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Theory of holographic models for linear Regge trajectories and its applications

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Introduction

Disclaimer: only bottom-up holographic models!

- AdS/QCD models — a set of phenomenological approaches, inspired by the ideas of gauge/gravity duality and AdS/CFT-correspondence.
- Based on a conjecture: observables in strongly coupled gauge theories can be determined from fields that are weakly coupled through gravity in higher dimension AdS space.
- Most AdS/QCD models are based on a Soft Wall (SW) holographic model, which had many variations over the years.
- Today: relation between various SW-like models + applications.

5D Scalar Soft-Wall Model

$$S = \frac{1}{2} \int d^4x \, dz \sqrt{g} \, e^{cz^2} \left(\partial^M \Phi \partial_M \Phi - m_5^2 \Phi^2 \right)$$

The dilaton parameter c can be either positive or negative.

$$g \equiv | \det g_{MN} |, \quad g_{MN} dx^M dx^N = \frac{R^2}{z^2} (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2), \quad z > 0$$

The 5D mass of the scalar field:

$$m_5^2 R^2 = \Delta(\Delta - 4).$$

4D modes have a discrete spectrum:

$$m_n^2 = 2|c| \left(2n + 1 + \sqrt{4 + m_5^2 R^2} - \frac{c}{|c|} \right)$$

Tensor SW Model

The SW model can be generalized for the case of an arbitrary spin:

$$S = \frac{1}{2} \int d^4x\, dz \sqrt{g}\, e^{cz^2} \left(\partial^M \Phi^J \partial_M \Phi_J - m_5^2 \Phi^J \Phi_J \right)$$

J is a multi-index, $\Phi_J \equiv \Phi_{M_1 \ldots M_J}$

This action is so simple due to the condition:

$$\Phi_{z\ldots} = 0$$

The condition on 5D mass is also generalized:

$$m_5^2 R^2 = (\Delta - J)(\Delta + J - 4)$$

Tensor SW Model

Next, we use the similar procedure and equations of motion:

$$\frac{\delta S}{\delta \Phi_{\mu_1 \dots \mu_J}} = 0.$$

For twist-2 operators, $\Delta = J + 2$, the spectrum takes the form:

$$m_n^2 = 2|c| \left(2n + J + 1 + \frac{c}{|c|}(J - 1) \right).$$

Interestingly, for $c > 0$ it takes the string form:

$$m_n^2 = 4|c|(n + J),$$

and for $c < 0$ it doesn't depend on the spin J.

Maximally Extended Model

$$S = \frac{1}{2}\int d^5x\sqrt{g}e^{cz^2}\left[\partial^M\Phi^J\partial_M\Phi_J - \left(m_5^2 + a_1 z^2 + a_2 z^4\right)\Phi^J\Phi_J + \right.$$
$$\left. + \, b g^{M_1N_1}\dots g^{M_JN_J} g^{zz}\Phi_{M_1\dots M_J} z\partial_z \Phi_{N_1\dots N_J}\right].$$

All of these new terms result in the following effective 5D mass:

$$m_{\text{eff}}^2(z) = m_5^2 + z^2\left(a_1 + \frac{(J-1)(2c-b)}{R^2}\right) + z^4\left(a_2 + \frac{c^2 - bc}{R^2}\right)$$

Essentially, only two parameters!

Maximally Extended Model

Nevertheless, we still get the Regge form of the spectrum:

$$m_n^2 = 2|\tilde{c}| \left(2n + 1 + J + \frac{\tilde{b}}{2|\tilde{c}|} \right),$$

$$\tilde{b} \equiv a_1^2 R^2 + (2c - b)(J - 1), \quad \tilde{c} \equiv a_2 R^2 + c(c - b).$$

In addition:

- $O(z^4)$ term contributes to the slope (and can be absorbed into background).
- $O(z^2)$ term dictates the intercept. But can intercept be defined by a background?

SW Models with Generalized Background

Consider the case of the arbitrary spin:

$$S = \frac{1}{2}\int d^5x\, f^2(z)(\partial_M \Phi_{M_1\dots M_J})^2,$$

where $f(z)$ is the background function that we need to find, with UV asymptotics:

$$f(z) \underset{z\to 0}{\sim} z^{(2J-3)/2}.$$

We would like to have the Regge spectrum shifted by an intercept:

$$m_n^2 = 2|c|\left(2n+1+|J-2|+\frac{c}{|c|}(J-1)+2b\right).$$

This leads to an equation for a background function $f(z)$:

$$\frac{\partial_z^2 f(z)}{f(z)} = c^2z^2 + 2c(J-1) + \frac{(J-2)^2 - 1/4}{z^2} + 4b|c|.$$

SW Models with Generalized Background

Two independent solutions can be interpreted as the generalizations of the SW model with either positive or negative dilaton.

For $J \leq 1$:

$$S = \frac{\Gamma(2-J+b)^2}{2\Gamma(2-J)^2} \int d^5x \sqrt{g} e^{-|c|z^2} U^2 \left(b, J-1, |c|z^2\right) \partial^M \Phi^J \partial_M \Phi_J$$

For $J > 1$:

$$S = \frac{1}{2} \int d^5x \sqrt{g} e^{|c|z^2} M^2 \left(-b, J-1, -|c|z^2\right) \partial^M \Phi^J \partial_M \Phi_J$$

NB: Introduction of a constant mass term doesn't affect the result!

Applications: Two-point Vector Correlator

The correlator is defined as (in Eucledian space, $Q^2 = -q^2$):

$$\int d^4x\, e^{iqx} \langle J_\mu(x) J_\nu(0) \rangle = (q_\mu q_\nu - q^2 g_{\mu\nu}) \Pi_V(Q^2)$$

Following the standard recipes of AdS/QCD:

$$\Pi_V(Q^2) \sim - \left. \frac{\partial_z V(Q^2, z)}{Q^2 z} \right|_{z \to 0}$$

$V(Q^2, z)$ is the so-called "bulk-to-boundary propagator" with the boundary condition:

$$V(Q^2, 0) = 1$$

Applications: Two-point Vector Correlator

$$2\Pi_V(Q^2) = \frac{1+\frac{c}{|c|}}{2\tilde{Q}^2} - \psi(1+\tilde{Q}^2) + \text{Const}, \; \tilde{Q}^2 \equiv \frac{Q^2}{4|c|}$$

The poles of the digamma function yield the mass spectrum:

$$-\tilde{Q}^2 = \frac{m_n^2}{4|c|} = n+1$$

In case of a positive dilaton, an unphysical massless pole appears in the vector correlator!

Applications: Two-point Vector Correlator

We can redefine the model with $O(c^2 z^4)$ (and $O(bz^2)$) contributions to the mass term instead of the dilaton background ("No-Wall" model).

The massless pole still remains and emerges even for the negative dilaton!

These issues can be solved by imposing the condition:

$$Q^2 \Pi_V(Q^2)|_{Q^2=0} = 0,$$

which also allows to obtain predictions for the intercept b.

See also: S. S. Afonin, T. D. Solomko, J. Phys. G **48** (2021) no.6, 065003, [arXiv:2006.14439].

Applications: Pion Form Factor

Space-like electromagnetic form factors are given by:

$$F(Q^2) = e \int_0^\infty \frac{dz}{z^3} e^{cz^2} V(Q^2, z) \Phi_0(z)^2$$

where $\Phi_0(z)$ is the holographic wave function of the ground scalar state.

If the mass of the ground state is zero, $\Phi_0(z)$ can be interpreted as a holographic wave function of pion. In case of the negative dilaton the FF can be computed exactly:

$$F_\pi(Q^2) = \frac{1}{1 + \frac{Q^2}{4|c|}}$$

Applications: Pion Form Factor

For other cases, form factor can be written as:

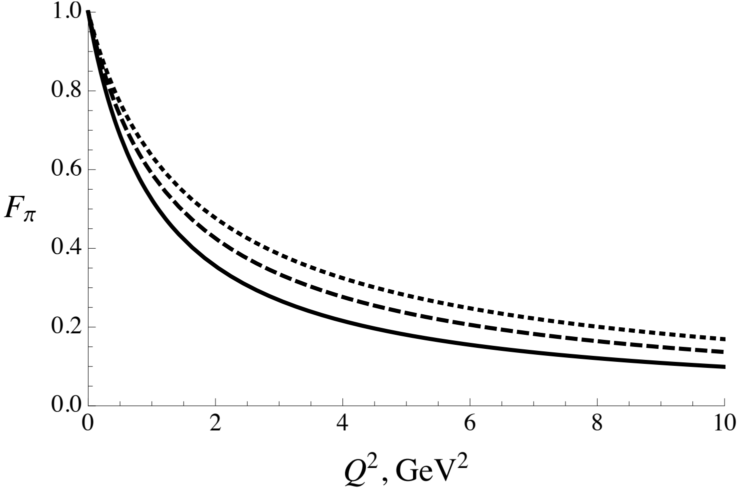
$$F_\pi(Q^2) = e(k)\Gamma(1+\tilde{Q}^2) \int_0^\infty dy\, e^{-(1+k)y} U(\tilde{Q}^2, 0, y), \quad y \equiv |c|z^2$$

For SW$^+$ ($k = 1$) and NW ($k = 1/2$) models:

$$F_\pi(Q^2) = 1 + \frac{1}{k} - \frac{\tilde{Q}^2}{k}\Phi\left(\frac{k}{1+k}, 1, \tilde{Q}^2\right)$$

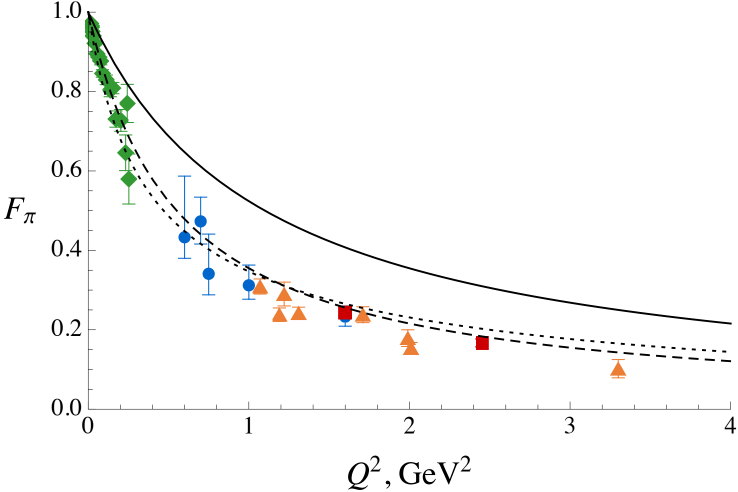
$$F_\pi(Q^2) = e(k, b)\left[1 + \frac{1}{k} - \frac{\tilde{Q}^2 + b}{k}\Phi\left(\frac{k}{1+k}, 1, \tilde{Q}^2 + b\right)\right]$$

Applications: Pion Form Factor



Solid — Soft Wall, negative dilaton; Dotted — Soft Wall, positive dilaton ($k = 1$); Dashed — No Wall ($k = 1/2$).

Applications: Pion Form Factor



Solid — $b = 0$; Dashed — $b = -1/2$; Dotted — Soft Wall, positive dilaton, $b \approx -0.73$.

Applications: What else?

1. **Confining Behavior:** Study of a confining behavior, using holographic Wilson criterion:

$$\partial_z g_{00}|_{z=z_0} = 0, \quad g_{00}|_{z=z_0} \neq 0.$$

   This criterion can be satisified for spin-one and spin-two field models.

2. **Chiral Symmetry Breaking:** Effects of CSB in meson spectra; this phenomenon seems to affect the entire leading trajectories.

See more details in the paper!

Summary

Today we have discussed:

- Various forms of bottom-up holographic models, and relations between them.
- Vector two-point correlators and the problem of the unphysical massless pole (and how to solve it!).
- Different results for the elecromagnetic pion form factor, produced by various forms of SW models.
- Confining behavior and CSB within bottom-up holographic models.

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