SELF-CONSISTENT CALCULATIONS OF SOLAR CNO NEUTRINO CAPTURE-RATES FOR ^{115}In.

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PHYSICS A

Gamow–Teller strength functions of superfluid odd-*A* nuclei and neutrino capture reactions

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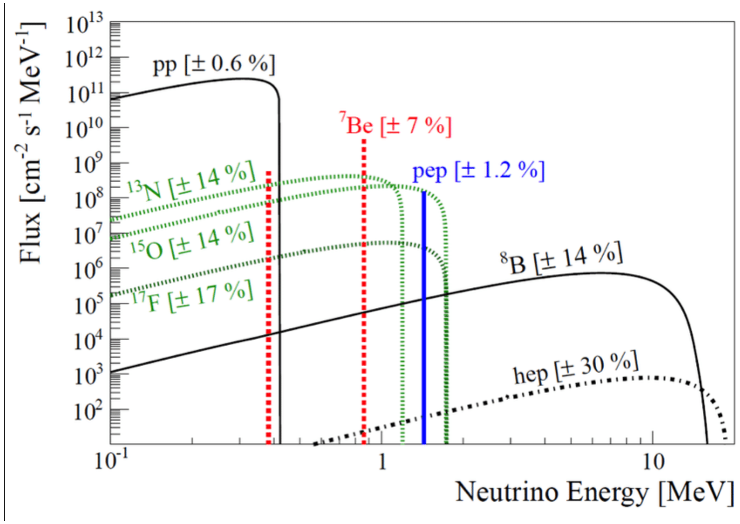
SELF-CONSISTENT APPROACH TO NEUTRINO CAPTURE

The practical aspect of the problem concerns solar-neutrino [15] and reactor-anti-neutrino capture rates in detectors based on the inverse $\beta^{\pm}$-decay processes, i.e. on the $\nu(\bar{\nu})$-capture reactions, which take place due to the electroweak charge currents,

$$\nu_e + A_Z(I^\pi) \rightarrow e^- + A_{Z+1}(I'^{\pi'}) , \quad (1)$$

$$\bar{\nu}_e + A_Z(I^\pi) \rightarrow e^+ + A_{Z-1}(I'^{\pi'}) . \quad (2)$$

A number of nuclei were suggested for neutrino detectors. The most interesting nuclei for theoretical analysis are the odd-A nuclei with lowest reaction thresholds: ^{71}Ga ($Q_\beta = -236$ keV), ^{81}Br (-471 keV), ^{115}In (-120 keV) which are sensitive both to the low- and to high-energy portions of the solar-neutrino spectrum.



Motivation

- *The first direct detection of the neutrinos from carbon-nitrogen-oxygen (CNO) fusion cycle in the Sun by the BOREXINO Collaboration [1].*
- *An estimate of "CNO-like" events from geo-antineutrino of ^{40}K decay in the Hydride model of the Earth [2].*
- *Additional experimental prospects given by improved ^{115}In detector system (LENS Project [3]).*

1. *M. Agostini M., K. Altenmuller, S. Appel et al. for BOREXINO Collaboration , arXiv:2006.15115. 2020.*
2. *L.B. Bezrukov et.al., Izvestia RAN **85,** (4), 566 (2021).*
3. *R.S. Raghavan, The LENS Experiment: Spectroscopy of Low Energy Solar Neutrinos, Neutrino 2010, Athens, Greece, 2010*

Theory background

- *Ground states of odd-A and odd-odd nuclei are described with the latest version of the Fayans density functional.*
- *The main feature is self-consistent CQRPA for odd-A and odd-odd nuclei within full SO(8) framework including the isoscalar (T=0) effective pp-interaction.*
- *The latter enables more precise description of the low-energy tail of the GT and FF strength functions in the daughter nuclei. That is important for estimating low energy pp and CNO neutrino rates in 82Se, 100Mo as well as 115In.*

Specific problems in odd-A and odd-odd nuclei:

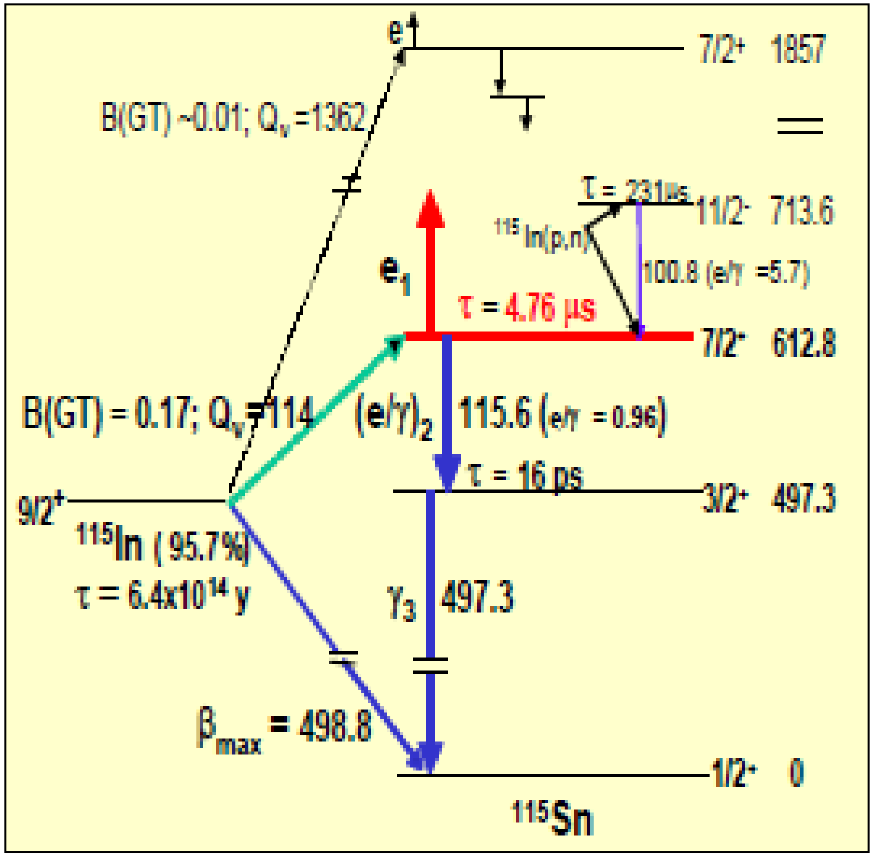
- *blocking of paring*
- *odd quasiparticle in the g.s. with correct J^{π}*

Unpleasant feature - violation of time-reversal invariance by odd nucleon !

The usual way out: Equal Filling Approximation (EFA)

$^{115}In \rightarrow ^{115}Sn$ LENS Project

No-threshold, retarded
υ -induced decay
9/2+ → 1/2+ 6.4 (14)y

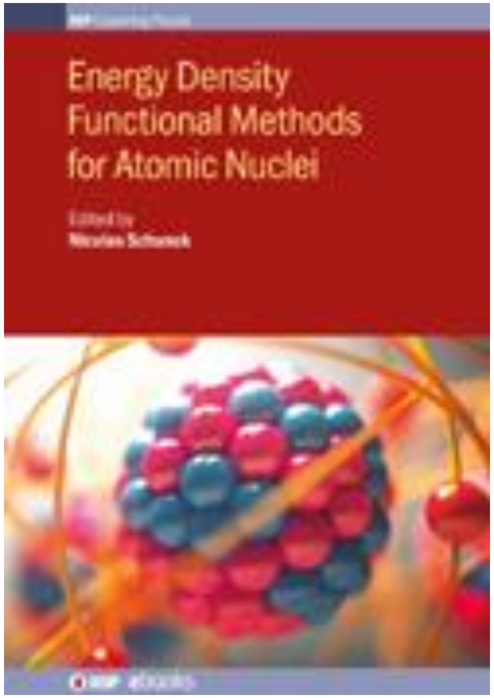


Low-threshold *inverse GT β-decay*
9/2+ → 7/2+
T12=3.26 μsec
Qυ=114 keV ,,
Ex=0.6128 MeV
B(GT)= 0.17

A unique delayed-coincidence signature

A low-threshold, high-efficiency, direct-counting detector for solar neutrinos from the p−p fusion reaction , pep as well as CNO neutrinos.

R. S. Raghavan
Phys. Rev. Lett. 37, 259



Three main families of phenomenological EDF

DF3… -a, -b, -f ,…FANDF^0

S.A. Fayans + collaborators, KI ,Moscow

BCPM *- Barcelona–Catania–Paris–Madrid*
(originating from an early work by Baldo et al.)

SeaLL *- Seattle–Livermore .*

- *directly parametrize the nuclear EoS by series of powers of the density ;*
- *add corrective terms to account for finite-size and many-body effects ;*
- *add terms accounting for the Coulomb potential and pairing corrections.*

Fayans and SeaLL functionals : the Kohn-Sham type EDF

independent-particle kinetic energy , m*=1

A detailed explanation of Skyrme and Fayans EDF as used here is given in Ref. [1]. We summarize here the essential features for the present paper. Both functionals are composed from volume, surface, spin-orbit, pairing, Coulomb and center-of-mass terms as

$$\mathcal{E} = \mathcal{E}^{\rm v}(\rho,\tau) + \mathcal{E}^{\rm s}(\rho) + \mathcal{E}^{\rm ls}(\rho,\vec{J}) + \mathcal{E}^{\rm Coul}(\rho) + \mathcal{E}^{\rm pair}(\rho) + \mathcal{E}^{\rm c.m.}(\rho) \quad .$$

These read in detail

	Skyrme	Fayans
volume:	$\mathcal{E}^{\rm v}_{\rm Sk} = \sum_{t=0}^{1}\left[(C^{\rho\rho}_{t0} + C^{\rho\rho}_{t\rm D}\rho_0^{\alpha})\rho_t^2 + C^{\rho\tau}_t\rho_t\tau_t\right]$	$\mathcal{E}^{\rm v}_{\rm Fy} = \frac{1}{3}\varepsilon_F\rho_{\rm sat}\left[a^{\rm v}_{+}\frac{1-h^{\rm v}_{1+}x_0^{\sigma}}{1+h^{\rm v}_{2+}x_0^{\sigma}}x_0^2 + a^{\rm v}_{-}\frac{1-h^{\rm v}_{1-}x_0}{1+h^{\rm v}_{2-}x_0}x_1^2\right]$
	$C^{\rho\rho}_{t0}, C^{\rho\rho}_{t\rm D}, \alpha, C^{\rho\tau}_t \leftrightarrow E/A_{\rm eq},\ \rho_{\rm eq},\ K,\ J,\ L,\ \frac{m^*}{m},\ \kappa_{\rm TRK}$	$a^{\rm v}_{\pm},\ h^{\rm v}_{1\pm},\ h^{\rm v}_{2\pm} \leftrightarrow E/A_{\rm eq},\ \rho_{\rm eq},\ K,\ J,\ L,\ h^{\rm v}_{2-}$
surface :	$\mathcal{E}^{\rm s}_{\rm Sk} = \sum_{t=0}^{1} C^{\rho\Delta\rho}_t\rho_t\Delta\rho_t$	$\mathcal{E}^{\rm s}_{\rm Fy} = \frac{1}{3}\varepsilon_F\rho_{\rm sat}\frac{a^{\rm s}_{+}r_s^2(\vec{\nabla}x_0)^2}{1+h^{\rm s}_{+}x_0^{\sigma}+h^{\rm s}_{\nabla}r_s^2(\vec{\nabla}x_0)^2}$
spin-orbit:	$\mathcal{E}^{\rm ls}_{\rm Sk} = \sum_{t=0}^{1} C^{\rho\nabla J}_t\rho_t\boldsymbol{\nabla}\cdot\boldsymbol{J}_t$	$\mathcal{E}^{\rm ls}_{\rm Fy} = \sum_{t=0}^{1} C^{\rho\nabla J}_t\rho_t\boldsymbol{\nabla}\cdot\boldsymbol{J}_t$
pairing:	$\mathcal{E}^{\rm pair}_{\rm Sk} = \frac{1}{4}\sum_{q\in\{p,n\}} V_{{\rm pair},q}\left(1-\frac{\rho_0}{\rho_{\rm pair}}\right)\breve{\rho}_q^2$	$\mathcal{E}^{\rm pair}_{\rm Fy} = \frac{2\varepsilon_F}{3\rho_{\rm sat}}\breve{\rho}_q^2\left[f^{\xi}_{\rm ex} + h^{\xi}_{+}x^{\gamma}_{\rm pair} + h^{\xi}_{\nabla}r_s^2(\vec{\nabla}x_{\rm pair})^2\right]$

where $x_t = \rho_t/\rho_{\rm sat}$ and $x_{\rm pair} = \breve{\rho}_q/\rho_{\rm sat}$. The γ, $\rho_{\rm sat} = 0.16\ {\rm fm}^{-3}$ and $\varepsilon_F = \varepsilon_F(\rho_{\rm sat})$ are given, fixed values. The non-linear surface coefficient is fixed as $h^s_+ = h^v_{2+}$. Coulomb term and c.m. correction are irrelevant here. Note that the parameters for the volume terms are handled in term of nuclear matter parameters $E/A_{\rm eq}$ etc as is indicted in the line below the volume terms.

The groups of model parameters for the MCC in Fig. 3 are defined as follows:

	Skyrme	Fayans
pairing:	$V_{{\rm pair},p}$, $V_{{\rm pair},n}$, $\rho_{\rm pair}$	$f^{\xi}_{\rm ex}$, h^{ξ}_{+}, h^{ξ}_{∇}
surface:	$C^{\rho\Delta\rho}_0$, $C^{\rho\Delta\rho}_1$	$a^{\rm s}_{+}$, $h^{\rm s}_{\nabla}$, h^{ξ}_{∇}
spin-orbit:	$C^{\rho\nabla J}_0$, $C^{\rho\nabla J}_1$	$C^{\rho\nabla J}_0$, $C^{\rho\nabla J}_1$
symmetry energy:	J, L	J, L

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I. Self-consistent approach to nuclear ground state .

Thus the total interaction energy of the superfluid nucleus, $E_{\text{int}}[\rho, \nu] = \int \mathrm{d}\boldsymbol{r}\, \varepsilon_{\text{int}}(\boldsymbol{r})$, where $\varepsilon_{\text{int}}(\boldsymbol{r})$ is defined as above, is a functional of two densities, the normal, $\rho(\boldsymbol{r})$, and the anomalous, $\nu(\boldsymbol{r})$. Self-consistent calculation with such a functional looks like the standard variational procedure in which the single-particle hamiltonian takes the form

$$\mathcal{H} = \begin{pmatrix} h - \mu & -\Delta \\ -\Delta & \mu - h \end{pmatrix}, \tag{15}$$

where

$$h = \frac{p^2}{2m} + \frac{\delta E[\rho, \nu]}{\delta \rho}, \qquad \Delta = -\frac{\delta E[\rho, \nu]}{\delta \nu}. \tag{16}$$

These equations have been solved iteratively as follows. For given densities $(\rho^{(i)}, \nu^{(i)})$, from the above functional, Eq. (4), the elements of the hamiltonian $\mathcal{H}^{(i)}$ were derived, through its eigenvalues and wave functions $(u^{(i)}, v^{(i)})$ the new densities were calculated, and then, as an input $(\rho^{(i+1)}, \nu^{(i+1)})$ for the next iteration, superpositions of previous densities and these new ones were used with the weights of 0.85 and 0.15, respectively.

IAS.
Fully self-consistent calculation.

$$E_0 = \int \mathcal{E}[\rho(\mathbf{r}), \nu(\mathbf{r})] d^3 r,$$

$$h(1,2) = \frac{\delta E[\rho, \nu]}{\delta \rho(2,1)}$$

1 ≡ (**r**1, *s*1, *t*1).

$$\Delta(\mathbf{r}) = \frac{\delta E[\rho, \nu]}{\delta \nu(\mathbf{r})}$$

$$\mathcal{F}^{-} = \frac{\delta^2 \mathcal{E}}{(\delta \rho_{-})^2},$$

$$\rho_{-} = \rho_n - \rho_p .$$

GT and FF.
Self-consistent g.s.calculation

GT and FF.
Effective NN-interaction.

U: Particle-hole channel:
δ-interaction
with Landau-Migdal constant
g' >0 (repulsion)
+ π-meson + ρ-meson exchange

V: Particle-particle channel:
T=0, δ-interaction
with one parameter:
g'_{pp}<0 (attraction)

Neglecting the spin-isospin dependent components in E_0 causes ~ 100 KeV error in masses

II. Continuum CQRPA approach to nuclear excited states

Δ=0

$$\rho_{pn} = L_{pn}(\omega)V_{pn} + M_{pn}(\omega)V^{h}_{pn} + N^{1}_{pn}(\omega)d^{1}_{pn} + N^{2}_{pn}(\omega)d^{2}_{pn},$$
$$\rho^{h}_{pn} = M_{pn}(\omega)V_{pn} + L_{pn}(-\omega)V^{h}_{pn} + N^{2}_{pn}(-\omega)d^{1}_{pn} + N^{1}_{pn}(-\omega)d^{2}_{pn},$$
$$\varphi^{1}_{pn} = N^{1}_{pn}(\omega)V_{pn} + N^{2}_{pn}(-\omega)V^{h}_{pn} + K_{pn}(\omega)d^{1}_{pn} - M_{pn}(\omega)d^{2}_{pn},$$
$$\varphi^{2}_{pn} = N^{2}_{pn}(\omega)V_{pn} + N^{1}_{pn}(-\omega)V^{h}_{pn} - M_{pn}(\omega)d^{1}_{pn} + K_{pn}(-\omega)d^{2}_{pn}. \quad (26)$$

Here $\omega = \tilde{\omega} - \delta\mu$, where $\delta\mu = \mu^{p} - \mu^{n}$ is the difference between the proton and the neutron chemical potentials, and $\tilde{\omega}$ is the nuclear excitation energy. Propagators obtained by integrating various products of the normal and abnormal Green functions G and F

Even-even core:

$$L_{pn}(\omega) = \int G_{p}(\varepsilon)G_{n}(\varepsilon+\tilde{\omega})\frac{d\varepsilon}{2\pi i} = -\left[\frac{v_p^2 u_n^2}{E_{pn}-\omega} + \frac{v_n^2 u_p^2}{E_{pn}+\omega}\right],$$

Odd nucleon:

$$L_{p_0 n}(\omega) = \int G_{p_0}(\varepsilon)G_{n}(\varepsilon+\tilde{\omega})\frac{d\varepsilon}{2\pi i} = -\left[\frac{v_{p_0}^2 u_n^2}{(E_n+E_{p_0})-\omega} + \frac{u_{p_0}^2 u_n^2}{(E_n-E_{p_0})-\omega}\right],$$

Continuum+pairing:

$$L(\boldsymbol{r},\boldsymbol{r}';\omega) = A(\boldsymbol{r},\boldsymbol{r}';\omega) + \sum[L_{pn}(\omega) - \tilde{A}_{pn}(\omega)] \times\varphi^{*}_{n}(\boldsymbol{r}_1)\varphi_{p}(\boldsymbol{r}_1)\varphi_{n}(\boldsymbol{r}_2)\varphi^{*}_{p}(\boldsymbol{r}_2),$$

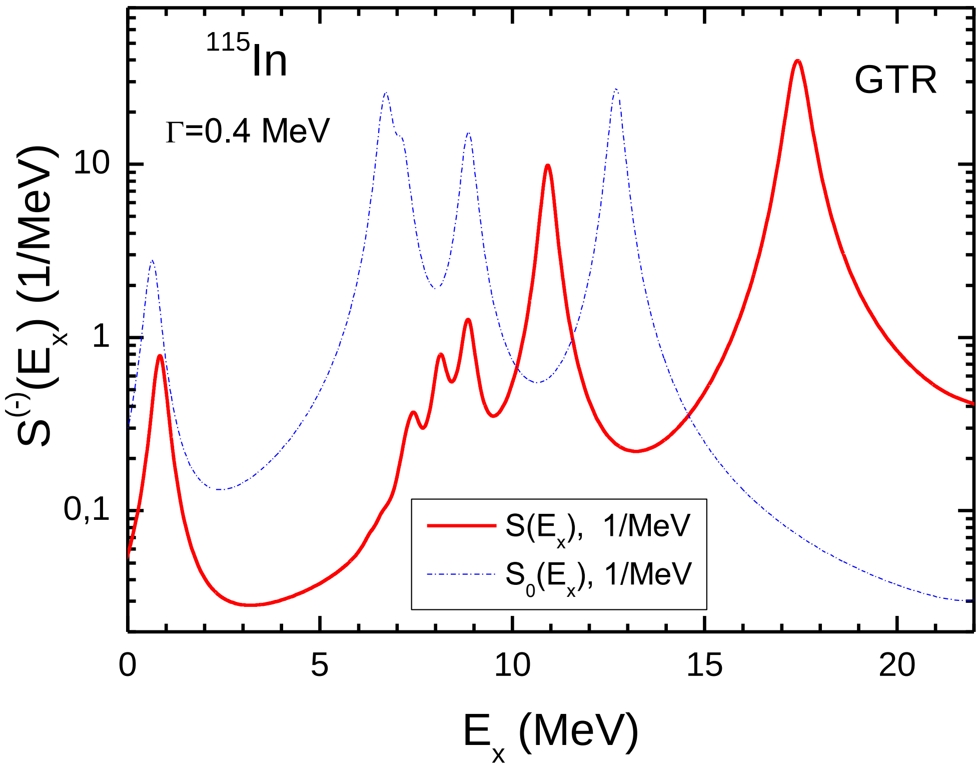
Low-lying tail of GT-excitation spectrum
is defined by competition of

$g'>0$ (shifts Ex upward) ↑

and $g'_{pp}<0$ (shifts Ex downward) ↓

^{115}In GT strength functions

The structure of the low-lying GT tail is simple



The capture cross section is sensitive to position of the 1st 1+ state. Its B(GT) is amplified by the Fermi function F(Z,w)!

DF3a

The role played by pairing correlations and F^{pp} !

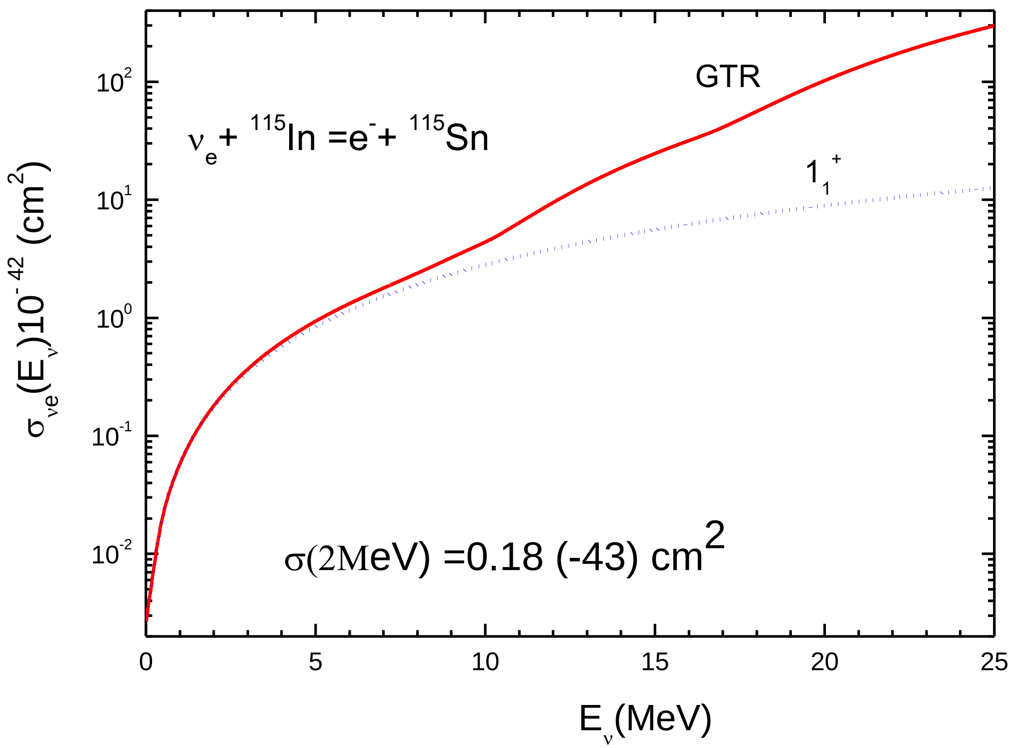
The strength functions in the $d = 0$, $F^{pp} = 0$ approximation:

For the first GT excited state corresponding to the transition lg9/2 -1g7/2 in 115In-115Sn:

- Ex(GT $_1$) =0.825 MeV
- B(GT) is higher than the exp. **by 2.5 times**.

d # 0, F^{pp} # 0 Ex(GT $_1$) =0.628 MeV
B(GT) =0.12

Neutrino capture cross sections, 71Ga, 115In



Solar-neutrino fluxes and capture rates (in SNU) for indium detector are not very sensitive to the DF

Neutrino source	E_ν^{max} $[\Phi_i]$	^{115}In this work	^{115}In Ref. [15]
1. ^{1}H(p,e$^+\nu_e$)^{2}H	0.420 [6.00]	460	468
2. ^{1}H(pe$^-$, ν_e)^{2}H	1.442 (d) [0.014]	7.7	8.1
3. ^{3}He(p,e$^+\nu_e$)^{4}He	18.773 [7.6×10^{-7}]	0.05	0.05
4. ^{7}Be(e$^-$, ν_e)^{7}Li	0.862 (d) 0.384 (d) [0.47]	119.8	116
5. ^{8}B(e$^+\nu_e$)^{8}Be*	15.00 [5.8×10^{-4}] (2.87×10^{-4})	14.9 (7.5)	14.4 (7.2)
6. ^{13}N(e$^+\nu_e$)^{13}C	1.199 [0.061]	13.4	13.6
7. ^{15}O(e$^+\nu_e$)^{15}N	1.732 [0.052]	18.0	18.5
total		634	639

DF3a result (with no fitting of the s.p. energies!) is quite close to DF1

[15] J.N. Bahcall, Phys. Rev. 135 (1964) B137;

Conclusions

1. *SELF-CONSISTENT calculations based on the new Fayans energy density functional (DF3a) and CQRPA for odd-A nuclei with pairing.*

2. *The total neutrino capture rates are not very sensitive to the EDF chosen. The rates from DF1 are rather close to the ones from DF3a.*

Perspectives and challenges.

1. *The odd-A nuclei . Time reversal symmetry violation.*

Framework is needed beyond the Equal filling approximation (EFA).

Schemes enriching the low-lying charge-exchange excitation spectra:

The odd-A nuclei beyond the QRPA (quasiparticle-phonon formalism).

A practical framework –FRSA approach extended for odd-A and odd-odd nuclei:

1. Complex configuration effects on beta-decay rates *A. P. Severyukhin, V. V. Voronov, I. N. Borzov, N. N. Arsenyev, Nguyen Van Giai,* Journal of Physics: conference series, 580, 012051-1-6, 2015.

2. Proton-neutron quasiparticle RPA with separable Gamow-Teller forces K. Muto, E. Bender, T. Oda & H. V. Klapdor-Kleingrothaus *Zeitschrift für Physik A Hadrons and Nuclei* **volume 341**, pages407–415 (1992)

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Self-Consistent Ground State. Fayans EDF.

$$\mathcal{E}[\rho(\mathbf{r}),\nu(\mathbf{r})] = \tau + \varepsilon_v + \varepsilon_s + \varepsilon_{\rm Coul} + \varepsilon_{sl} + \varepsilon_{ss} + \varepsilon_{\rm pair}\,.$$

$$E_0^{\rm int}[\rho] = \int \mathcal{E}(\rho(\mathbf{r}))d^3r = \int \frac{\alpha\rho^2}{2}\left(1+\alpha\rho^\sigma\right)d^3r,$$

Skyrme EDF

VS.

$$\mathcal{E}(\rho) = \frac{\alpha\rho^2}{2}\frac{1+\alpha\rho^\sigma}{1+\gamma\rho}.$$

m=m*

new ρ – dependent terms

Fayans EDF

DF3a + Deformed HFBTHO :

S.V. Tolokonnikov, I.N. Borzov, M. Kortelainen, Yu.S. Lutostansky, E.E.Saperstein
J. Phys.G 35, 291, 2014.

Neutrino capture cross sections, 71Ga, 115In

Δ # 0

σ^{115}In (2 MeV) ~0.4*10(-43) cm^2

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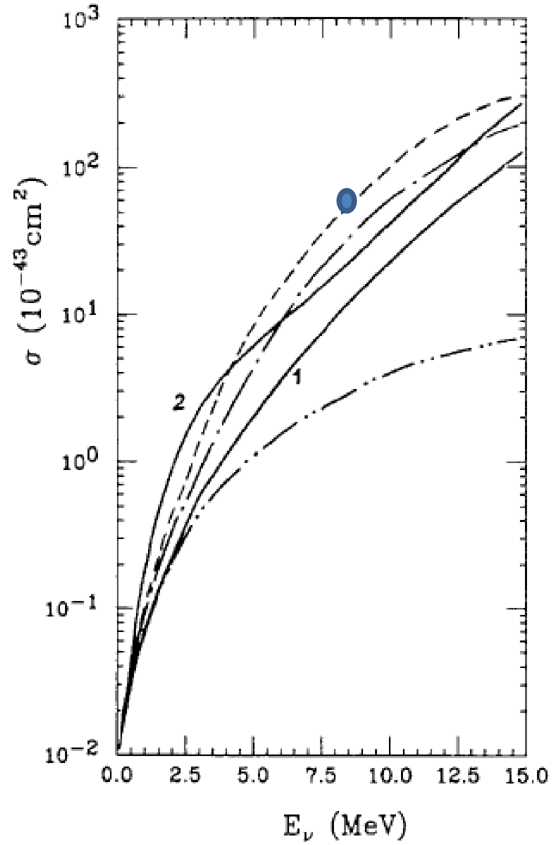
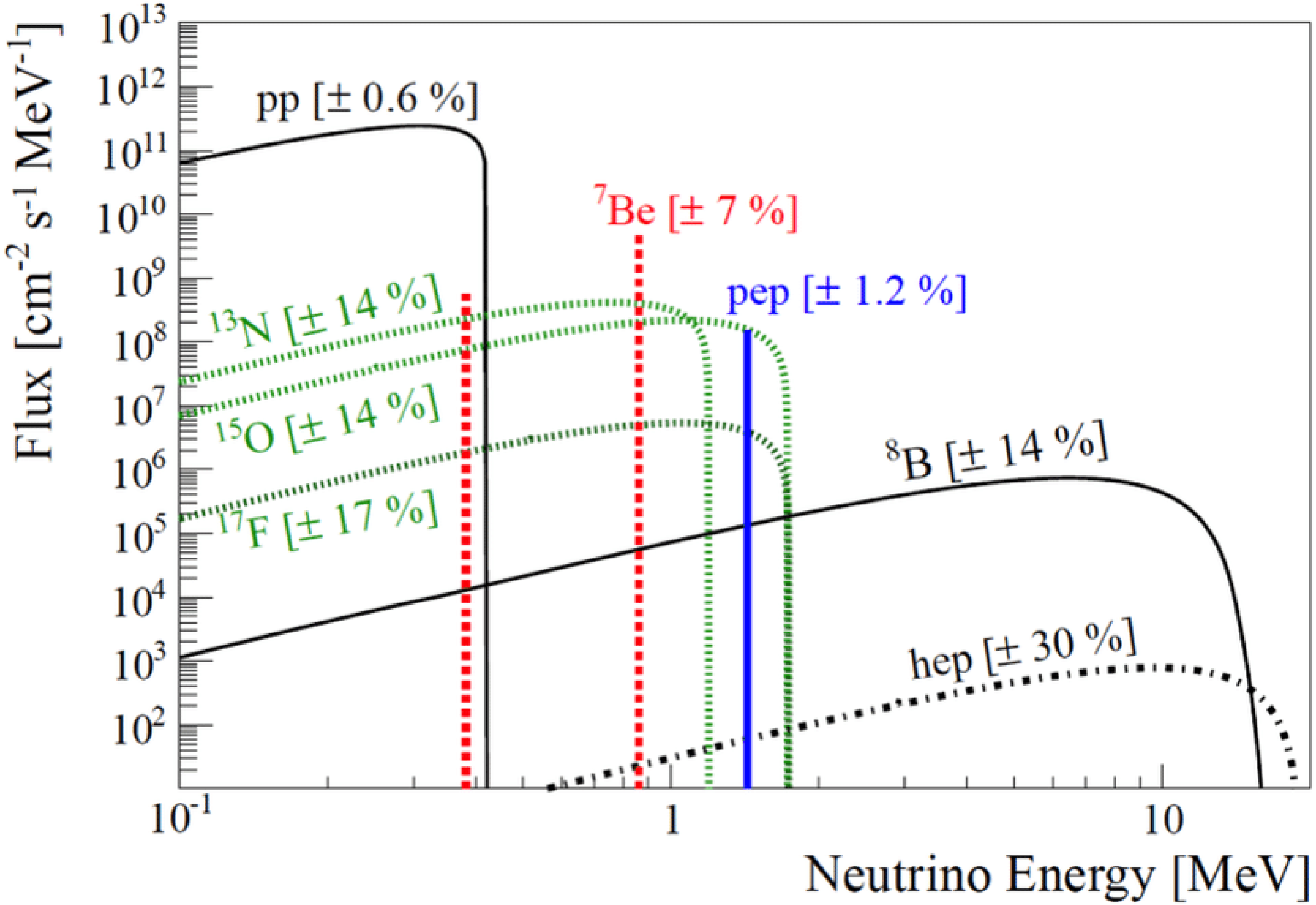


Fig. 2. Neutrino capture cross sections for ^{71}Ga and ^{115}In as functions of E_ν. ^{71}Ga: for the g.s.→g.s (dash–double-dotted line); including all GT transitions up to the neutron separation threshold B_n in ^{71}Ge (curve 1); calculated using the strength function from Ref. [11] (dash-dotted line); taken from Ref. [21] (curve 2). ^{115}In: including all GT transitions up to E_x = 20 MeV (dashed line).

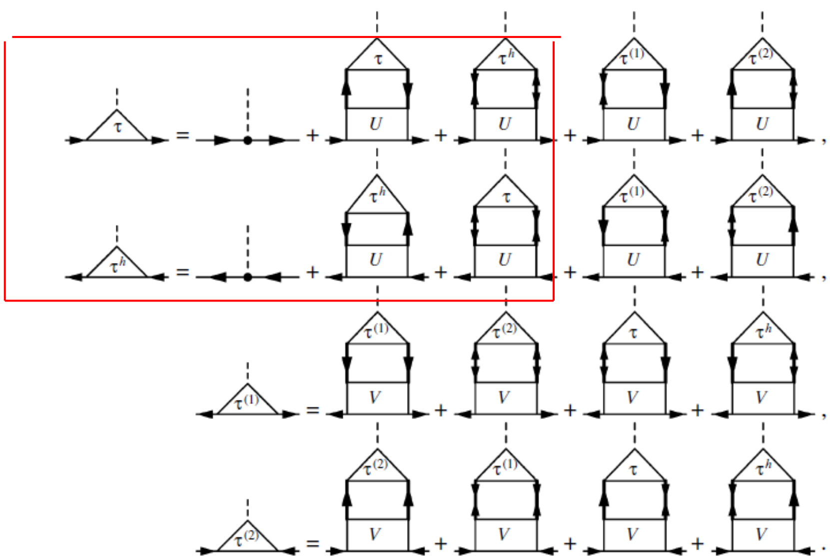


pp [± 0.6 %]
^{7}Be [± 7 %]
pep [± 1.2 %]
^{13}N [± 14 %]
^{15}O [± 14 %]
^{17}F [± 17 %]
^{8}B [± 14 %]
hep [± 30 %]
Flux [cm^{-2} s^{-1} MeV^{-1}]
Neutrino Energy [MeV]

Continuum pnQRPA. Full ph-basis, SO(8) symmetry

$$\overset{\wedge}{[}I - \begin{pmatrix} F^{\omega} & -F^{\xi} & F^{\omega\xi} & F^{\xi\omega} \\ -F^{\xi} & F^{\omega} & F^{\xi\omega} & F^{\omega\xi} \\ F^{\omega\xi} & F^{\xi\omega} & F^{\xi} & -F^{\omega} \\ F^{\xi\omega} & F^{\omega\xi} & -F^{\omega} & F^{\xi} \end{pmatrix} \begin{pmatrix} L(\omega) & M(\omega) & N^{1}(\omega) & N^{2}(\omega) \\ M(\omega) & L(-\omega) & N^{2}(-\omega) & N^{1}(-\omega) \\ N^{1}(\omega) & N^{2}(-\omega) & K(\omega) & -M(\omega) \\ N^{2}(\omega) & N^{1}(-\omega) & -M(\omega) & K(-\omega) \end{pmatrix}] \begin{pmatrix} V \\ V^{h} \\ d^{(1)} \\ d^{(2)} \end{pmatrix} = \begin{pmatrix} e_q V_0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$L(r,r';\varpi) = A(r,r';\varpi) + \sum \left| L_{pn} - A^{\sim\sim}_{pn} \right| \varphi_p \varphi_n \varphi_{n*} \varphi_{p*}$$



SO(8)

Self-consistent pnQRPA

SU(4)

Standard beta decay Tables

P. Moeller et al., Phys. Rev. (1996, 2003, 2012)