WHY DO WE NEED the FORMULA for the ENERGY RESOLUTION of a SCINTILLATION SPECTROMETER with SEVERAL PHOTODETECTORS?

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The main drawback of all current theories of scintillation spectrometers is in introducing various terms into the formula for the energy resolution of scintillation spectrometers, without giving specific formulae for the relationship of these terms with characteristics of scintillation detectors. Such insertion of various contributions by hands is not only wrong but also counterproductive, since it does not allow comparing the results obtained by different scientific groups. In this work, the microscopic mathematical model was formulated, which serves as the basis for the standard theory of scintillation spectrometers. The standard theory allows obtaining the formulae for arbitrary moments of the signal distribution function at the output of the scintillation spectrometer. In particular, the formulae for the average value and the variance of the signal at the output of the photodetector are obtained. The structure of the formula for the energy resolution of a scintillation spectrometer reveals the contributions of the processes that take place at converting the energy of a primary particle into the output signal. It was shown that in the developed standard theory of scintillation spectrometers there are no drawbacks to the currently existing theories of scintillation spectrometers. Thus, the developed standard theory of scintillation spectrometers creates a solid basis for linking theoretical and experimental researches in this field.

1. INTRODUCTION

Now, the physics of scintillators represents a wide area of theoretical and experimental researches in physics and chemistry of scintillators, in the field of technologies for creating new crystals and their use in physical experiments. However, all features and advantages of a new scintillator are revealed only in experimental studies of its properties in using it as a crystal of a scintillation spectrometer. Therefore, one of the goals of the theory of scintillation spectrometers is to clarify the conditions under which the characteristics of the processes occurring in the scintillator can be extracted from the photodetector signal.

The first attempts to apply the theory of random processes to the process occurring in scintillation spectrometers was made by Ernst Breitenberger [1]. Breitenberger's work, the most significant of its time in the theory of scintillation detectors, has fundamental drawbacks. First, it is a macroscopic theory in which the description of successive cascade processes is carried out through the moments of the distribution functions of the average values of the corresponding stages. Second, it assumes that each primary particle with energy E interacting with the scintillator generates on the average $\overline{N} = E/\varepsilon$ light photons, where ε is the average energy of a light photon generation. Thus, it lacks the intermediate stages which take place in the scintillator, i.e. the conversion of the energy of a primary particle into the energy of secondary charged particles, the generation of electron-hole pairs, and the excitation of luminescent centers. Third, it assumes that fluctuations in the number of light photons are described by the Poisson distribution.

Breitenberger's formulae for the average value and the relative variance of the output signal of the

scintillation spectrometer have forms

$$\overline{Q} = \overline{N} \cdot \overline{p} \cdot \overline{M} \qquad (1)$$

$$\eta_Q^2 = \frac{\sigma_Q^2}{\overline{Q}^2} = \eta_p^2 + (1+\eta_p^2)\left(\eta_N^2 - \frac{1}{\overline{N}}\right) + \frac{1+\eta_M^2}{\overline{N} \cdot \overline{p}}. \qquad (2)$$

where $\overline{N}$ and η_N^2 are the average value and the relative variance of the number of photons generated by registered particles; $\overline{p}$ and η_p^2 are the average value and the relative variance of the probability for a photon to produce an electron on the first dynode of a photomultiplier.

From the formula (2), it follows that the minimum value of the relative variance is reached, when fluctuations in the number of light photons are described by the Poisson distribution and there are no fluctuations in the probability for a photon to produce an electron on the first dynode of a photomultiplier

$$\eta_{Q\min}^2 = \frac{1+\eta_M^2}{\overline{N} \cdot p} \qquad (3)$$

It should be noticed that this formula occurs in many theoretical works, and is interpreted as the intrinsic resolution, i.e. the minimum limiting resolution which can be achieved in the scintillation detector.

2. MACROSCOPIC THEORIES OF SCINTILLATION DETECTORS AFTER BREITENBERGER'S WORK

In subsequent works, to explain the discrepancy between the experimental results and the formula (3), authors began to include various terms, reflecting, from their point of view, the contribution of certain factors to the energy resolution of scintillation detectors. For illustration, I can cite two publications with formulae that include contributions to the energy resolution of scintillation detectors, differing not only in names but also in the physical meaning of the processes taken into account. So, in [2], Moszyński M. give the formula

$$\left(\Delta E / E\right)^2 = \delta_{sc}^2 + \delta_{\mathrm{p}}^2 + \delta_{\mathrm{st}}^2 + \delta_{\mathrm{n}}^2, \qquad (4)$$

where δ_{sc}^2 is the intrinsic resolution of the scintillator, δ_{p}^2 is the contribution associated with the collection of light by the photomultiplier or photodiode, δ_{st}^2 is the contribution of statistical processes of electron multiplication in the photomultiplier or fluctuation processes in the photodiode, and δ_{n}^2 is the contribution of electronic noise. The only formula for the contribution of statistical processes of electron multiplication in a photomultiplier, the is given

$$\delta_{st} = 2.35\sqrt{\left(1+\varepsilon\right)/N}, \qquad (5)$$

where N is the number of photoelectrons, ε is the relative variance of the multiplication factor of the photomultiplier that coincide with (3).

A similar formula for the energy resolution is given in [3]

$$R^2 = R_{np}^2 + R_{\mathrm{inh}}^2 + R_{\mathrm{tr}}^2 + R_{\mathrm{lim}}^2, \qquad (6)$$

where R_{np}^2 is the contribution associated with the nonproportionality of the light yield, R_{inh}^2 is the contribution associated with the inhomogeneity of the scintillation crystal, R_{tr}^2 is the contribution associated with the light collection on the photomultiplier photocathode, R_{lim}^2 is the limiting resolution of the detector. In [5], the formula for the limiting resolution of the scintillation detector coincides with the formula for the

contribution of statistical processes of electron multiplication in a photomultiplier (5).

In the book by Glen Knoll [4], which is classical in the field of detectors and their application for registration of radiation, the formula for the energy resolution of scintillation detectors is given

$$(FWHM)^2_{overall} = (FWHM)^2_{statistical} + (FWHM)^2_{noise} + (FWHM)^2_{drift} + \dots , \quad (7)$$

and possible contributions affecting the energy resolution of scintillation detectors are only mentioned without giving specific formulae.

In the book [5] devoted to elementary particles detectors, a formula for the energy resolution of scintillation detectors has the form

$$\frac{\sigma_{E_{dep}}}{E_{dep}} = \sqrt{\frac{f}{N_{pe}} + \left(\frac{\sigma_e}{E_{dep}}\right)^2 + \Delta^2} , \quad (8)$$

where f is the excess noise factor; σ_e is the contribution of electronics noise; Δ is the contribution from the nonproportionality of the scintillator light yield. In the formula (8),

$$N_{pe} = L_{ph} E_{dep} \eta_c Q_s , (9)$$

E_{dep} is the energy absorbed in the scintillator; L_{ph} is the light output of the scintillation crystal; η_c is the light collection efficiency; η_c is the quantum efficiency of the photodetector.

In the paper [6], an attempt was made to take into account the contribution of the nonproportionality of the light yield in the scintillator by separating the contributions from ionization losses and from delta-electrons. However, as a result, the authors summed up the relative variances of the contributions, thereby nullifying all their efforts.

In [7], the authors, as a result of the analysis of factors affecting the energy resolution of the scintillation detector, gave a formula that takes into account, from their point of view, all sources of fluctuations that determine the intrinsic energy resolution of the scintillator:

$$\frac{\sigma^2_{N_{ph}}}{\langle N_{ph}\rangle^2} = \frac{\sigma^2_{N_{eh}}}{\langle N_{eh}\rangle^2} + \frac{\int \langle q(n)\rangle\left(1-\langle q(n)\rangle\right)\langle w(n)\rangle d\log n}{\langle N_{eh}\rangle\left(\int \langle q(n)\rangle\langle w(n)\rangle d\log n\right)^2} + \frac{\int \sigma^2_{q(n)} \langle w(n)\rangle^2 d\log n}{\left(\int \langle q(n)\rangle\langle w(n)\rangle d\log n\right)^2} + \frac{\iint \langle q(n)\rangle\langle q(n')\rangle \operatorname{cov}\left(w(n)w(n')\right) d\log n \cdot d\log n'}{\left(\int \langle q(n)\rangle\langle w(n)\rangle d\log n\right)^2} . \quad (10)$$

In the formula (10), $w(n)$ is the distribution function of the concentration of electron-hole pairs in the track with the normalization condition

$$\int w(n) d\log n = 1 . \quad (11)$$

In (10), $q(n)$ is the fraction of excitation that produce the light photon, and

$$\langle N_{ph}\rangle = \langle N_{eh}\rangle \int \langle q(n)\rangle\langle w(n)\rangle d\log n , \quad (12)$$

where $\langle N_{eh}\rangle$ is the average number of electron-hole pairs created by the detected particle.

The first term in (11) describes the contribution to the intrinsic energy resolution of fluctuations in the number of electron-hole pairs produced by the ionizing particle

$$\frac{\sigma^2_{N_{eh}}}{\langle N_{eh}\rangle^2} = \frac{F_{eh}}{\langle N_{eh}\rangle}. \quad (13)$$

The authors expressed the second term in (10)

$$\frac{\int \langle q(n)\rangle\left(1-\langle q(n)\rangle\right)\langle w(n)\rangle\, d\log n}{\langle N_{eh}\rangle\left(\int \langle q(n)\rangle\langle w(n)\rangle\, d\log n\right)^2} = \frac{F_{ph}}{\langle N_{ph}\rangle}, \qquad (14)$$

by introducing the Fano factor for photons

$$F_{ph} = \frac{\int \langle q(n)\rangle\left(1-\langle q(n)\rangle\right)\langle w(n)\rangle\, d\log n}{\int \langle q(n)\rangle\langle w(n)\rangle\, d\log n} < 1. \qquad (15)$$

The authors attributed the third term in (10) with the inhomogeneous distribution of defects and impurities of the crystal $\sigma^2_{in\,\mathrm{hom}}$, and the last term with fluctuations of track topology σ^2_{track}. The authors note that fluctuations $w(n)$ can be accounted for using covariance $\mathrm{cov}\left(w(n)w(n')\right)$ which can be estimated using Monte Carlo simulations.

As a result, they obtained the formula for the intrinsic energy resolution of a scintillator

$$R_{\mathrm{int}} = 2.355\frac{\sigma_{N_{ph}}}{\langle N_{ph}\rangle} = 2.355\sqrt{\frac{F_{eh}}{\langle N_{eh}\rangle} + \frac{F_{ph}}{\langle N_{ph}\rangle} + \sigma^2_{in\,\mathrm{hom}} + \sigma^2_{track}}, \qquad (16)$$

and taking into account the contribution of the photomultiplier, their formula for the total energy resolution has the final form

$$R_{\mathrm{int}} = 2.355\sqrt{\frac{F_{eh}}{\langle N_{eh}\rangle} + \frac{F_{ph}}{\langle N_{ph}\rangle} + \frac{1+\varepsilon}{\langle N_{pe}\rangle} + \sigma^2_{in\,\mathrm{hom}} + \sigma^2_{track}} \qquad (17)$$

It should be noted the ambiguity in the separation of contributions to the energy resolution of scintillation detectors by various authors, and the lack of information on the relationship of the corresponding contributions with the characteristics of the scintillator, scintillator-photodetector interface, characteristics of the photodetector, and the characteristics of spectrometer electronics.

The main disadvantage of all existing works is in the possibility of introducing various terms into the formula for the energy resolution of scintillation spectrometers, as a rule, without giving specific formulae for their relationship with the characteristics of the detector. However, such insertion of various contributions by hands is not only wrong but also counterproductive, since it does not allow comparing the results obtained by different scientific groups.

The correct approach to obtaining the formula for the energy resolution of scintillation spectrometers is in creating the theoretical model that includes all possible processes occurring during the transformation of the registered particle energy into the output signal of the scintillation detector. Only after the creation of the theoretical model, it should be translated, using an adequate formalism, into an appropriate mathematical form. Since the process of transforming the energy of the registered particle into the output signal of a scintillation detector is a random branching cascade process, the formalism of probability generating functions should be used for its mathematical description. Only in this case, the formulae for any moments of the distribution function of the output signal will strictly follow from the theory. In accordance with the theoretical model, these formulae will contain all information about the dependencies of all contributions to

the energy resolution on the characteristics of the scintillation material and other parameters of the detector. Only these formulae have predictive power. It should be emphasized that changing in the theory is possible only at the stage of the theoretical model since mathematical formalism guarantees to obtain all the necessary formulae. Only after obtaining the formulae for the moments of the distribution function of the output signal, one can make the necessary approximations that take into account the conditions of the experiment, under which the characteristics of the output signal can be used to extract from experimental data information about the characteristics of the processes occurring in the detector during the registration of radiation.

3. MICROSCOPIC MODEL OF A SCINTILLATION SPECTROMETER WITH SEVERAL PHOTODETECTORS

The main drawback of Breitenberger's theory, and all subsequent works, is that they are macroscopic theories that take into account branching cascade processes through the moments of the distribution functions of quantities, which are the average values of the corresponding stages. In [8], the mathematical model for registering a primary particle with a scintillation spectrometer with several photodetectors was formulated. The mathematical model takes into account that the process of transforming the energy of a registered particle into the output signals of photodetectors of a scintillation spectrometer includes the following successive stages.

1. The stage of the interaction of a registered particle with the scintillation crystal.
2. The stage of electron-hole pairs generation.
3. The stage of electron-hole pairs recombination.
4 The stage of diffusion of carriers (electrons, holes, and excitons) in the scintillator.
5. The stage of luminescent centers activation.
6. The stage of light photon emission by a luminescent center.
7. The stage of a light photon collection onto the photocathode of a photodetector.
8. The stage of a light photon conversion into a photoelectron in the photocathode of a photodetector.
9. The stage of signal amplification by the electronic amplifier, taking into account the electronics noise.

The microscopic approach consists of the detailed description of processes of transformation of registered particle energy into the output signals of photodetectors through the moments of the distribution functions of stages, in particular, through the joint distribution function of the secondary particles in the elements of the phase space $d\Gamma = dVdEd\vec{\Omega}$.

It should be noted that the mathematical model is applicable not only to inorganic scintillators but also to organic scintillators if we consider that the stage of electron-hole pair generation corresponds to the stage of ionization and excitation of organic molecules; the stage of recombination of electron-hole pairs - the stage of "quenching" of luminescence; the stage of diffusion of carriers - the stage of migration of the excitation energy to other molecules; the stage of activation of the luminescent center - the stage of transition of the excitation energy to the corresponding radiative transition; the stage of light photon emission by a luminescent center - the stage of the radiative transition with emission of a light photon.

For the case when the energy E_0 of primary monoenergetic particles is totally absorbed in the detector volume, the microscopic theory gives the formula for the mean value of the sum signal of scintillation detector with N photodetectors

$$\langle Q\rangle = \sum_{\alpha=1}^{A} \int_V \int_E \int_\Omega dV dE d\vec{\Omega} u(E - E_{\alpha\ \min}(\vec{r}, E, \vec{\Omega})) \left\langle \frac{w_\alpha^c(E_0, \vec{r}, E, \vec{\Omega})}{\varepsilon_{\text{e-h}\,\alpha}(\vec{r}, E, \vec{\Omega})} \left(\sum_{n=1}^{N} \left\langle \chi_{n\,\alpha}(\vec{r}, E, \vec{\Omega}) \right\rangle_D \langle g_n \rangle \right) \right\rangle_c \qquad (18)$$

where the Heaviside unit function, ($u(x) = 0$ for $x < 0$ and $u(x) = 1$ for $x \geq 0$) accounts for the threshold energy of electron-hole pair creation by secondary particles. The subscript c of the angle brackets denotes averaging over all possible distributions of secondary particles in the phase space elements; $w_\alpha^c(E_0, \vec{r}, E, \vec{\Omega})$ is the differential density of absorbed energy at the scintillator point $\vec{r}$ provided that a certain configuration of charged particles of the type α appears in the phase space element $d\Gamma = dV dE d\vec{\Omega}$; $\varepsilon_{\text{e-h}\,\alpha}(\vec{r}, E, \vec{\Omega})$ is the average energy of the electron-hole pair creation by the secondary particle of the type α belonging to the phase space element $d\Gamma$; $\langle g_n \rangle$ is the mean value of the gain of the n-th photodetector electronics.

In (18)

$$\left\langle \chi_{n\,\alpha}(\vec{r}, E, \vec{\Omega}) \right\rangle_D = \sum_{l=1}^{L} p_l(\lambda_l)(1 + \lambda_D^2 \Delta) \int_{\Omega'} d\vec{\Omega}' \varepsilon_{r\,\alpha}(\vec{r}, E, \vec{\Omega}) \varepsilon_a(\vec{r}) \varepsilon_e(\vec{r}, \lambda_l, \vec{\Omega}') \int_{S_n} \int_{\vec{\Omega}_n''} dS_n d\vec{\Omega}_n'' \tau_n(\vec{r}, \lambda_l, \vec{\Omega}', S_n, \vec{\Omega}_n'') \eta_n(\lambda_l, S_n, \vec{\Omega}_n''), \qquad (19)$$

The subscript D of the angle brackets denotes averaging over the diffusion probability density, with the characteristic diffusion length λ_D; Δ is the Laplace operator; p_l is the probability of the l radiative transition mode; $\varepsilon_{r\,\alpha}(\vec{r}, E, \vec{\Omega})$ is the probability of an electron-hole pair generated at the scintillator point $\vec{r}$ to survive in recombination or to form an exciton, which strongly depends on the stopping power of the secondary particle of the type α belonging to the phase space $d\Gamma$; $\varepsilon_a(\vec{r})$ is the probability of the luminescent center activation at the scintillator point $\vec{r}$; $\varepsilon_e(\vec{r}, \lambda_l, \vec{\Omega}')$ is the probability of emission of the photon with a wavelength λ_l, in the solid angle element $d\vec{\Omega}'$ by the luminescent center at the scintillator point $\vec{r}$; $\tau_n(\vec{r}, \lambda_l, \vec{\Omega}', S_n, \vec{\Omega}_n'')$ is the probability of a photon with the wavelength λ_l, emitted in the solid angle element $d\vec{\Omega}'$ by the luminescent center at the scintillator point $\vec{r}$ to reach the element dS_n of the entrance window of the n-th photodetector in the direction belonging to the solid angle element $d\vec{\Omega}''$ relative to the normal to the photocathode surface element; $\eta_n(\lambda_l, S_n, \vec{\Omega}_n'')$ is the quantum efficiency of the n-th photodetector surface element dS_n to the light photon with the wavelength λ_l, crossing the entrance window in the direction belonging to the solid angle element $d\vec{\Omega}''$.

The microscopic theory gives the formula for the variance of the sum signal of scintillation detector with N photodetectors

$$\sigma_Q^2 = \sigma_{\text{cov}}^2 + \sigma_{\text{pair}}^2 + \sigma_{tr}^2 + \sigma_{gain}^2 + \sigma_{\text{noise}}^2 \qquad (20)$$

where

$$\sigma^2_{\text{cov}} = \sum_{\alpha=1}^{A}\sum_{\alpha'=1}^{A}\int_V\int_E\int_\Omega\int_{V'}\int_{E'}\int_{\Omega'} dVdEd\vec{\Omega}dV'dE'd\vec{\Omega}' u(E - E_{\alpha\ \min}(\vec{r},E,\vec{\Omega}))u(E' - E_{\alpha'\ \min}(\vec{r}',E',\vec{\Omega}'))\times$$

$$\left\langle \frac{w^c_\alpha(E_0,\vec{r},E,\vec{\Omega})}{\varepsilon_{\text{e-h}\,\alpha}(\vec{r},E,\vec{\Omega})}\left(\sum_{n=1}^{N}\left\langle \chi_{n\,\alpha}(\vec{r},E,\vec{\Omega})\right\rangle_D \langle g_n\rangle\right)\frac{w^c_{\alpha'}(E_0,\vec{r}',E',\vec{\Omega}')}{\varepsilon_{\text{e-h}\,\alpha'}(\vec{r}',E',\vec{\Omega}')}\left(\sum_{n'=1}^{N}\left\langle \chi_{n'\,\alpha'}(\vec{r}',E',\vec{\Omega}')\right\rangle_D \langle g_{n'}\rangle\right)\right\rangle_c , \qquad (21)$$

$$-\left(\sum_{\alpha=1}^{A}\int_V\int_E\int_\Omega dVdEd\vec{\Omega}u(E - E_{\alpha\ \min}(\vec{r},E,\vec{\Omega}))\left\langle \frac{w^c_\alpha(E_0,\vec{r},E,\vec{\Omega})}{\varepsilon_{\text{e-h}\,\alpha}(\vec{r},E,\vec{\Omega})}\left(\sum_{n=1}^{N}\left\langle \chi_{n\,\alpha}(\vec{r},E,\vec{\Omega})\right\rangle_D \langle g_n\rangle\right)\right\rangle_c\right)^2$$

is the variance of the sum signal of the spectrometer due to the covariances between the secondary particles in the phase space;

$$\sigma^2_{\text{pair}} = \sum_{\alpha=1}^{A}\int_V\int_E\int_\Omega dVdEd\vec{\Omega}u(E - E_{\alpha\ \min}(\vec{r},E,\vec{\Omega}))\times$$

$$\left\langle F_\alpha(\vec{r},E,\vec{\Omega})\frac{w^c_\alpha(E_0,\vec{r},E,\vec{\Omega})}{\varepsilon_{\text{e-h}\,\alpha}(\vec{r},E,\vec{\Omega})}\left(\sum_{n=1}^{N}\left\langle \chi_{n\,\alpha}(\vec{r},E,\vec{\Omega})\right\rangle_D \langle g_n\rangle\right)^2\right\rangle_c , \qquad (22)$$

is the variance of the sum signal of the spectrometer due to fluctuations in the number of electron-hole pairs, where $F_\alpha(\vec{r},E,\vec{\Omega})$ is the Fano factor for the electron–hole pairs generation at the scintillator point $\vec{r}$ by the secondary particle of the type α belonging to the phase space element $d\Gamma$;

$$\sigma^2_{tr} = \sum_{\alpha=1}^{A}\int_V\int_E\int_\Omega dVdEd\vec{\Omega}u(E - E_{\alpha\ \min}(\vec{r},E,\vec{\Omega}))\times$$

$$\left\langle \frac{w^c_\alpha(E_0,\vec{r},E,\vec{\Omega})}{\varepsilon_{\text{e-h}\,\alpha}(\vec{r},E,\vec{\Omega})}\left(\sum_{n=1}^{N}\left\langle \chi_{n\,\alpha}(\vec{r},E,\vec{\Omega})\right\rangle_D \langle g_n\rangle^2 - \left(\sum_{n=1}^{N}\left\langle \chi_{n\,\alpha}(\vec{r},E,\vec{\Omega})\right\rangle_D \langle g_n\rangle\right)^2\right)\right\rangle_c \qquad (23)$$

is the variance of the sum signal of the spectrometer caused by fluctuations in the processes occurring in the detector from the formation of an electron-hole pair in the scintillator to the formation of a photoelectron in one of the photodetectors;

$$\sigma^2_{gain} = \sum_{\alpha=1}^{A}\int_V\int_E\int_\Omega dVdEd\vec{\Omega}u(E - E_{\alpha\ \min}(\vec{r},E,\vec{\Omega}))\left\langle \frac{w^c_\alpha(E_0,\vec{r},E,\vec{\Omega})}{\varepsilon_{\text{e-h}\,\alpha}(\vec{r},E,\vec{\Omega})}\sum_{n=1}^{N}\left\langle \chi_{n\,\alpha}(\vec{r},E,\vec{\Omega})\right\rangle_D \sigma^2_{g\,n}\right\rangle_c \qquad (24)$$

is the variance of the sum signal of the spectrometer caused by fluctuations in the gains of electronic amplifiers, where $\sigma^2_{g\,n}$ is the variance of the n-th photodetector electronics gain;

$$\sigma^2_{\text{noise}} = \sum_{n=1}^{N}\sigma^2_{\text{noise}\,n} \qquad (25)$$

is the variance due to electronic noise, where $\sigma^2_{\text{noise}n}$ is the electronic noise at the output of n-th amplifier.

The expressions (18) – (25) are the most general formulae for the average and the variance of a scintillation detector with several photodetectors and are the basis for various approximations. These expressions include the formulae for the average and the variance of any photodetector signal if we leave in the sum only one term for the given photodetector.

All the formulae for the energy resolution existing in the literature are applicable only to scintillation spectrometers with one photodetector at registration of monoenergetic X-rays of low energy E_0, when all secondary particles are electrons. Therefore, for comparison with the existing formulae, the formulae of the

microscopic theory for the mean value and the relative variance of the output signal of the microscopic theory for this case will be given. As in all existing works always assumed the uniform and isotropic scintillator with the only one radiative transition mode the expression (21) is simplified to

$$\left\langle \chi(\vec{r},E,\vec{\Omega})\right\rangle_D = S(E)QT(\vec{r}), \qquad (26)$$

where $S(E)=\varepsilon_r(E)\varepsilon_a$ is the standard notation of the probability of the luminescent center activation by one electron-hole pair that can depend only on the electron energy; Q is the standard notation of the quantum efficiency of the luminescence process defined as $Q/4\pi=\varepsilon_e(\vec{r},\lambda,\vec{\Omega}')$;

$$T(\vec{r})=(1+\lambda_D^2\Delta)\int_{\Omega'}\frac{d\vec{\Omega}'}{4\pi}\int_S\int_{\Omega''}dSd\vec{\Omega}''\tau(\vec{r},\lambda,\vec{\Omega}',S,\vec{\Omega}'')\eta(\lambda,S,\vec{\Omega}'') \qquad (27)$$

is the probability of a light photon emitted by a luminescent center at the point $\vec{r}$ of the scintillator volume to form a photoelectron in the photodetector.

For the scintillation spectrometer with the one photodetector, for comparing with existing in literature formulae, it is more convenient to use the relative variance related to the energy resolution of the spectrometer by the expression

$$\frac{\Delta E}{E_0}=2.36\eta_Q, \qquad (28)$$

where ΔE is the full width at half maximum (FWHM) of the line with the energy E_0.

The formulae for the mean value and the relative variance of the output signal of the microscopic theory of scintillation spectrometers with one photodetector take forms

$$\left\langle Q(E_0)\right\rangle=\left\langle Y^c(E_0)\right\rangle_c\left\langle T(\vec{r}_c)\right\rangle_c\left\langle g\right\rangle, \qquad (29)$$

$$\eta_Q^2=\eta_{\text{cov}}^2+\eta_{\text{pair}}^2+\eta_{tr}^2+\eta_{gain}^2+\eta_{\text{noise}}^2, \qquad (30)$$

$$\eta_{\text{cov}}^2=\frac{\int_E\int_{E'}dEdE'u(E-E_{\min})u(E'-E_{\min})\left\langle\frac{\partial Y^c(E_0,E)}{\partial E}\frac{\partial Y^c(E_0,E')}{\partial E'}\right\rangle_c\left\langle T^2(\vec{r}_c)\right\rangle_c}{\left\langle Y^c(E_0)\right\rangle_c^2\left\langle T(\vec{r}_c)\right\rangle_c^2}-1, \qquad (31)$$

is the relative variance of the output signal of the spectrometer due to the covariances between the secondary particles in the phase space;

$$\eta_{\text{pair}}^2=\frac{\int_E dEu(E-E_{\min})\left\langle F(E)\frac{\partial Y^c(E_0,E)}{\partial E}S(E)Q\right\rangle_c\left\langle T^2(\vec{r}_c)\right\rangle_c}{\left\langle Y^c(E_0)\right\rangle_c^2\left\langle T(\vec{r}_c)\right\rangle_c^2}, \qquad (32)$$

is the relative variance of the output signal of the spectrometer due to fluctuations in the number of electron-hole pairs;

$$\eta_{tr}^2=\frac{1}{\left\langle Y^c(E_0)\right\rangle_c\left\langle T(\vec{r}_c)\right\rangle_c}-\frac{\int_E dEu(E-E_{\min})\left\langle\frac{\partial Y^c(E_0,E)}{\partial E}S(E)Q\right\rangle_c\left\langle T^2(\vec{r}_c)\right\rangle_c}{\left\langle Y^c(E_0)\right\rangle_c^2\left\langle T(\vec{r}_c)\right\rangle_c^2}, \qquad (33)$$

is the relative variance of the output signal of the spectrometer caused by fluctuations of the processes

occurring in the detector from the formation of an electron-hole pair in the scintillator to the formation of a photoelectron in the photodetector;

$$\eta_{gain}^2 = \frac{1}{\left\langle Y^c(E_0)\right\rangle_c \left\langle T(\vec{r}_c)\right\rangle_c} \frac{\sigma_g^2}{\left\langle g\right\rangle^2}, \quad (34)$$

is the relative variance of the output signal of the spectrometer caused by fluctuations in the gain of the photodetector electronic amplifier;

$$\eta_{\text{noise}}^2 = \frac{\sigma_{\text{noise}}^2}{\left\langle Q(E_0)\right\rangle^2} \quad (35)$$

is the relative variance of the output signal of the spectrometer caused by the noise of the photodetector and electronics.

In all the above formulae,

$$\frac{\partial Y^c(E_0,E)}{\partial E} = \frac{w^c(E_0,E)}{\varepsilon_{\text{e-h}}(E)} S(E)Q, \quad (36)$$

is the differential light output of the scintillator for the energy of an electron E generated by an X-ray quantum with energy E_0 during its energy loss in the scintillator, and $w_\alpha^c(E_0,E)$ is the differential density of the absorbed energy for a certain configuration c of the absorbed energy distribution in the elements of the phase space $d\Gamma = dVdEd\vec{\Omega}$; $\varepsilon_{\text{e-h}}(E)$ is the average energy of electron-hole pair creation by the electron with the energy E; $S(E)$ is the probability of a luminescent center activation, which depends on the stopping power of the electron with the energy E; Q is the quantum efficiency of the luminescence center;

$$Y^c(E_0) = \int_E dEu(E-E_{\min}) \frac{\partial Y^c(E_0,E)}{\partial E} = E_0 L \qquad (37)$$

is the light output of the scintillator for X-rays with the energy E_0; L is the specific light output. The formulae take into account the commutativity of the integration and averaging operations, and the multiplicativity of averaging of the product of independent quantities.

In contrast to all existing in the literature formulae, the formulae of the microscopic theory contain information of the dependences of all contributions to the energy resolution on the characteristics of the scintillator, the scintillator-photodetector interface, the characteristics of photodetectors, and electronics. The formulae reflect the cascade nature of the random processes of the primary particle energy transformation into the output signal of the spectrometer since each subsequent contribution to the energy resolution decreases by the factor equal to the product of the mean values of the previous stages. It should be emphasized that all existing theories do not account for the cascade nature of the processes and, as a rule, sum up only the relative variances.

4. ANALYSIS OF EXISTING WORKS FROM THE POINT OF VIEW OF MICROSCOPIC THEORY

From the microscopic theory of scintillation spectrometers, it follows that the formula (5) in [2] and formula (17) in [7] cannot represent the statistical contribution of the PMT or photodiode, since it also contains the positive part of the relative variance (33). This is unacceptable because the negative term of the relative variance (33) cannot separately enter into the relative variance of the output signal since all contributions must be strictly positive. Only the relative variance of the output signal caused by fluctuations

of the processes occurring in the detector from the creation of an electron-hole pair in the scintillator to the formation of a photoelectron in the photodetector (35) is always positive since it is the relative variance of the binomial process. In [7], this positive contribution is also taken into account in the second term in (10), that is, twice.

The formulae (5) and (17) are also unacceptable because they are valid only if fluctuations in the number of light photons are described by the Poisson distribution. This is unacceptable because the relative variance of the output signal of the scintillation spectrometer should not contain the Fano factor for light photons. This is because the processes of emission of light photons by various luminescent centers in the scintillator are independent. Since the process of emission of a light photon by a luminescent center is described by the binomial distribution, its fluctuations are taken into account in the formula (33) which describes the fluctuations associated with the compound binomial processes of light photon emission and its registration by the photodetector.

The Fano factor for light photons, introduced in [7], is actually determined by fluctuations in the compound process of converting an electron-hole pair into a photoelectron in the photodetector, which depends on many factors: the scintillator geometry, its transparency, the quantum efficiency of the photodetector, etc. Therefore, the Fano factor for light photons, introduced in the second term in the expression (10), which corresponds to the term (33), is not fundamental and its introduction is meaningless. Moreover, the Fano factor introduced in [7], may be misleading since, from the point of view of quantum optics, the value of the Fano factor less than one corresponds to non-classical quantum statistics [9].

The only fundamental factor is the Fano factor for electron-hole pairs, which characterizes the fluctuations in the process of generating electron-hole pairs in the scintillator. It should be noted that the contribution of fluctuations in the process of generating electron-hole pairs can be represented in the form (13) only for the absolutely transparent scintillation crystal, otherwise it should be represented by the formula (32).

The structure of the third term in (10), which the authors identified with the inhomogeneity of the crystal, contradicts the theory of random branching processes since it follows from the cascade nature of the random processes of converting the energy of the primary particle into the output signal of the spectrometer that the fluctuations of each subsequent process are always decreased by the factor equals to the product of the average values of the previous stages. The structure of the third term in (10) is also doubtful from the mathematical meaning of the probability density function. The last term in (10), that the authors associated with fluctuations in the track topology σ^2_{track}, has the covariance of the probability density function, which is incomprehensible from the point of view of the probability theory.

The main conclusion of the microscopic theory is that it is impossible to separate the contributions from ionization losses, from delta-electrons, and from the nonproportionality of the scintillator light yield, since they are all taken into account in one contribution to the relative variance of the detector output signal caused by the covariances between the secondary particles in the phase space (31). This explains the erroneous approach in [6], where the authors separated the contributions from ionization losses and from delta-electrons and then summed up their relative fluctuations.

5. COVARIANCES BETWEEN SIGNALS OF SCINTILLATION SPECTROMETER PHOTODETECTORS.

In [10], the formula for the covariance between the signals of photodetectors of a scintillation spectrometer was derived, and using of the relative covariance between the signals of the photodetectors was

proposed

$$\frac{\operatorname{cov}(Q_n,Q_{n'})}{\langle Q_n\rangle\langle Q_{n'}\rangle}=\frac{\int\limits_E\int\limits_{E'}dEdE'u(E-E_{\min})u(E'-E_{\min})\left\langle\frac{\partial Y^c(E_0,E)}{\partial E}\frac{\partial Y^c(E_0,E')}{\partial E'}\right\rangle_c}{\left\langle Y^c(E_0)\right\rangle_c^2}\frac{\left\langle T_n(\vec{r}_c)T_{n'}(\vec{r}_c)\right\rangle_c}{\left\langle T_n(\vec{r}_c)\right\rangle_c\left\langle T_{n'}(\vec{r}_c)\right\rangle_c}-1$$
$$+\frac{\int\limits_E dEu(E-E_{\min})\left\langle\frac{\partial Y^c(E_0,E)}{\partial E}\big(F(E)-1\big)S(E)Q\right\rangle_c}{\left\langle Y^c(E_0)\right\rangle_c^2}\frac{\left\langle T_n(\vec{r}_c)T_{n'}(\vec{r}_c)\right\rangle_c}{\left\langle T_n(\vec{r}_c)\right\rangle_c\left\langle T_{n'}(\vec{r}_c)\right\rangle_c} \quad (38)$$

The formula (38) becomes the simplest when the particles interact with the scintillator near the scintillator point $\vec{r}_0$, and the volume element in which the energy of the detected particles is converted into light photons is small enough that the coefficients $\langle T_n(\vec{r}_0)\rangle_c=T_n(\vec{r}_0)$ and $\langle T_{n'}(\vec{r}_0)\rangle_c=T_{n'}(\vec{r}_0)$ be almost constant. In this case, $\langle T_n(\vec{r}_0)T_{n'}(\vec{r}_0)\rangle_c=T_n(\vec{r}_0)T_{n'}(\vec{r}_0)$, and (38) takes the form

$$\frac{\operatorname{cov}(Q_n,Q_{n'})}{\langle Q_n\rangle\langle Q_{n'}\rangle}=\frac{\int\limits_E\int\limits_{E'}dEdE'u(E-E_{\min})u(E'-E_{\min})\left\langle\frac{\partial Y^c(E_0,E)}{\partial E}\frac{\partial Y^c(E_0,E')}{\partial E'}\right\rangle_c}{\left\langle Y^c(E_0)\right\rangle_c^2}-1$$
$$+\frac{\int\limits_E dEu(E-E_{\min})\left\langle\frac{\partial Y^c(E_0,E)}{\partial E}\big(F(E)-1\big)S(E)Q\right\rangle_c}{\left\langle Y^c(E_0)\right\rangle_c^2} \quad (39)$$

If the Fano factor, the average energy of an electron-hole pair creation and the probability of a luminescent center activation do not depend on the energy of electron E, the relative covariance between the signals of the photodetectors reduced to

$$\frac{\operatorname{cov}(Q_n,Q_{n'})}{\langle Q_n\rangle\langle Q_{n'}\rangle}=\eta_Y^2+\frac{\varepsilon_{\text{e-h}}\big(F-1\big)}{E_0}. \quad (40)$$

The first term in (40) is connected with the covariances of the scintillator differential light yield associated with the nonproportionality of the scintillator.

The covariance between the signals from the photodetectors of the spectrometer underlies the experimental method for determining the Fano factor. An important advantage of this method is its independence from the gain and noise of the photodetectors electronics. This is an important advantage of the proposed experimental method, compared with the existing methods for determining the Fano factor, based on the subtraction of electronic noise from the variance of the photodetector signal [11].

The dependence of the last term on the inverse energy of the registered particles makes it possible to separate the contributions to the covariance from the scintillator light output and the Fano factor and allows their experimental determination. Since the contribution of the second term in (40) is inversely proportional to the number of electron-hole pairs, then for sufficiently high energies, the relative covariance determines the relative variance connected with the covariances of the scintillator differential light yield. As the relative covariance between the signals of photodetectors does not depend on the gain and noise, to improve the local absorption condition, it is necessary to direct a thin beam of radiation perpendicular to the middle of a long bar of scintillation material with two photodetectors at the ends.

6. WHY DO WE NEED A STANDARD THEORY OF SCINTHILLATION SPECTROMETERS?

The purpose of the standard theory of scintillation spectrometers is to ensure the uniformity of obtaining the characteristics of the processes occurring in scintillation spectrometers from experimental data, and on the methods of achieving the required accuracy. It should be emphasized that the theory of scintillation spectrometers does not replace either the physics and optics of scintillators or the physics of photodetectors and the methods of nuclear electronics. The standard theory of scintillation spectrometers and the general formulae obtained in it should show theorists in scintillator physics what probabilities they should calculate, in accordance with their models, and what requirements should be imposed on experiments in order to compare their calculations with experimental data. The standard theory of scintillation spectrometers should indicate to experimenters under what conditions in their experiments they can compare their results with the theory and with the results of other experimental groups.

Thus, the main goal of the standard theory of scintillation spectrometers is not to replace extensive research in fields related to scintillator physics, but to create a solid basis for linking theoretical and experimental research in this field. This work appeared in connection with my understanding of the absence of this connection.

7. CONCLUSION

In this work, it was shown that the existing macroscopic theories of scintillation spectrometers with one photodetector have a number of fundamental drawbacks. In this work, the microscopic mathematical model was formulated, which serves as the basis for the standard theory of scintillation spectrometers with several photodetectors. The standard theory allows obtaining the formulae for arbitrary moments of the signal distribution function at the outputs of the scintillation spectrometer. It was shown that in the developed standard theory of scintillation spectrometers there are no drawbacks of the currently existing theories of scintillation spectrometers with one photodetector.

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