



## Study of Fusion, Transfer, and Breakup Reactions with Weakly Bound Nuclei ${}^6,8\text{He}$

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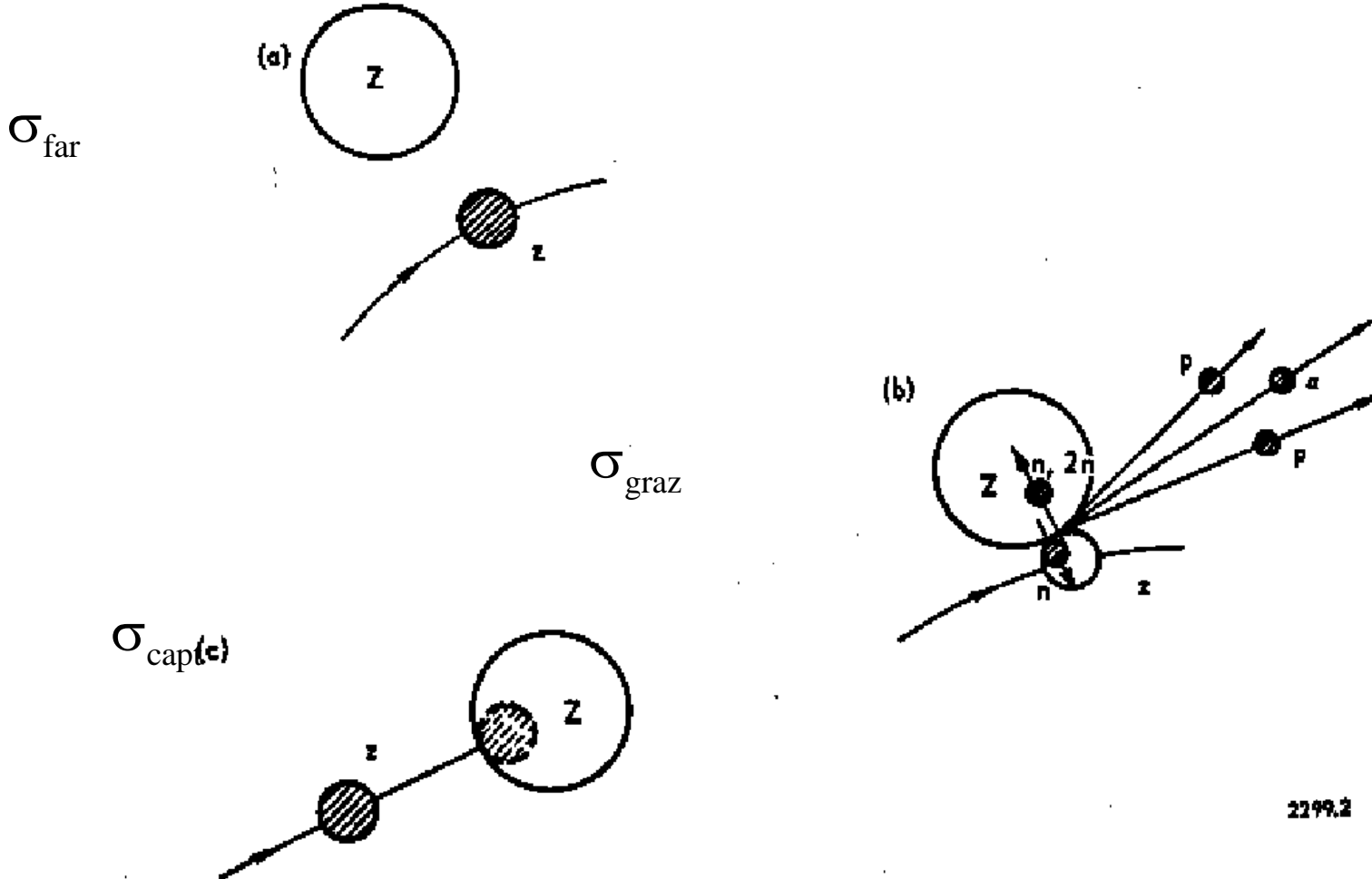
# Outline

- 1. Role of neutron transfer in the dynamics of nucleus-nucleus collisions.**
- 2. Time-dependent approach of nucleon rearrangement in nucleus-nucleus collisions.**
- 3. Time-dependent analysis of the neutron transfer reactions with  ${}^{6,8}\text{He}$  nuclei.**
- 4. Time-dependent analysis of the neutron removal reactions with  ${}^{6,8}\text{He}$  nuclei.**

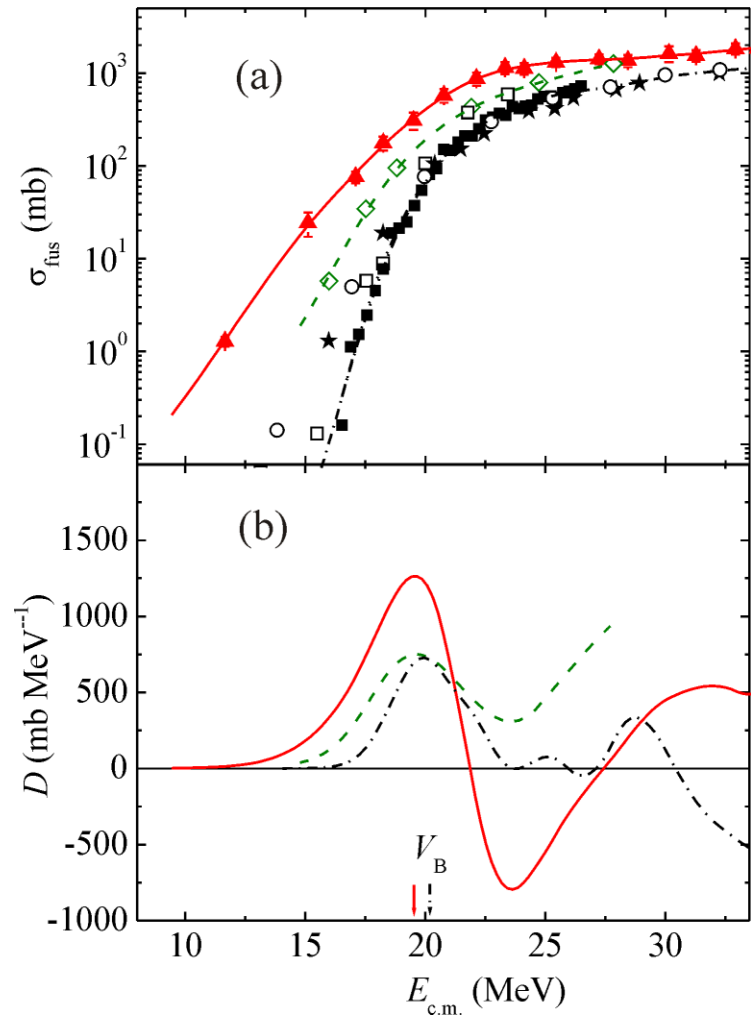
# Main processes in nucleus-nucleus collisions

Far interaction (a), grazing interaction (b), and fusion (capture) (c).

Total reaction cross section:  $\sigma_R = \sigma_{\text{capt}} + \sigma_{\text{graz}} + \sigma_{\text{far}}$



# Fusion reactions: experiment



(a) Experimental fusion cross sections (symbols) and the result of their smoothing (curves) for the reactions with the  ${}^{197}\text{Au}$  target nucleus and the projectile nuclei  ${}^6\text{He}$  (triangles and solid curve),  ${}^8\text{He}$  (diamonds and dashed curve),  ${}^4\text{He}$  (other symbols and dash-dotted curve). Data for  ${}^6\text{He}+{}^{197}\text{Au}$  and  ${}^8\text{He}+{}^{197}\text{Au}$  are taken from [1] and [2, 3]. Data for  ${}^4\text{He}+{}^{197}\text{Au}$  are taken from [4] (circles), [5] (empty squares), [6] (stars), and [7] (filled squares).

(b) The barrier distribution functions

$$D(E) = d^2(E\sigma_{\text{fus}})/dE^2$$

for the reactions with the  ${}^{197}\text{Au}$  target nucleus and the projectile nuclei  ${}^6\text{He}$  (solid curve),  ${}^8\text{He}$  (dashed curve),  ${}^4\text{He}$  (dash-dotted curve). Arrows indicate the energies of the maxima of the functions  $D(E)$  for  ${}^6,8\text{He}$  (solid arrow) and  ${}^4\text{He}$  (dash-dotted arrow; they may be interpreted as the tops of the effective Coulomb barriers).

[1] Yu. E. Penionzhkevich *et al.*, *Eur. Phys. J. A* **31**, 185 (2007).

[2] A. Lemasson *et al.*, *Phys. Lett. B* 697, **454** (2011).

[3] A. Lemasson *et al.*, *Phys. Rev. Lett.* **103**, 232701 (2009)

[4] A. A. Kulko *et al.*, *Phys. At. Nucl.* **70**, 613 (2007).

[5] M. S. Basunia *et al.*, *Phys. Rev. C* **75**, 01582 (2007).

[6] H. E. Kurz *et al.*, *Nucl. Phys. A* **168**, 129 (1971).

[7] A. Calboreanu *et al.*, *Nucl. Phys. A* **383**, 251 (1982).

# Fusion reactions: theory

## Quantum coupled-channels model of nuclear fusion with a semiclassical consideration of neutron rearrangement [1]

The fusion cross section is calculated **by** formula:

$$\sigma_{\text{fus}}(E) = \frac{\pi \hbar^2}{2\mu E} \sum_{l=0}^{\infty} (2l+1) T_l(E) P_{CN}(E^*, l) \quad T_l(E) = N_{\text{tr}}^{-1} \int_{-E}^{\max\{Q_{xn}\}} [\delta(Q) + \alpha_{\text{tr}}(E, l, Q)] \times T_l^{\text{CC}}(E+Q) dQ$$

$$N_{\text{tr}} = 1 + \int_{-E}^{Q_{xn}} \alpha_{\text{tr}}(E, l, Q) dQ. \quad \alpha_{\text{tr}}(E, l, Q) = \sum_{x=1}^{x_{\text{max}}} \alpha_x(E, l, Q)$$

$$T_l^{\text{CC}} \otimes T_l^Q = \frac{j_l(E+Q)}{j} \text{ for He+Au,}$$

The neutron rearrangement probability is defined as

$$\alpha_x(E, l, Q) \approx N_x^{-1} \exp(-Q^2/2\sigma_x^2) \exp(-2\kappa_x [R_{\text{min}}(E, l) - R_c]),$$

$$\sigma_x = \sqrt{\frac{2\hbar^2 \kappa_x B}{\mu R_B}}, \quad \kappa_x = \prod_{i=1}^{x_{\text{max}}} \kappa(\varepsilon_i), \quad \kappa(\varepsilon_i) = \sqrt{2\mu_n \varepsilon_i / \hbar^2}, \quad R_c = r_0 (A_1^{1/3} + A_2^{1/3}),$$

where  $x_{\text{max}}$  is the maximum number of the included neutron transfer channels with  $Q$  values of the ground-to-ground transfer of  $x$  neutrons.  $R_{\text{min}}(E, l)$  is the distance of the closest approach along the Coulomb trajectory with the angular momentum  $l$ ,  $\varepsilon_i$  is the binding energy of the  $i$ th transferred neutron, and  $\mu_n$  is the neutron reduced mass.

$$j = \frac{\hbar k}{\mu}, \quad j_l = -i \frac{\hbar}{2\mu} \left[ \psi_l \frac{d\psi_l^*}{dr} - \psi_l^* \frac{d\psi_l}{dr} \right]_{r=R_{\text{fus}}}$$

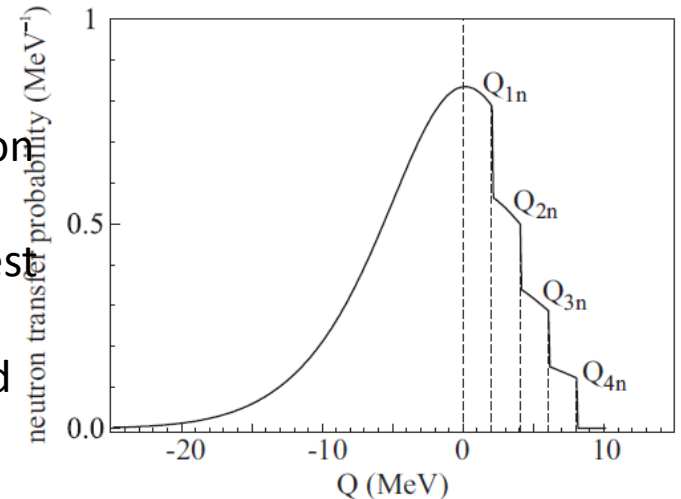


FIG. 1. Typical behavior of neutron transfer probability function  $\alpha_{\text{tr}}$  entering Eq. (12). The  $Q_{xn}$  values are assumed to be equal to  $2x$  (MeV).

# Fusion reactions: theory

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$$N_{\text{tr}} = 1 + \int_{-E}^{Q_{\text{xn}}} \alpha_{\text{tr}}(E, l, Q) dQ. \quad \alpha_{\text{tr}}(E, l, Q) = \sum_{x=1}^{x_{\text{max}}} \alpha_x(E, l, Q)$$

$$T_l^{\text{CC}} \otimes T_l^Q = \frac{j_l(E+Q)}{j} \text{ for He+Au,}$$

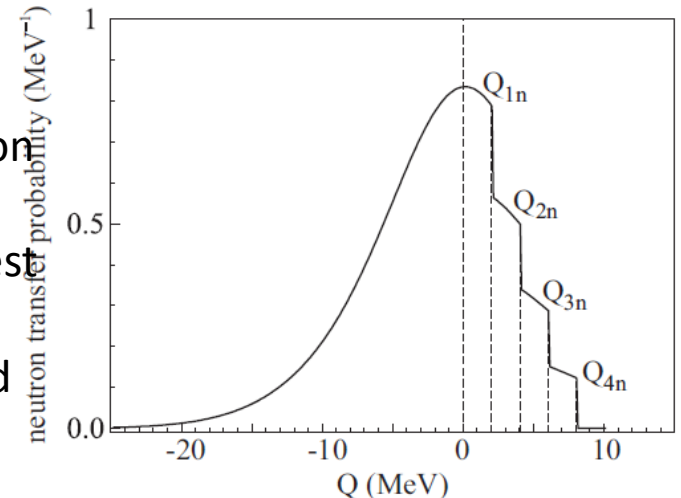
The neutron rearrangement probability is defined as

$$\alpha_x(E, l, Q) \approx N_x^{-1} \exp(-Q^2/2\sigma_x^2) \exp(-2\kappa_x [R_{\text{min}}(E, l) - R_c]),$$

$$\sigma_x = \sqrt{\frac{2\hbar^2 \kappa_x B}{\mu R_B}}, \quad \kappa_x = \prod_{i=1}^{x_{\text{max}}} \kappa(\varepsilon_i), \quad \kappa(\varepsilon_i) = \sqrt{2\mu_n \varepsilon_i / \hbar^2}, \quad R_c = r_0 (A_1^{1/3} + A_2^{1/3}),$$

where  $x_{\text{max}}$  is the maximum number of the included neutron transfer channels with  $Q$  values of the ground-to-ground transfer of  $x$  neutrons.  $R_{\text{min}}(E, l)$  is the distance of the closest approach along the Coulomb trajectory with the angular momentum  $l$ ,  $\varepsilon_i$  is the binding energy of the  $i$ th transferred neutron, and  $\mu_n$  is the neutron reduced mass.

$$j = \frac{\hbar k}{\mu}, \quad j_l = -i \frac{\hbar}{2\mu} \left[ \psi_l \frac{d\psi_l^*}{dr} - \psi_l^* \frac{d\psi_l}{dr} \right]_{r=R_{\text{fus}}}$$

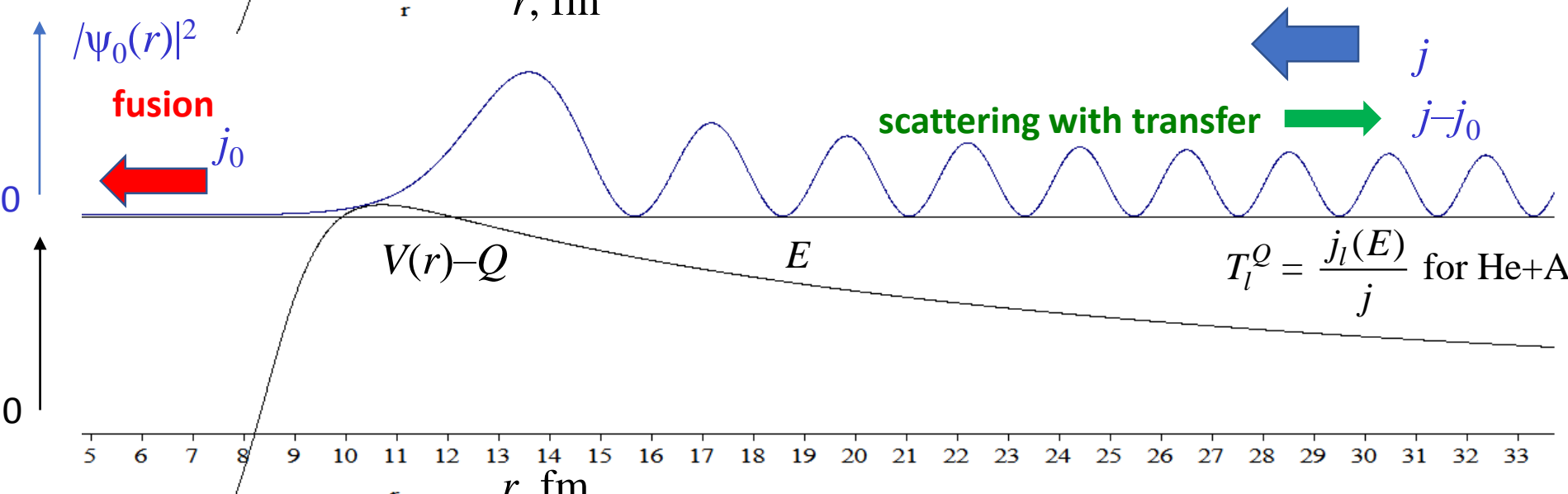
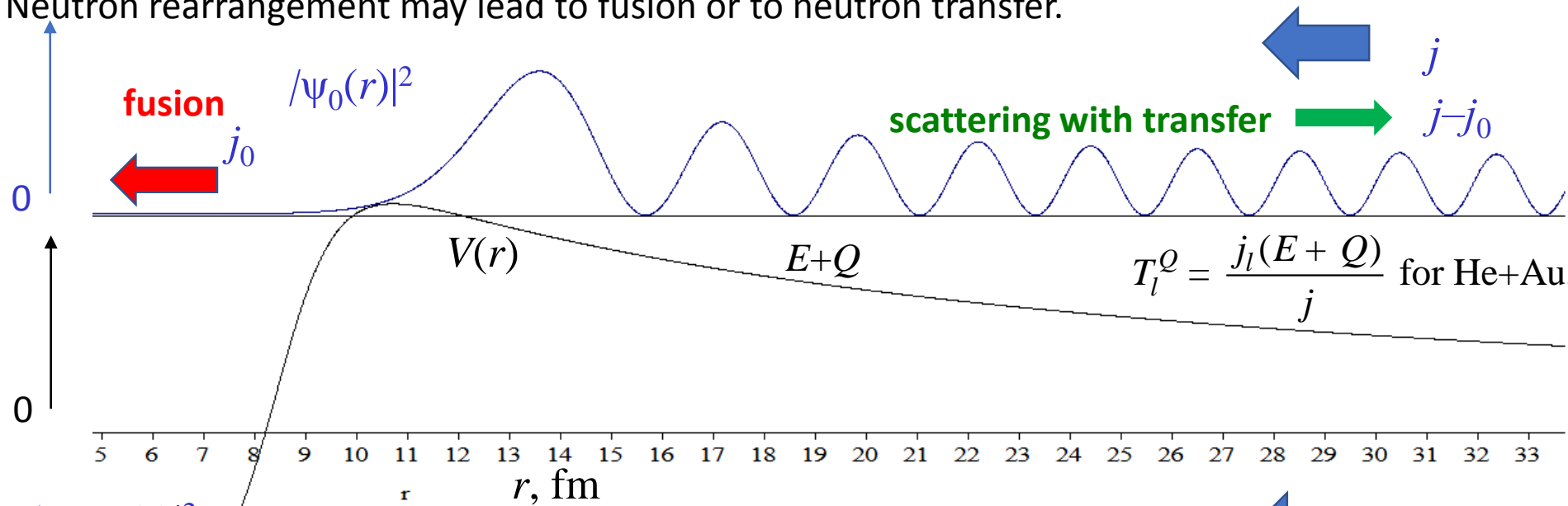


For  ${}^6\text{He} + {}^{197}\text{Au}$   $Q_{1n} = 4.65 \text{ MeV}$   $\sigma_{1n} = 2.67 \text{ MeV}$   
 $Q_{2n} = 13.12 \text{ MeV}$   $\sigma_{2n} = 2.56 \text{ MeV}$

# Fusion reactions: theory

$$j = \frac{\hbar k}{\mu}, j_l = -i \frac{\hbar}{2\mu} \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial \psi_l}{\partial r} \right) - \psi_l \frac{d}{dr} \left( \frac{1}{r} \right) \Big|_{r=R_{\text{fus}}}$$

Quantum one-channel model of nuclear fusion in the neutron rearrangement (transfer) channel with  $Q$  value. Neutron rearrangement may lead to fusion or to neutron transfer.

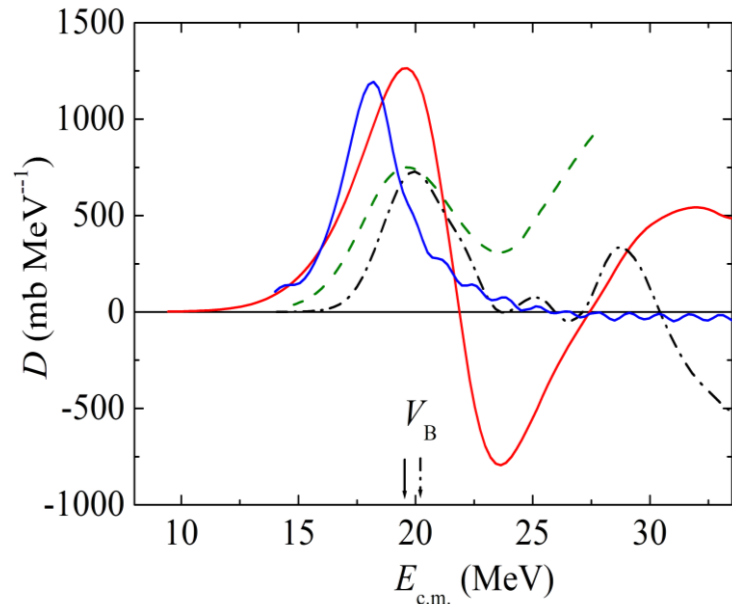
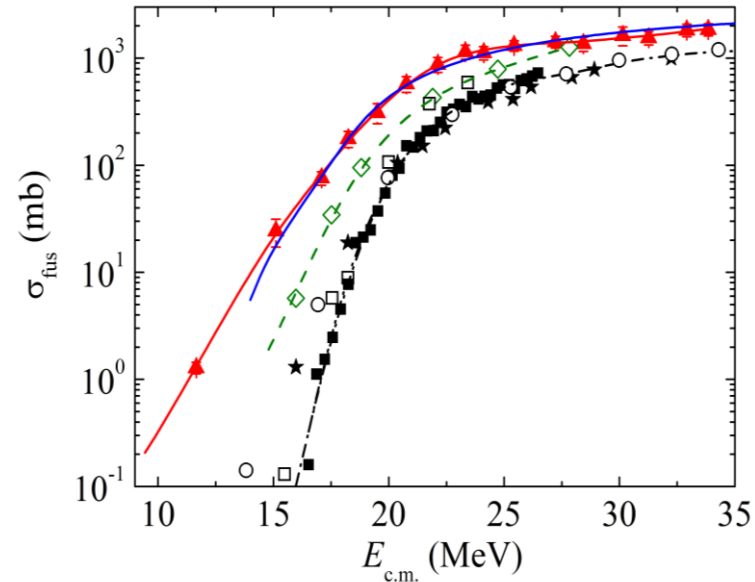


# Fusion reactions: theory

## Quantum coupled-channels model of nuclear fusion with a semiclassical consideration of neutron rearrangement

We use the [quantum coupled-channel model](#) of nuclear fusion with a semiclassical consideration of neutron rearrangement. Results are similar to the experimental barrier distribution functions and fusion cross section for the  ${}^6\text{He}+{}^{197}\text{Au}$  reaction.

$$V(r) = V_N(r) + V_C(r) \quad V_N(r) = \frac{V_0}{1 + \exp\left(\frac{r - r_v}{a}\right)}, \quad V_C(r) = \frac{Z_1 Z_2 e^2}{r}.$$



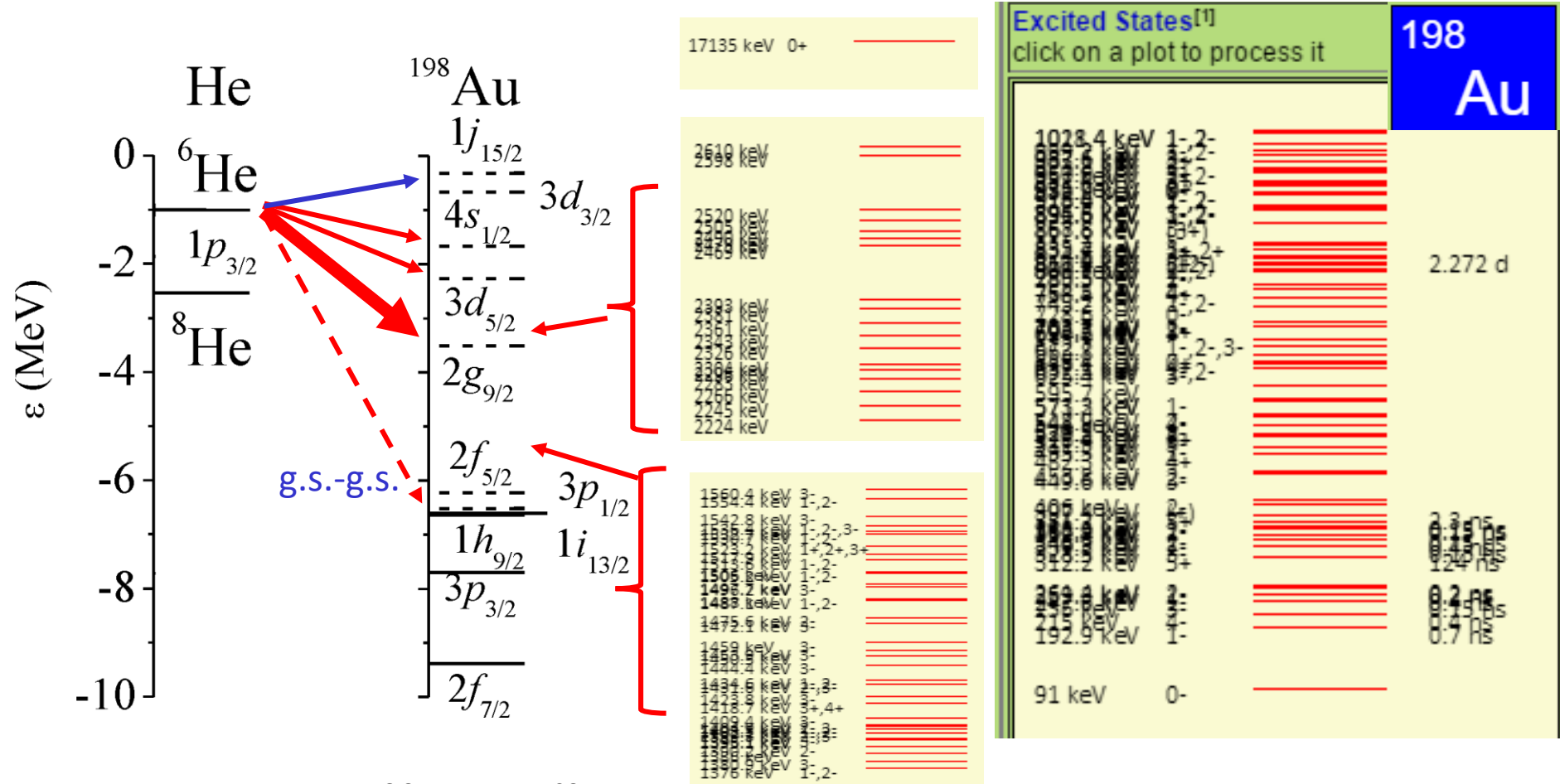
### Parameters of calculation for ${}^6\text{He} + {}^{197}\text{Au}$ fusion reaction within QCC+ENR model

$V_0$	-75.5 MeV
$r_v$	1.176 fm
$a$	0.67 fm
$Q_{1n}$	4.65 MeV
$Q_{2n}$	13.12 MeV



# Fusion and transfer reactions: theory

Neutron rearrangement may lead to fusion or to neutron **transfer**.



Neutron levels in the  ${}^6, {}^8\text{He}$  and  ${}^{198}\text{Au}$  nuclei in the simple shell model and the neutron transfer channels with  $Q$  values  $Q > 0$  and  $Q < 0$ . The dashed arrow is the ground-to-ground transfer of 1 or 2 neutrons.

Excitation levels in the  ${}^{198}\text{Au}$  nuclei (experimental data) [1]

[1] V. I. Zagrebaev *et al.*, NRV web knowledge base on low-energy nuclear physics, <http://nrv.jinr.ru/nrv/>.

# Time-dependent model of ${}^6\text{He}+{}^{197}\text{Au}$ and two-center probability densities.

$$1p_{3/2}^2({}^6\text{He})$$

$\Omega=1/2$

$$2g_{9/2}(\text{Au})$$

$$1p_{3/2}(\text{He})$$

$$3d_{5/2}(\text{Au})$$

$$1j_{15/2}(\text{Au})$$

$$1i_{11/2}(\text{Au})$$

$$4s_{1/2}(\text{Au})$$

$\Omega=3/2$

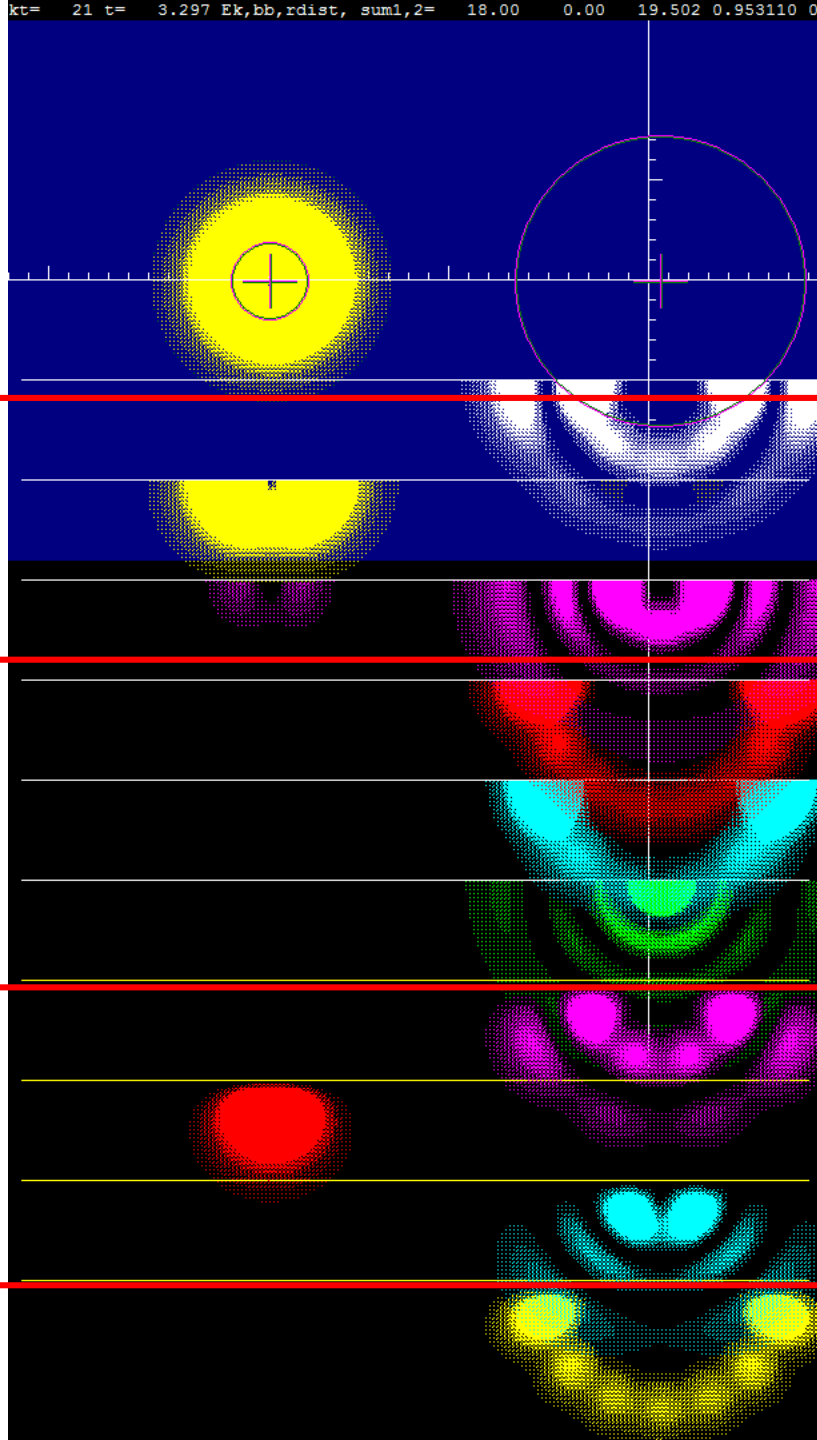
$$2g_{9/2}(\text{Au})$$

$$1p_{3/2}(\text{He})$$

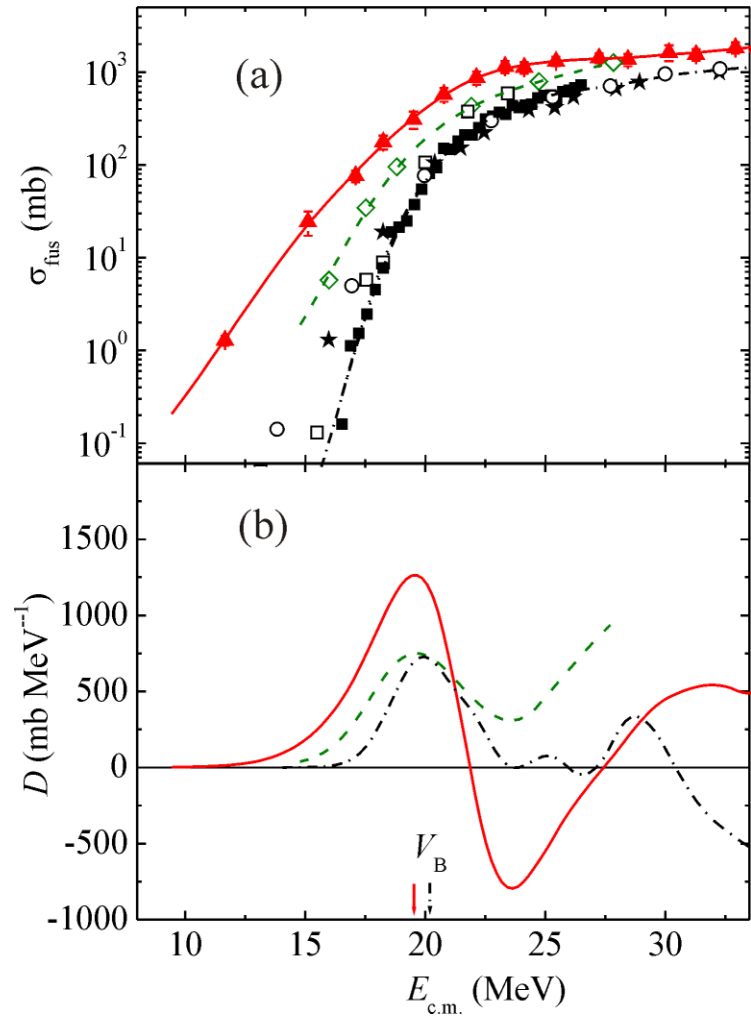
$$3d_{5/2}(\text{Au})$$

$$1j_{15/2}(\text{Au})$$

The strong overlapping of some two-center wave functions lead to the neutron rearrangement from  ${}^6\text{He}$  to  ${}^{197}\text{Au}$  with  $Q>0$ .



# Fusion and transfer reactions: theory



We use the model of discretization of barrier distributions and multi-dimensional potential barriers taking into account neutron rearrangement from  ${}^6\text{He}$  to  ${}^{197}\text{Au}$  with  $Q > 0$ . We assume that the cross sections for fusion  $\sigma_{\text{fus}}$  may be represented as combinations of the cross sections for the finite numbers of one-dimensional radial Coulomb barriers  $V_i(r)$  with  $Q_i$  values determined by fitting of experimental barrier distribution functions and fusion cross sections

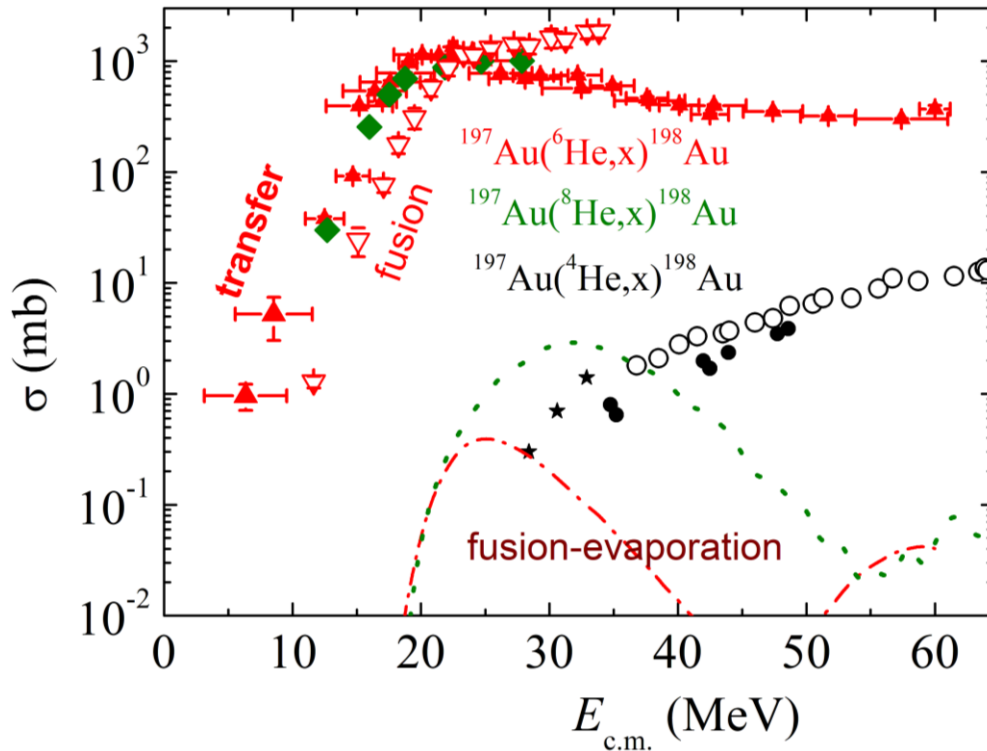
$$\sigma_{\text{fus}} = \sum_i w_i(E, Q_i) \sigma_{\text{fus},i}(Q_i),$$

Here weights  $w_i(E, Q_i)$  are the average probabilities of neutron transition from  ${}^6\text{He}$  to  ${}^{197}\text{Au}$  with  $Q_i$ ,  $Q_1 = Q_{\text{g.s.}-3p}$ ,  $Q_2 = Q_{\text{g.s.}-2f}$ ,  $Q_3 = Q_{\text{g.s.}-2g}$ ,  $Q_4 = Q_{\text{g.s.}-3d}$ , ...

The transfer  $\sigma_{\text{tr}}$  cross section may be represented as combinations of the cross sections

$$\sigma_{\text{tr}} = \sum_{i, Q_i \neq 0} \sigma_{\text{tr},i}(E, Q_i)$$

# Transfer reactions: experiments



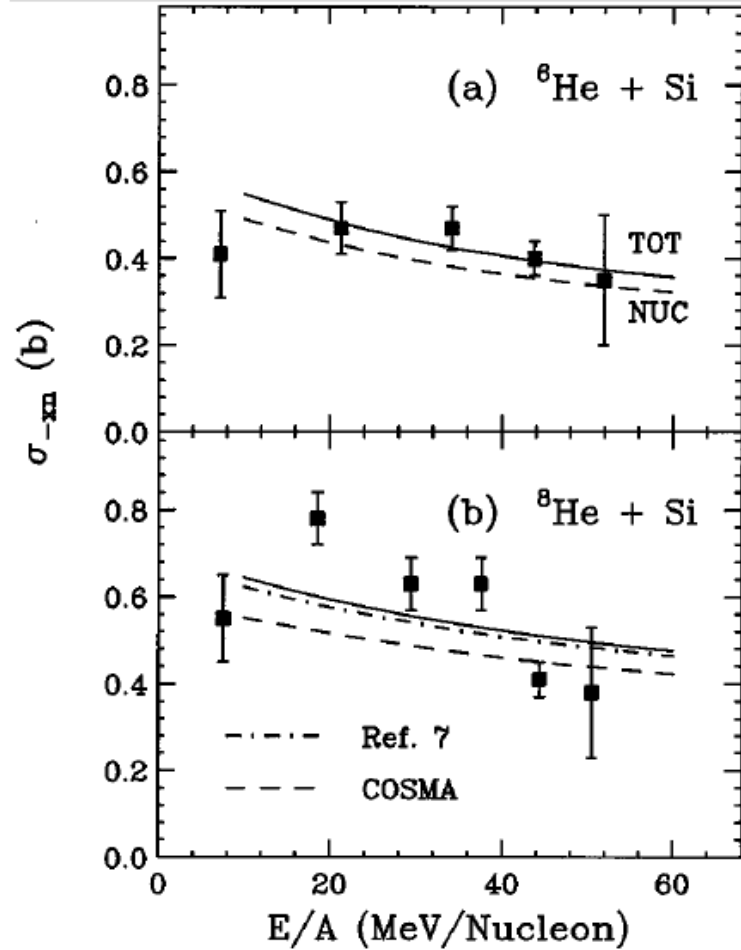
Experimental fusion cross section in reaction  ${}^6\text{He}+{}^{197}\text{Au}$  (empty triangles up) [1]. Formation cross sections of  ${}^{198}\text{Au}$  in reactions  ${}^3,4,6,8\text{He}+{}^{197}\text{Au}$ . Data for  ${}^4\text{He}+{}^{197}\text{Au}$  (filled and empty circles and stars),  ${}^6\text{He}+{}^{197}\text{Au}$  (filled red triangles),  ${}^8\text{He}+{}^{197}\text{Au}$  (filled diamonds) are taken from [1-8]. The results of the fusion-evaporation channel for  ${}^6\text{He}+{}^{197}\text{Au}$  (dash-dotted curve),  ${}^8\text{He}+{}^{197}\text{Au}$  (dotted curve) were calculated using the NRV web knowledge base [9]. The large difference between the experimental cross sections and the results of the calculations for fusion-evaporation channels indicates that neutron transfer channels are the main contribution to the experimental data. Evaporation channels are suppressed in this case because of the high Coulomb barrier for evaporation of protons and alpha particles from the formed compound nucleus.

- [1] Yu. E. Penionzhkevich *et al.*, *Eur. Phys. J. A* **31**, 185 (2007).
- [2] N. K. Skobelev *et al.*, *Phys. Part. Nucl. Lett.* **11**, 114 (2014).
- [3] Y. Nagame *et al.*, *Phys. Rev. C* **41**, 889 (1990).
- [4] A. A. Kulko *et al.*, *Phys. At. Nucl.* **70**, 613 (2007).
- [5] F. M. Lanzafame and M. Blann, *Nucl. Phys. A* **142**, 545 (1970).
- [6] N. Chakravarty, *Phys. Rev. C* **45**, 1171 (1992).
- [7] C. Necheva and D. Kolev, *Appl. Radiat. Isot.* **48**, 807 (1997).
- [8] A. Lemasson *et al.*, *Phys. Lett. B* 697, **454** (2011).
- [9] V. I. Zagrebaev *et al.*, NRV web knowledge base on low-energy nuclear physics, <http://nrv.jinr.ru/nrv/>.

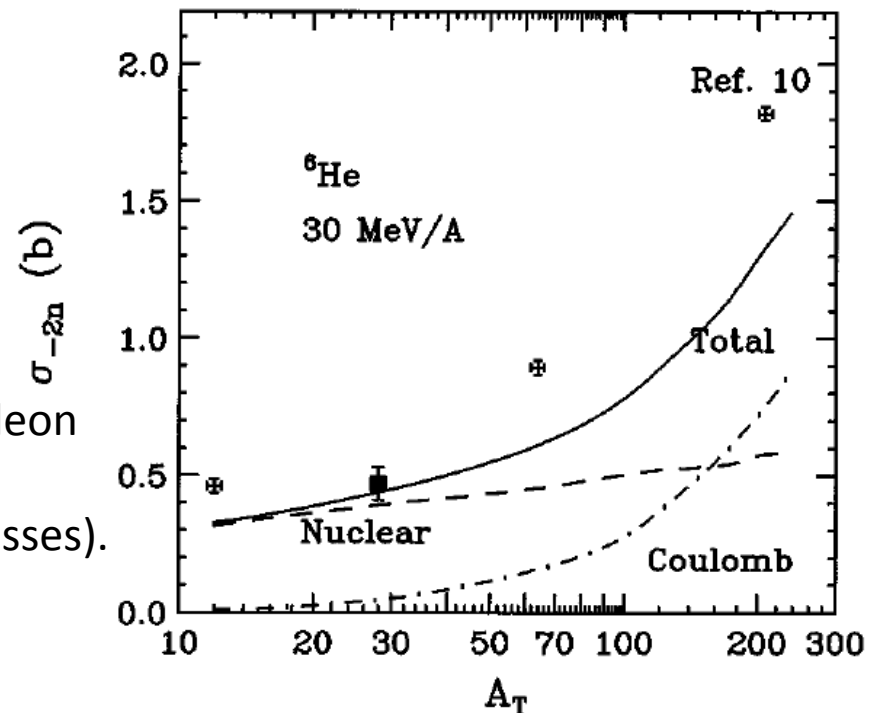
# Breakup (neutron removal) reactions: experiments

Measured cross sections for removal by Si of two neutrons from  ${}^6\text{He}$  (a) and two or four neutrons from  ${}^8\text{He}$ , compared with calculations described in text. Dashed curves show nuclear cross sections obtained with empirical matter densities; the dot-dashed curve is for COSMA density. Solid curves show total (nuclear plus Coulomb) cross sections; that for  ${}^8\text{He}$  applies to the COSMA nuclear calculation.

[1] R. E. Warner, Phys. Rev. C **55**, 298 (1997).



Target-A dependence of  $\sigma_{-2n}$  for  ${}^6\text{He}$  at 30 MeV/nucleon from calculations described in text (smooth curves), compared with predictions from [10] (shown by crosses). The measurement (solid data point) is from Ref. [5].



[5] R. E. Warner *et al.*, Phys. Rev. C **54**, 1700 (1996).

[10] L. S. Ferreira *et al.*, Phys. Lett. B **316**, 23 (1993).

# Transfer and breakup reactions: theoretical model basics

The microscopic approach based on the numeric solution of the time-dependent Schrödinger equation [1-5] for the outer and inner neutrons and protons of colliding nuclei.

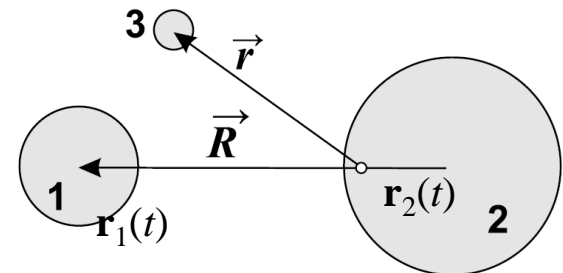
- Classical motion of cores.
- Time-dependent Schrödinger equation (**TDSE**) to describe neutron rearrangement and modification of the barrier of the nucleus-nucleus potential

$$i\hbar \frac{\partial}{\partial t} \Psi(\mathbf{r}, t) = \left\{ -\frac{\hbar^2}{2m} \Delta + V_1(|\mathbf{r} - \mathbf{r}_1(t)|) + V_2(|\mathbf{r} - \mathbf{r}_2(t)|) + \hat{V}_{LS}^{(1)}(\mathbf{r} - \mathbf{r}_1(t)) + \hat{V}_{LS}^{(2)}(\mathbf{r} - \mathbf{r}_2(t)) \right\} \Psi(\mathbf{r}, t), \quad \Psi(\mathbf{r}, t) = \begin{pmatrix} \psi(\mathbf{r}, t) \\ \phi(\mathbf{r}, t) \end{pmatrix}$$

- The initial wave functions were determined from the spherical shell model with parameters providing reasonable values of charge radius and separation energies of outer neutrons.
- The center of mass system was used for energies near and above the Coulomb barrier.
- The moving reference system was used for energies near and above 10 A · MeV.

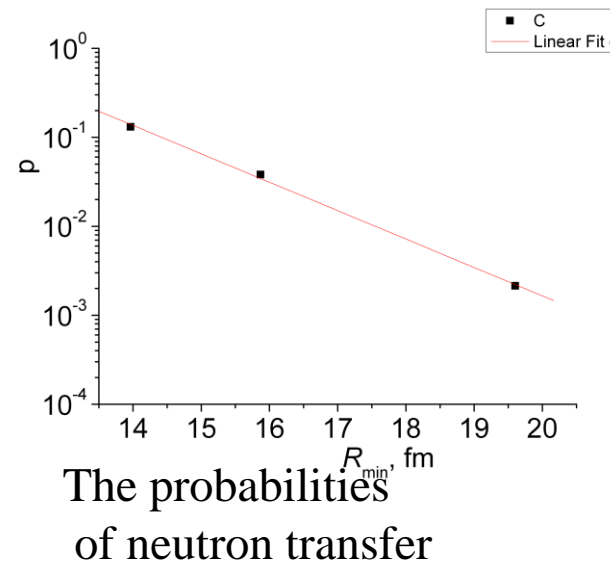
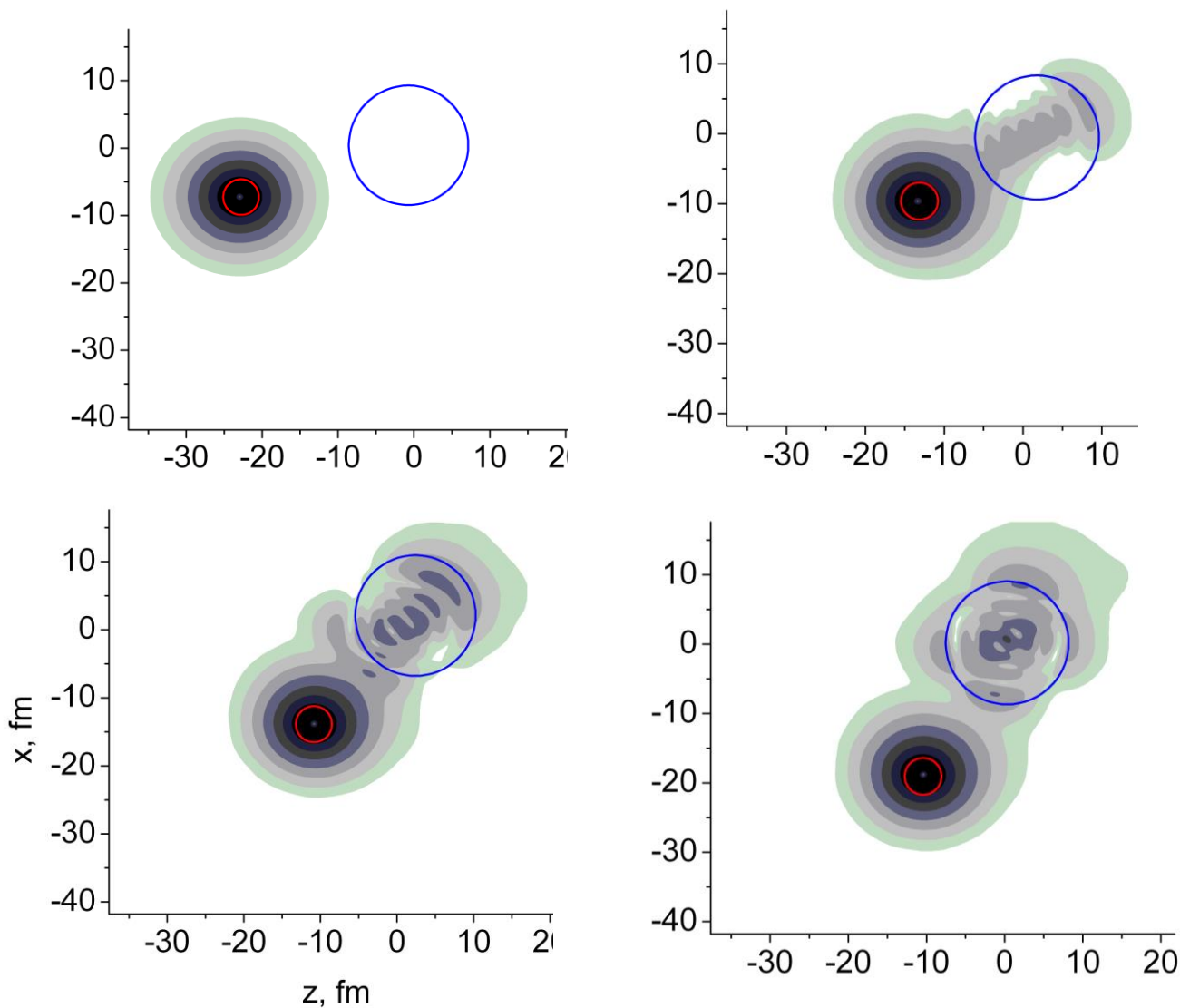
- [1] V. Samarin. Phys. Atom. Nucl. **78**, 128 (2015).  
 [2] V. Samarin, EPJ Web Conf. **66**, 03075 (2014).  
 [3] V. Samarin, EPJ Web Conf. **86**, 00040 (2015).  
 [4] V. Samarin. Phys. Atom. Nucl. **81**, 486 (2018).  
 [5] V. Samarin. Bull. Russ. Acad. Sci.: Phys., **84**, 990 (2020).

Light quantum particle (nucleon) 3



Two heavy classical particles (cores) 1 and 2

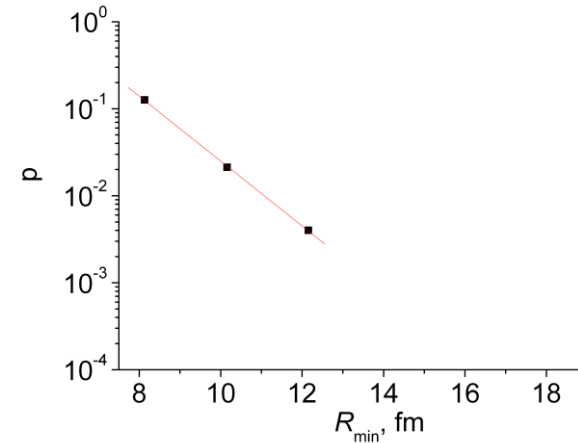
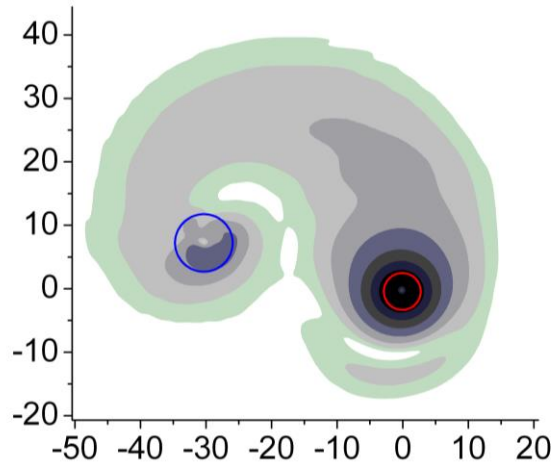
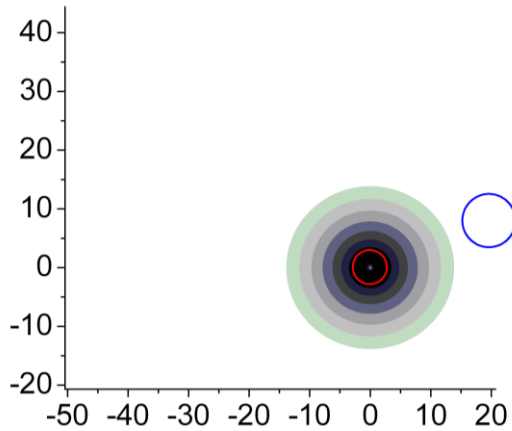
# Transfer reactions: theory ${}^6\text{He}+{}^{197}\text{Au}$



The probabilities  
of neutron transfer

# Breakup reactions: theory

Time-dependent microscopic description of outer neutrons of  ${}^8\text{He}$  during the collision  ${}^8\text{He} + {}^{28}\text{Si}$  in the moving reference frame



The probabilities of neutron transfer to states of continuous spectrum

Evolution of the probability density for the outer neutrons of  ${}^6\text{Be}$  with the initial state  $1p_{3/2}$  in the collision  ${}^8 + {}^{28}\text{Si}$  at  $E_{\text{c.m.}} = 183$  MeV and impact parameter  $b=8$  fm in the reference frame moving relative to the laboratory system with a constant velocity equal to that of a projectile at a fairly large distance from the target nucleus. The course of time corresponds to the panel locations (a), (b), (c), (d). Greyscale gradation in the logarithmic scale is used. The radii of the circles correspond to those of nuclear cores, 2.8 fm and 4.5 fm, respectively.



# Conclusions

- The quantum coupled-channel model of nuclear fusion with a semiclassical consideration of neutron rearrangement is applied to the description of the fusion reactions  ${}^6\text{He} + {}^{28}\text{Si}$ ,  ${}^8\text{He} + {}^{28}\text{Si}$ .
- The physical mechanism of the neutron rearrangement with  $Q$  values,  $Q > 0$ , at the near barrier energies consists in the transitions between two-center levels in the dinuclear systems  ${}^6\text{He} + {}^{28}\text{Si}$ ,  ${}^8\text{He} + {}^{28}\text{Si}$  because of the strong overlapping of some two-center wave functions.
- The numerical solution of the time-dependent Schrödinger equation is applied to the analysis of dynamics of neutron transfer and rearrangement at energies near and above the Coulomb barrier and neutron removal at energies near and above  $10 A \cdot \text{MeV}$ . The calculations of transfer and breakup cross sections for the reactions  ${}^6\text{He} + {}^{28}\text{Si}$ ,  ${}^8\text{He} + {}^{28}\text{Si}$  are in progress.

Thank you for your attention!

