DESCRIPTION OF ATOMIC NUCLEUS COLLISIONS IN THE NON-EQUILIBRIUM HYDRODYNAMIC APPROACH AS COLLISIONS OF KORTEWEG-DE VRIES SOLITONS

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Within the framework of the non-equilibrium hydrodynamic approach, a soliton-like analytical solution of the equations for the collision of nuclear layers-slabs is found. The prospects of the hydrodynamic approach to the description of collisions of heavy ions of medium energies and the importance of taking into account non-equilibrium processes are noted. Within the framework of a single formula, the stages of compression, expansion and expansion of layers of nuclear matter with energies of the order of ten MeV per nucleon are considered. The reduction of solutions of hydrodynamic equations in this case to the solution of two Korteweg – de Vries equations has not been previously considered and is of independent interest for a wide range of applied problems.

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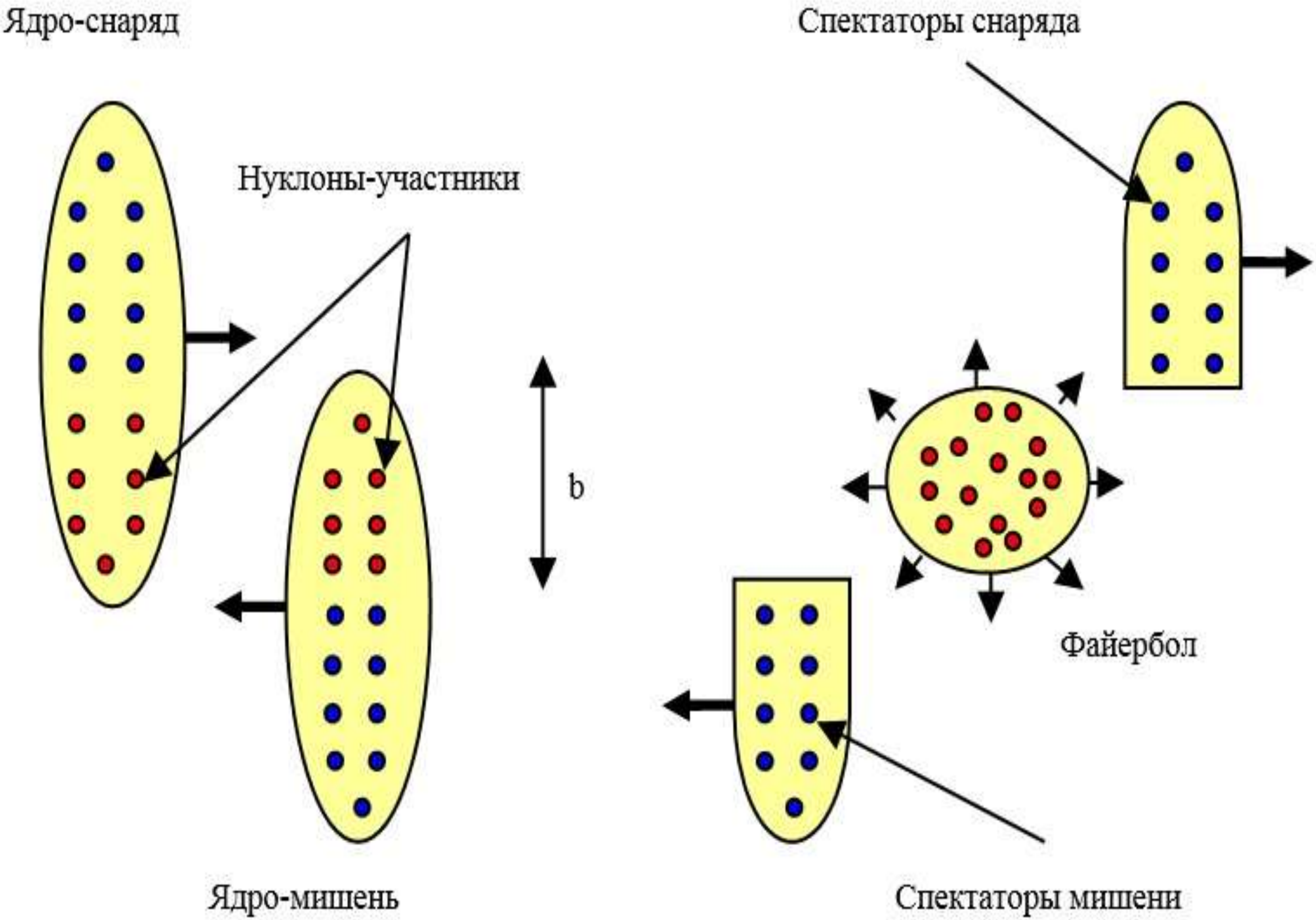
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1.KORTEVEG-DE VRIES SOLITONS

$$\frac{\partial \rho}{\partial t} + \upsilon \frac{\partial \rho}{\partial x} + \rho \frac{\partial \upsilon}{\partial x} = 0 \qquad P(\rho) = a\rho + b\rho^2 + \acute{\alpha}\frac{d^2\rho}{dx^2} \qquad a\rho_0 + b\rho_0^2 = 0 \qquad I = I_1\left(\frac{\rho}{\rho_0}\right)^3$$

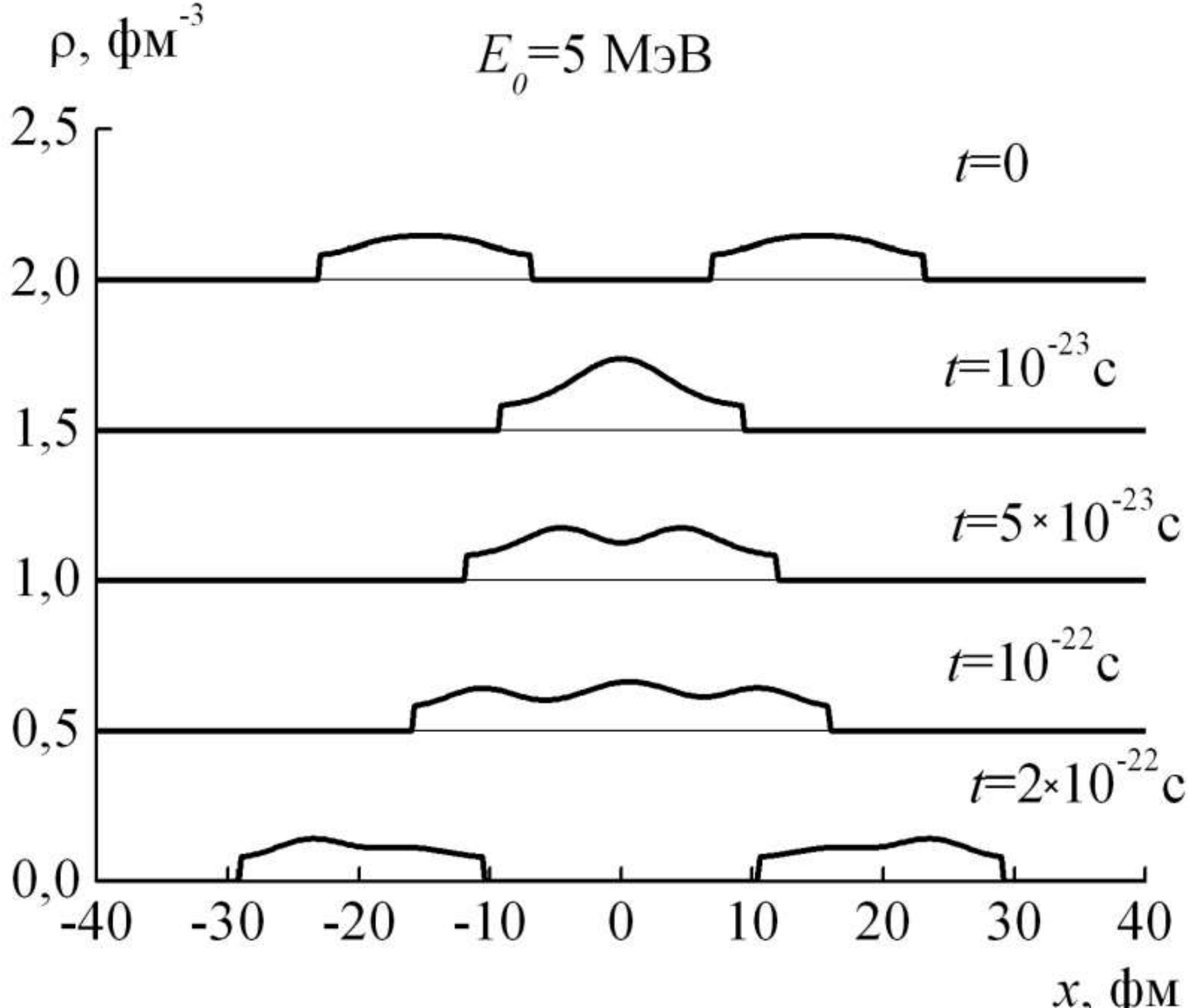
$$\frac{\partial \upsilon}{\partial t} + \upsilon \frac{\partial \upsilon}{\partial x} + \frac{1}{m\rho}\frac{\partial P}{\partial x} = 0 \qquad \rho\upsilon'(\rho) = \pm\sqrt{\frac{\partial P}{m\partial\rho}} \approx \pm\left[c_{so} + \beta(\rho-\rho_0) + \frac{\alpha}{2mc_{so}}\frac{\partial}{\partial\rho}\left(\frac{\partial^2\rho}{\partial x^2}\right)\right] = \pm c_s(\rho)$$

$$\frac{\partial I}{\partial t} + \upsilon \frac{\partial I}{\partial x} + 3I\frac{\partial \upsilon}{\partial x} = 0 \qquad \upsilon = \pm\int_{\rho_0}^{\rho}\frac{c_s(\rho)}{\rho}d\rho + \upsilon_0 \qquad c_{s0} = \sqrt{\frac{a+2b\rho_0}{m}} \approx 1/3c \approx$$

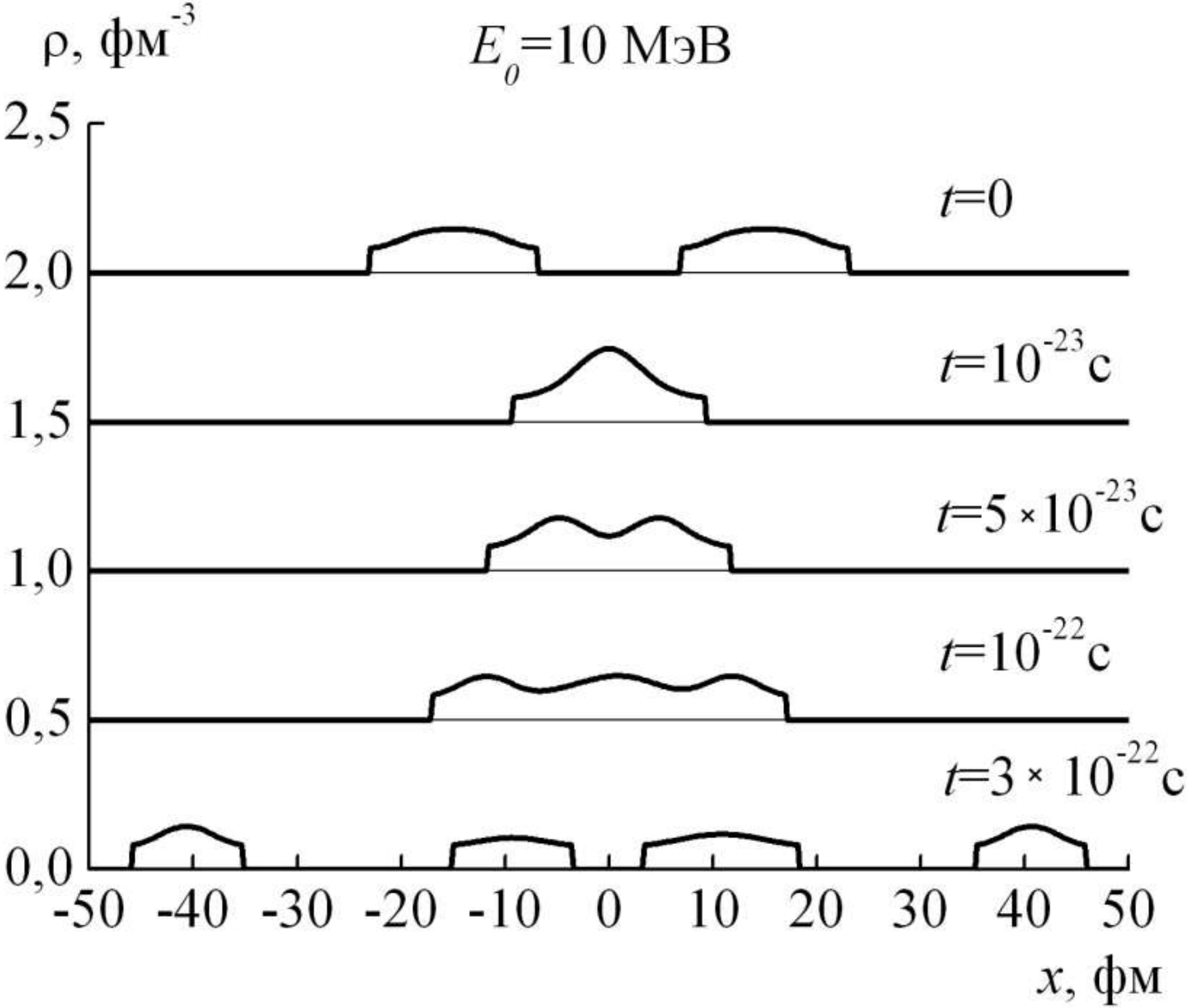
$$\frac{\partial \zeta}{\partial t'} + 6\zeta\frac{\partial \zeta}{\partial x'} + \frac{\partial^3\zeta}{\partial x'^3} = 0 \qquad \zeta = \left[\pm\upsilon_0 + c_{so} + 2c_{so}\frac{(\rho-\rho_0)}{\rho_0}\right]\frac{1}{A} \qquad \zeta = 2\frac{\partial^2 \ln s}{\partial x'^2}$$

$$s = 1 + \exp(\omega t' + k(x' - x_1)) \qquad Z = \int_0^L \xi \frac{dx_1}{L}$$

$$\omega = -k^3$$



ρ, фм⁻³
E_0=5 МэВ
2,5
2,0
1,5
1,0
0,5
0,0
t=0
t=10^-23 c
t=5×10^-23 c
t=10^-22 c
t=2×10^-22 c
-40
-30
-20
-10
0
10
20
30
40
x, фм



ρ, фм$^{-3}$
E_0=10 МэВ
2,5
2,0
1,5
1,0
0,5
0,0
t=0
t=10^{-23}с
t=5×10^{-23}с
t=10^{-22}с
t=3×10^{-22}с
-50 -40 -30 -20 -10 0 10 20 30 40 50
x, фм

2. FEATURES OF NON-EQUALIBRIUM HYDRODYNAMIC APPROACH

- To describe the collisions of heavy ions we use the non-equilibrium hydrodynamic approach, in which the kinetic equation for the nucleon distribution function $f(\mathbf{r},\mathbf{p},t)$ is solved jointly with the equations of hydrodynamics, which are essentially local laws of conservation of mass, momentum and energy. $\int \frac{d^3\vec{p}}{(2\pi\hbar)^3} \quad 1, \vec{p}, p^2$
- $$\frac{df}{dt} = \frac{f_0 - f}{\tau} \qquad , \qquad (1)$$
- where $f_0(\mathbf{r},\mathbf{p},t)$ is the locally equilibrium distribution function and
- ‘ τ is the relaxation time
- $$W(\rho) = \alpha\rho + \beta\rho^{\chi} \qquad , \qquad (2)$$

- The self-consistent potential appearing in the interaction term is specified in just the same way as this is done in the case of density-dependentive forces belonging to the Skyrme type

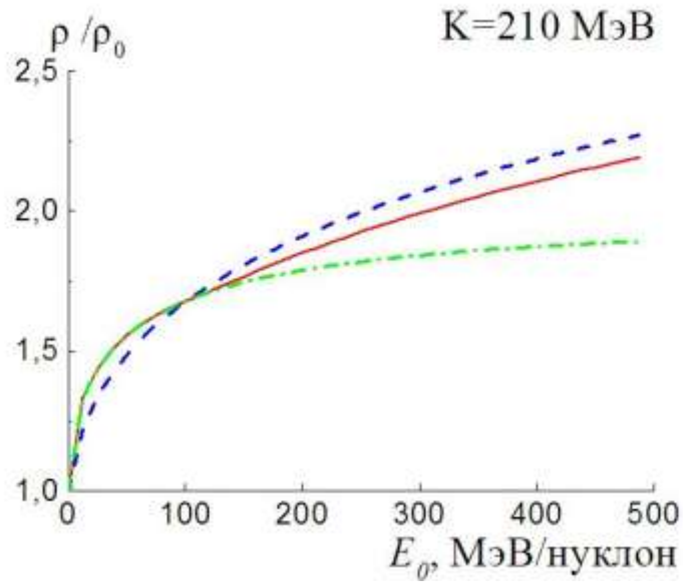
- $\rho = \int \frac{f d^3\vec{p}}{(2\pi\hbar)^3}$ $\tau = \lambda / \upsilon_T$ $W(\rho) = \alpha\rho + \beta\rho^\gamma,$
- $$f(\vec{r}, \vec{p}, t) = f_1 \cdot q + f_0 \cdot (1-q)$$ (3)
-
-
- $P_{kin}^{||} = P_{(kin)11} = 2(\varepsilon_1 + I_1)q + \frac{2}{3}(\varepsilon + I)(1-q)$ $\varepsilon_1 = \frac{\hbar^2}{10m}\left(\frac{3}{2}\pi^2\rho_0\right)^{2/3}\frac{\rho^3}{\rho_0^2}$
- $P_{kin}^{\perp} = P_{(kin)22} = P_{(kin)33} = 2\varepsilon_2 + \frac{2}{3}(\varepsilon + I)(1-q)$ $\varepsilon_2 = \frac{\hbar^2}{10m}\left(\frac{3}{2}\pi^2\rho_0\right)^{2/3}\rho$
- $\varepsilon = \frac{3}{10}\frac{\hbar^2}{m}\left(\frac{3}{2}\pi^2\rho\right)^{2/3} \cdot \rho$ $I = \int \frac{p^2}{2m}\delta f \frac{d^3\vec{p}}{(2\pi\hbar)^3}$ $I_1 = \int \frac{p^2}{2m}\delta f_1 \frac{d^3\vec{p}}{(2\pi\hbar)^3}$
-
- $P_{ij} = P_{(kin)ij} + P_{int}\delta_{ij}$ $e = \varepsilon + I + e_{int}$ $e_{int} = \int_0^\rho W(\rho)d\rho$ $P_{int} = \rho^2 \frac{d(e_{int}/\rho)}{d\rho}$
- $$\frac{\partial}{\partial t}((\varepsilon_1 - \varepsilon_2 + I_1)q) + \frac{\partial}{\partial x_1}(\upsilon_1(3\varepsilon_1 - \varepsilon_2 + 3I_1)q) + \sum_{i=2,3}\frac{\partial}{\partial x_i}(\upsilon_i(\varepsilon_1 - \varepsilon_2 + I_1)q) +$$
- $$\rho\upsilon_1\frac{\partial W}{\partial x_1} - \sum_{i=2,3}\frac{\rho\upsilon_i}{2}\frac{\partial W}{\partial x_i} = -\frac{(\varepsilon_1 - \varepsilon_2 + I_1)q}{\tau}$$

(4)

$p_1^2 - (p_2^2 + p_3^2)/2$

3.THE HYDRODYNAMIC STAGE

- After selecting the region of the local heating, hot spot - the overlap region of the colliding nuclei, we analyze the stages of compression, expansion and freeze-out of matter during the collision of heavy ions. At the compression stage, a collisionless shock wave with a changing front is formed. At the expansion stage, when the shock wave reaches the boundaries of hot spot, the initially compressed system is expanded, we describe it in the relaxation approximation taking into account the nuclear viscosity. As the relaxation time we take $\tau = \lambda / \upsilon_T$ where $\lambda = 1/\sigma\rho$ is the mean free path, $\sigma \approx$ 40mb is the total nucleon-nucleon cross section, ρ is the nucleon density, and
- υ_T is the average velocity of the thermal motion of the nucleons. At the freeze-out stage, when the system reaches a critical density also called the freeze-out density, the system does not "hold itself" and the secondary particles are formed.



- Fig. 1. Dependence on the collision energy of the maximum compression ratio
- ρ/ρ_0 achieved in the central collision of nuclei for the case of the relaxation factor
- q (solid line) calculated by us, for the case when the factor $q=0$
- (dashed line), and for the case when $q=1$ (a dashed-dot line)

4.STATISTICAL FRAGMENTATION MECHANISM

- To describe the soft part of the spectrum of emitted protons, one can use the statistical model of fragmentation of colliding heavy ions proposed by Feshbach, Huang, and Goldhaber . According to this model, the probability of the escape of fragments from a compound nucleus is proportional to , $\exp\left(-\frac{p^2}{2\sigma_K^2}\right)$
- where p is the momentum of the fragment in the rest frame of the nucleus, and the variance
- $\sigma_K^2 = \sigma_0^2 \frac{K(A-K)}{A-1}$ 1) $\sigma_0^2 = \frac{< p^2 >}{3} = \frac{1}{3}\frac{3}{5} p_F^2, \quad p_F = \sqrt{2m(E^* - 3/2T)}$ 2) $\sigma_0^2 = mT$
- As a result, we find the contribution we need to the cross section for protons during fragmentation (*b* is the impact parameter)

$$E\frac{d^2\sigma}{p^2 dp d\Omega} = \frac{2\pi}{(2\pi\hbar)^3}\int bdb \int Cd\mathbf{r} r\gamma(E - \vec{p}\vec{\upsilon})\exp\left(-\frac{(\mathbf{p}-\mathbf{p}_0)^2}{2\sigma_0^2}\right)$$

5. A COMPARISON WITH EXPERIMENTAL DATA

- As a result, the double differential cross-section of proton emission has the form (where *b* is the impact parameter, $\hbar$ is a Planck constant, $\vec{r}$ is the radius vector):
- $$E\frac{d^2\sigma}{p^2dpd\Omega}=\frac{2\pi}{(2\pi\hbar)^3}\int G(b)bdbd\vec{r}\gamma(E-\vec{p}\vec{\upsilon})f(\vec{r},\vec{p},t) \qquad (5)$$
- where the distribution function of emitted protons
- $$f(\vec{r},\vec{p},t)=g\left[\exp\left(\frac{\gamma(E-\vec{p}\vec{\upsilon}-\mu)+T\delta}{T}\right)+1\right]^{-1} \qquad (6)$$
- Here the spin factor $g=2$, $E=\sqrt{p^2+m^2}$, γ and $\vec{p}$ are respectively the total energy, the Lorentz factor and the proton momentum; $\vec{\upsilon}(\vec{r},t)$ is the velocity field, $G(b)$ is the factor taking into account that the cross section of the hot spot formation is always greater than the geometric one, μ is the chemical potential, which is found from the conservation of the average number of particles for a grand canonical ensemble, T is the temperature, δ is correction for the microcanonical distribution, which for the kinetic energy $\varepsilon=E-m>E_1$ is equal to

$$\delta = \left[-M \ln\left(1 - \frac{\gamma(E - \vec{p}\vec{\upsilon}) - m}{MT} \right) - \frac{\gamma(E - \vec{p}\vec{\upsilon}) - m}{T} \right] \qquad (7)$$

where $M = 3N/2$, N is the number of nucleons in the thermostat, $E_1 (E_1 >> T)$ is the energy that is close to the energy of the thermostat, i.e. close to the kinematic limit for the energy of the system. We also chose the energy value $E_2 (E_2 < E_1)$, when the distribution function decreases by an order of magnitude compared to its maximum. When $\varepsilon < E_2$ the amendment δ was supposed equal to zero. In the energy interval $E_2 > \varepsilon > E_1$ it was a linear interpolation between zero and expression (7). Here the correction δ is found for the Boltzmann limit of an ideal gas, since deviations from a grand canonical distribution of the Fermi gas are manifested on the "tails" of the energy spectra when the Fermi distribution coincides with the Boltzmann limit.

- The probability of a microcanonical distribution in the limit of the Boltzmann limit of an ideal gas is
- $$w(\vec{r}, \vec{p}) = C_M \left(1 - \frac{\varepsilon}{U} \right)^M = C_M \exp\left(M \ln\left(1 - \frac{\varepsilon}{MT} \right) \right) , \qquad (8)$$

- where ε is the kinetic energy of the system, $U = MT$ is the energy of the thermostat, C_M is the normalization factor. As a result, in the limit of a large number of particles N at $M = \frac{3}{2}N \to \infty$, expression (8) becomes a grand canonical distribution
- $$w_0(\vec{r}, \vec{p}) = C_M \exp\left(-\frac{\varepsilon}{T}\right)$$
- Thus, on the tails of the energy distributions, using formula (7), we find an amendment for the microcanonical distribution (6), which changes the usual Fermi-Dirac distribution, describing the system well away from the tails of the proton spectrum. Moreover, in formulas (5) - (6) it is taken into account that the energy of the system is recalculated in accordance with the Lorentz transformations. The energy in the distribution (6) is reckoned from the value of the self-consistent mean field with allowance for the surface energy, since the nucleons are "locked" by the mean field.
- In addition to the contribution of (1) to the cross section for the emission of protons from the hot spot, we also took into account the contribution from the fusion of the non-overlapping parts of the colliding nuclei so called "spectators".

Fig.2. $^{12}C+^{9}Be$ 300MeV/nucl. (protons) $3,5^0$

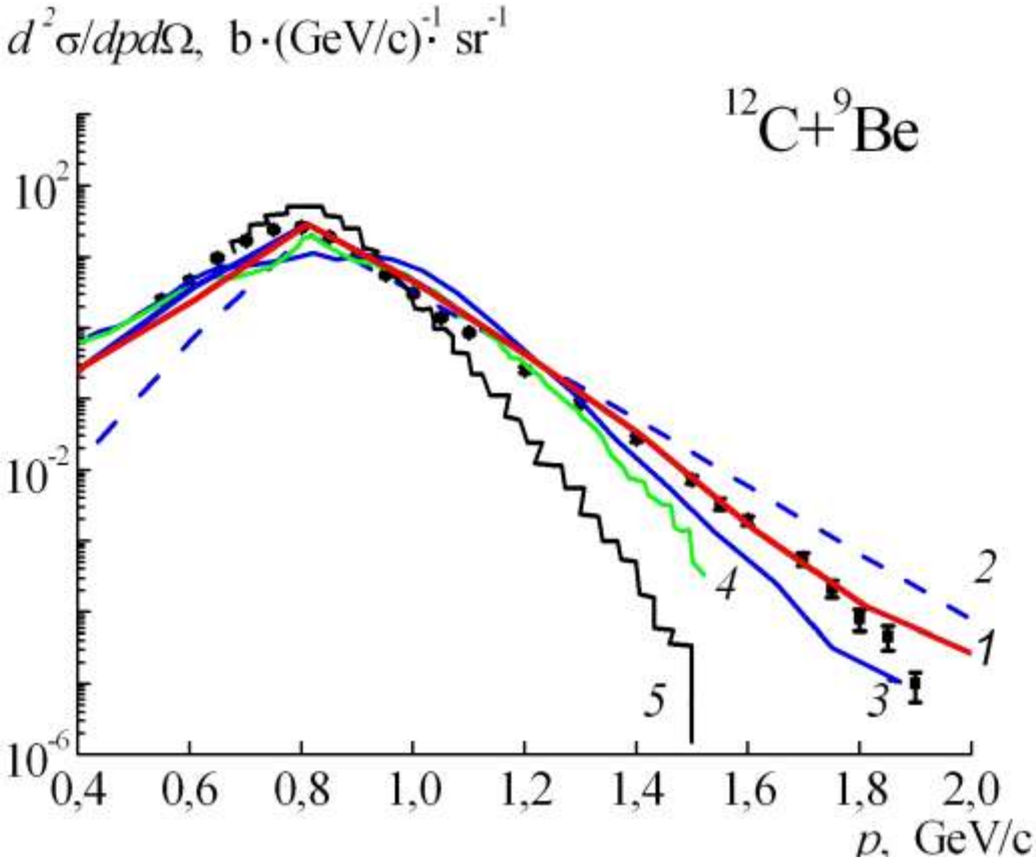


Fig.3. $^{12}C+^{9}Be$ 600 MeV/nucl. (protons) $3{,}5^0$

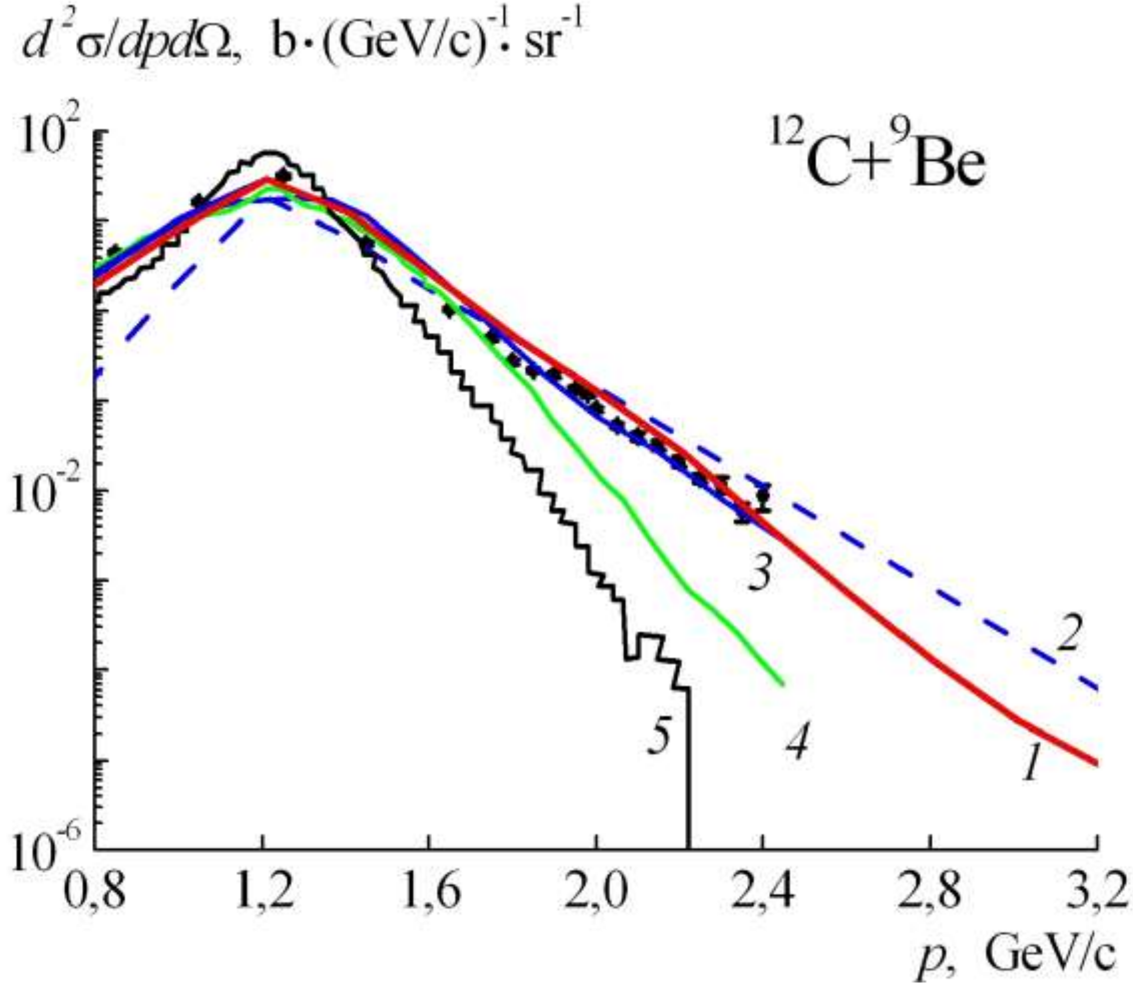


Fig.4. $^{12}C+^{9}Be$ 950 MeV/nucl. (protons) $3,5^0$

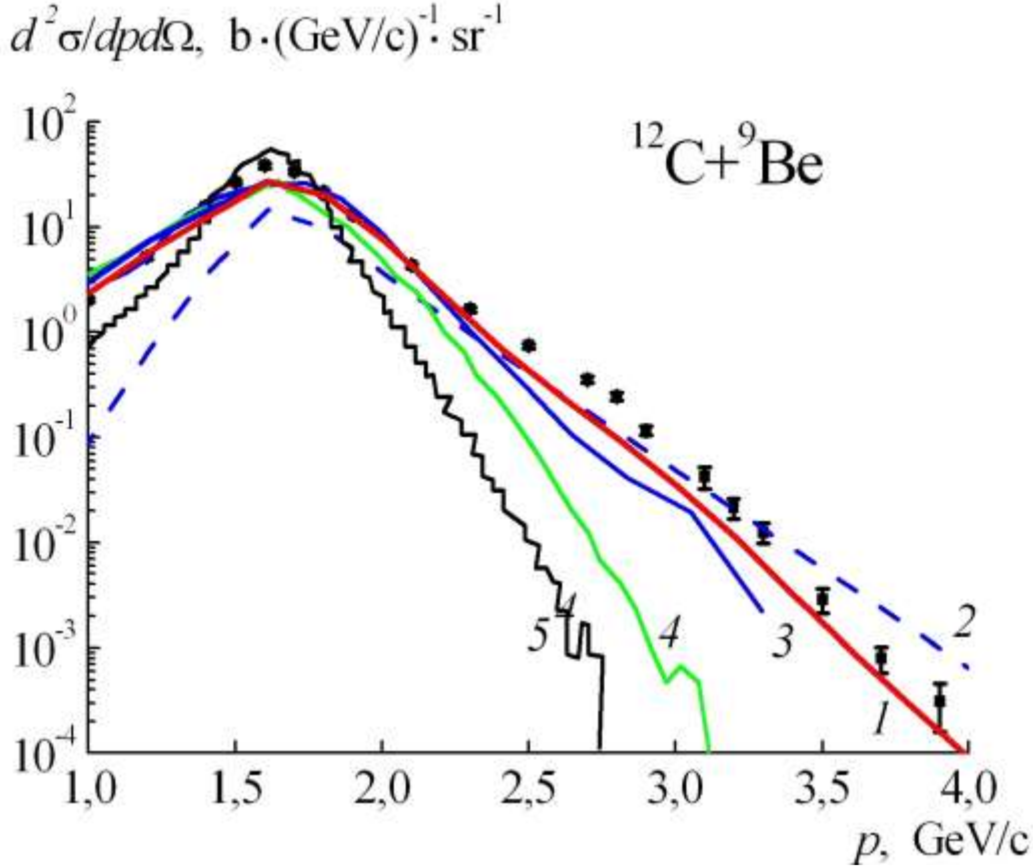


Fig.5. $^{12}C+^{9}Be$ 2 GeV/nucl. (protons) $3,5^0$

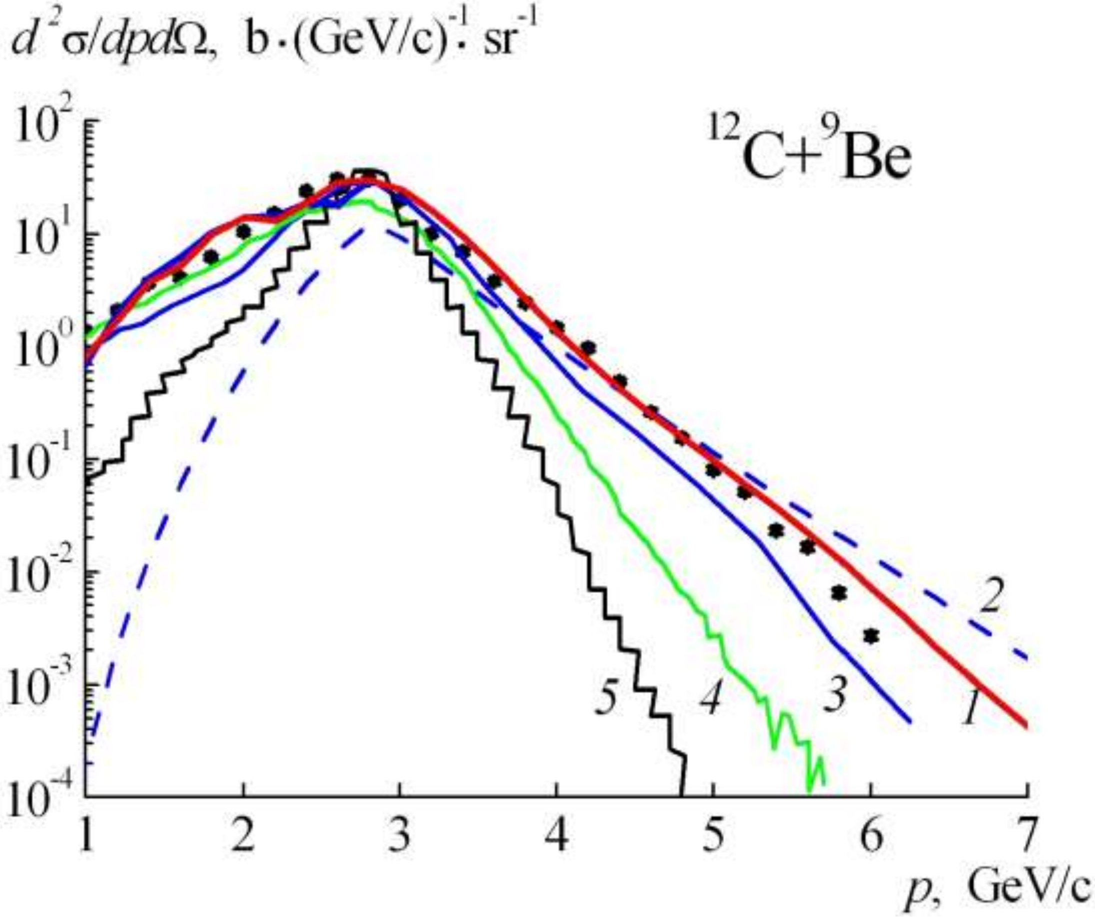


Fig.6. $^{12}C+^{9}Be$ 3.2 GeV/nucl. (protons) $3{,}5^0$

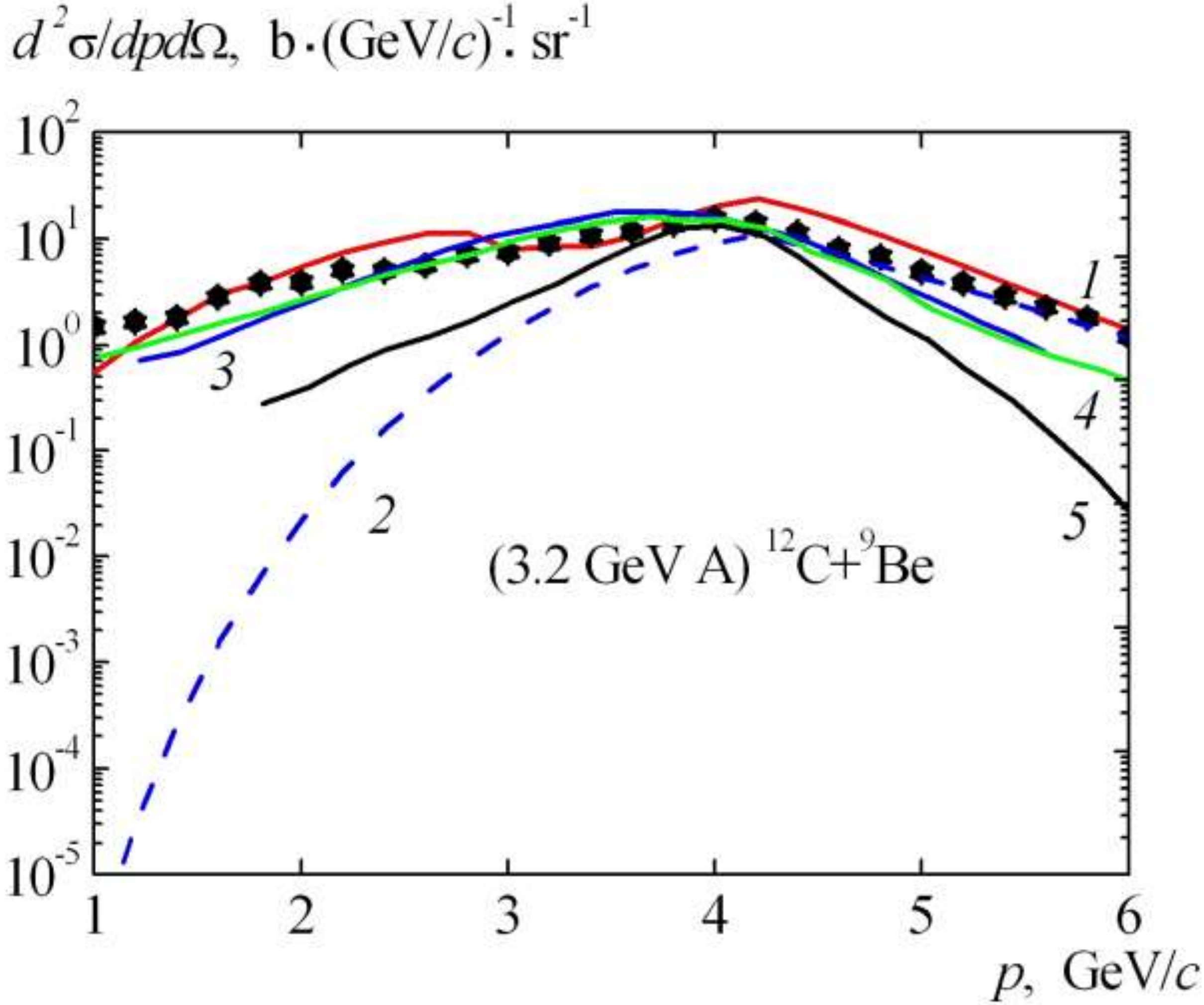


Fig.7. $^{12}C+^{9}Be$ 3.2 GeV/nucl. (negative pions) $3,5^0$

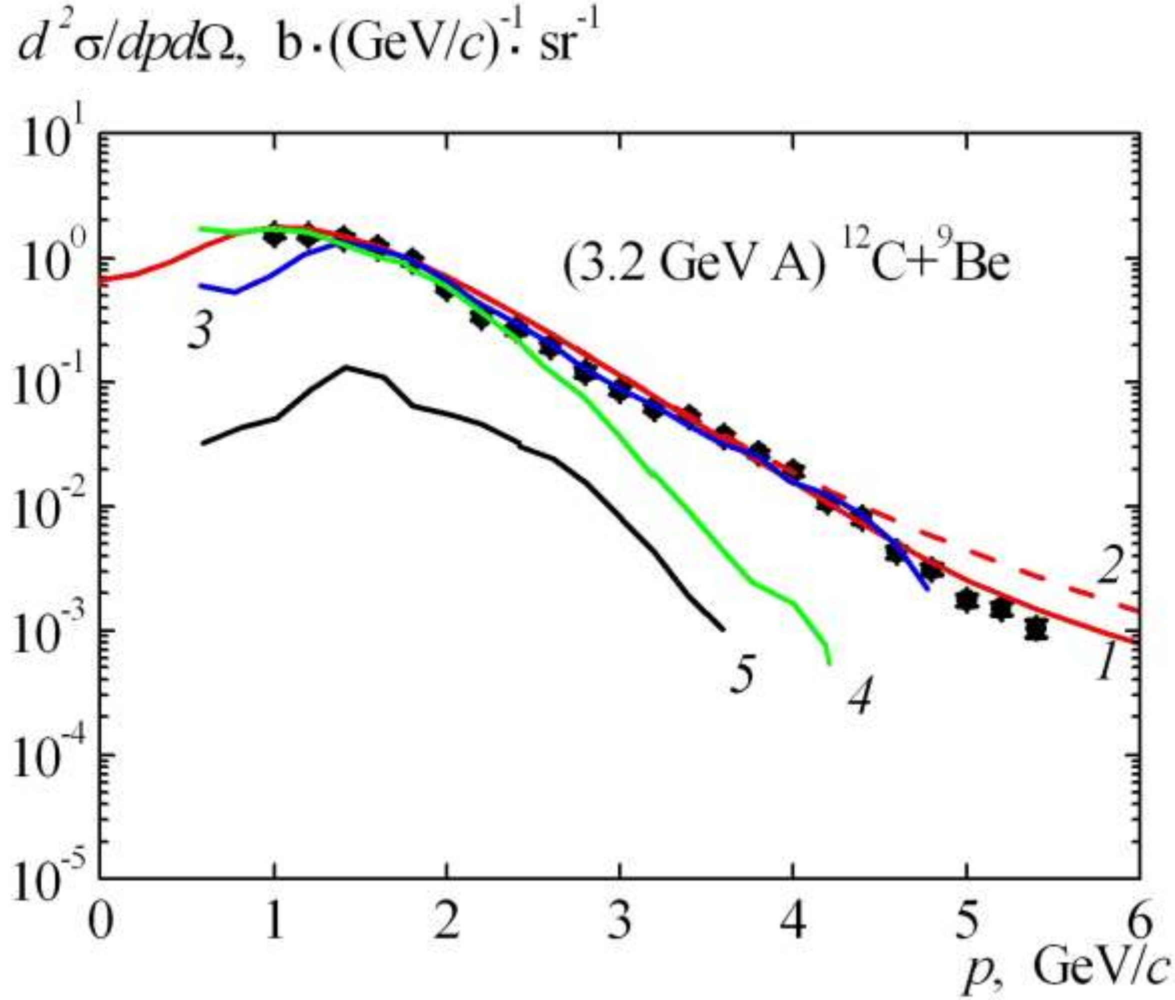


Fig.8. $^{12}C+^{9}Be$ (photons) 38^0

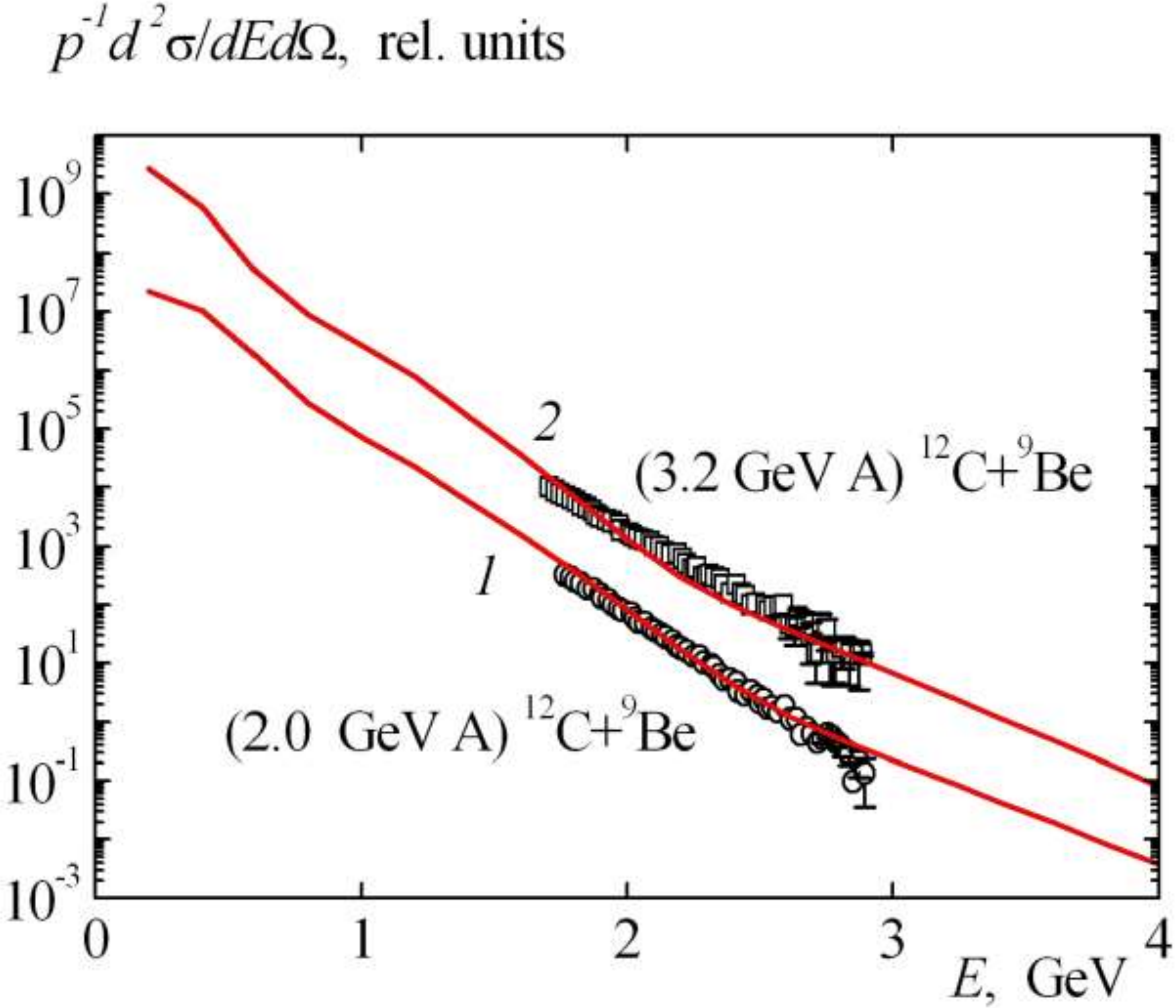


Fig.9. (1.76 GeV A) Ar+KCl (protons)

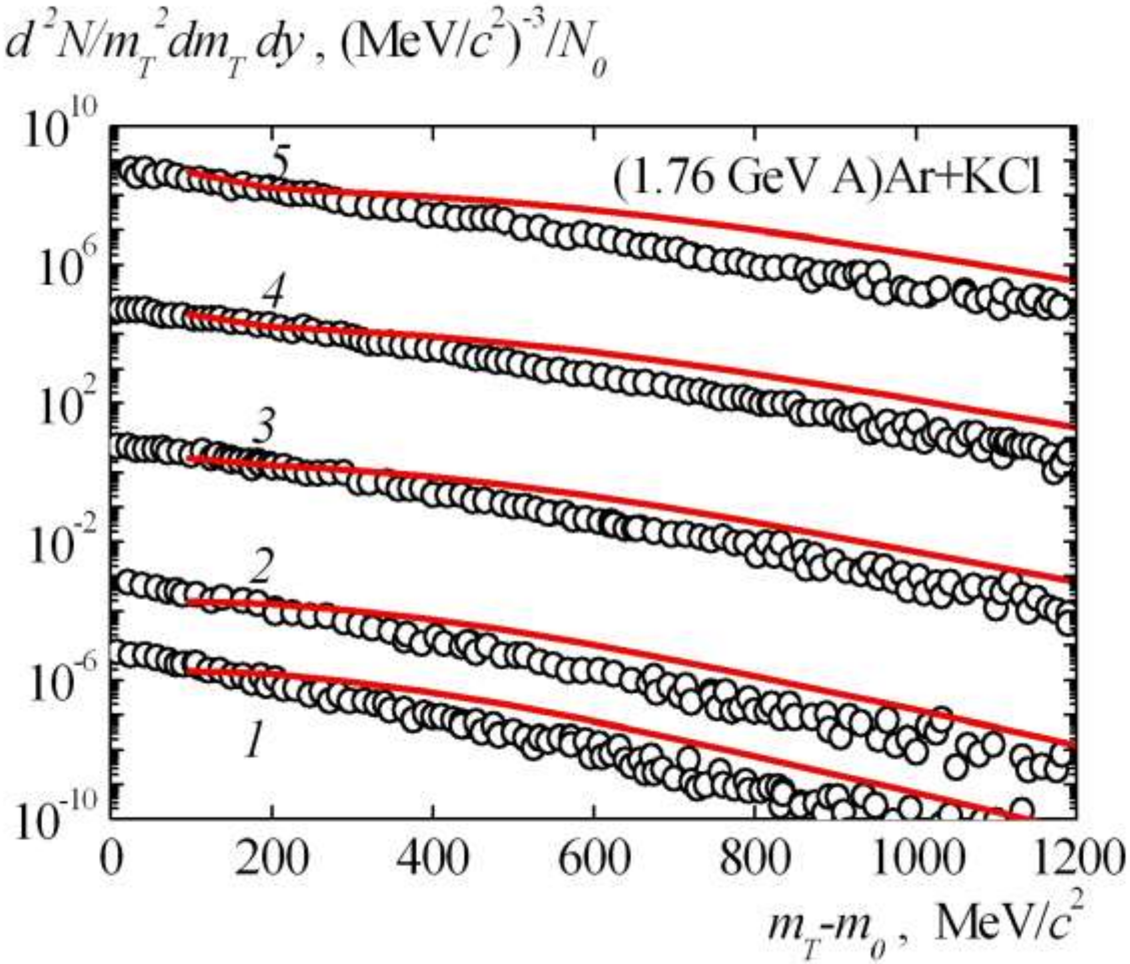


Fig. 10. $^{40}Ar+^{40}Ca$ 92 MeV/A (protons)

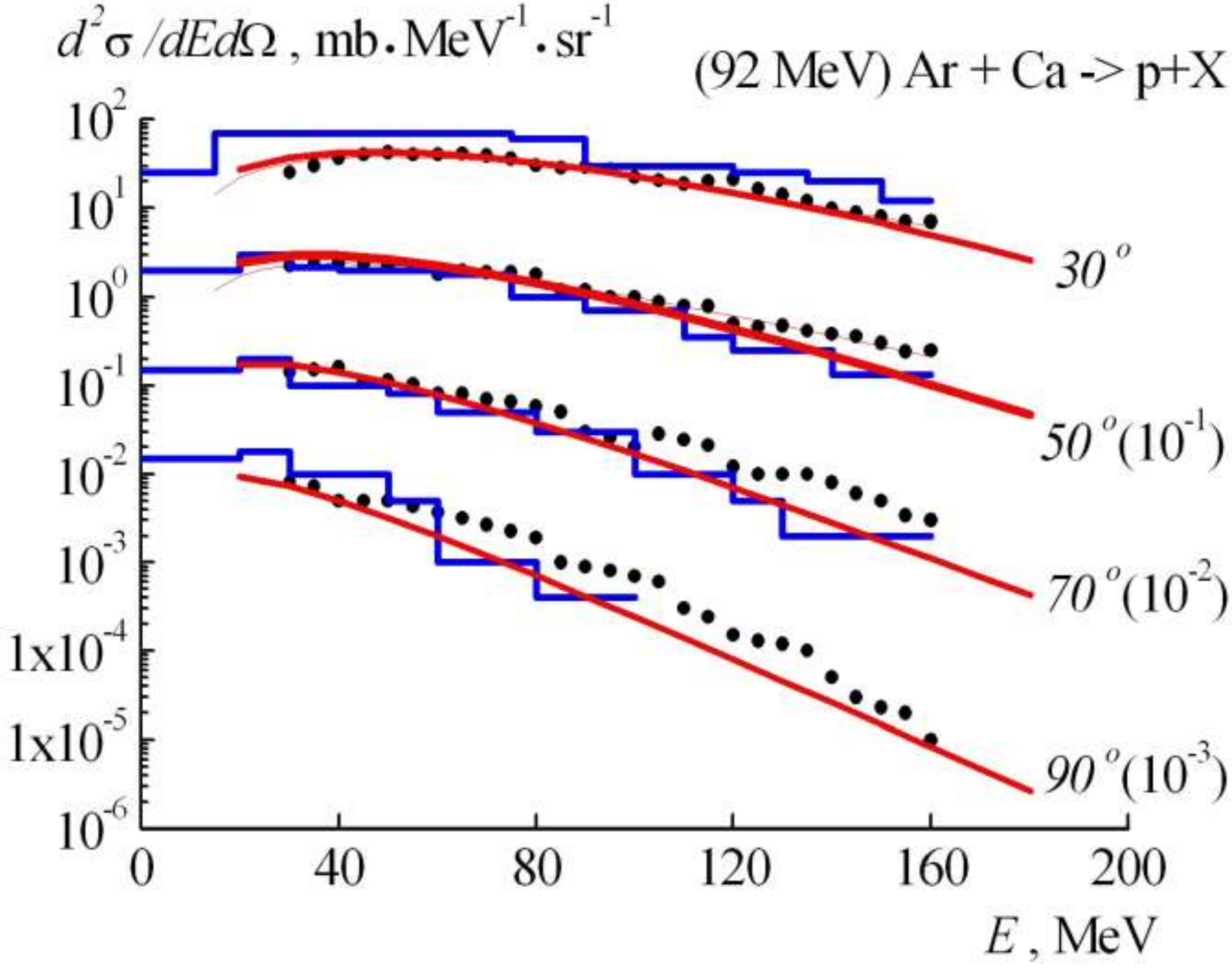


Fig.11. ^{16}O 94 MeV/A 90^o

(subthreshold pions)

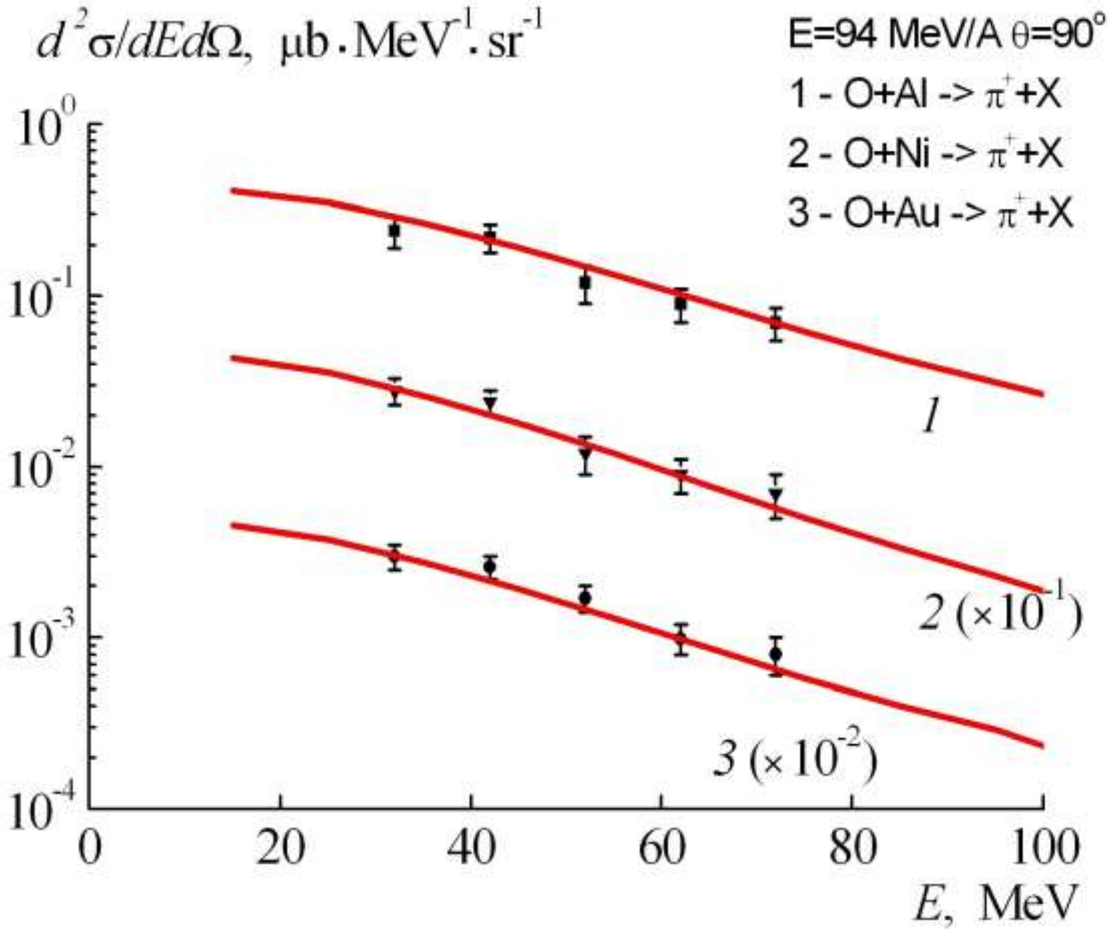


Fig. 12. $^{16}O+^{27}Al$ 94 MeV/A (subthreshold pions)

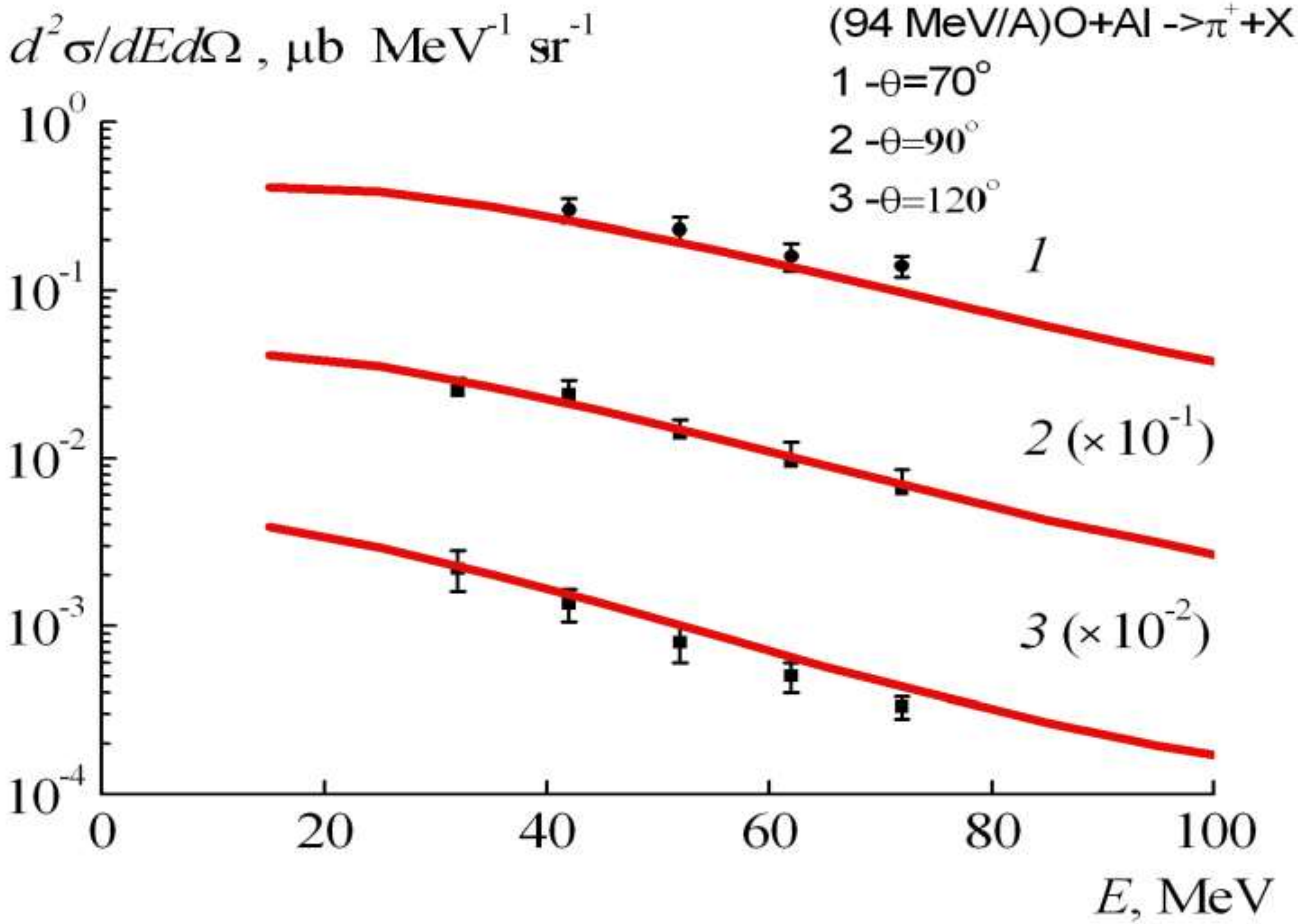


Fig. 13. $^{20}Ne+^{208}Pb$ 2.1 GeV/A (protons)

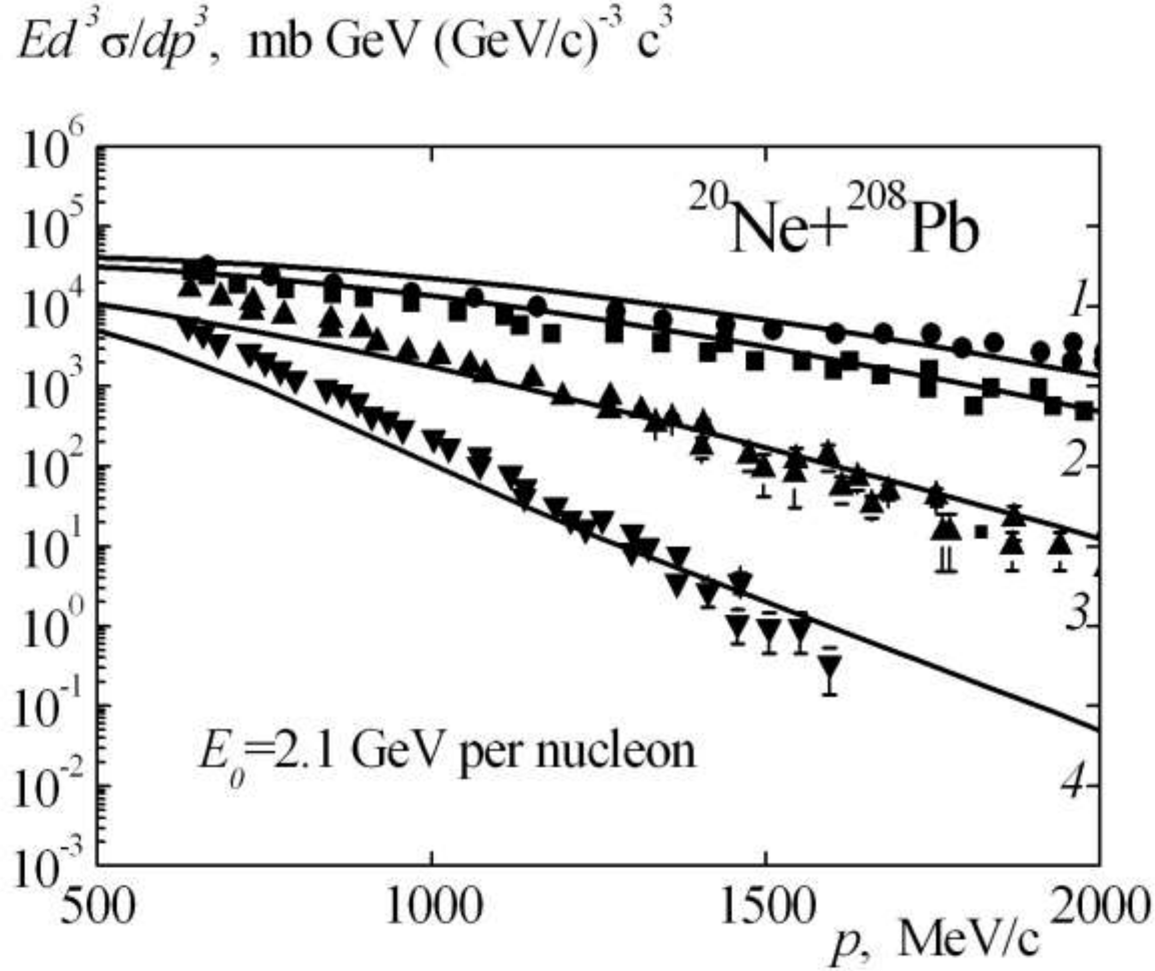


Fig. 14. $^{20}Ne+^{208}Pb$ 2.1 GeV/A (negative pions)

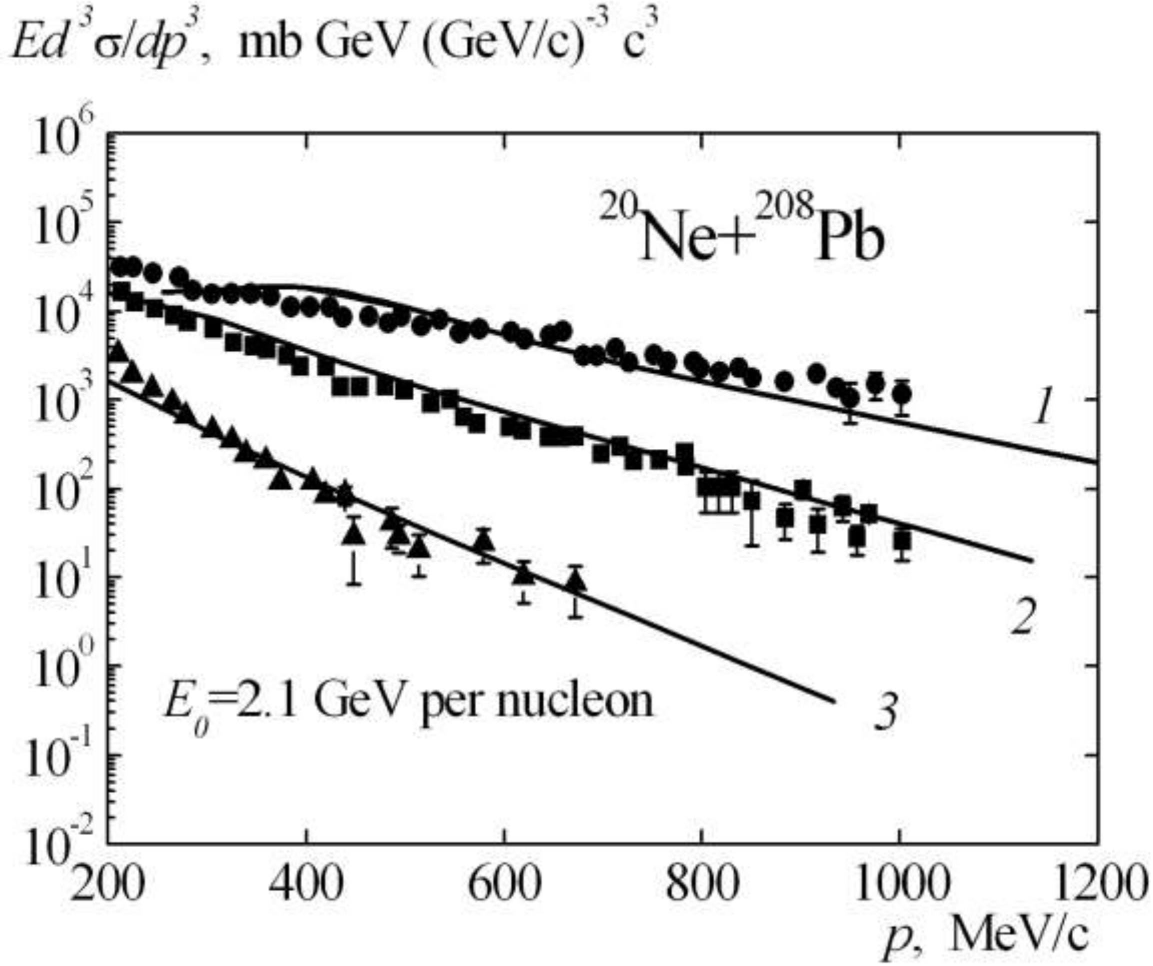
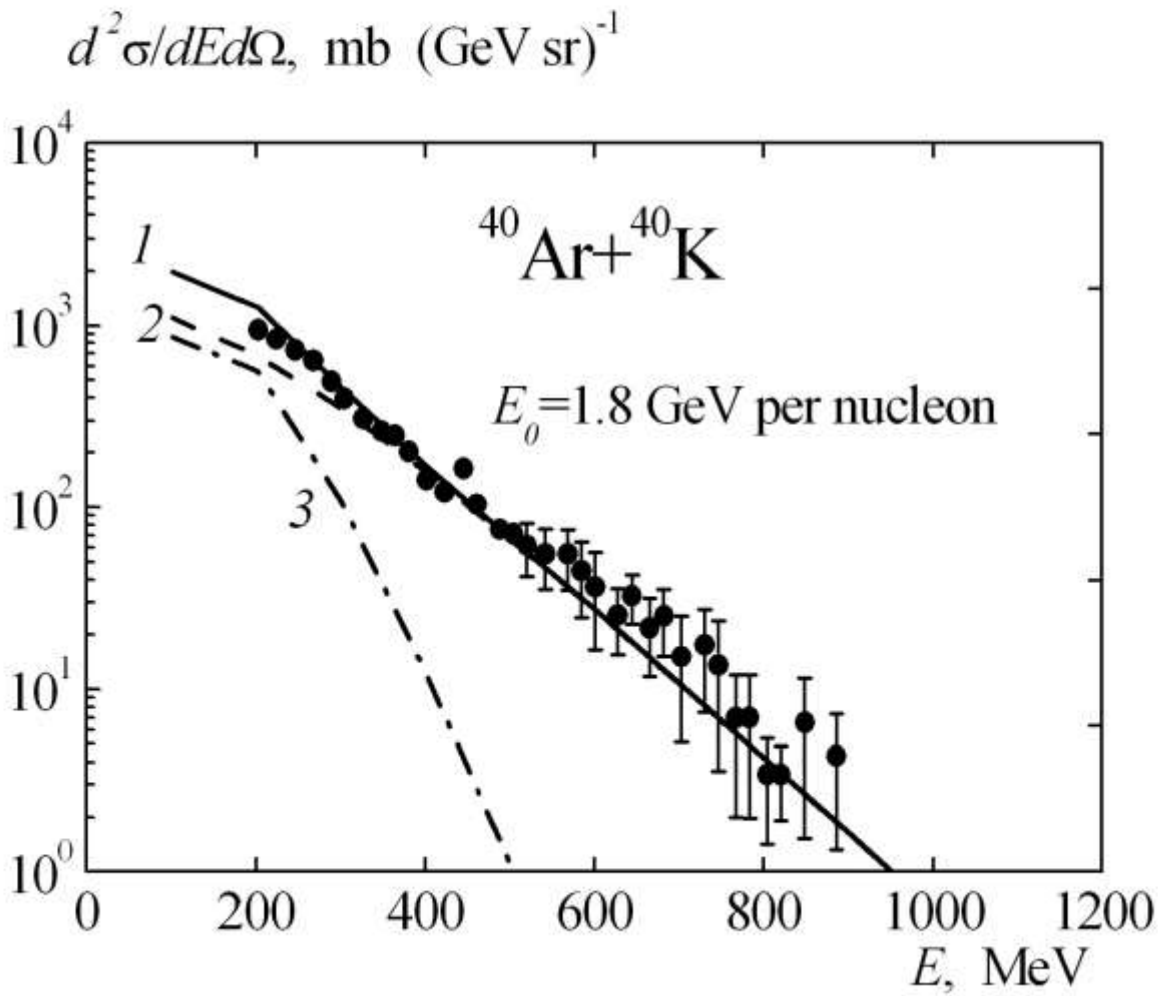
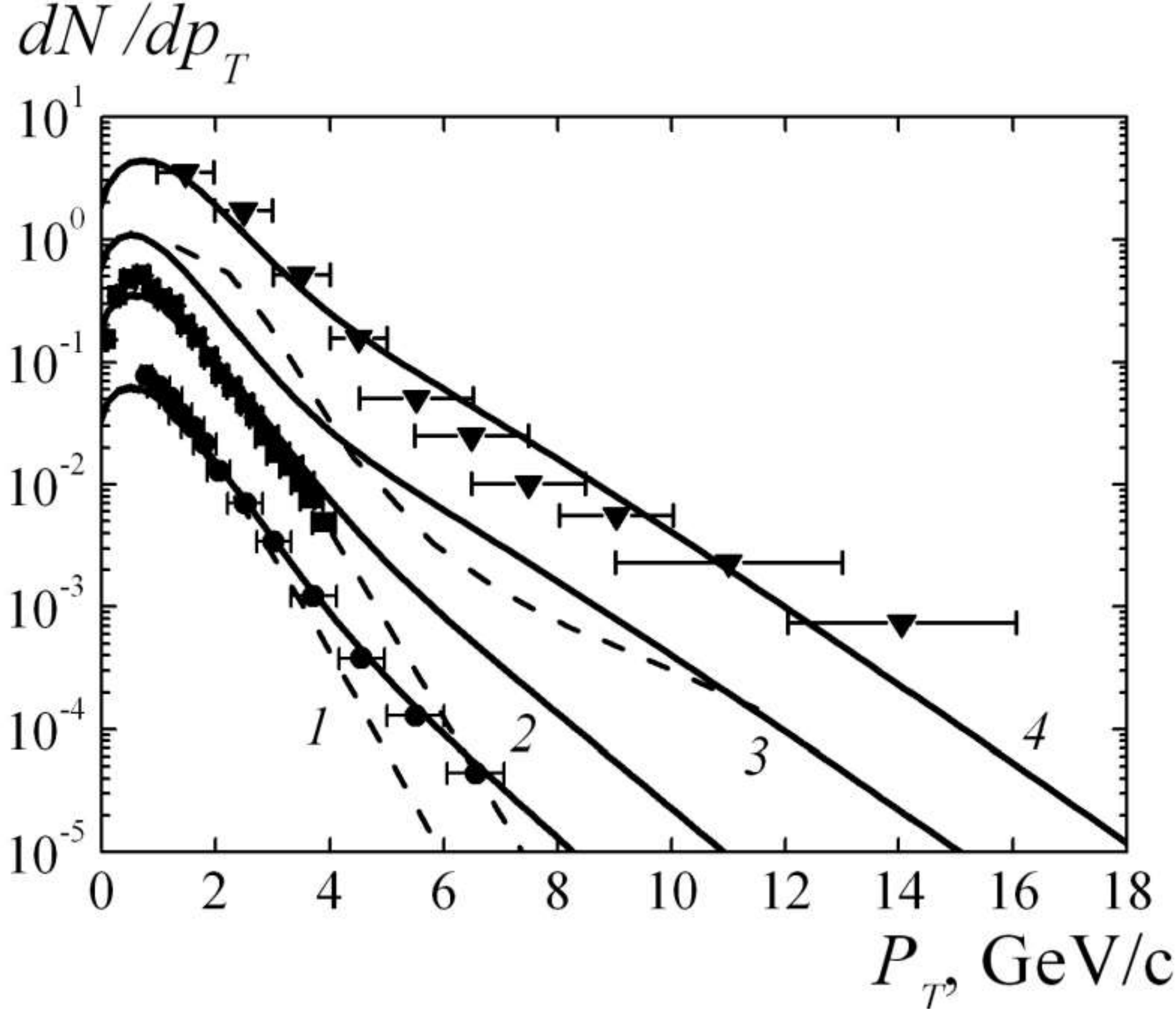
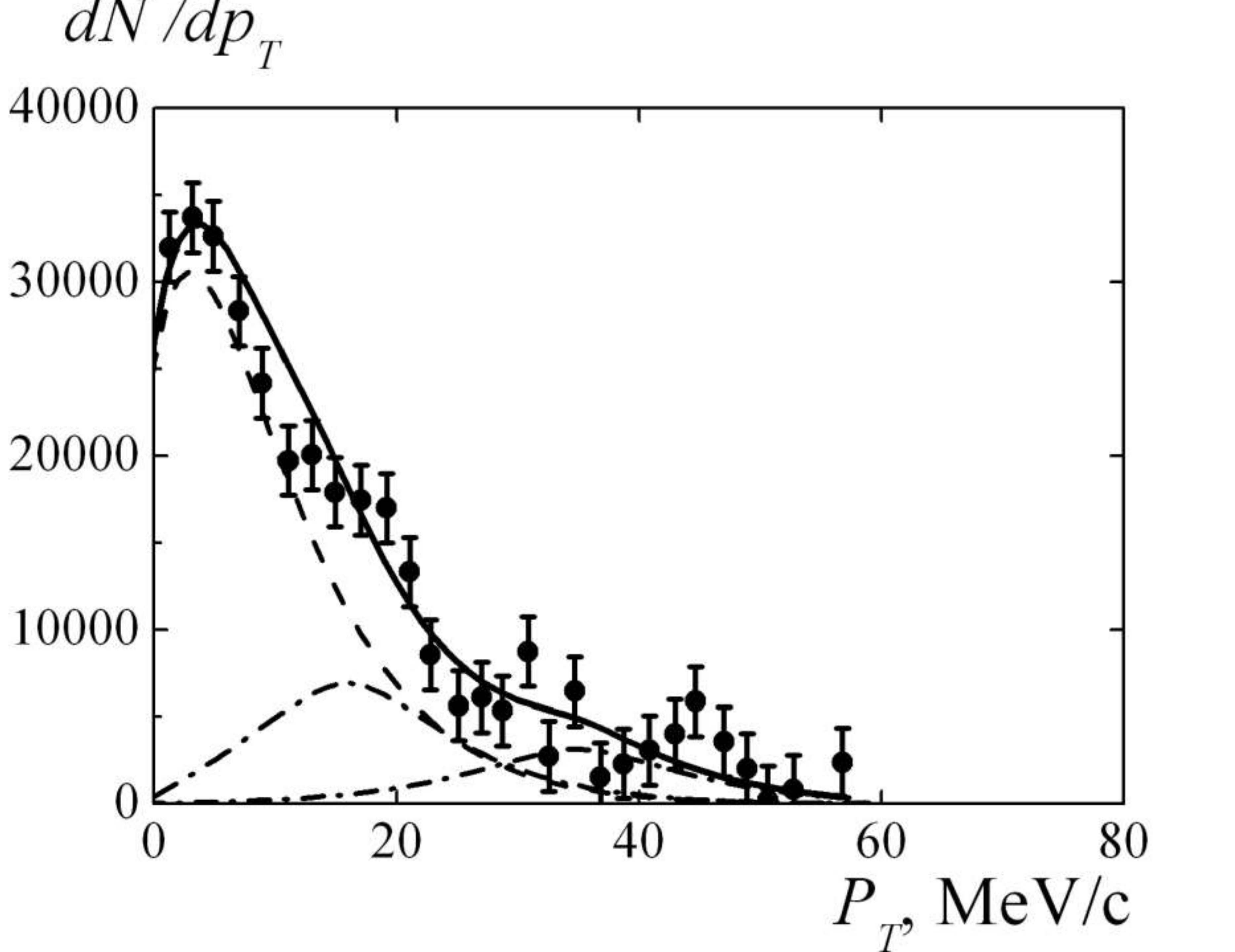


Fig. 15. $^{40}Ar+^{40}K$ 1.8 GeV/A (negative pions)

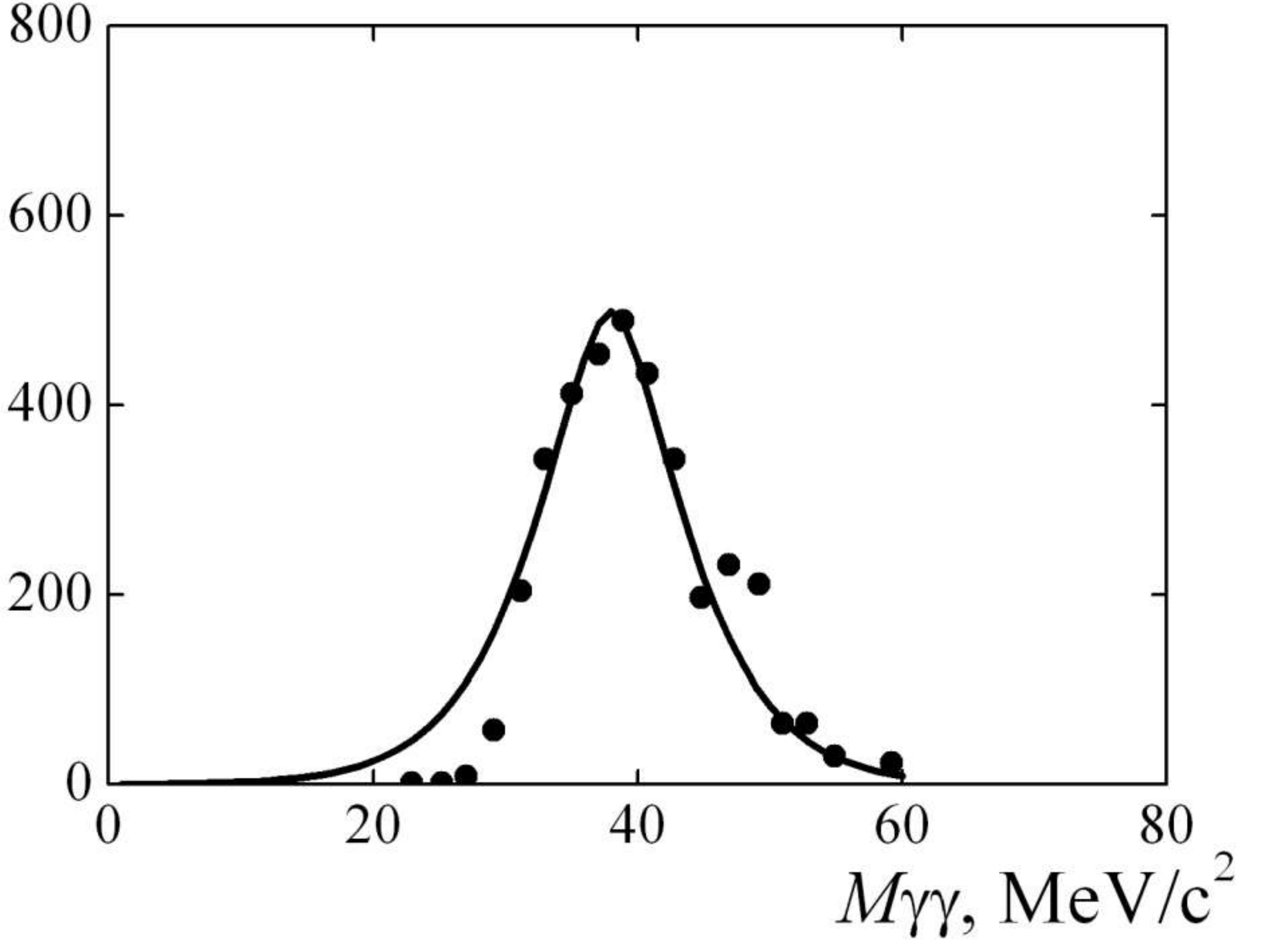




dN/dp_T
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
0
2
4
6
8
10
12
14
16
18
P_T, GeV/c
1
2
3
4



dN /dp_T
40000
30000
20000
10000
0
0
20
40
60
80
P_T, MeV/c


800
600
400
200
0
0
20
40
60
80
$M\gamma\gamma$, MeV/c^2

Conclusion

- Thus, in this paper, the idea of using of a non-equilibrium equation of state in the hydrodynamic approach with hot spot to describe the high-momentum proton, pion and photon spectra emitted in heavy-ion collisions over a wide energy range has been further developed. We also succeded in the description of the subthreshold pion energy spectra.
- The experimental shoulder in the cross section for the production of protons in the cumulative region is reproduced by our calculations, and sometimes by cascade models.The cumulative gamma-quanta is reproduced by ours too. Perhaps this may be due to the contribution of the rescattering of pions to the cumulative production of protons, considered earlier in [35].

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THANK YOU !