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TRANSMISSION EFFICIENCY OF ISOTROPICALLY EMITTED NUCLEAR DECAY AND REACTION PRODUCTS FROM THE RADIOACTIVE SOURCE

PhD Student: Simonovski Dimitar

24.09.2021

Scientific Problem

Thermalized ion distribution formula inside the radioactive source medium

- No available analytical formulas
  - Only for monoenergetic singly directed ion beam from a point source placed inside the media
  - Gaussian distribution
  - Not available for more realistic cases:
    - 1D, 2D or 3D source media
    - Isotropicaly emmited particles (alpha, beta, gamma decay...)
    - Anisotropicaly emmited partecles

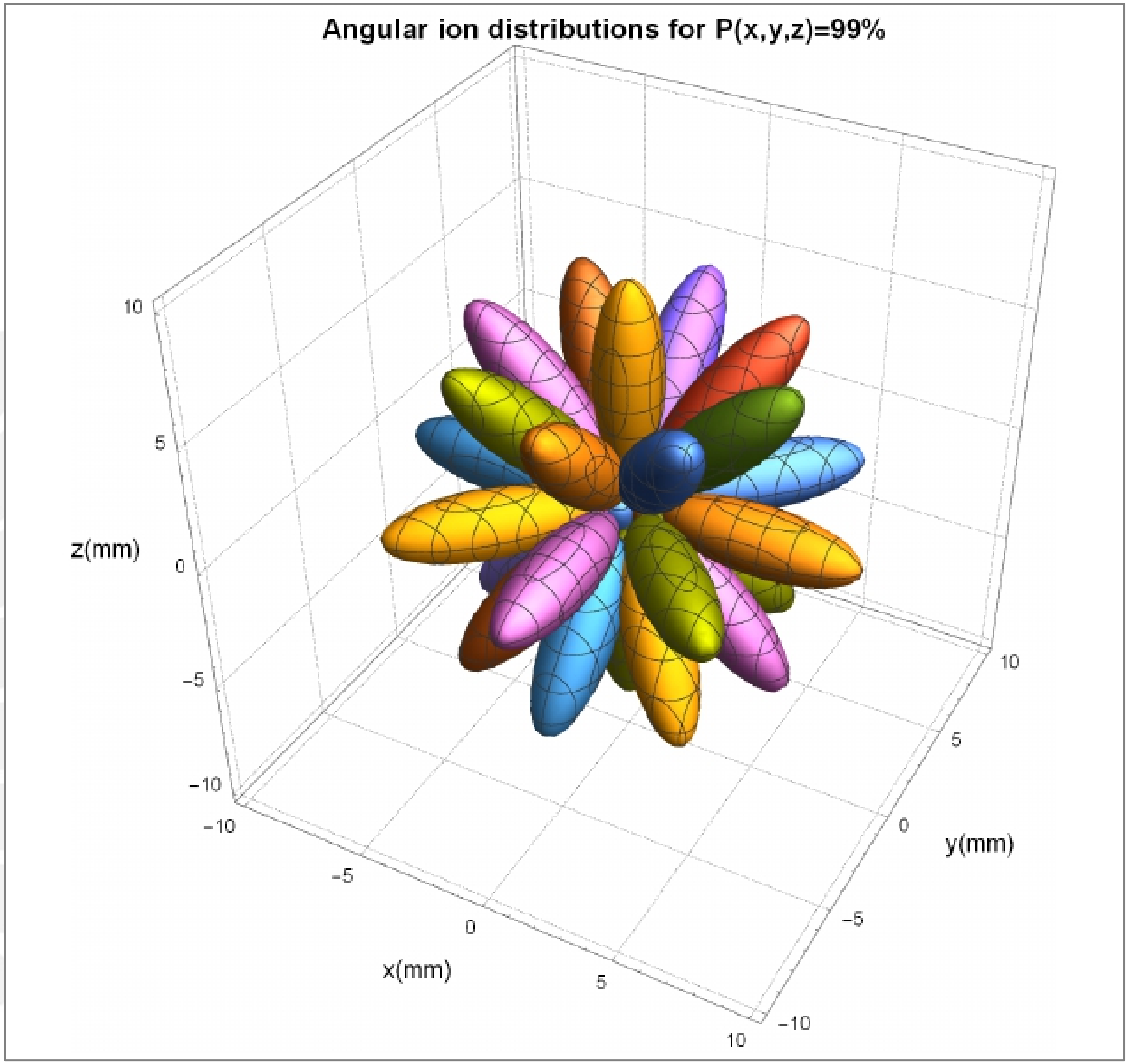
- Numerical simulations (TRIM)
  - Take long time to calculate

Simplest Case

- Thermalized ion distribution function isotropicaly emmited from a single point source

- Numerical calcul-ations for the more general analytic idea

- Rotated Gaussian 3D distributions



Analytical Results

Thermalized ion distribution function isotropicaly emmited from a single point source

1. General distribution formula

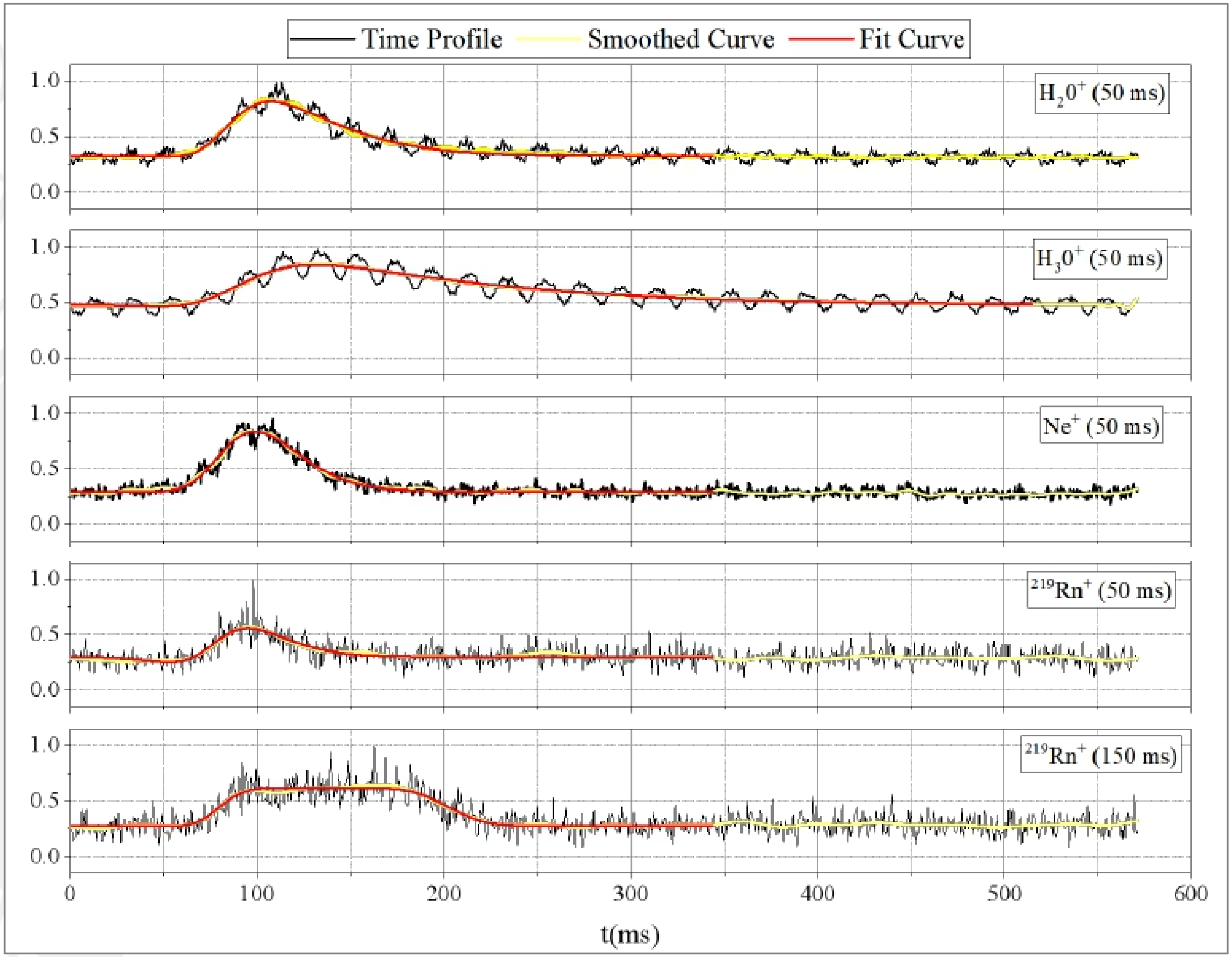
$$G(x,y,z;z_0,\alpha_y,\alpha_z)=\frac{e^{-\left(\frac{\left(\mathrm{R}_{zy}(\alpha_z,\alpha_y)\cdot\vec{r}\right)^T\cdot\vec{k}-z_0}{\sqrt{2}\sigma_z}\right)^2}}{(2\pi)^{\frac{3}{2}}\sigma_y^2\sigma_z}e^{-\left(\frac{\left(\mathrm{R}_{zy}(\alpha_z,\alpha_y)\cdot\vec{r}\right)^T\cdot\vec{i}}{\sqrt{2}\sigma_y}\right)^2-\left(\frac{\left(\mathrm{R}_{zy}(\alpha_z,\alpha_y)\cdot\vec{r}\right)^T\cdot\vec{j}}{\sqrt{2}\sigma_y}\right)^2} \tag{1.9}$$

2. Analytical solution for the point source isotopic emmiter

$$p(r)=\frac{e^{-\frac{r^2}{2\sigma_\perp^2}+\frac{1}{2}\left(\frac{\overline{R}_\parallel^2}{\sigma_\perp^2-\sigma_\parallel^2}\right)}}{8\pi\sigma_\perp\sqrt{\sigma_\perp^2-\sigma_\parallel^2}r}\left\{\mathrm{erf}\left(\frac{\left(\sigma_\perp^2-\sigma_\parallel^2\right)r+\sigma_\perp^2\overline{R}_\parallel}{\sigma_\perp\sigma_\parallel\sqrt{2\left(\sigma_\perp^2-\sigma_\parallel^2\right)}}\right)+\mathrm{erf}\left(\frac{\left(\sigma_\perp^2-\sigma_\parallel^2\right)r-\sigma_\perp^2\overline{R}_\parallel}{\sigma_\perp\sigma_\parallel\sqrt{2\left(\sigma_\perp^2-\sigma_\parallel^2\right)}}\right)\right\}$$

Experimental Results

Fitted time profiles of detected radioactive alpha-decay recoils generated from a radioactive needle point source



Transmission Efficiency

Transmission efficiency analytical formulas

1. Analytical difficulties with the higher order integrals in the general distribution formula

$$p(x,y,z) = \frac{\iiint_{x,y,z\in V} \iiint_{x_s,y_s,z_s\in V_s} \iint_{\alpha_y,\alpha_z\in\Omega} G(\vec{r}, \vec{r}_s, \alpha_y, \alpha_z)\, n(\vec{r}_s)\, \eta(\alpha_y, \alpha_z)\, d\Omega dV_s dV}{\iiint_{x,y,z\in V} dV \iiint_{x_s,y_s,z_s\in V_s} n(\vec{r}_s)\, dV_s \iint_{\alpha_y,\alpha_z\in\Omega} \eta(\alpha_y,\alpha_z)\, d\Omega}$$

2. Possible (most recent) solution

$$\eta = 1 - C \sum_{n=0}^{\infty} \frac{2(-1)^n A^{2n+1}}{(2n+1)\sqrt{\pi}\Gamma(n+1)} \left\{ 2 \sum_{j=0}^{n} \binom{2n+1}{2j+1} \left(\frac{B}{A}\right)^{2(n-j)} \left(\mathcal{F}_\beta\left[F(r)\right] * I(\beta)\right)\Big|_{\beta=1} \right\}$$

Thank you for your attention!

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