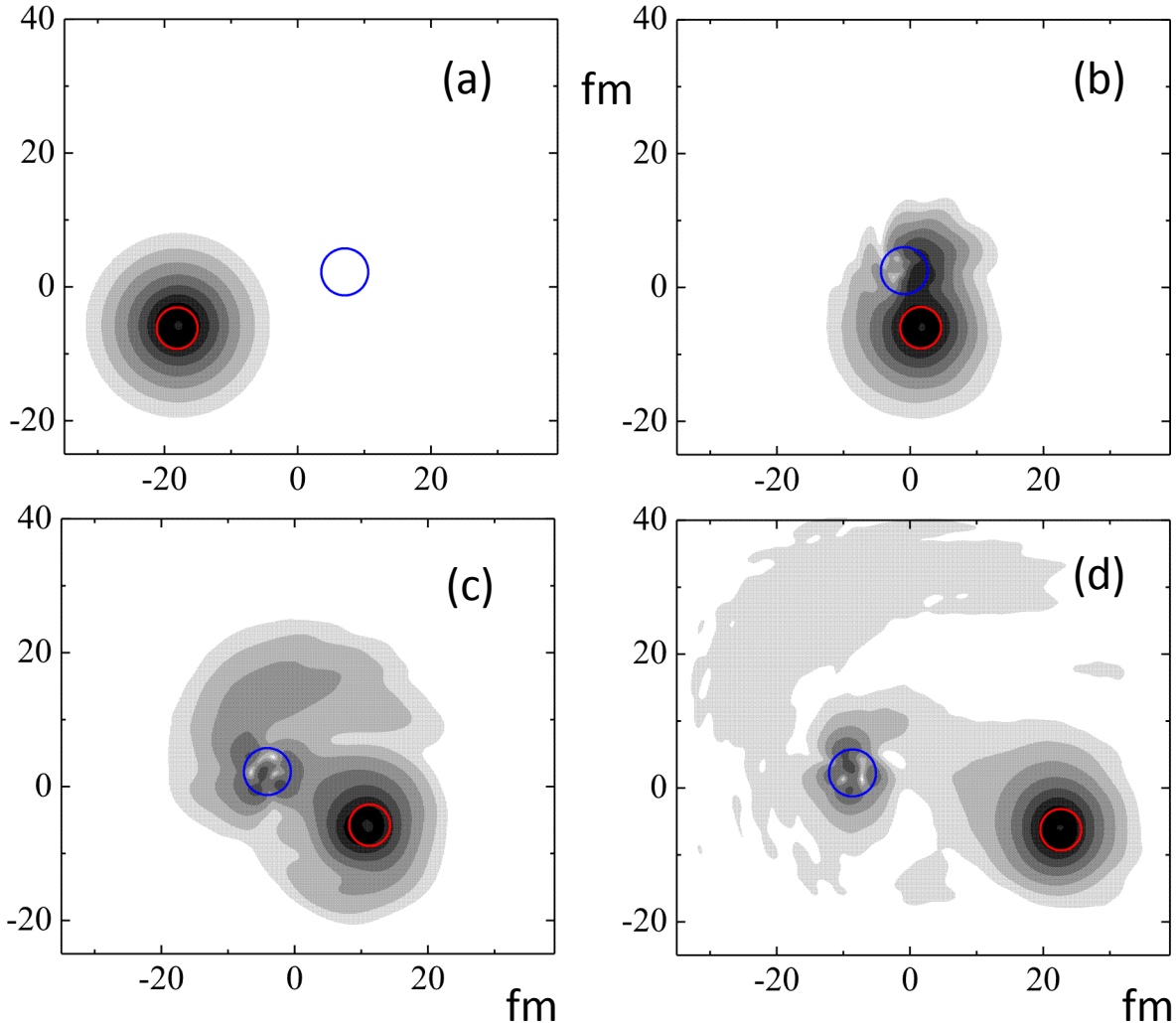
breakup of projectile and escape of neutrons forward



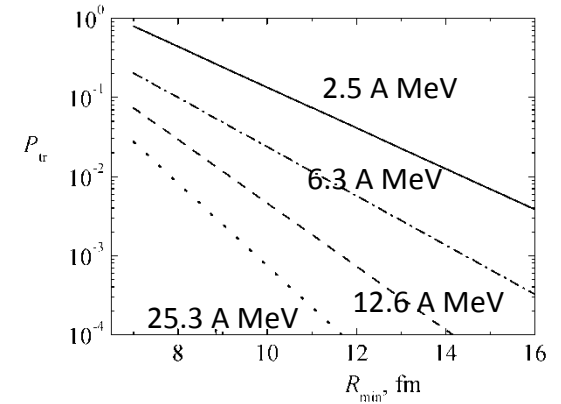
The probabilities of neutron transfer to states of continuous spectrum

FIG. 11. The total cross section for the  $^{11}\text{Li} + ^{28}\text{Si}$  reaction. Symbols are experimental data. Filled (red) squares are the results of this work, filled diamonds (Warner 1996 [5]), empty circle (Villari 1991 [16]), and empty diamonds (Li Chen 2007 [17]). Curves are the results of calculations for the values of the adjustable parameter  $C = 2$  (solid curve) and  $C = 1$  (dashed curve) with the probability  $P_c$  of transition to the states of the continuous spectrum calculated

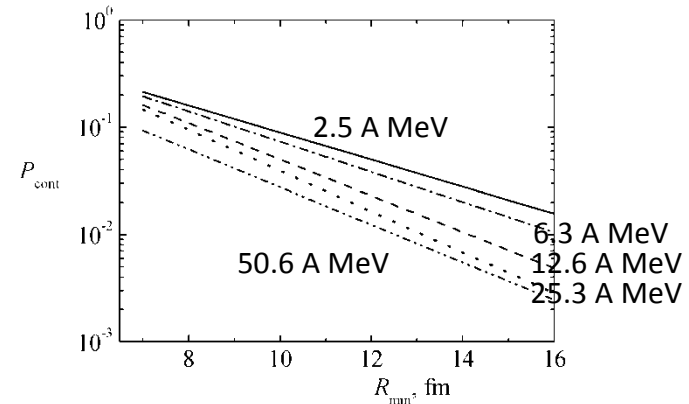
# Time-dependent microscopic description of outer neutrons of $^{11}\text{Li}$ during the collision $^{11}\text{Li} + ^{28}\text{Si}$ in the center of mass



An example of the evolution of the probability density  $\rho(\mathbf{r}, t)$  of external neutrons of the  $^{11}\text{Li}$  nucleus in its collision with the  $^{28}\text{Si}$  nucleus at energy  $E_{\text{lab}} = 12.6 \text{ A MeV}$  ( $E_{\text{cm}} = 100 \text{ MeV}$ ). The course of time corresponds to panel locations (a, b, c, d).

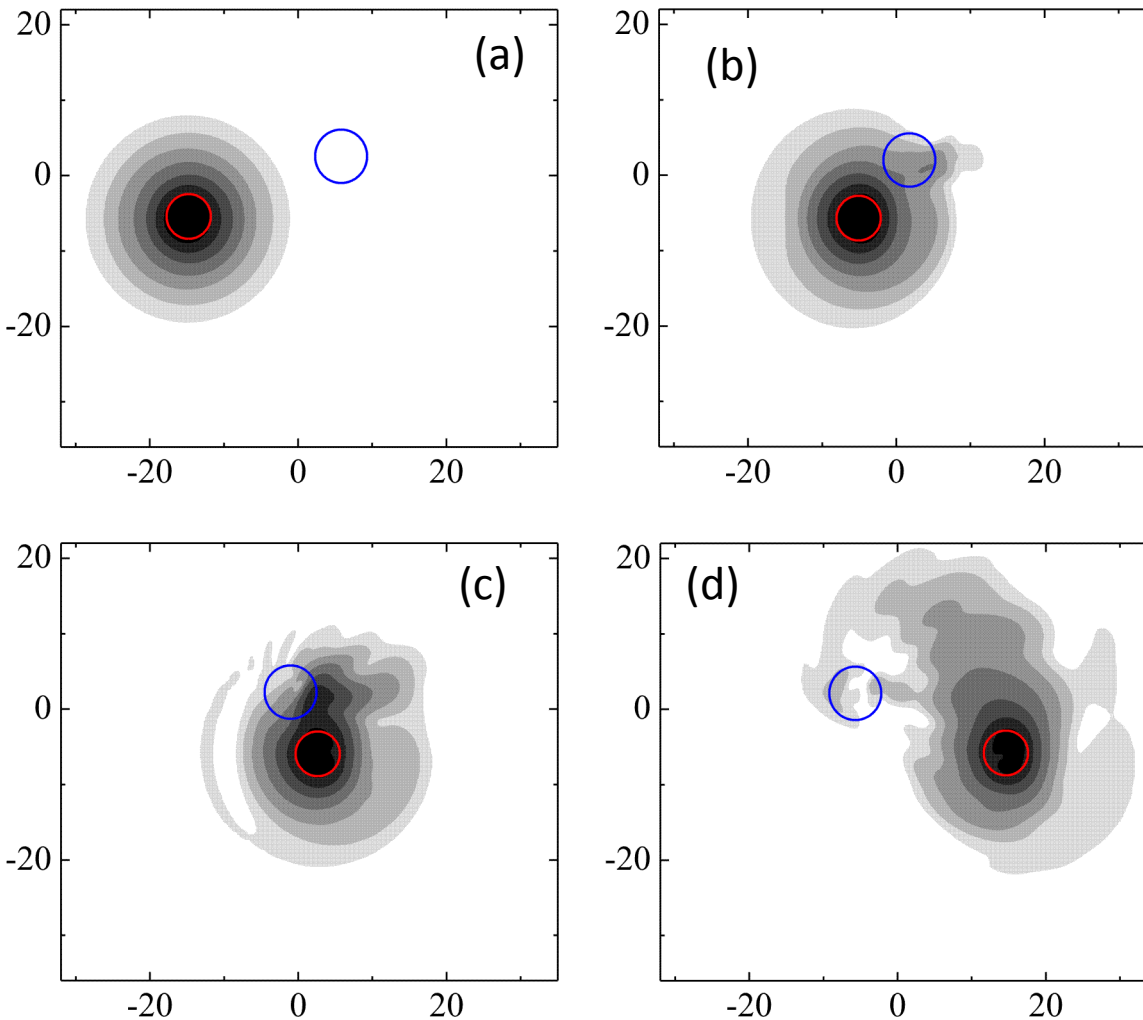


The probabilities of neutron transfer to unoccupied bound states of discrete spectrum of the  $^{28}\text{Si}$

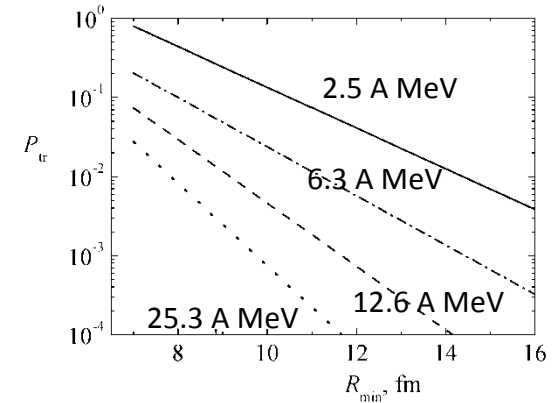


The probabilities of neutron transfer to states of continuous spectrum

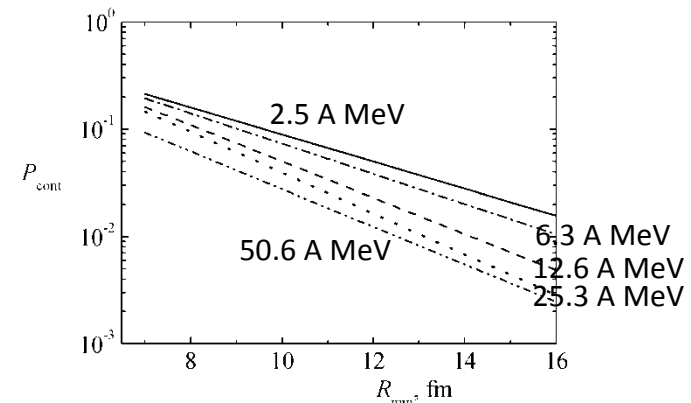
# Time-dependent microscopic description of outer neutrons of $^{11}\text{Li}$ during the collision $^{11}\text{Li} + ^{28}\text{Si}$ in the center of mass



An example of the evolution of the probability density  $\rho(\mathbf{r}, t)$  of external neutrons of the  $^{11}\text{Li}$  nucleus in its collision with the  $^{28}\text{Si}$  nucleus at energy  $E_{\text{lab}} = 50.6 \text{ A MeV}$  ( $E_{\text{cm}} = 400 \text{ MeV}$ ). The course of time corresponds to panel locations (a, b, c, d).

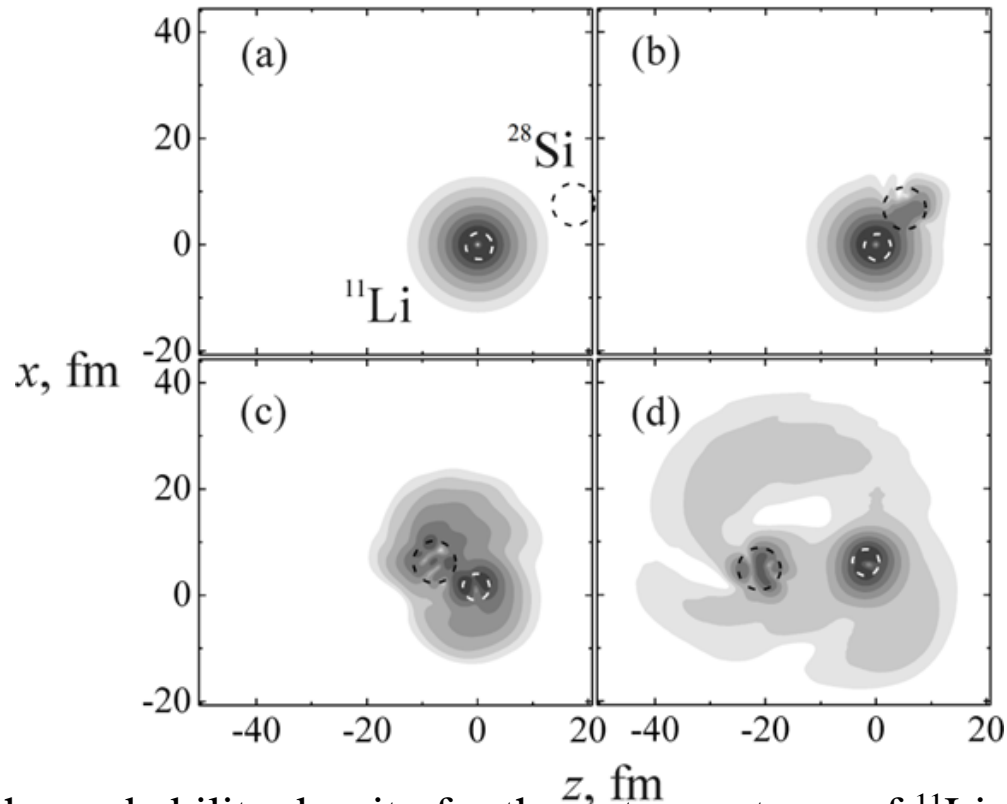


The probabilities of neutron transfer to unoccupied bound states of discrete spectrum of the  $^{28}\text{Si}$



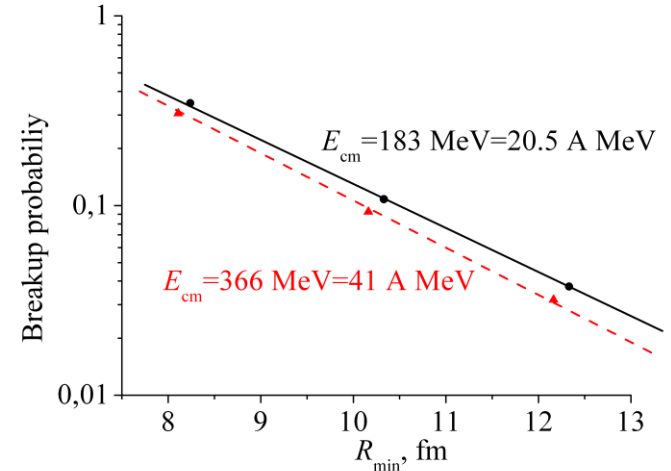
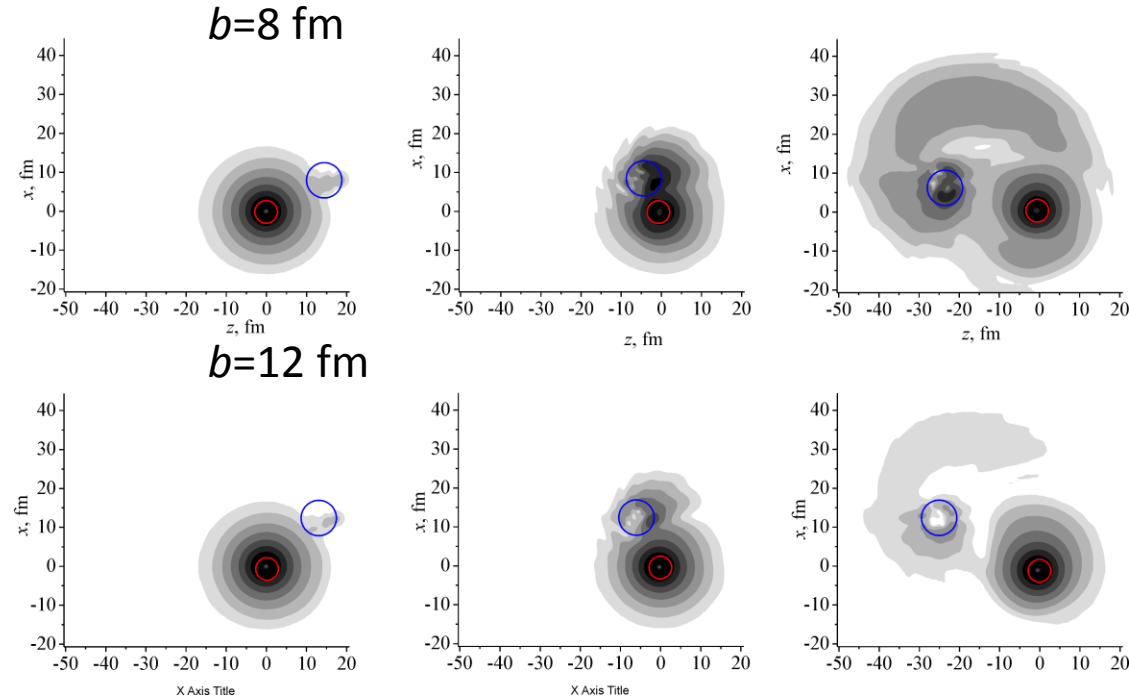
The probabilities of neutron transfer to states of continuous spectrum

# Time-dependent microscopic description of outer neutrons of $^{11}\text{Li}$ during the collision $^{11}\text{Li} + ^{28}\text{Si}$ in the moving reference system



Evolution of the probability density for the outer neutrons of  $^{11}\text{Li}$  with initial state  $1p_{1/2}$  in collision  $^{11}\text{Li} + ^{28}\text{Si}$  for  $E_{\text{c.m.}} = 79$  MeV, along with impact parameter  $b=7$  fm in a reference system moving relative to the laboratory system with a constant velocity equal to that of a projectile at a fairly large distance from the target nucleus. The course of time corresponds to the panel locations (a), (b), (c), (d). Greyscale gradation in the logarithmic scale is used. The radii of the circles correspond to those of nuclear cores of 2.4 fm and 3.8 fm respectively.

# Time-dependent microscopic description of outer neutrons of $^{11}\text{Be}$ during the collision $^{11}\text{Li} + ^{48}\text{Ti}$ in the moving reference system



The probabilities of neutron transfer to states of continuous spectrum

Evolution of the probability density for the outer neutrons of  $^{11}\text{Be}$  with initial state  $1p_{1/2}$  in collision  $^{11}\text{Li} + ^{48}\text{Ti}$  for  $E_{\text{c.m.}} = 183$  MeV, along with impact parameter  $b=8$  fm in a reference system moving relative to the laboratory system with a constant velocity equal to that of a projectile at a fairly large distance from the target nucleus. The course of time corresponds to the panel locations (a), (b), (c), (d). Greyscale gradation in the logarithmic scale is used. The radii of the circles correspond to those of nuclear cores of 2.8 fm and 4.5 fm respectively.

# Conclusions

- The numerical solution of the time-dependent Schrödinger equation is applied to analysis of dynamics of nucleon transfer and rearrangement at energies near and above the Coulomb barrier.
- The evolution of wave functions of outer nucleons is used for the description of neutron transfer and breakup in reactions  $^{11}\text{Li} + ^{28}\text{Si}$ ,  $^{11}\text{Be} + ^{48}\text{Ti}$ . The calculations of transfer and breakup cross sections are in progress.



Thank you  
for attention!

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