Theoretical limitations of amplitudes and their decisive influence on the parameters of resonances

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First predictions

M. Gell-Mann, Phys. Lett. 8 (1964) 214-215

"A Schematic Model of Baryons and Mesons"

... **Barions can now be constructed from quarks by using the combinations** (qqq)**,** $(qqqq\bar{q})$**, etc., while mesons are made out of** $(q\bar{q})$**,** $(qq\bar{q}\bar{q})$**, etc.** ...

also:
$(qq\bar{q}\bar{q})$ *states:*
R.L. Jaffe, Phys. Rev. D15, 267 (1977);
R.L. Jaffe, Phys. Rev. D15, 281 (1977)

Is what we observe

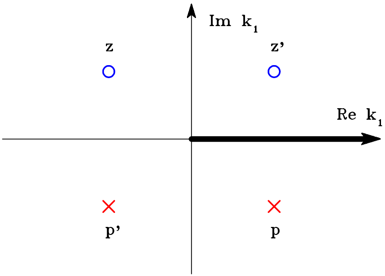
exactly that what

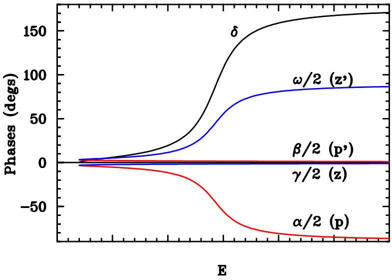
really exists in nature?

One channel scattering

- $S(k) = \frac{D(-k)}{D(k)} = e^{2i\delta}$, $|S(k)| = 1$
- $D(k) = (k - k_j)$
- $D(-k) = (-k - k_j)$
- But $|S(k)| \neq 1$ so
- $D(k) = (k - k_j)(k + k_j^*)$
- $D(-k) = (-k - k_j)(-k + k_j^*)$
- then $|S(k)| = 1$
- and $\delta = (-\alpha - \beta + \gamma + \omega)/2$

$$angle = ArcTan(\frac{-Imk_j}{k - Rek_j})$$





Breit Wigner approximation

- $BW(E) = \frac{\Gamma/2k}{M_{BW} - E - i\Gamma/2}$
- $\sigma_l(E) = 4\pi(2+1)|BW(E)|^2 = \frac{\pi}{k^2}(2l+1)\frac{\Gamma^2}{(M_{BW}-E)^2+\Gamma^2/4}$
- $BW(E) = \frac{S_l(E)-1}{2ik}$
- $S_l(E) = \frac{M_{BW}-E+i\Gamma/2}{M_{BW}-E-i\Gamma/2}$
- Defining $E_j = M_{BW} - i\Gamma/2$ we get
- $S_l(E) = \frac{E-E_j^*}{E-E_j}$ and
- because $E = \sqrt{(\pm k)^2/4 + m^2}$ there are two poles and two zeroes and
- $|S_l(E)| = 1$
- so Breit Wigner approximation is unitary!!!

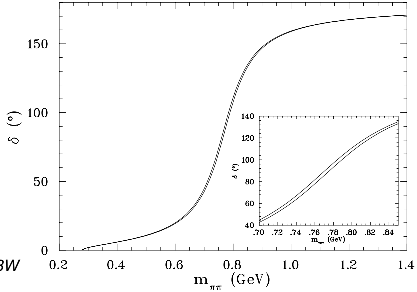
Pole and mass of a resonance

- Let's imagine good fit of an amplitude to the data —> mass M_{BW} at $\delta = 90^o$
  - Let's fit amplitude $A_{S_{notU}}$ to the same data. $A_{S_{notU}}$ has a single pole at $k_j = a - ib$ then $\delta = ArcTan(\frac{-b}{k-a}) + ArcTan(\frac{b}{-k-a})$ and $M_{BW} \neq 2\sqrt{a^2+m^2}$, additionally $|S| \neq 1$
  - Let's fit amplitude A_{S_U} to the same data. A_{S_U} has a two symmetric poles at $k_j = a - ib$ and $k = -a - ib$ then $\delta = ArcTan(\frac{2bk}{k^2-a^2-b^2}) + ArcTan(\frac{-2bk}{-k^2-a^2-b^2})$ and again $M_{BW} \neq 2\sqrt{a^2+m^2}$, but now $|S| = 1$

Let's check it for ρ(770):
$M_{BW} = 775.26 \pm 0.25$ MeV (PDG'2016),
$\Gamma = 149.1 \pm 0.8$ MeV (PDG'2016),
$2\sqrt{a^2+m^2} < M_{BW}$ by ≈ 9 MeV !!!

Left (upper) line:
A_{BW} fitted to the data

Right (lower) line:
A_S fitted to the data with $2\sqrt{a^2+m^2} = M_{BW}$



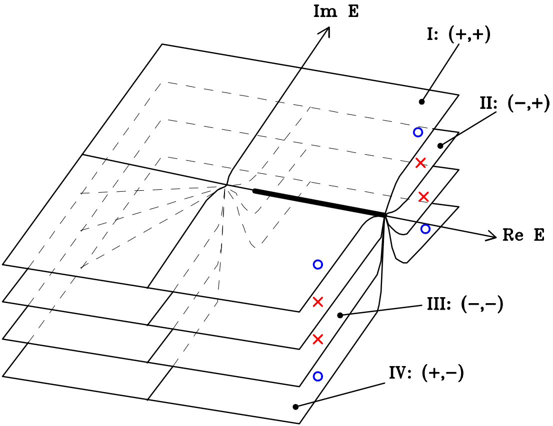
More resonances (but still one channel)

Adding resonances (for simplicity two resonances, both with $S = e^{2i\delta}$):

- Isobar model: adding amplitudes (even unitary ones) violates unitarity:
  $T_{1,2} = T_1 + T_2 = \frac{S_1-1}{2ik} + \frac{S_1-1}{2ik} \to S_1 + S_2 = e^{2i\delta_1} + e^{2i\delta_2}$
  of course $|S_1 + S_2| \neq 1$,
- Product of S matrices: $|S_1 S_2| = 1$ in elastic case and $|S_1 S_2| < 1$ in inelastic case ($S = \eta e^{2i\delta}$)
  For example $S_{1,2} = \frac{(-k-k_1)(-k+k_1^*)(-k-k_2)(-k+k_2^*)}{(k-k_1)(k+k_1^*)(k-k_2)(k+k_2^*)}$
  Of course $T_{1,2} = \frac{S_{1,2}-1}{2ik}$
- Sum of K matrices: $S = 1 + 2iT = (1 + iK)/(1 - iK)$ does noit violate unitarity, for example $T_{1,2} = \frac{1}{k}\frac{K_1+K_2}{1-iK_1-iK_2}$

More channels: $k_2 = \pm\sqrt{k_1^2 + m_1^2 - m_2^2}$

$(\mathrm{Im}(k_1), \mathrm{Im}(k_2))$: (+,+), (-,+) .... 1 pole $\longrightarrow 2^{(n-1)}$ poles (n-number of channels)



Multiplication and displacement of S matrix poles

- Multiplication:

  1 pole $\longrightarrow 2^{n-1}$ poles due to $(\pm k)^2$ ambiguity and

  2^n Riemann sheets

- Displacement:

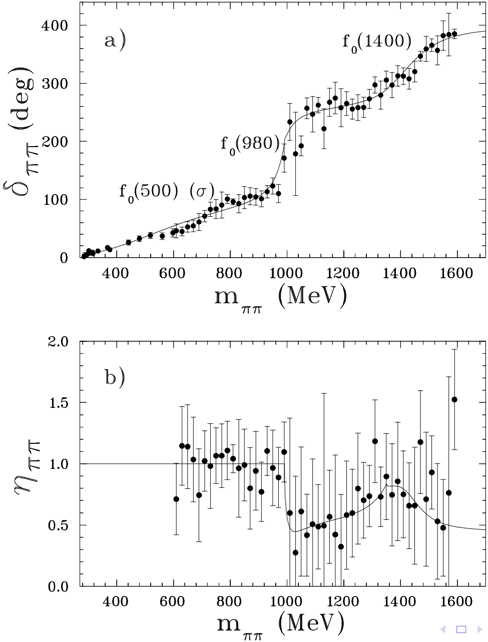
  $S_{11} = \frac{D_1(-k_1)D_2(k_2)}{D_1(k_1)D_2(k_2)}$ in decoupled case

  $S = \frac{D_1(-k_1)D_2(k_2)+C(-k_1,k_2)}{D_1(k_1)D_2(k_2)+C(k_1,k_2)}$ in coupled case

$$S = \begin{pmatrix} \eta e^{2i\delta_1} & i\sqrt{1-\eta^2}e^{i(\delta_1+\delta_2)} \\ i\sqrt{1-\eta^2}e^{i(\delta_1+\delta_2)} & \eta e^{2i\delta_2} \end{pmatrix} = \begin{pmatrix} \frac{D(-k_1,k_2)}{D(k_1,k_2)} & S_{12} \\ S_{21} & \frac{D(k_1,-k_2)}{D(k_1,k_2)} \end{pmatrix}$$

where $S_{12}^2 = S_{21}^2 = S_{11}S_{22} - \frac{D(-k_1,-k_2)}{D(k_1,k_2)}$

Example for two channels: $JI = S0$ wave

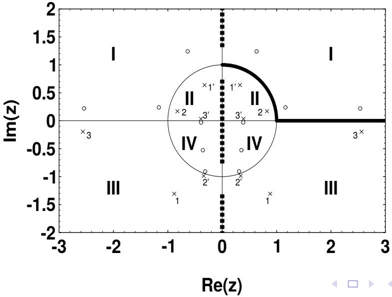


Example for two channels: $JI = S0$ wave

Pole	ReE_{pole} MeV	ImE_{pole} MeV	R. sheet
1	639.6	-323.9	$(-,-)$: *III*
1’	511.4	-230.6	$(-,+)$: *II*
2	982.0	-36.9	$(-,+)$: *II*
2’	432.4	-8.4	$(-,-)$: *III*
3	1431.7	-79.3	$(-,-)$: *III*
3’	1394.9	-120.6	$(-,+)$: *II*

$$z = \frac{k_1 + k_2}{\sqrt{m_K^2 - m_\pi^2}}$$

Rysunek 16: Położenie biegunów (krzyże) i zer (kółka) elementu macierzowego S_{11} macierzy rozpraszania dla dopasowania do zestawu D_{CKM} A. Gruba linia ciągła oznacza obszar fizyczny rozpraszania w kanałach sprzężonych $\pi\pi$ i $K\overline{K}$. Grubą linią przerywaną przedstawione jest położenie cięć funkcji Josta. Cienką linią zaznaczony jest okrąg $|z| = 1$. Numeracja poszczególnych płatów i biegunów została wyjaśniona w tekście.



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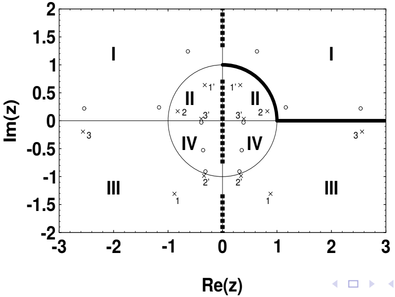
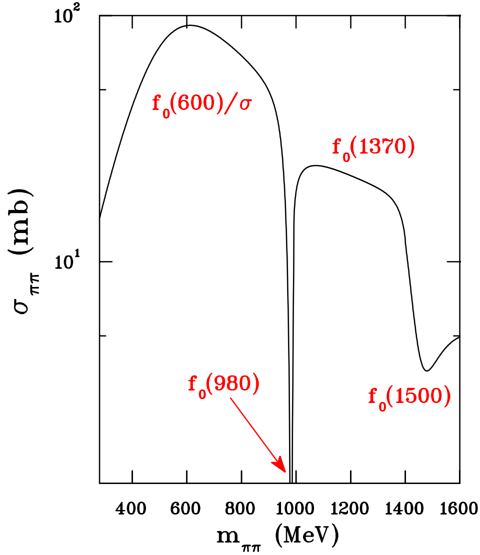


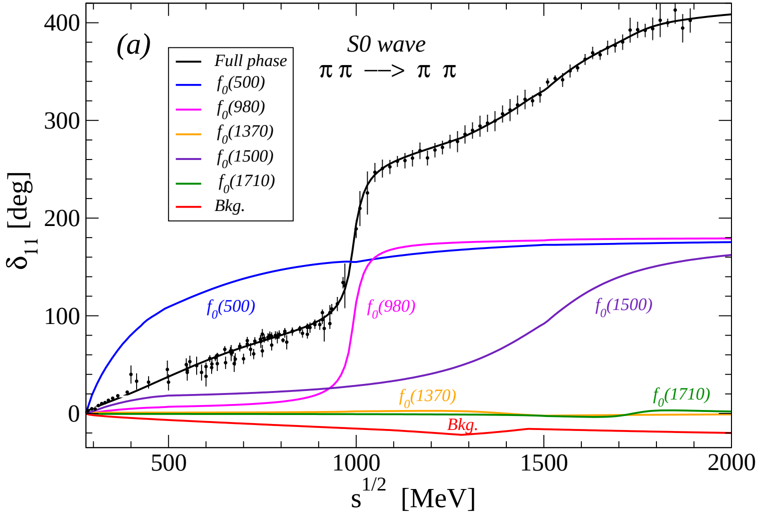
2^n Riemann sheets for n channels

channel	$C=0$		$C=1$		sign	sheet
	ReE	*ImE*	*ReE*	*ImE*	Imk_π, Imk_K, Imk_3	
$\pi\pi$	658	-607	564	-279	$-,-,-$	VI
			518	-261	$-,+,+$	II
			211	0	$-,+,-$	VII
			532	-315	$-,-,+$	III
			235	0	$+,+,-$	VIII
$\pi\pi$	1346	-275	1405	-74	$-,-,-$	VI
			1445	-116	$-,+,+$	II
			1424	-94	$-,+,-$	VII
			1456	-47	$-,-,+$	III
$K\bar{K}$	881	-498	170	0	$+,-,-$	V
			159	0	$-,-,-$	VI
			418	-10	$-,-,+$	III
			1038	-204	$-,+,-$	VII
			988	-31	$-,+,+$	II
$\sigma\sigma$	118	-2227	4741	-4688	$-,-,-$	VI
			3687	-2875	$-,+,-$	VII
			3626	-3456	$+,-,-$	V
			3533	-579	$+,+,-$	VIII

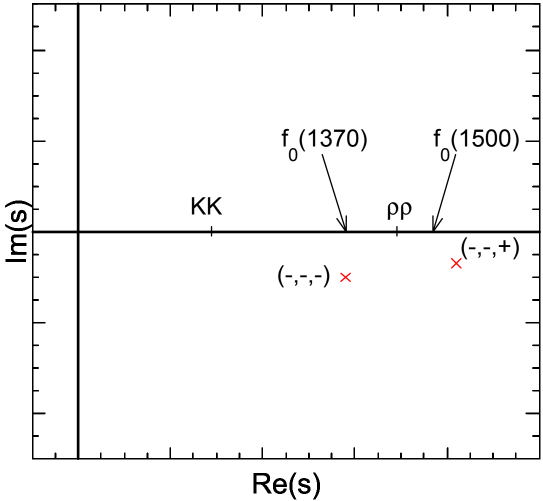
Puzzling (*JI*) *S*0 wave $\pi\pi$ cross section



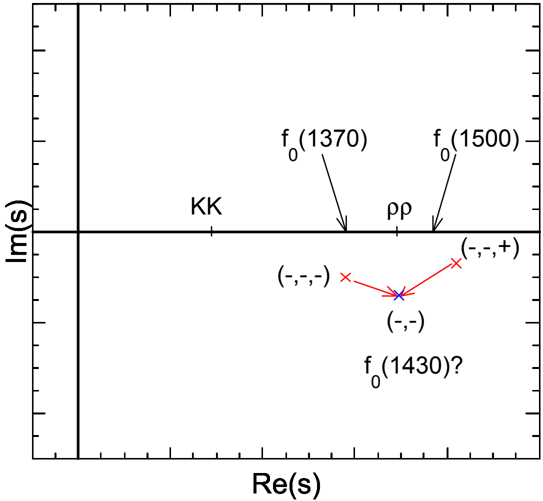
phase shifts of components in the *S*0 wave



$f_0(1370)$ and $f_0(1500)$: positions of poles, $C = 1$



$f_0(1370)$ and $f_0(1500)$: positions of poles, $C = 0$



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$\pi\pi$	658	-607	564	-279	$-,-,-$	VI
			518	-261	$-,+,+$	II
			211	0	$-,+,-$	VII
			532	-315	$-,-,+$	III
			235	0	$+,+,-$	VIII
$\pi\pi$	1346	-275	1405	-74	$-,-,-$	VI
			1445	-116	$-,+,+$	II
			1424	-94	$-,+,-$	VII
			1456	-47	$-,-,+$	III
$K\bar{K}$	881	-498	170	0	$+,-,-$	V
			159	0	$-,-,-$	VI
			418	-10	$-,-,+$	III
			1038	-204	$-,+,-$	VII
			988	-31	$-,+,+$	II
$\sigma\sigma$	118	-2227	4741	-4688	$-,-,-$	VI
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	ReE	*ImE*	*ReE*	*ImE*	Imk_π, Imk_K, Imk_3		
$\pi\pi$	658	-607	564	-279	$-,-,-$	VI	
			518	-261	$-,+,+$	II	$\leftarrow f_0(500)$
			211	0	$-,+,-$	VII	
			532	-315	$-,-,+$	III	
			235	0	$+,+,-$	VIII	
$\pi\pi$	1346	-275	1405	-74	$-,-,-$	VI	$\leftarrow f_0(1370)?$
			1445	-116	$-,+,+$	II	
			1424	-94	$-,+,-$	VII	
			1456	-47	$-,-,+$	III	$\leftarrow f_0(1500)?$
$K\bar{K}$	881	-498	170	0	$+,-,-$	V	
			159	0	$-,-,-$	VI	
			418	-10	$-,-,+$	III	
			1038	-204	$-,+,-$	VII	
			988	-31	$-,+,+$	II	$\leftarrow f_0(980)$
$\sigma\sigma$	118	-2227	4741	-4688	$-,-,-$	VI	
			3687	-2875	$-,+,-$	VII	
			3626	-3456	$+,-,-$	V	
			3533	-579	$+,+,-$	VIII	

$\rho(770)$

$I^G(J^{PC}) = 1^+(1^{--})$

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$\rho(770)$ MASS

We no longer list *S*-wave Breit-Wigner fits, or data with high coml background.

NEUTRAL ONLY, e^+e^-

VALUE (MeV)	EVTS	DOCUMENT ID		TECN	COMM
775.26±0.25 OUR AVERAGE					
775.02±0.35		[1] LEES	12G	BABR	e^+e^-
775.97±0.46±0.70	900k	[2] AKHMETSHIN	07		e^+e^-
774.6 ±0.4 ±0.5	800k	[3,4] ACHASOV	06	SND	e^+e^-
775.65±0.64±0.50	114k	[5,6] AKHMETSHIN	04	CMD2	e^+e^-
775.9 ±0.5 ±0.5	1.98M	[7] ALOISIO	03	KLOE	1.02 e π^+
775.8 ±0.9 ±2.0	500k	[7] ACHASOV	02	SND	1.02 e π^+
775.9 ±1.1		[8] BARKOV	85	OLYA	e^+e^-
• • • We do not use the following data for averages, fits, limits, etc. • •					
775.8 ±0.5 ±0.3	1.98M	[9] ALOISIO	03	KLOE	1.02 e π^+
775.9 ±0.6 ±0.5	1.98M	[10] ALOISIO	03	KLOE	1.02 e π^+
775.0 ±0.6 ±1.1	500k	[11] ACHASOV	02	SND	1.02 e π^+
775.1 ±0.7 ±5.3		[12] BENAYOUN	98	RVUE	e^+e^- μ^+
770.5 ±1.9 ±5.1		[13] GARDNER	98	RVUE	0.28–(π^+
764.1 ±0.7		[14] O'CONNELL	97	RVUE	e^+e^-
757.5 ±1.5		[15] BERNICHA	94	RVUE	e^+e^-
768 ±1		[16] GESHKEN...	89	RVUE	e^+e^-

CHARGED ONLY, τ DECAYS and e^+e^-

VALUE (MeV)	EVTS	DOCUMENT ID		TECN	CHG	COM
775.11±0.34 OUR AVERAGE						
774.6 ±0.2 ±0.5	5.4M	[17,18] FUJIKAWA	08	BELL	±	τ^-
775.5 ±0.7		[18,19] SCHAEL	05C	ALEP		τ^-
775.5 ±0.5 ±0.4	1.98M	[7] ALOISIO	03	KLOE		1.02 π
775.1 ±1.1 ±0.5	87k	[20,21] ANDERSON	00A	CLE2		τ^-
• • • We do not use the following data for averages, fits, limits, etc. • •						
774.8 ±0.6 ±0.4	1.98M	[10] ALOISIO	03	KLOE	−	1.02 π
776.3 ±0.6 ±0.7	1.98M	[10] ALOISIO	03	KLOE	+	1.02 π
773.9 ±2.0 $^{+0.3}_{-1.0}$		[22] SANZ-CILLERO	03	RVUE		τ^-
774.5 ±0.7 ±1.5	500k	[7] ACHASOV	02	SND	±	1.02 π
775.1 ±0.5		[23] PICH	01	RVUE		τ^-

MIXED CHARGES, OTHER REACTIONS

VALUE (MeV)	EVTS	DOCUMENT ID		TECN	CHG	COMMENT
763.0±0.3±1.2	600k	[24] ABELE	99E	CBAR	0±	0.0 $\bar{p}p \to \pi^+\pi^-\pi^0$

CHARGED ONLY, HADROPRODUCED

VALUE (MeV)	EVTS	DOCUMENT ID		TECN	CHG	COMMENT
766.5±1.1 OUR AVERAGE						
763.7±3.2		ABELE	97	CBAR		$\bar{p}n \to \pi^-\pi^0\pi^0$
768 ±9		AGUILAR-...	91	EHS		400 pp
767 ±3	2935	[25] CAPRARO	87	SPEC	−	200 π^-Cu $\to \pi^-\pi^0$Cu
761 ±5	967	[25] CAPRARO	87	SPEC	−	200 π^-Pb $\to \pi^-\pi^0$Pb
771 ±4		HUSTON	86	SPEC	+	202 $\pi^+A \to \pi^+\pi^0A$
766 ±7	6500	[26] BYERLY	73	OSPK	−	5 π^-p
766.8±1.5	9650	[27] PISUT	68	RVUE	−	1.7–3.2 π^-p, $t<10$
767 ±6	900	[25] EISNER	67	HBC	−	4.2 π^-p, $t<10$

NEUTRAL ONLY, PHOTOPRODUCED

VALUE (MeV)	EVTS	DOCUMENT ID		TECN	COMMENT
769.0± 1.0 OUR AVERAGE					
771 ± 2 $^{+2}_{-1}$	63.5k	[28] ABRAMOWICZ	12	ZEUS	$ep \to e\pi^+\pi^-p$
770 ± 2 ±1	79k	[29] BREITWEG	98B	ZEUS	50–100 γp
767.6± 2.7		BARTALUCCI	78	CNTR	$\gamma p \to e^+e^-p$
775 ± 5		GLADDING	73	CNTR	2.9–4.7 γp
767 ± 4	1930	BALLAM	72	HBC	2.8 γp
770 ± 4	2430	BALLAM	72	HBC	4.7 γp
765 ±10		ALVENSLEB...	70	CNTR	γA, $t<0.01$
767.7± 1.9	140k	BIGGS	70	CNTR	<4.1 γC $\to \pi^+\pi^-$C
765 ± 5	4000	ASBURY	67B	CNTR	$\gamma +$ Pb
• • • We do not use the following data for averages, fits, limits, etc. • • •					
771 ± 2	79k	[30] BREITWEG	98B	ZEUS	50–100 γp

NEUTRAL ONLY, OTHER REACTIONS

VALUE (MeV)	EVTS	DOCUMENT ID		TECN	CHG	COMMENT
769.0±0.9 OUR AVERAGE	Error includes scale factor of 1.4. See the ideogram below.					
765 ±6		BERTIN	97C	OBLX		0.0 $\bar{p}p \to \pi^+\pi^-\pi^0$
773 ±1.6		WEIDENAUER	93	ASTE		$\bar{p}p \to \pi^+\pi^-\omega$
762.6±2.6		AGUILAR-...	91	EHS		400 pp
770 ±2		[31] HEYN	81	RVUE		Pion form factor
768 ±4		[32,33] BOHACIK	80	RVUE	0	

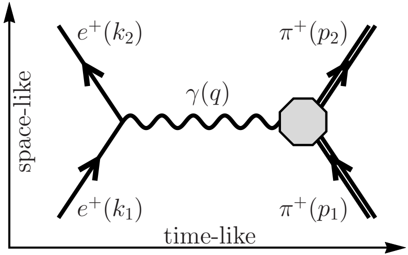
Pion electromagnetic form factor in the *P* wave

What are the correct $\rho(770)$ meson mass and width values?

PRD 96, 113004 (2017)

Erik Bartoŝ, Stanislav Dubniĉka, Andrej Liptaj Anna Zuzana Dubniĉkova and RK

$$\langle \pi^+(p_2) | J^\mu_\pi(0) | \pi^-(p_1) \rangle = e\,(p_1 + p_2)^\mu \, F_\pi(q^2)$$



Gounaris-Sakurai pion EM FF and $\rho(770)$

Gounaris-Sakurai pion electromagnetic form factor at the elastic region.

What are the correct $\rho(770)$ *meson mass and width values?*
Erik Bartoŝ, Stanislav Dubnička, Andrej Liptaj Anna Zuzana Dubničkova and RK
PRD 96, 113004 (2017)

$$\sigma_{tot}(e^+e^- \to \pi^+\pi^-) = \frac{\pi\alpha^2(0)}{3s}\beta_\pi^3(s)\left|F_\pi^{EM,I=1}(s) + Re^{i\phi}\frac{m_\omega^2}{m_\omega^2 - s - im_\omega\Gamma_\omega}\right|^2$$

where pion "velocity" $\beta_\pi(s) = \sqrt{\frac{s-4m_\pi^2}{s}}$, R - amplitude for $\rho - \omega$ interference (free parameter), phase $\phi = ArcTan\frac{m_\rho\Gamma_\rho}{m_\rho^2 - m_\omega^2}$ is the $\delta_1^1 = \delta_\rho$ fixed at $s = m_\omega^2$

Fit to data for $\sigma_{tot}(e^+e^- \to \pi^+\pi^-)$
- M. Ablikin et al. (BESIII Collaboration), Phys. Lett. B 753, 629 (2016).
- J. P. Lees et al. (BABAR Collaboration), Phys. Rev. D 86, 032013 (2012).

Gounaris-Sakurai pion electromagnetic form factor

G. J. Gounaris and J. J. Sakurai, Phys. Rev. Lett. 21, 244 (1968)

Assumption: $\frac{q^3}{\sqrt{s}} Cotg\delta_1^1(s) = a + bq^2 + h(s)q^2$ where $h(s) = \frac{2q}{\pi\sqrt{s}} Log\left(\frac{\sqrt{s}+2q}{2m_\pi}\right)$
Then $F_\pi^{GS}(s) = \frac{\sqrt{s}}{q^3}\frac{1}{Cotg\delta_1^1(s)-i}$ - no direct dependence on $\rho(770)$ parameters, however taking into account two conditions:

- $Cotg\delta_1^1(s)\Big|_{s=m_\rho^2} = 0$ and
- $F_\pi^{BW}(s) = \frac{m_\rho^2}{m_\rho^2 - s - im_\rho\Gamma_\rho} \to \delta_1^1(s) = ArcTan\frac{m_\rho\Gamma_\rho}{m_\rho^2 - s} \to \frac{d\delta_1^1(s)}{ds}\Big|_{s=m_\rho^2} = \frac{1}{m_\rho\Gamma_\rho}$

$$a = \frac{4q_\rho^5}{m_\rho^2\Gamma_\rho} + 4q_\rho^4 h'(m_\rho^2)$$

$$b = -\frac{4q_\rho^3}{m_\rho^2\Gamma_\rho} - 4q_\rho^4 h'(m_\rho^2) - h(m_\rho^2)$$

$$F_\pi^{GS}(s) = \frac{m_\rho^2 + m_\rho\Gamma_\rho\left[\frac{3m_\pi^2}{\pi q_\rho^2} log\left(\frac{m_\rho+2mq_\rho}{2m_\pi}\right) + \frac{m_\rho}{2\pi q_\rho} - \frac{m_\pi^2 m_\rho}{\pi q_\rho^3}\right]}{(m_\rho^2 - s) + \Gamma_\rho\frac{m_\rho^2}{q_\rho^3}\left[q^2\left(h(s) - h(m_\rho^2)\right) + q_\rho^2 h'(m_\rho^2)(m_\rho^2 - s)\right] - im_\rho\Gamma_\rho\left(\frac{q}{q_\rho}\right)^3\frac{m_\rho}{\sqrt{s}}}$$

Generalisation of Gounaris-Sakurai model to excited ρ mesons $\rho(1450)$ and $\rho(1700)$

- $F_\pi = \frac{1}{1+\beta+\gamma}\left[F^{GS}_{\rho(770)}(s)\left(1+\delta\frac{s}{m^2_\omega}BW_\omega(s)\right)+\beta F^{GS}_{\rho(1450)}(s)+\gamma F^{GS}_{\rho(1700)}(s)\right]$
- $\chi^2 = 0.98$ pdf

Generalisation of U&A model to excited ρ mesons

- $F_\pi = \frac{\Pi(q-q_i)}{\Pi(q+q_i^*)}$
- $\chi^2 = 1.84$ pdf

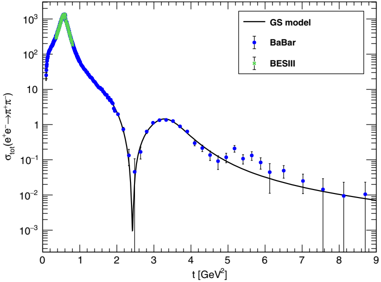


FIG. 6. Optimal description of the unified BESIII-*BABAR* complete data on $\sigma_{\mathrm{tot}}(e^+e^- \to \pi^+\pi^-)$ by the generalized pion EM FF G.-S. model.

determined by the original pion EM FF G.-S. model (13) to be valid only at the elastic region.

A totally different situation is in a generalization of the U&A pion EM FF model. Here, the contribution of all three vector mesons is at an equal level. Only now, the effective inelastic threshold, which is left as a free parameter of the model, has to be taken into account explicitly. Therefore, instead of the q variable, the W variable

TABL
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e^+e^-
Analy

Param

m_ρ
$m_{\rho'}$
$m_{\rho''}$
Γ_ρ
$\Gamma_{\rho'}$
$\Gamma_{\rho''}$
χ^2/nd

$W(t)$

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Parameter	PDG MeV	G.S. MeV	U&A MeV
m_ρ	775.26 ± 0.25	774.81 ± 0.01	763.88 ± 0.04
$m_{\rho'}$	1465.00 ± 25.00	1497.70 ± 1.07	1326.35 ± 3.46
$m_{\rho''}$	1720.00 ± 20.00	1848.40 ± 0.09	1770.54 ± 5.49
Γ_ρ	149.10 ± 0.80	149.22 ± 0.01	144.28 ± 0.01
$\Gamma_{\rho'}$	400.00 ± 60.00	442.15 ± 0.54	324.13 ± 12.01
$\Gamma_{\rho''}$	250.00 ± 100.00	322.48 ± 0.69	268.98 ± 11.40
χ^2 pdf		0.98	1.84
		14 param.	11 param.

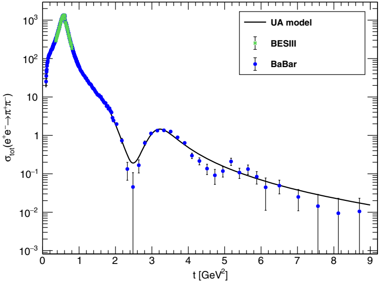

FIG. 7. Optimal description of the unified BESIII-*BABAR* complete data on $\sigma_{tot}(e^+e^- \to \pi^+\pi^-)$ by the generalized pion

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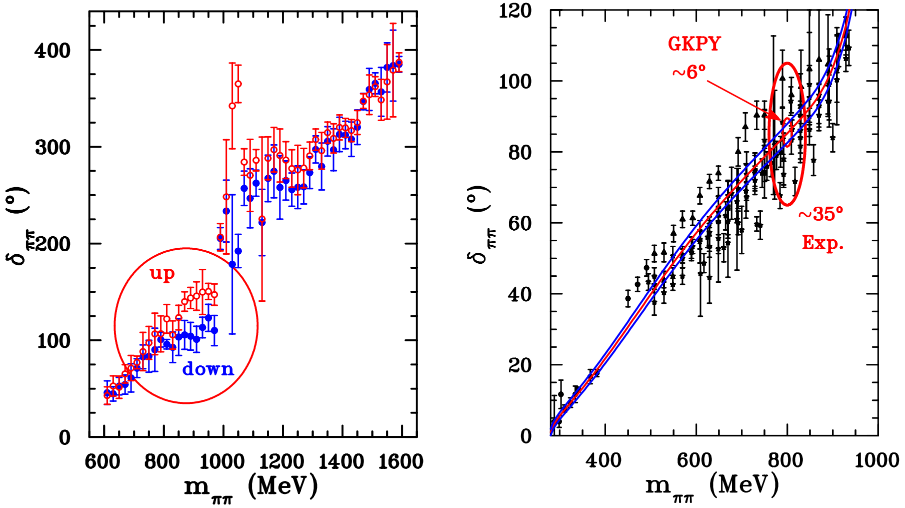
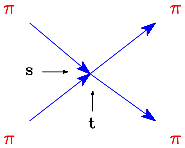
Experimental data for the $\pi\pi$ in the S0 wave (JI)

In PWA (CERN-Munich group'74) $A(s,t) \sim Cos(\theta_S - \theta_P)$



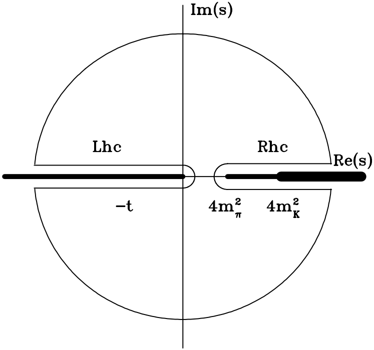
Dispersion relations with imposed crossing symmetry condition for $\pi\pi$ interactions theory $\longleftrightarrow$ experiment

crossing symmetry:



$\rightarrow \vec{T}_s(s,t) = \hat{C}_{st}\vec{T}_t(t,s)$

$\vec{T}(s,t)$ + crossing symmetry $\rightarrow$ dispersion relations for $\underline{4m_\pi^2 < s < \sim (1150\ \text{MeV})^2}$

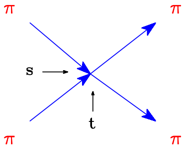


Once subtracted DR:

$$
\begin{aligned}
\text{Re}\,\vec{F}(s,t) &= \text{Re}\,\vec{F}(s_0,t) + \frac{s-s_0}{\pi} \\
&\times \left[\int_{4m_\pi^2}^{\infty} ds' \frac{\text{Im}\,\vec{F}(s',t)}{(s'-s_0)(s'-s)} \right. \\
&+ \left. \int_{-t}^{-\infty} ds' \frac{\text{Im}\,\vec{F}(s',t)}{(s'-s_0)(s'-s)} \right]
\end{aligned}
$$

Dispersion relations with imposed crossing symmetry condition for $\pi\pi$ interactions theory $\longleftrightarrow$ experiment

crossing symmetry:



$\rightarrow \vec{\boldsymbol{T}}_{\boldsymbol{s}}(s,t) = \hat{\boldsymbol{C}}_{st}\vec{\boldsymbol{T}}_{\boldsymbol{t}}(t,s)$

$\vec{\boldsymbol{T}}(\boldsymbol{s},\boldsymbol{t})$ + crossing symmetry $\rightarrow$ dispersion relations for $4m_\pi^2 < s < \sim (1150\ \text{MeV})^2$

Once subtracted dispersion relations ("GKPY" for the S and P waves):

$$\text{Re}\ t_\ell^{I(OUT)}(s) = \sum_{I'=0}^{2} C_{st}^{II'} a_0^{I'} + \sum_{I'=0}^{2}\sum_{\ell'=0}^{4} \int_{4m_\pi^2}^{\infty} ds' K_{\ell\ell'}^{II'}(s,s')\ \text{Im}\ t_{\ell'}^{I'(IN)}(s')$$

$a_0^{I'}$ - subtraction constant $= \vec{\boldsymbol{T}}_{\boldsymbol{s}}(s = 4m_\pi^2, t = 0)$ - scattering lengths from only S wave

due to $\text{Re}\ t_\ell^I(k) = k^{2\ell}(a_\ell^I + b_\ell^I k^2 + O(k^4))$ $\qquad$ $\text{Re}\ t_\ell^{I(OUT)}(s) - \text{Re}\ t_\ell^{I(IN)}(s) \rightarrow 0$

GKPY equations and $\pi\pi$ amplitudes

partial waves: *JI*

experiment

F1 D2

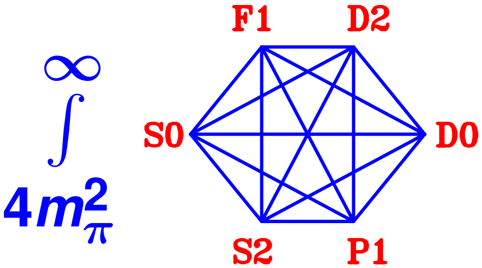
S0 D0

S2 P1

GKPY equations and $\pi\pi$ amplitudes

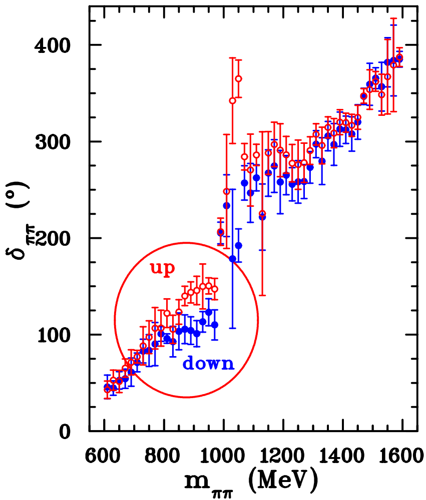
partial waves: *Jl*

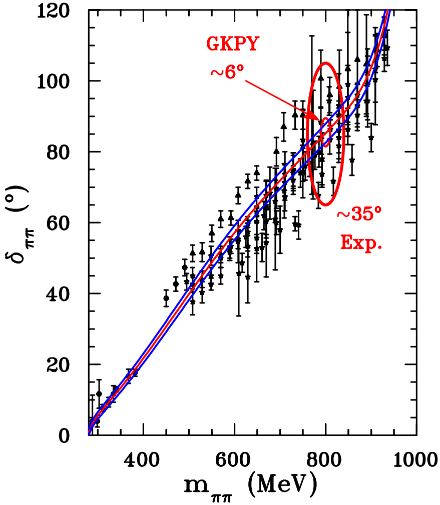
experiment + theory (GKPY)



Experimental data for the $\pi\pi$ in the S0 wave (JI)

In PWA (CERN-Munich group'74) $A(s,t) \sim Cos(\theta_S - \theta_P)$

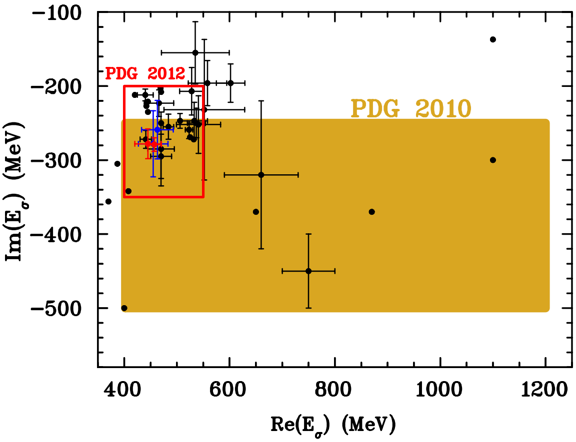




precise determination of $f_0(500)$ (σ) meson and threshold parameters

$f_0(500)$ (σ)

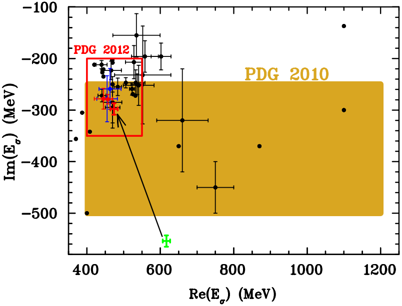
- PDG 2010:
  M = 400 – 1200 MeV
  $\Gamma = 2 \times (250 - 500)$ MeV
- PDG 2012:
  M = 400 – 550 MeV
  $\Gamma = 2 \times (200 - 350)$ MeV
- GKPY:
  $E_\sigma = 457 \pm 14 - i279^{+11}_{-7}$ MeV



threshold parameters, e.g. a_0^0:

- ChPT + Roy eqs (Bern group):
  $0.220 \pm 0.005\ m_\pi^{-1}$
- GKPY:
  $0.220 \pm 0.008\ m_\pi^{-1}$

what forces GKPY eqs to pull up-left the sigma pole?



Two things: trigonometry and crossing symmetry algebra lead to narrower and lighter σ.

Modified $\pi\pi$ *amplitude with* σ *pole* PRD 90, 116005 (2014) P. Bydzovský, 1, R. Kamiński, V. Nazari

Nothing more and nothing instead of it is needed.

Resonance is near the threshold

1976 S. M. Flatté analyses the $\pi\eta$ and the $K\bar{K}$ coupled channel systems

$$A_i \sim \frac{M_R\sqrt{\Gamma_0\Gamma_i}}{M_R^2 - E^2 - iM_R(\Gamma_1+\Gamma_2)}, \quad i = 1, 2.$$

$\Gamma_i = g_i k_i$ and $\Gamma_0 = g_1 q$ with $q = k_1(2m_K)$. So THREE free parameters: M_R, g_1, g_2.

One channel case:

$$T_{22} = \frac{\sin\delta_2}{k_2} e^{i\delta_2} \equiv \frac{1}{k_2\cot\delta_2 - ik_2},$$

$$k_2\cot\delta_2 \approx \frac{1}{a} + \frac{1}{2} r\, k_2^2 \longrightarrow T_{22} = \frac{1}{\frac{1}{a} - i\,k_2 + \frac{1}{2}\,r\,k_2^2}$$

where a is the scattering length and r is the effective range (both real).
Two channel case: A and R are complex so FOUR free parameters

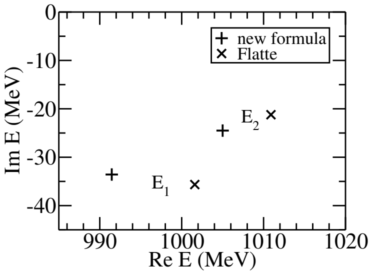
$$T_{22} = \frac{1}{2ik_2}(\eta e^{2i\delta_2} - 1) \longrightarrow T_{22} = \frac{1}{\frac{1}{A} - i\,k_2 + \frac{1}{2}\,R\,k_2^2}.$$

$$A = -i\left(\frac{1}{z_1} + \frac{1}{z_2}\right), \qquad R = \frac{2i}{z_1+z_2}\,.$$

where z_1 and z_2 are zeroes of the S_{22} matrix element and are related with resonance.
Flatte approach: $ImR = 0$ so $Rez_1 = -Rez_2$

For $a_0(980)$

L. Leśniak, AIP Conf.Proc. 1030 (2008) 238



Conclusions

- there is still many known mesons with not enough well determined parameters,
- low lying exotics, gleaballs and hybrids need confirmation and theoretical explanation which is possible only when we know other resonances very precisely,
- mixing of the standard mesons is probably one of the biggest problems,
- the main condition for carrying out mathematically correct analyzes is unitarity of amplitudes,
- before starting the analyzes, you should correctly construct the amplitudes and look for their poles, and do not rely only on what we observe in experiments,
- this holds true for both experimentalists and theorists