Some aspects of Fermi-functions and neutrino capture cross-sections

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What is Fermi-function?

Neutrino capture cross-section

$$\sigma_{tot} = \sigma_{diskr} + \sigma_{res}$$

$$\sigma_{diskr} = \frac{1}{\pi} \sum_k G_F^2 \cos^2 \theta_C \, p_e W \, \underline{F(Z, W)} [B(F)_k + \left(\frac{g_A}{g_V}\right)^2 B(GT)_k]$$

$$\sigma_{res} = \frac{1}{\pi} \int_{\epsilon_{min}}^{\epsilon_{max}} G_F^2 \cos^2 \theta_C \, p_e W \, \underline{F(Z, W)} S(W) dW$$

$$W - m_{ec}^2 = E_\nu - Q_{EC} - E_{excitation}$$

Where by def. $F(Z, W) = \frac{|\psi_E(0)|_Z^2}{|\psi_E(0)|_{Z=0}^2}$

takes into account Coulomb interaction between the emitted beta and the final state nucleus

Why it's important?

Beta-spectrum shape

$$N(W)dW \sim C_L(W)\underline{F(Z,W)}p_e W(E_0 - W)^2 dW$$

Neutrino capture cross-section

$$\sigma_{tot} \sim C_L(W)\underline{F(Z,W)}p_e W$$

Fermi-function calculation

E. Fermi, “An attempt of a theory of beta radiation. 1.”, Z. Phys.88, 161–177(1934). (point-like nuclei)

$$F_0(Z,A,W) = 4(2pR)^{2(\gamma-1)} \frac{|\Gamma(\gamma+iy)|^2}{(\Gamma(1+2\gamma))^2} e^{\pi y}, \gamma = \sqrt{1-(\alpha Z)^2}, y = \pm \alpha Z W/p \quad (11)$$

Point-like nuclei + finite size correction

$$F(Z,A,W) = F_0 \cdot L_0.$$

$$L_0 = 1 + \frac{13}{60}(\alpha Z)^2 \mp \frac{\alpha Z W R(41-26\gamma)}{[15(2\gamma-1)]} \mp \frac{\alpha Z R\gamma(17-2\gamma)}{[30W(2\gamma-1)]} + \Omega$$

H. Behrens and J. Janecke,*Numerical Tables for Beta-Decay and Electron Capture, Landolt-Boernstein - Group I Elementary Particles,Nuclei and Atoms (Springer, 1969).*

B. S. Dzhelepov and L. N. Zyrianova, *Influence of atomic electric fields on beta decay(Moscow: Akad. Nauk SSSR, 1956).*

Y. P. Suslov, *Izv. Akad. Nauk SSSR, Ser. Fiz.32, 213 (1968).*

More corrections ...

$$F(W) = F_0(Z,W)\cdot L_0(Z,W)\cdot U(Z,W)\cdot R(Z,W,A)\cdot S(Z,W)\cdot D_{FC}(Z,W)\cdot \ldots$$

Effect	Formula	Magnitude
Traditional Fermi function	F_0	Unity or larger
Finite size of the nucleus	L_0	10^{-1}-10^{-2}
Radiative corrections	R	
Atomic screening	S	
Diffuse nuclear surface	U	10^{-3}-10^{-4}
Nuclear deformation	D_{FS}	

More details:
L. Hayen, N. Severijns, K. Bodek, D. Rozpedzik, and X. Mougeot, "High precision analytical description of the allowed β spectrum shape", Rev. Mod.Phys.90, 015008 (2018)

Fermi-functions

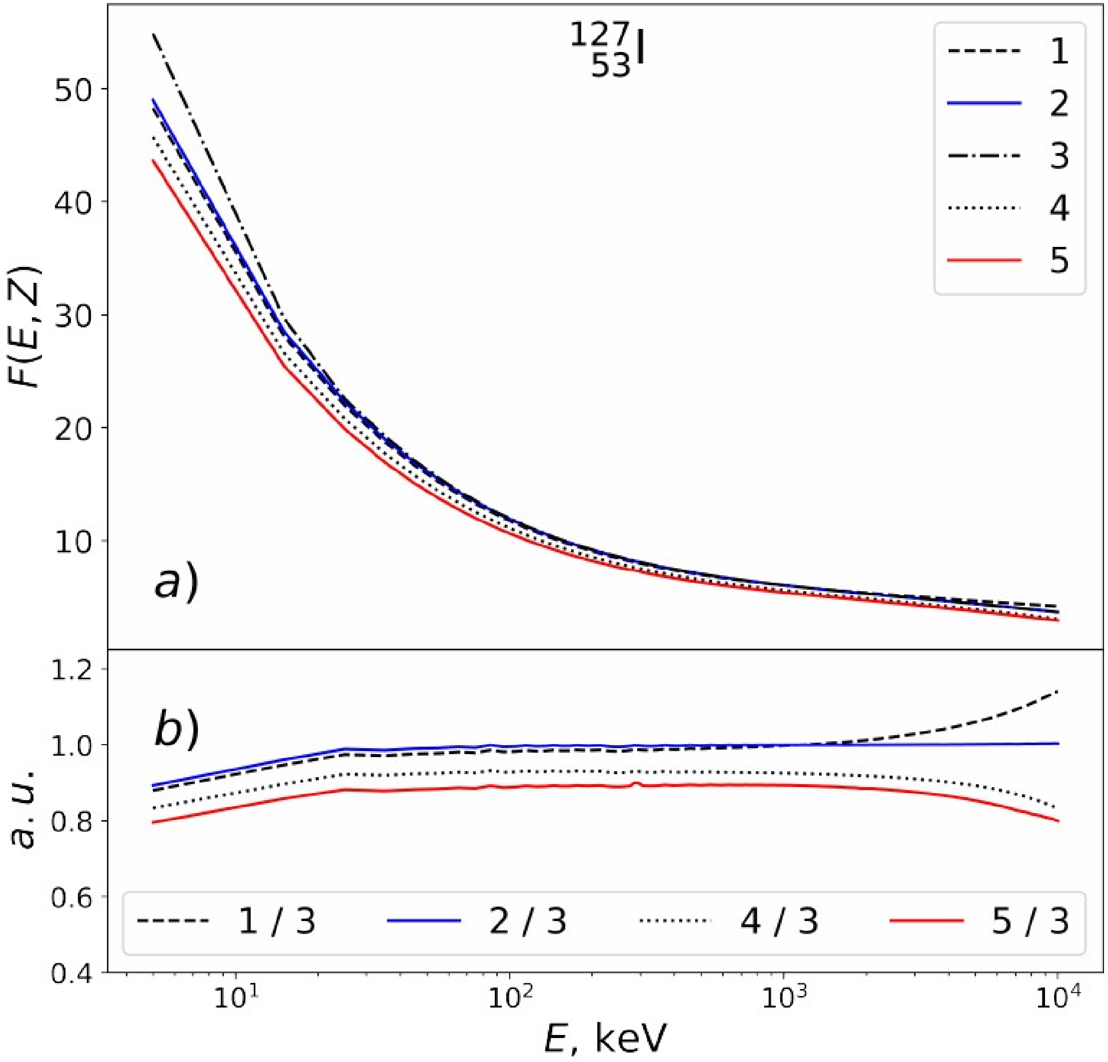
1 - **E. Fermi**, *“An attempt of a theory of beta radiation. 1.”, Z. Phys.88, 161–177(1934).*

2 - **L. Hayen, N. Severijns, K. Bodek, D. Rozpedzik, and X. Mougeot**, *“High precision analytical description of the allowed β spectrum shape”, Rev. Mod.Phys.90, 015008 (2018) (Fermi 2017)*

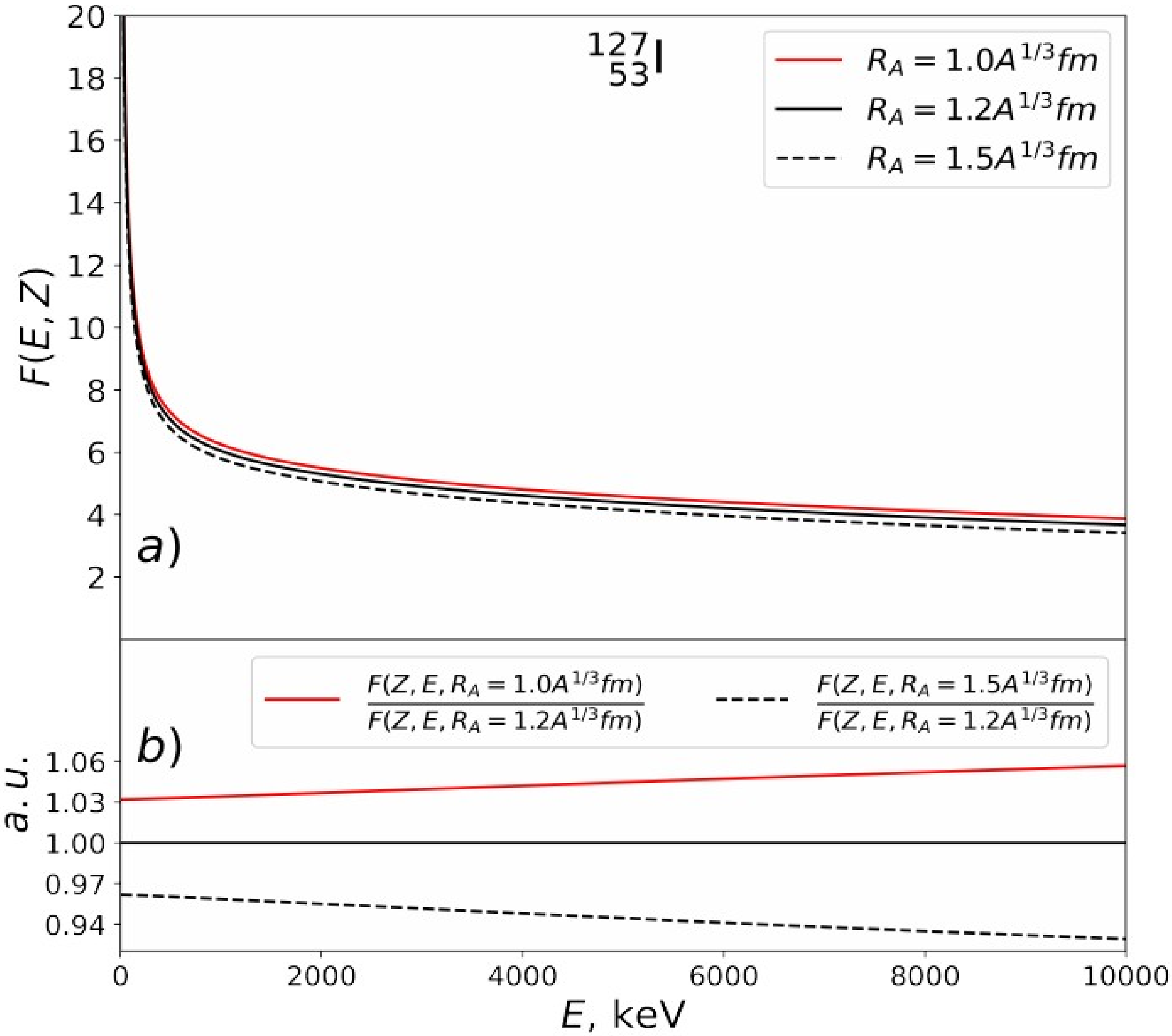
3 - **H. Behrens and J. Janecke**,*Numerical Tables for Beta-Decay and Electron Capture, Landolt-Boernstein - Group I Elementary Particles,Nuclei and Atoms (Springer, 1969).*

4 - **B. S. Dzhelepov and L. N. Zyrianova,** *Influence of atomic electric fields on beta decay(Moscow: Akad. Nauk SSSR, 1956).*

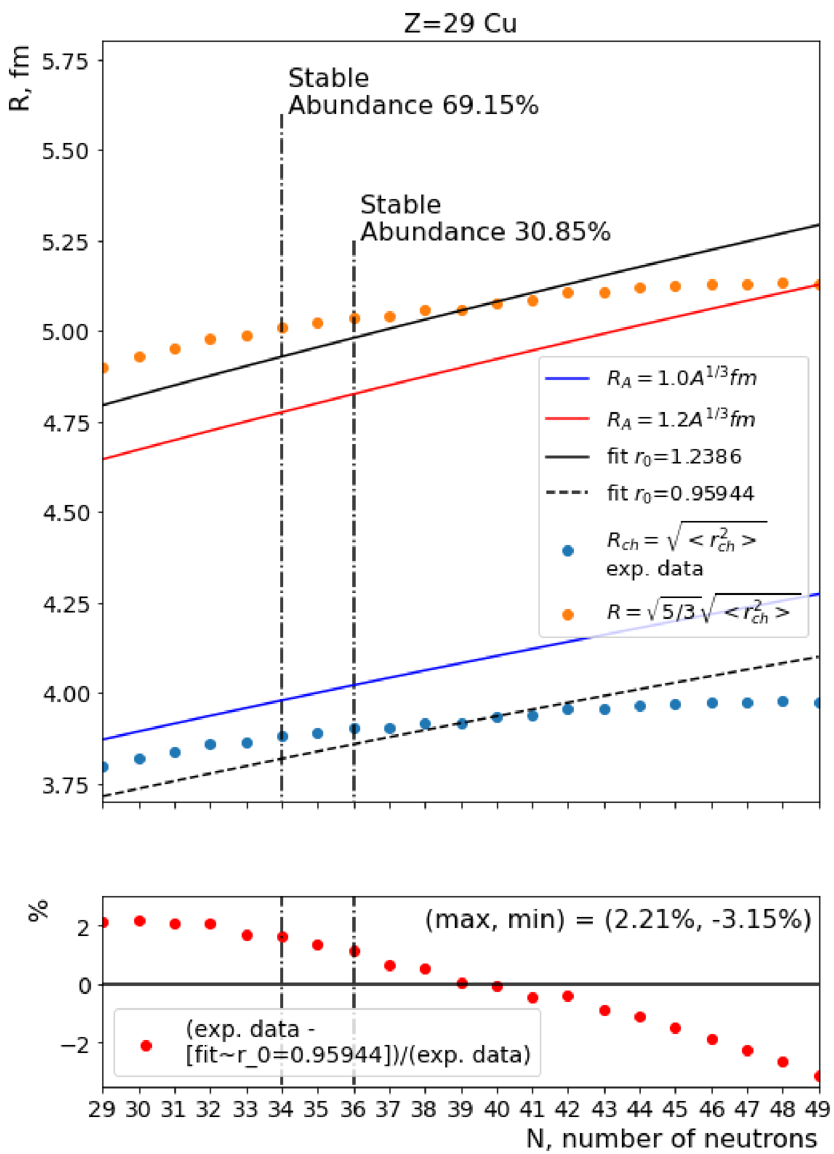
5 - **Y. P. Suslov**, *Izv. Akad. Nauk SSSR, Ser. Fiz.32, 213 (1968).*

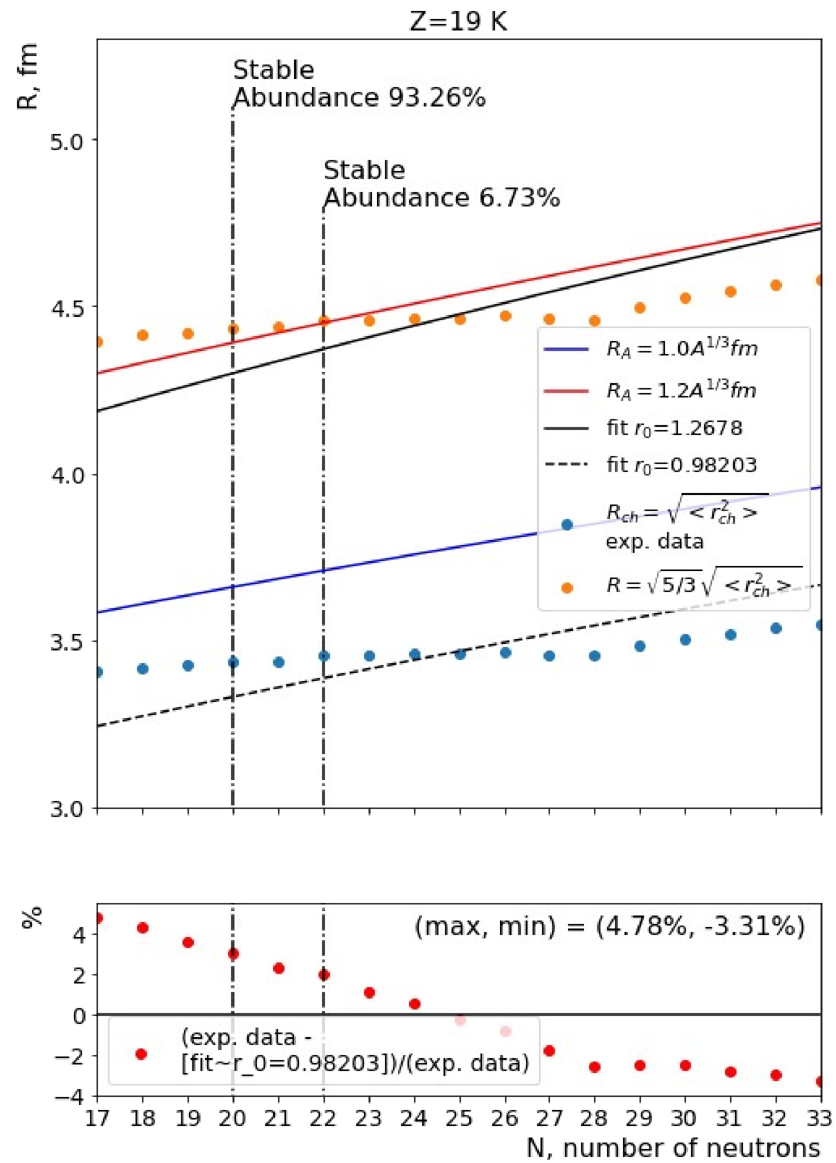


Radius coefficient dependance

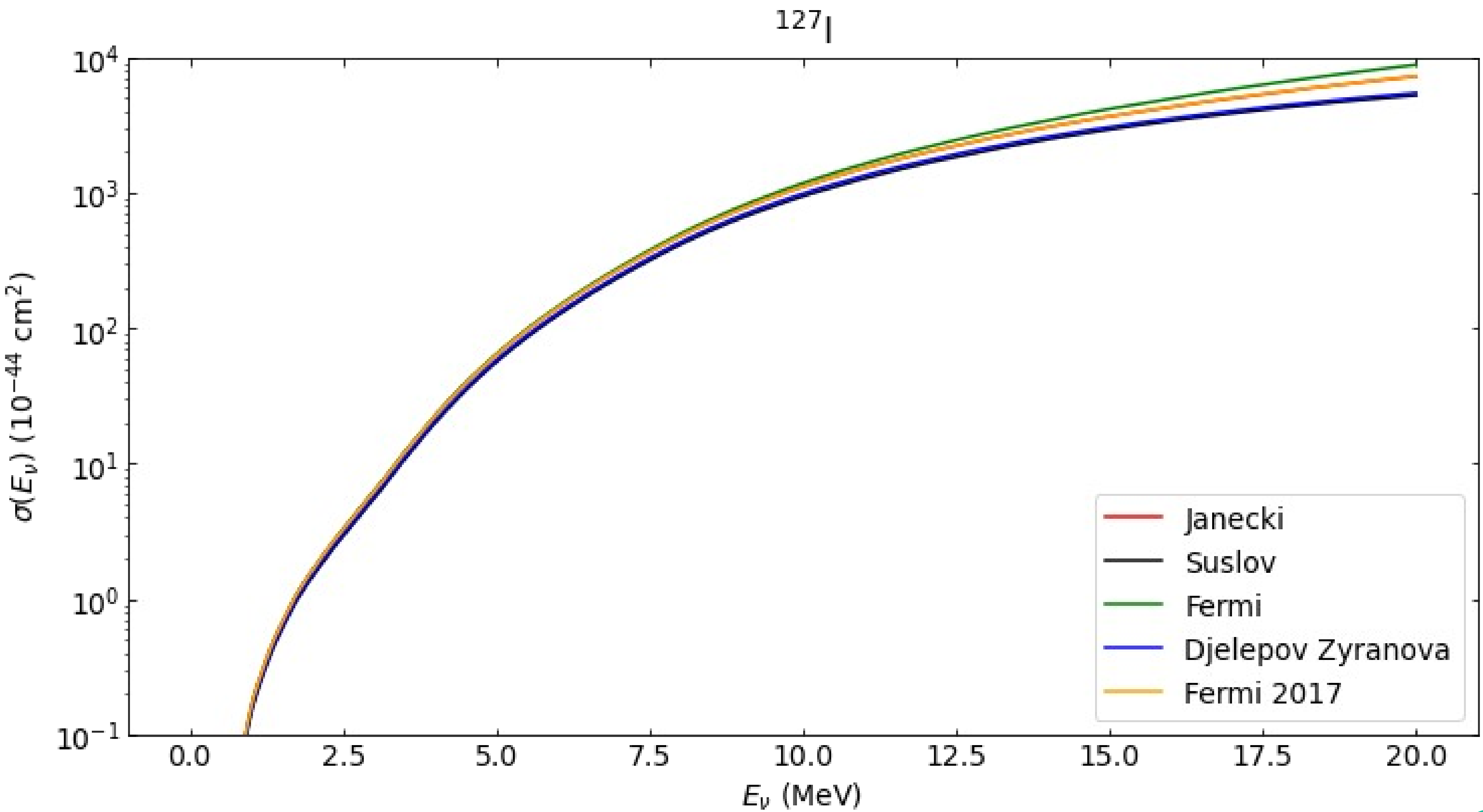


Nuclear radius fit





F(Z, W) influence on ν capture cross-section for ^{127}I



F(Z, W) influence on neutrino capture rate for ^{127}I

$$R = \int_0^{E_{max}} \rho_{\text{solar}}(E_\nu)\sigma_{total}(E_\nu)dE_\nu$$

Capture rate R given in units of SNU (10^{-36} interactions per target atom per second)

Capture rate for ^{127}I by solar component (in SNU)

Fermi-function	^{8}B	hep	^{13}N	^{15}O	^{17}F	pep	Be	Total
Janecki	25.706	0.108	0.164	0.551	0.013	0.828	2.923	30.29
Suslov	21.881	0.089	0.145	0.490	0.012	0.738	2.562	25.920
Fermi	27.286	0.120	0.161	0.543	0.013	0.818	2.850	31.795
Dzhelepov-Zyrianova	22.853	0.093	0.152	0.510	0.012	0.767	2.695	27.085
Fermi 2017	25.735	0.108	0.163	0.549	0.013	0.826	2.890	30.287

Solar flux taken from *J. N. Bahcall, A. M. Serenelli, S. Basu, Astrophys. J. 621 (1) (2005)*

Conclusion

- **Fermi-function plays significant role in determination of the beta-decay rate, neutrino capture cross-section, etc.**

- **Correct calculation requires considering many correction e.g. nuclear radius and charge distribution, screening from orbital electrons, radiative corrections, ...**

- **Depending on the choice of the fermi-function neutrino capture rate can change up to 10%-15%**

Thank you for your attention!