Correlation of cumulative particle production with strange and heavy-flavor particle yields in the string fusion model

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Nuclear physics and elementary particle physics.
Nuclear physics technologies”

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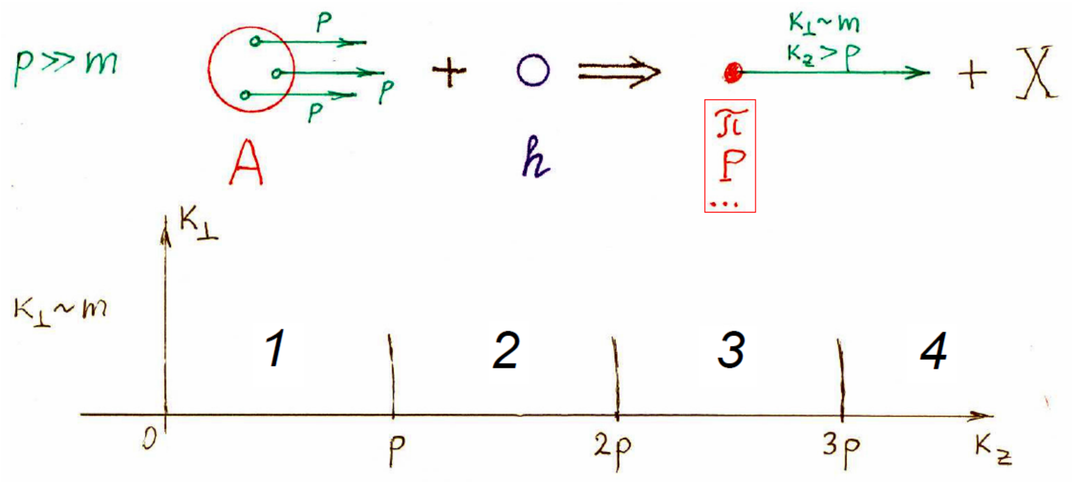
Kinematics of cumulative production

Fragmentation of projectile nucleus



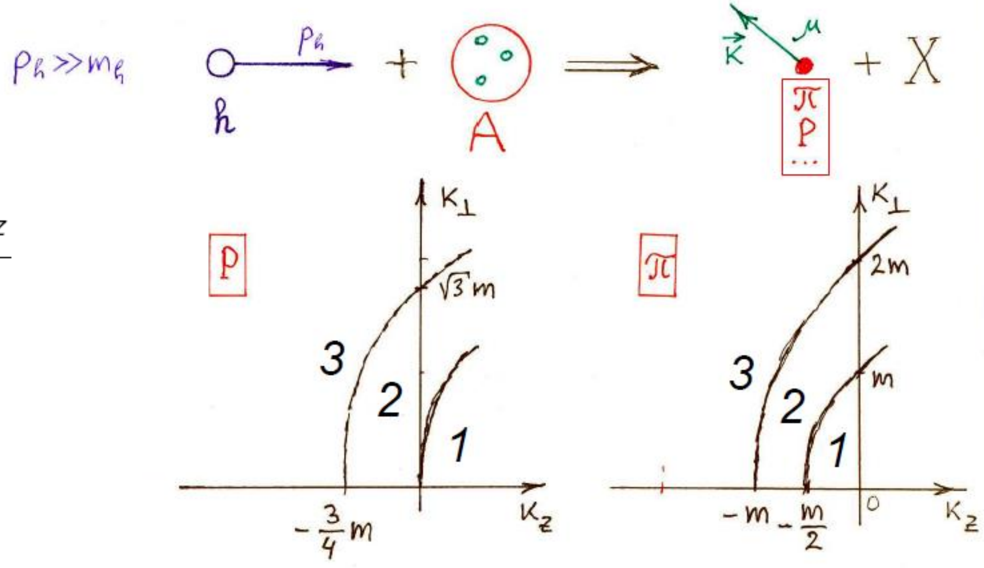
$$x \equiv \frac{k_+}{p_+} = \frac{k_0 + k_z}{p_0 + p_z} \approx \frac{k_z}{p}$$

$$k_z,\ p \gg m,\ \ m - \text{nucleon mass}$$

$$x = 1, 2, 3, \ldots, A$$

The borders increase with *p*

Fragmentation of target nucleus

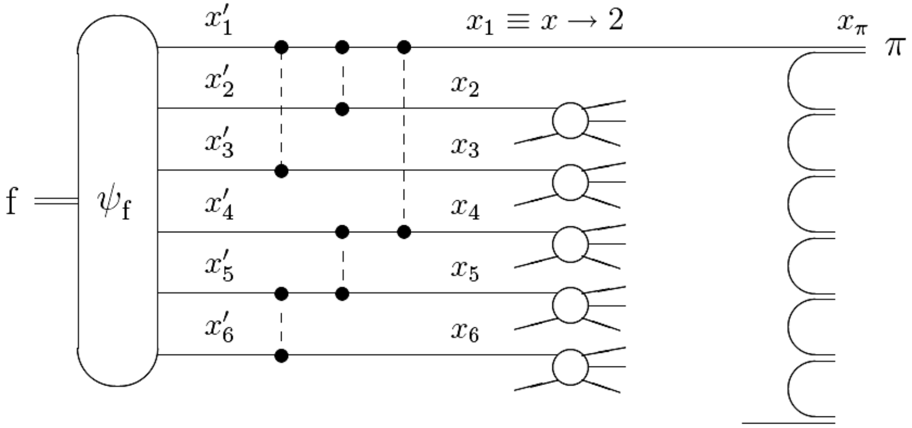


$$x \equiv \frac{k_-}{p_-} = \frac{\tilde{k}_0 - \tilde{k}_z}{m} = \frac{\sqrt{\tilde{k}_z^2 + k_\perp^2 + \mu^2} - \tilde{k}_z}{m}$$

$$\tilde{k}_z = -\frac{xm}{2} + \frac{k_\perp^2 + \mu^2}{2xm}$$

The borders are fixed at *p>>m*

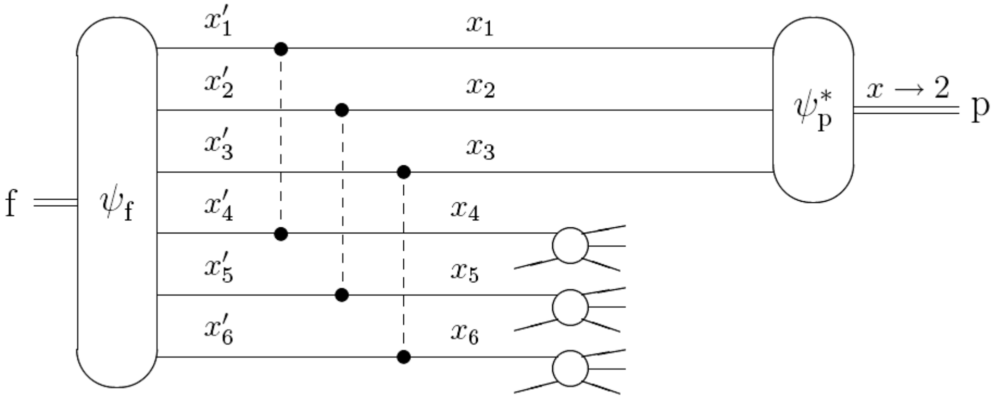
Coherent Quark Coalescence and Production of Cumulative Protons



- the cumulative pion production

by hadronization of one fast quark

M.A. Braun, V.V. Vechernin,
*Nucl.Phys.**B 427**, 614 (1994);*
*Phys.Atom.Nucl. **60**, 432 (1997);*
***63**, 1831 (2000)*



- the cumulative proton production

by **coherent** quark coalescence mechanism:

M.A. Braun, V.V. Vechernin,
*Nucl.Phys.**B 92**, 156 (2001);*
*Theor.Math.Phys **139**, 766 (2004);*
V.Vechernin,
AIP Conf.Proc.1701 (2016) 060020.

The last **recalls** the few nucleon **short-range correlations** in a nucleus
L.L. Frankfurt, M.I. Strikmann, Phys. Rep. 76, 215 (1981); ibid 160, 235 (1988).
But instead of using the relativistic generalization of non-relativistic NN wave function
the microscopic analysis of the flucton fragmentation process near cumulative thresholds on the base of the intrinsic diagrams of QCD in light-cone gauge
*Brodsky S.J., Hoyer P., Mueller A., Tang W.-K., Nucl. Phys. **B369** (1992) 519.* **(x–>1)**
was developed and applied.

W_j : $j=1,2,3.$

$n=n_1+n_2+n_3$

$p_1=n_1-1$

$p_2=n_2-1$

$p_3=n_3-1$

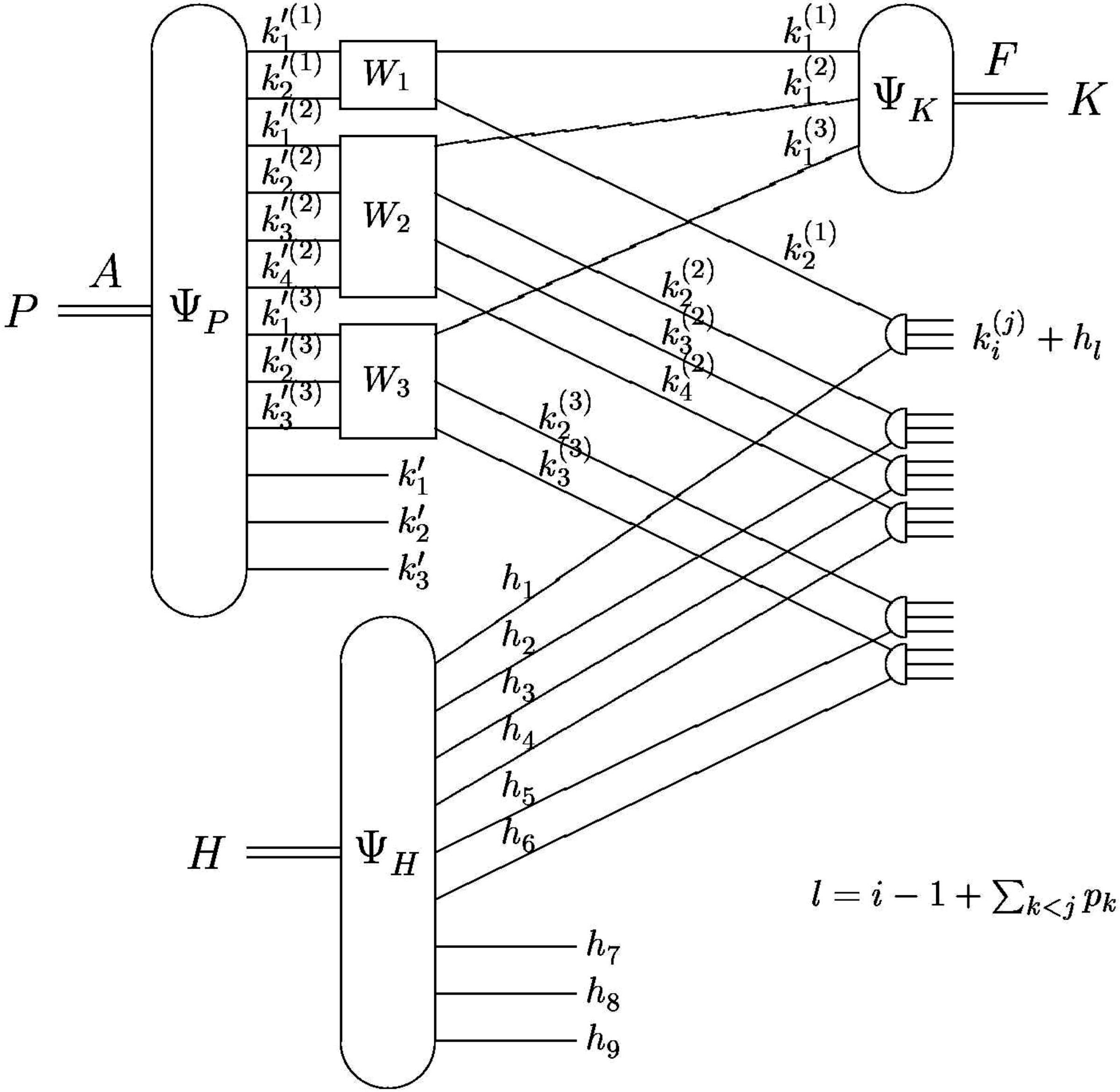
$p=p_1+p_2+p_3=1+3+2=6$

$n=p+3=9$

The analysis of the diagrams shows that all donor quarks:

p_1, p_2, p_3

have large virtuality and must to interact with the projectile *H*.



M.A. Braun, V.V. Vechernin, Theor.Math.Phys ***139****, 766 (2004);*

$$\sigma_{pion}(x, k_\perp; p) = C(p)\ (x_{frag} - x)^{2p-1}\ f_p\left(\frac{k_\perp}{m}\right)$$

$$x < x_{frag}(p) = 1/3 + p/3$$

Quark counting rules near the cumulative thresholds

p=n-1

M.A. Braun, V.V. V , Phys.Atom.Nucl. ***63****, 1831 (2000)*

$$\sigma_{prot}(x, k_\perp; p_1, p_2, p_3) = C(p_1, p_2, p_3)\,(x_{coal} - x)^{2p-1}\, f_{p_1}\left(\frac{k_\perp}{3m}\right)\ f_{p_2}\left(\frac{k_\perp}{3m}\right)\ f_{p_3}\left(\frac{k_\perp}{3m}\right)$$

$$x < x_{coal}(p) = 1 + p/3\ , \qquad p = p_1 + p_2 + p_3$$

p=n-3

M.A. Braun, V.V. V , Theor.Math.Phys. ***139****, 766 (2004)*

$$f_p(t) = 2\pi \int_0^\infty dz\, z J_0(tz)[zK_1(z)]^p$$

$J_0(z)$ - the Bessel function, $K_1(z)$ - the modified Bessel function.

$$(2\pi)^{-2} \int f_p(|\mathbf{b}|)\ d^2\mathbf{b} = (2\pi)^{-1} \int_0^\infty f_p(t)\ t\,dt = 1$$

$$e^{-b_s x} = 10^2,$$

$$b_s \approx 7, \quad x = -2/3$$

Note that for *p=1* it can be simplified to $f_1(t) = 4\pi/(t^2+1)^2$

$$\varphi_{pion}(k_\perp, p) \equiv \sigma_{pion}(x, k_\perp; p)/\sigma_{pion}(x, 0; p) = f_p\left(\frac{k_\perp}{m}\right)/f_p(0)$$

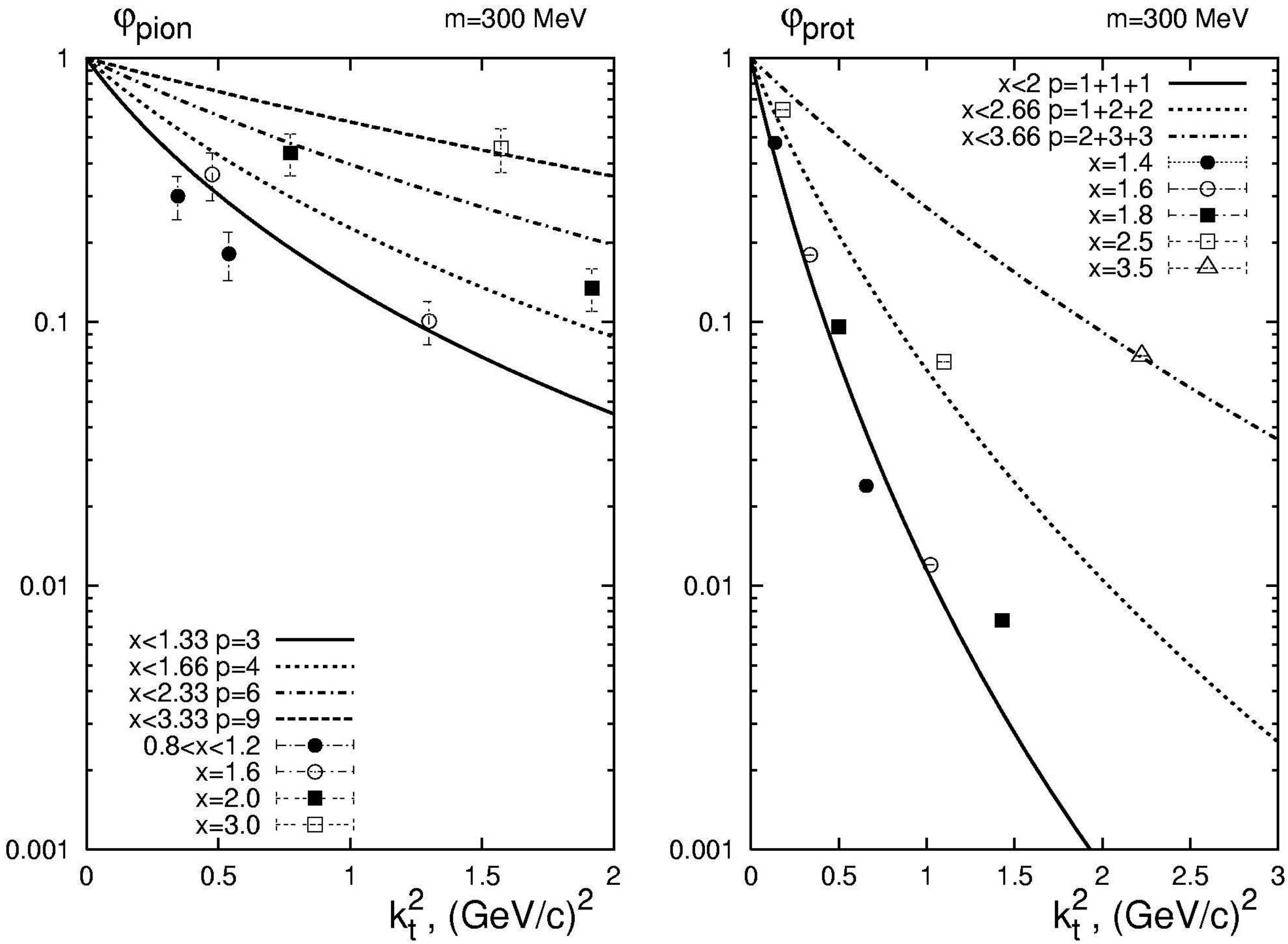
$$\varphi = \sigma(x, \theta)\,/\,\sigma(x, \theta{=}180^{o})$$

$$\varphi_{prot}(k_\perp, p) \equiv \sigma_{prot}(x, k_\perp; p)/\sigma_{prot}(x, 0; p)$$

$$\varphi_{prot}(k_\perp, p) = \frac{\sum_{p_1,p_2,p_3}\ \delta_{p\ \, p_1+p_2+p_3}\ C(p_1, p_2, p_3)\ f_{p_1}\left(\frac{k_\perp}{3m}\right)\ f_{p_2}\left(\frac{k_\perp}{3m}\right)\ f_{p_3}\left(\frac{k_\perp}{3m}\right)}{\sum_{p_1,p_2,p_3}\ \delta_{p\ \, p_1+p_2+p_3}\ C(p_1, p_2, p_3)\ f_{p_1}(0)\ f_{p_2}(0)\ f_{p_3}(0)}$$

$$\varphi_{prot}(k_\perp, p_1, p_3, p_3) \equiv \frac{\sigma_{prot}(x, k_\perp; p_1, p_2, p_3)}{\sigma_{prot}(x, 0; p_1, p_2, p_3)} = \frac{f_{p_1}\left(\frac{k_\perp}{3m}\right)}{f_{p_1}(0)}\,\frac{f_{p_2}\left(\frac{k_\perp}{3m}\right)}{f_{p_2}(0)}\,\frac{f_{p_3}\left(\frac{k_\perp}{3m}\right)}{f_{p_3}(0)}$$

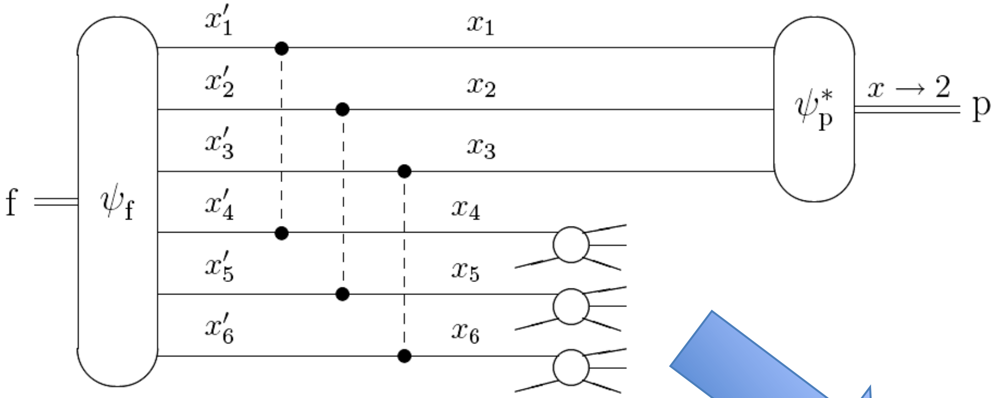
No free parameters (!) only *m* – the constituent quark mass: *m = 300 MeV.*



V.Vechernin,
AIP Conference Proceedings
1701 (2016) 060020.

S.V. Boyarinov et al., Sov.J.Nucl.Phys. ***46****, 871 (1987)*
S.V. Boyarinov et al., Physics of Atomic Nuclei ***57****, 1379 (1994)*
S.V. Boyarinov et al., Sov.J.Nucl.Phys. ***55****, 917 (1992)*

Study of correlations between cumulative particles and strangeness and/or charm forward production



- the cumulative proton production in backward hemisphere

Flucton fragmentation

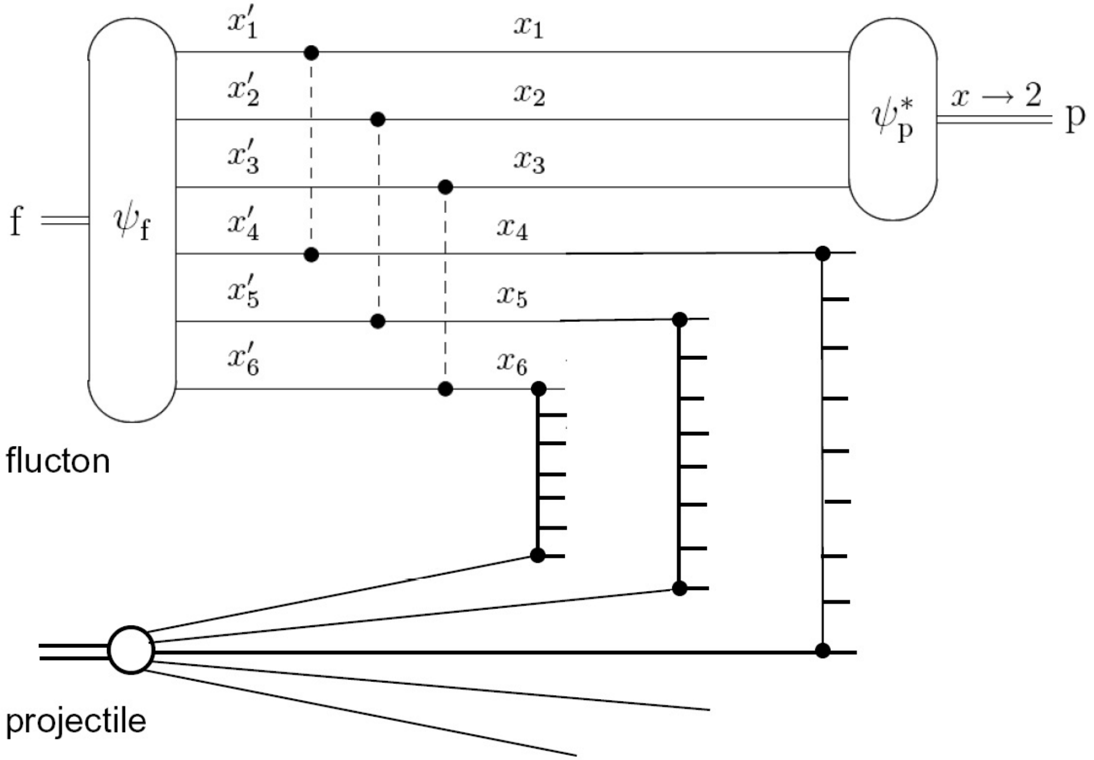
- enhanced strangeness and charm production in forward hemisphere

The analysis of the diagrams shows that all donor quarks have large virtuality and must to interact with the projectile:

M. A. Braun, V. V. V , Nucl. Phys. B 427, 614 (1994);
Phys.Atom.Nucl. 60, 432 (1997); Theor.Math.Phys. 139, 766 (2004).

So one can expect an enhanced yield of strange and charm particles in the process of their hadronization.

Flucton cumulative fragmentation



Formation of the donor strings

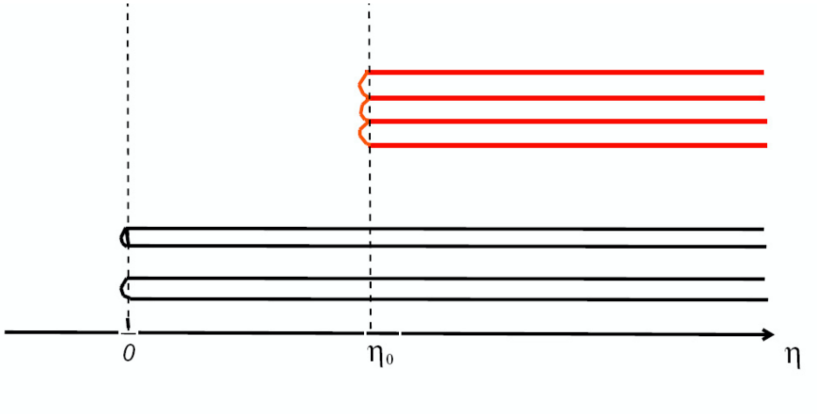
due to interaction of the flucton recoil with the projectile

(all donors have to interact with the projectile)

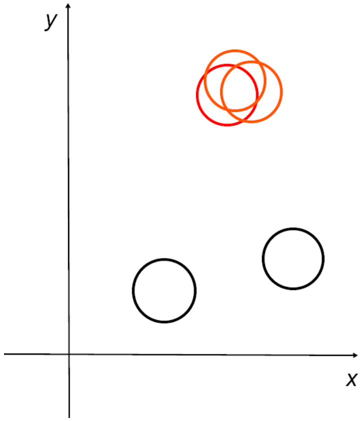
[M.A. Braun, V.V. V , Phys.Atom.Nucl. 60, 432 (1997))]

Flucton cumulative fragmentation

1) **Cumulative fragmentation of flucton needs *shrunk* flucton configuration in transverse plane *[M.A. Braun, V.V. V , Theor.Math.Phys. 139, 766 (2004)]***
   => Overlapping of donor strings
2) **Cumulative particle momentum needs to be compensated by longitudinal momenta of the donors => Donor strings are shifted to positive rapidities**



Rapidity distribution of the donor strings
(in laboratory frame)



- the donor strings

- general valence
quark strings

Transverse plane distribution
of the donor strings

- Overlapping of donor strings => *Enhanced production of heavy flavors*

e.g. enhanced strange production:

N. Armesto, M. Braun, E. Ferreiro, and C. Pajares, Phys.Lett.B344 (1995) 301,
"Strangeness enhancement and string fusion in nucleus-nucleus collisions".
E.G. Ferreiro, C. Pajares, J.Phys.G23 (1997) 1961,
"Strangeness enhancement in the String Fusion Model Code" .
String fusion influence on strange production:
- fused strings decay into more strange particles
- the overall dumping of multiplicities as a consequence of the string fusion

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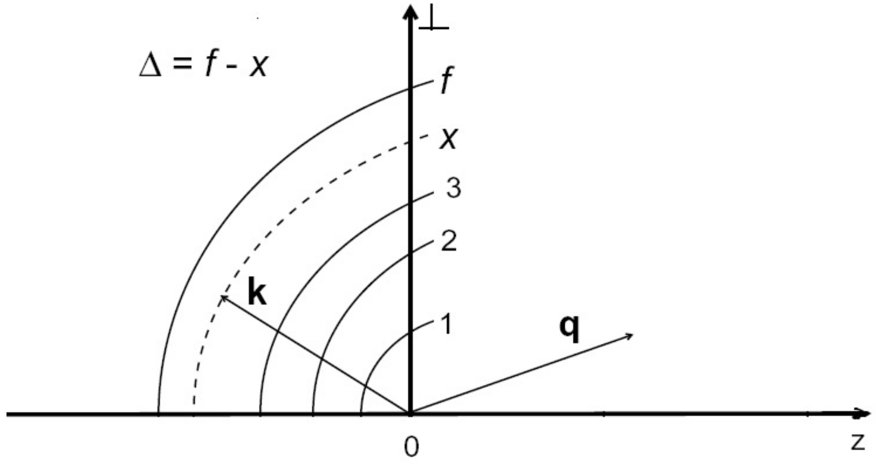
ALICE collaboration, Nature Physics 13 (2017) 535,
"Enhanced production of multi-strange hadrons in high-multiplicity proton–proton collisions":
-- The model which describes the data best, DIPSY, is a model where interaction between gluonic strings is allowed to form "color ropes" which are expected to produce more strange particles and baryons.

C. Bierlich, G. Gustafson, L. Lonnblad, A. Tarasov, JHEP 1503 (2015) 148,
"Effects of overlapping strings in pp collisions".
C. Bierlich and J. R. Christiansen,, Phys. Rev. D92 (2015) 094010,
"Effects of colour reconnection on hadron flavour observables"

Kinematics of simultaneous cumulative and heavy flavor particles production



In the rest frame of fragmentating nucleus at large initial energy:

$$-x_F \approx x \equiv \frac{k_-}{p_-} = \frac{\sqrt{k_z^2 + \mu_\perp^2} - k_z}{m_N} > 0$$

$$x_q \equiv \frac{q_-}{p_-} = \frac{\sqrt{q_z^2 + M_\perp^2} - q_z}{m_N} > 0$$

The kinematical border is given by the condition: $x + x_q = f$

For the given deviation *Δ* of the *x* from its maximal value for flucton with *f* nucleons, we have*:*

$$q_-^{max} = m_N\,(f - x) = m_N\,\Delta$$

Then by $q_z = \frac{1}{2}\left(\frac{M_\perp^2}{q_-} - q_-\right)$ we find $q_z^{min} = \frac{1}{2}\left(\frac{M_\perp^2}{m_N\,\Delta} - m_N\,\Delta\right)$

- min q_z depends only on *Δ* (not on *f* and **k** components)
- no dependence on initial energy (at large energies)
- dependence only on particle masses, not on their quantum numbers
  (that is not true at small initial energies)

That gives for the minimal rapidity of the heavy flavor particle:

$$y_{min} = -\ln \frac{m_N \Delta}{M_\perp}$$

where the rapidity is defined in a standard way: $y \equiv \frac{1}{2} \ln \frac{q_+}{q_-} = \ln \frac{M_\perp}{q_-}$

	M = 0.5 GeV		M = 1.87 GeV	
Δ	min q_z , GeV	y_{min}	min q_z , GeV	y_{min}
0.9	-0.275	-0.526	1.644	0.793
0.8	-0.210	-0.408	1.949	0.911
0.7	-0.139	-0.275	2.328	1.044
0.6	-0.060	-0.120	2.818	1.199
0.5	0.031	0.062	3.485	1.381
0.4	0.144	0.285	4.462	1.604
0.3	0.302	0.573	6.059	1.892
0.2	0.571	0.978	9.206	2.297
0.1	1.283	1.671	18.554	2.990

Dynamics of string decay

J. Schwinger, Phys. Rev. 82, 664 (1951).
A.I. Nikshov, Nucl. Phys. B21, 346 (1970).
T.D. Cohen and D.A. McGady, Phys.Rev.D 78, 036008 (2008).
Only *n=1* contribution.

$$W = \frac{\rho^2}{4\pi^3} \sum_{q} \sum_{n=1}^{\infty} \frac{1}{n^2} \exp(-n\pi m_q^2/\rho) \equiv \sum_{q} W_q \qquad m_q\ (q = u, d, s, c, \ldots)$$

$$W = \int d^2k_T\, \widetilde{W}(k_T^2) \qquad \widetilde{W} = \frac{\rho}{4\pi^3} \sum_{q} \sum_{n=1}^{\infty} \frac{1}{n} \exp[-n\pi(m_q^2 + k_T^2)/\rho]$$

E.G. Gurvich, Phys.Lett. 87B (1979) 386.
A. Casher, H. Neunberg and S. Nussinov, Phys. Rev. D20 (1979) 179.
M. Gyulassy and A. Iwazaki, Phys. Lett. B165 (1985) 157.

A. Bialas, Phys.Lett.B 466 (1999) 301. $$\frac{dn_\kappa}{d^2p_\perp} \sim e^{-\pi m_\perp^2/\kappa^2} \quad \Rightarrow \quad \frac{dn}{d^2p_\perp} \sim e^{-m_\perp/T}$$

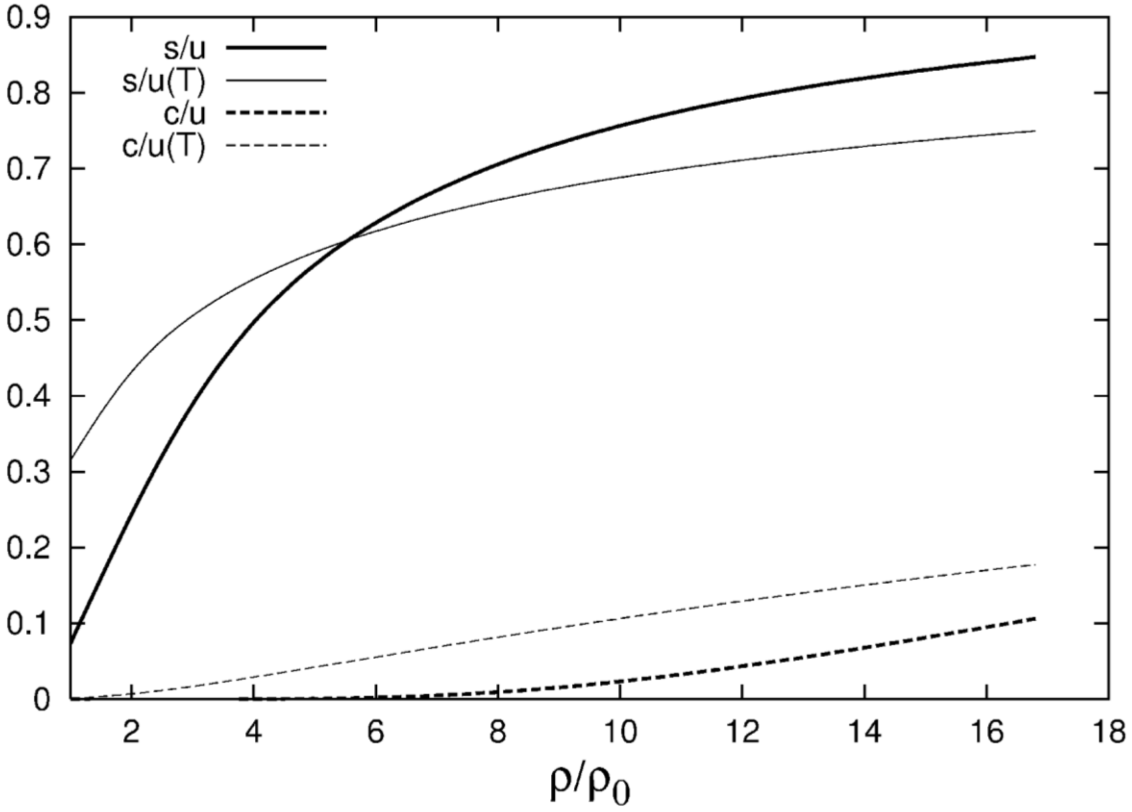
$$\langle\kappa^2\rangle = \rho = \frac{1}{2\pi\alpha'}\,, \quad \alpha' = 0.9 GeV^{-2}\,, \quad \rho = 0.18 GeV^2$$

(From the parameters of the potential connecting heavy quarks in nonrelativistic models one obtains the close value $\rho = 0.19\ GeV^2$.)

$$T = \sqrt{\frac{\langle\kappa^2\rangle}{2\pi}} \approx 170 MeV$$

Increase of the relative strangeness and charm production with string fusion

$$m_u = m_d = 0.3\,GeV\ , \quad m_s = 0.5\,GeV\ , \quad m_c = 1.5\,GeV$$

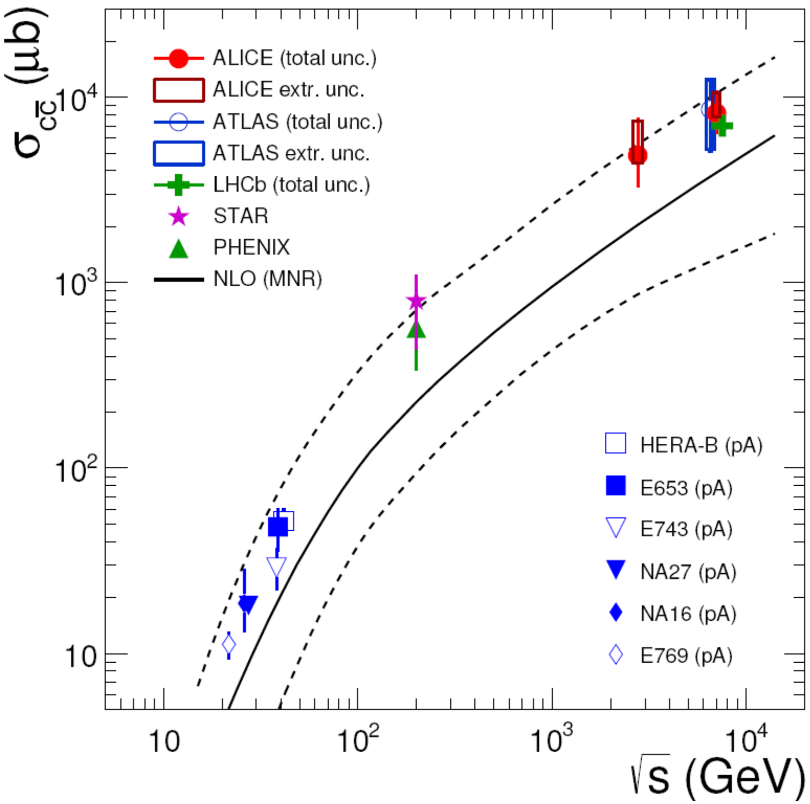


$$\langle K^-\rangle/\langle\pi^-\rangle/2 \quad = 0.24\ (\sqrt{s} = 0.9\ TeV) \quad = 0.2\ (\sqrt{s} = 0.2\ TeV)$$

ALICE Collaboration, Eur. Phys. J. C71,1594(2011) “Strange particle production in proton-proton collisions at 0.9 TeV with ALICE at the LHC”

*STAR Collaboration, Phys. Rev. C**75**, (2007) 064901.*

The non perturbative and pQCD contributions to charm production



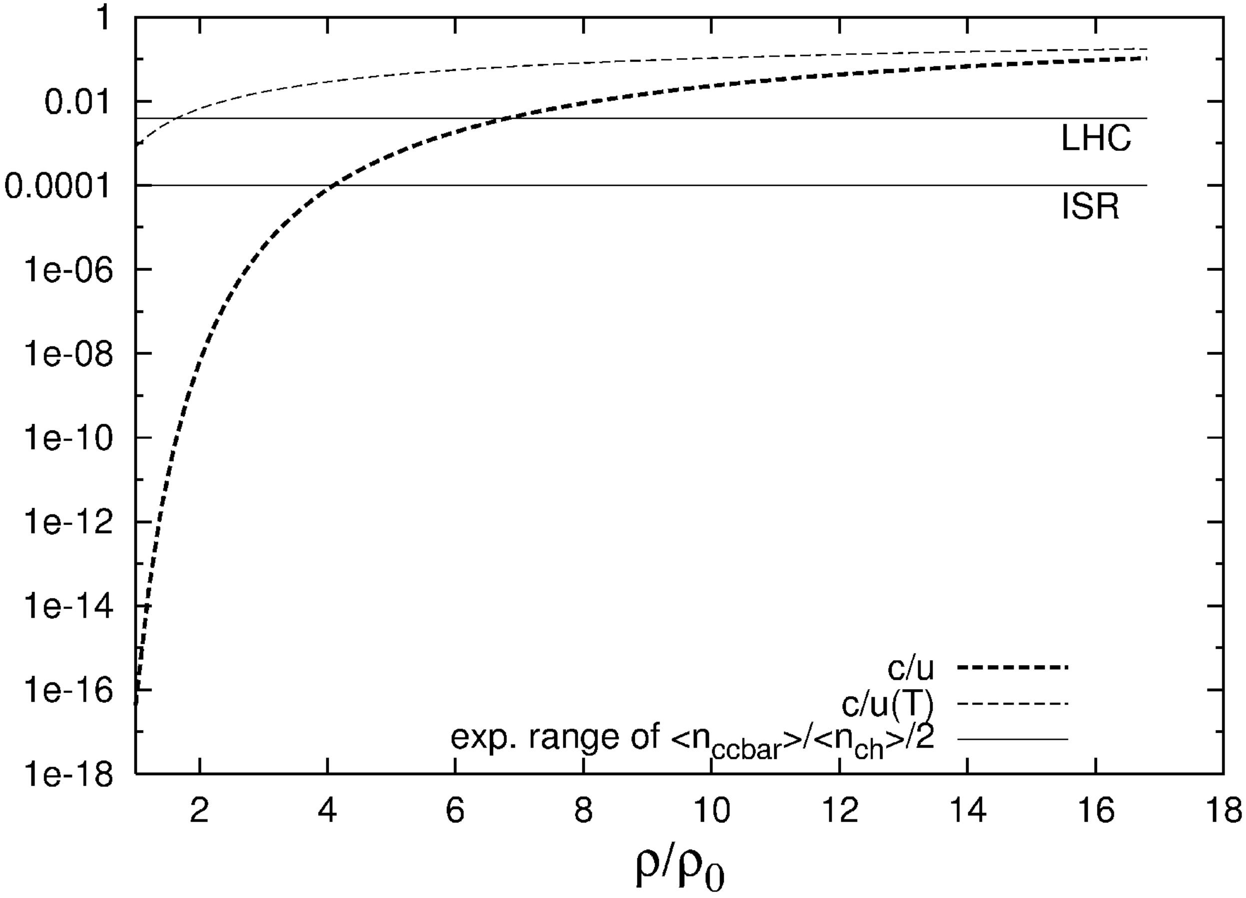
ALICE Collaboration, Phys. Rev. C 94, 054908 (2016)
"D-meson production in p-Pb collisions at 5.02 TeV and in pp collisions at 7 TeV".

C. Lourenco and H. K. Wohri,
Phys. Rept. 433 (2006) 127–180.
"Heavy flavour hadro-production from fixed-target to collider energies,"

M.L.Mangano, P.Nason,G. Ridolfi, Nucl. Phys. B373 (1992) 295 "Heavy quark correlations in hadron collisions at next-to-leading order"

Figure 10: Total inclusive charm production cross section in nucleon–nucleon collisions as a function of $\sqrt{s}$ [51, 68, 73, 80–82]. Data are from pA collisions for $\sqrt{s} < 100$ GeV and from pp collisions for $\sqrt{s} > 100$ GeV. Data from pA collisions were scaled by $1/A$. Results from NLO pQCD calculations (MNR [76]) and their uncertainties are shown as solid and dashed lines.

So in principle the experimental data lives room for the non perturbative contribution to charm production, e.g. due to the string fusion effects.



Increase of the relative charm production with string fusion

The non perturbative contribution, we are interested in (due to the additional string fusion effects in flucton recoil), can be more noticeable at small initial energies in fix-target pA experiments as in discussed future experiments with upgraded NA61 at SPS.

We also expect that this contribution will manifest itself more clear in p collisions with light nuclei (pD, pHe, pBe), due to minimization of other shadowing effects in this case, e.g. the absence of other string fusion effects.

The observables

We define the two class of events with and without the particle (proton or pion) with $x>x_0$ in cumulative region and introduce the ratio:

$$\gamma = \frac{\sigma_{h.f.}^{with\ cum.part.}(y > y_{min}(\Delta))}{\sigma_{h.f.}^{without\ cum.part.}(y > y_{min}(\Delta))}$$

In described approach we expect that $\gamma > 1$

The restriction to the rapidity region $y > y_{min}(\Delta)$
is necessary to suppress the increase of the phase volume
of the heavy flavor particle production in the case without cumulative particle.

$y_{min}(\Delta)$ was calculated above for the case of large initial energies,

$\Delta = [x_0] + 1 - x_0$

The importance of the registration of the particles formed from fragmentation of the flucton residue for the confirmation of the flucton mechanism of the cumulative particle production.
The need to use the ITS vertex detector to suppress of the contribution from the event pile-ups when registering the particles formed from fragmentation of the flucton residue.

V.I. Zherebchevsky, V.P. Kondratiev, V.V. Vechernin, S.N. Igolkin, NIM A 985 (2021) 164668.

Conclusion

- So in this approach based on the combination of two complementary models (flucton cumulative fragmentation + string fusion)
  we can expect the positive correlation between production of particle in the backward cumulative region and relative yield of strange (heavy) flavor in forward direction.
- The non perturbative contribution to the production of strange particles,
  due to the additional string fusion effects in flucton recoil,
  can be more noticeable at small initial energies in fix-target pA experiments
  as in discussed future experiments with upgraded NA61 at SPS.
- We also expect that this contribution will manifest itself more clear
  in p collisions with light nuclei (pD, pHe, pBe),
  due to minimization of other shadowing effects in this case,
  e.g. the absence of other string fusion effects.
- The vertex detector with high spatial resolution is needed to remove of all particle tracks coming from the other collision vertices which distort the particle spectrum from the fragmentation of the flucton residue.

The work was supported by the RFBR grant 18-02-40075.

Backup slides

Cumulative Particle Production

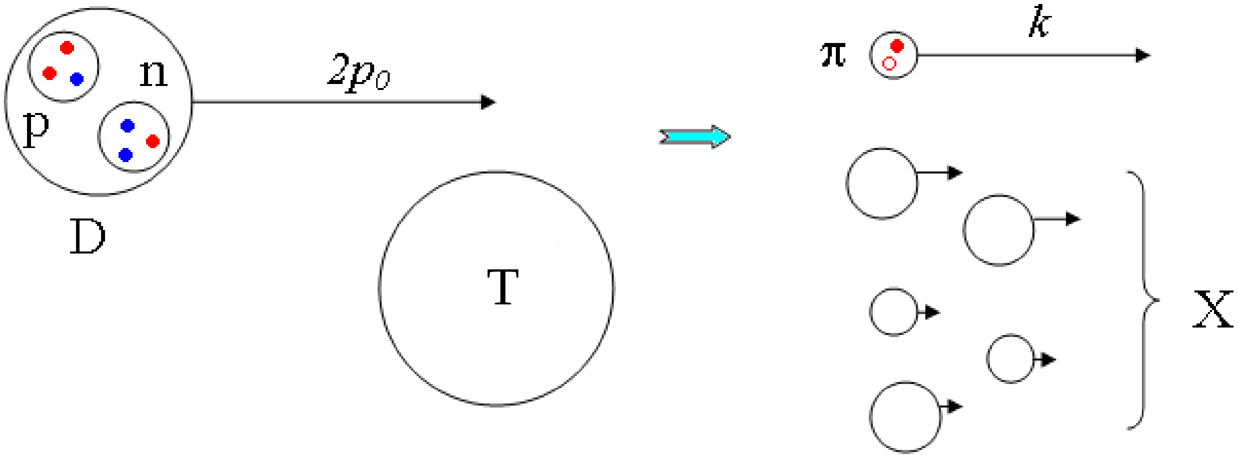
Production of particles from nuclei in a region, kinematically forbidden for reactions with free nucleons.

Cumulative Pion Production

1970 - Nuclotron@Dubna – beams of relativistic deutrons (p_0=5 *GeV/c/nucleon*)
Stavinskiy V.S. =>
Fragmentation of projectile deuterons, D, on some target, T.
*Baldin A.M. et al., Yad.Fiz.**18** (1973) 79*

$$D + T \Rightarrow \pi + X$$

$p_0 >> m_N$: $p_0 < k < 2p_0$ - cumulative pions



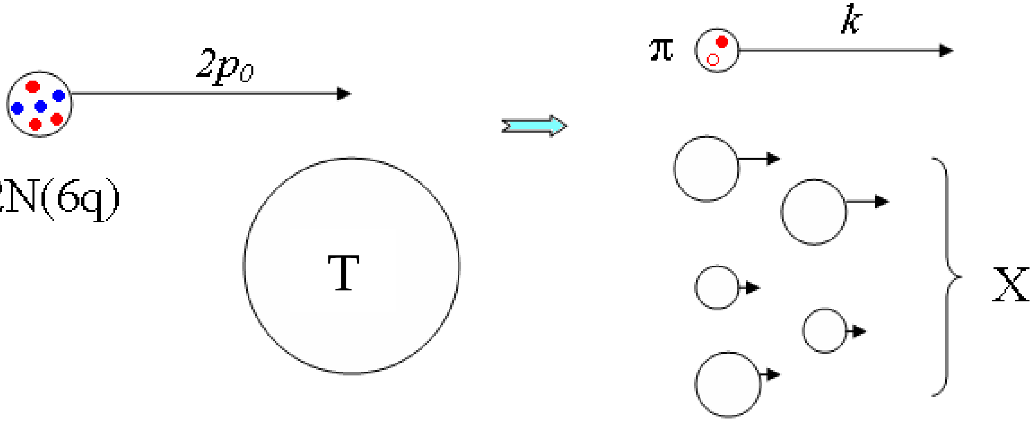
Flucton – intrinsic droplet of dense cold nuclear matter in a nucleus

*Blokhintsev D.I., JETP **33** (1957) 1295*

(2N flucton – 6 quark state)

~5% 2N(6q) – flucton admixture in D

Common 6q-bag



Fragmentation of projectile nucleus <=> Fragmentation of target nucleus

(the same phenomenon in different frames of reference)

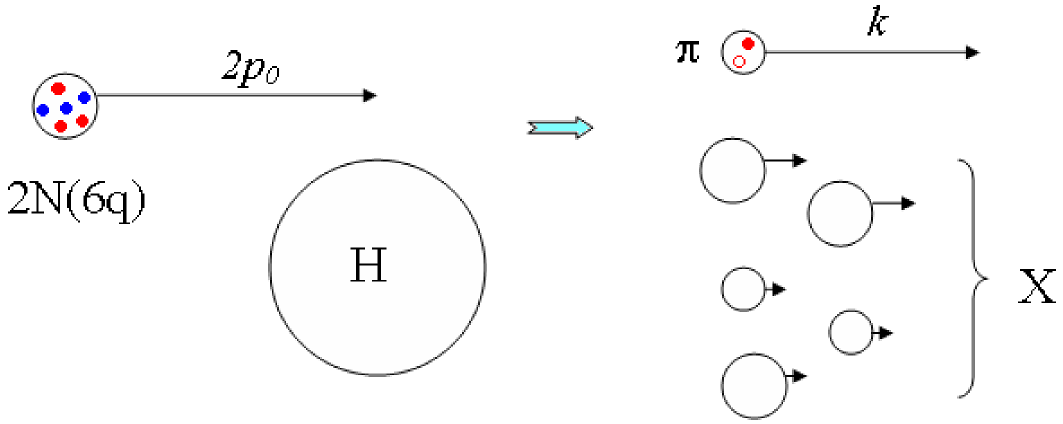
Cumulative fragmentation of target nucleus:

The 1st experimental observations of the backward particle production in p+A collisions on a fixed target nucleus:

G.A. Leksin et al., ZhETF 32, 445 (1957)
L.S. Azhgirej et al., ZhETF 33, 1185 (1957)
Yu.D. Bayukov et al., Izv. AN SSSR 30, 521 (1966)

The Reserford-like experiments indicating the presence of droplets of dense nuclear matter in a target nucleus (fluctons).

Limiting fragmentation of light nuclei. Quark counting rules.



$1 < x < 2$ - the cumulative region ($1 < x < f$ - *for the fN flucton*)

Theoretical description near upper threshold: for 2N(6q) flucton $k \to 2p_0$, $x = k/p_0 \to 2$ (Limiting fragmentation of a nucleus)

Quark counting rules: $I \sim \Delta^{2p-1}$

Δ – the deviation of x from its maximal value f, $\Delta = f - x$

p – the number of "donors", stopped quarks, $p = n - 1$

n – the number of constituents.

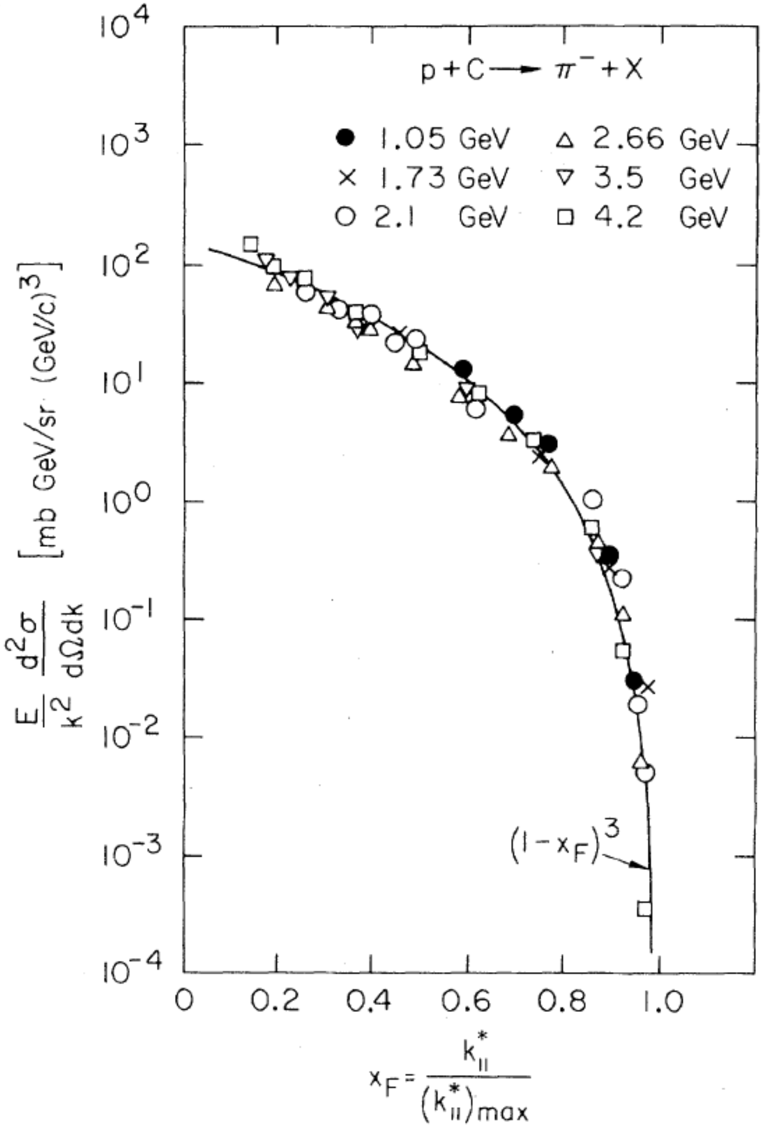
For 2N(6q) flucton $f = 2$, $n = 6$, $p = 5$, *then* $I \sim (2-x)^{2*5-1} = (2-x)^9 = \Delta^9$

Brodsky S., Farrar G. Phys.Rev.Lett. 31 (1973) 1153

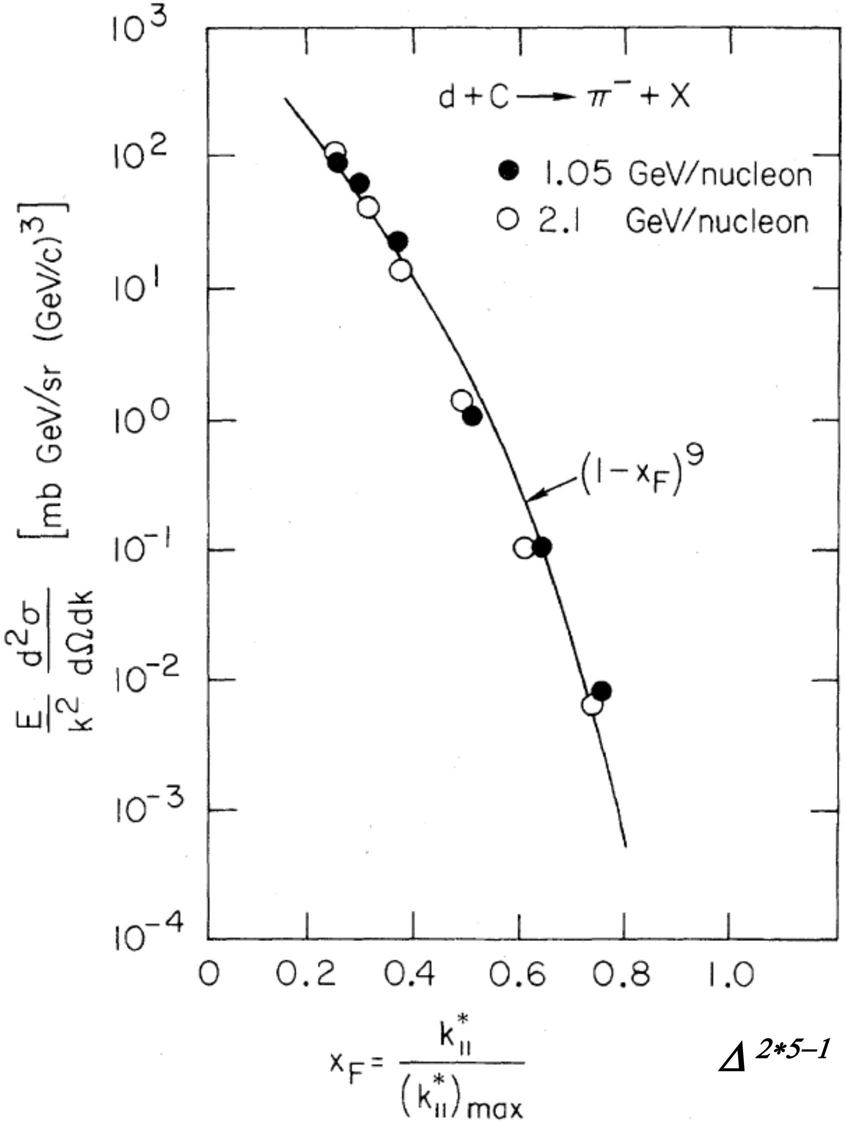
Matveev V.A., Muradyan R.M., Tavkhelidze A.N. Lett. Nuovo Cimento 7 (1973) 719

Brodsky S.J., Chertok B.T. Phys.Rev. ***D14*** *(1976) 3003; Phys.Rev.Lett.* ***37*** *(1976) 269*

*Schmidt I.A., Blankenbecler R. Phys.Rev. **D15** (1977) 3321*



Δ^{2*2-1}

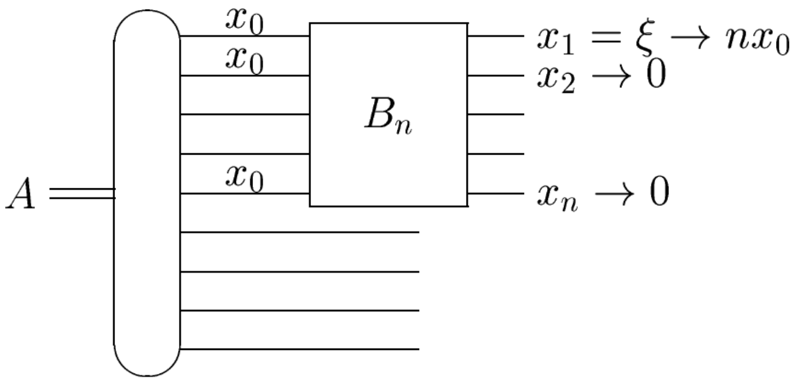


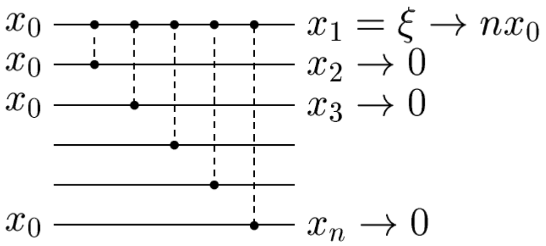
Δ^{2*5-1}

The experimental points from J. Papp et al., Phys.Rev.Lett. 34, 601 (1975).

Description of the hadron asymptotics at x->1
by the intrinsic diagrams of QCD in light-cone gauge
with low-x spectator quarks interact with the target
Brodsky S.J., Hoyer P., Mueller A., Tang W.-K., Nucl.Phys. ***B369*** *(1992) 519*

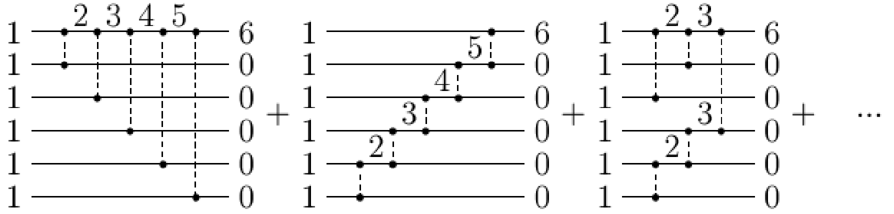
Description of the flucton asymptotic at *x->f*,
f - the number of nucleons in flucton, *n* - the number of quarks in flucton, $x_0 = f / n$ (=1/3).
M.A. Braun, V.V. V , Nucl.Phys. ***B427*** *(1994) 614.* (DIS in cumulative region)

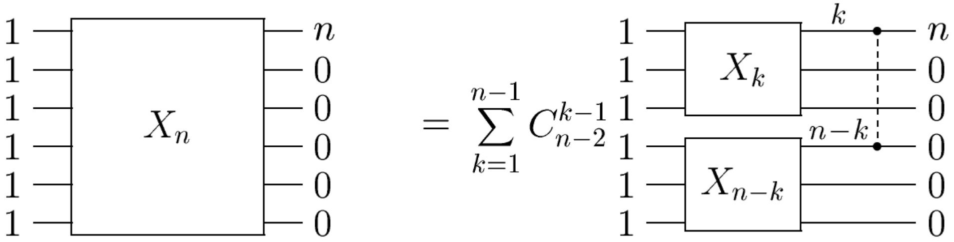




$\sim \Delta^{2p-1}$
$p=n-1$
$\Delta = f - \mathbf{x} = nx_0 - \xi$

At $x_1 \to f$ => all $x_2, \ldots, x_n \to 0$ => all $|q_i| \gg m$ =>
pQCD works => min.number of hard exchanges.
Simple instantaneous Coulomb part dominates
in light-cone gauge.

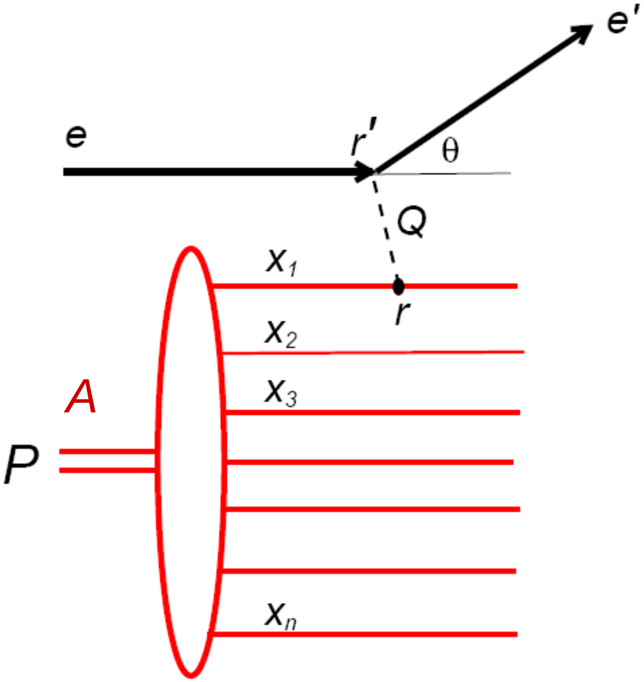




- the recurrence relation

Deep Inelastic Scattering (DIS) in cumulative region

Lehman E., Phys.Lett.62B (1976) 296 – connection of the limiting fragmentation of deutron into pions with deutron DIS structure function F_2 *(5% 2N(6q)-flucton admixture in D)*



$$|\mathbf{r}-\mathbf{r'}| \sim 1/|Q|, \quad |Q| \gg m$$

$$\xi \equiv \frac{-Q^2}{2(pQ)} = \frac{-Q^2}{2mQ_0}, \quad p = P/A$$

$$0 < \xi < A, \quad 1 < \xi < A - \text{cumulative region}$$

$$Q^2 = -4EE' \sin^2\frac{\theta}{2}, \quad Q_0 = E - E'$$

$$x_1 \equiv \frac{k_{1+}}{p_+} \approx \frac{k_{1z}}{p} \geq \xi \text{ - Bjorken scaling variable}$$

$$(x_1 = \xi \text{ for elastic } \gamma q)$$

Experimental observations of DIS in cumulative region:

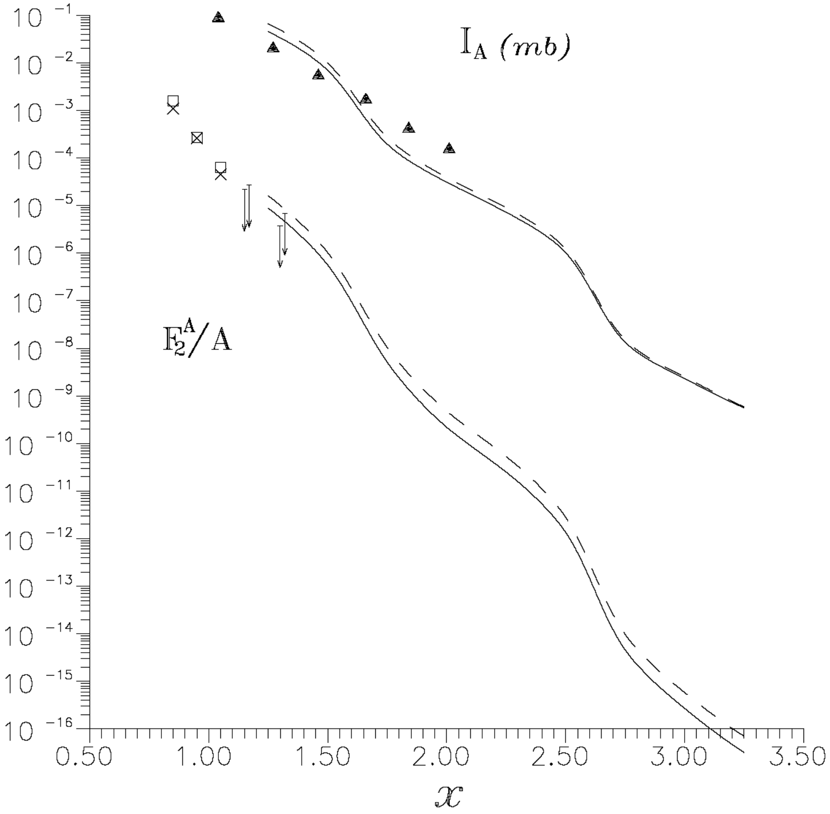
Shuetz W.P. et al., Phys.Rev.Lett., 38 (1977) 259 [D]

Filippone B.W. et al., Phys.Rev.C, 45 (1992) 1582 [Fe]

Benvenuti A.C. et al. (BCDMS collaboration) Z. Phys. C63 (1994) 29 [C]

Egiyan K.S., et al., Phys.Rev.Lett. 96 (2006) 082501 [^{3}He,^{4}He,C,Fe]

The different slopes of spectra for DIS and for particle production in cumulative region



$$F_2^A(x) \sim \exp(-b_0 x) \qquad b_0 \sim 16$$

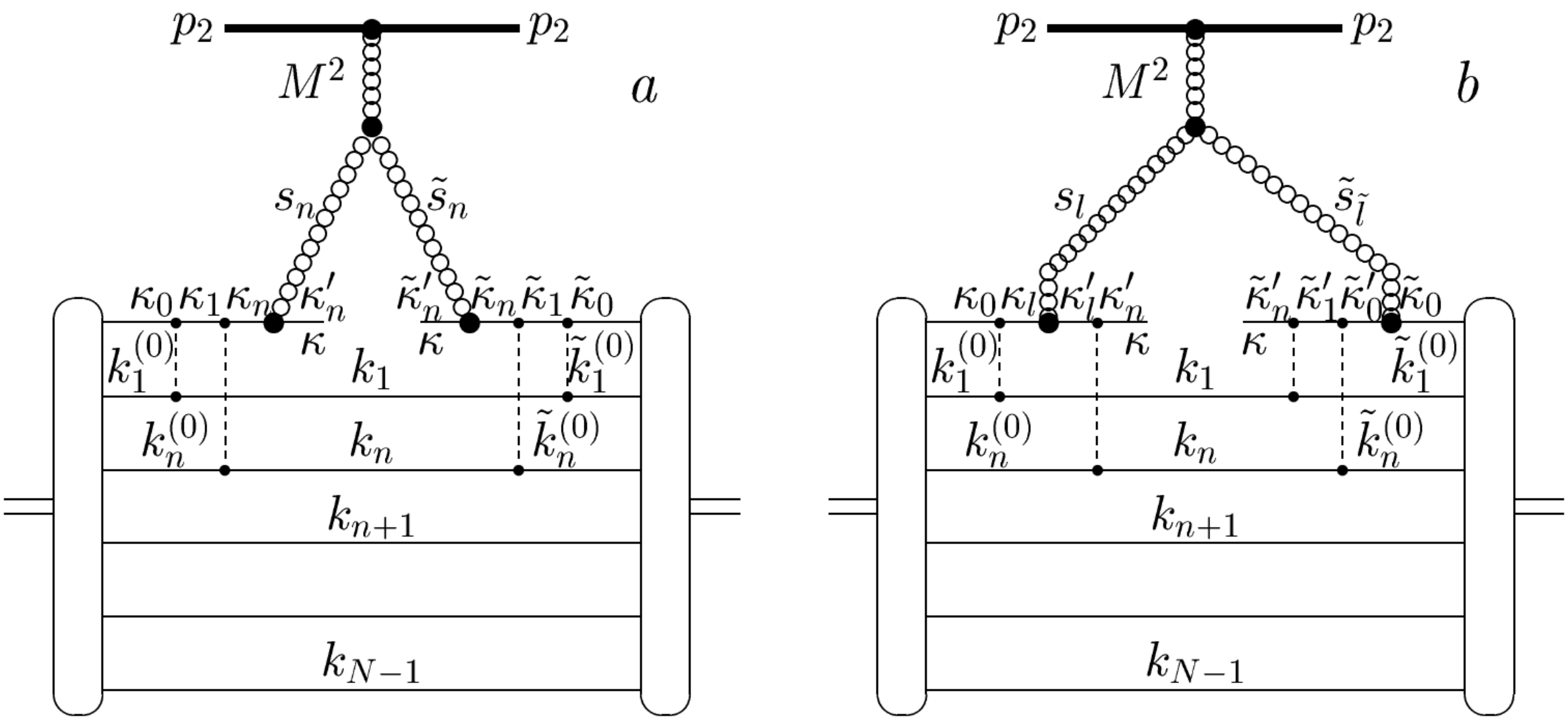
$$I_A(x) \sim \exp(-b_s x) \qquad b_s \sim 6 \div 8$$

The experimental points from

Benvenuti A.C. et al. (BCDMS collaboration)
Z. Phys. C63 (1994) 29
[^{12}C, q^2 = 61 GeV^2, 150 GeV^2].

Nikiforov N.A. et al. Phys. Rev. C22 (1980) 700
[p+181Ta->p +X, 400 GeV/c]

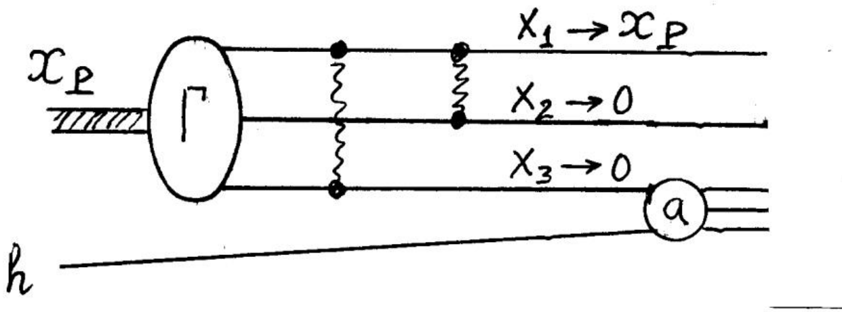
Cancellation of the direct contributions to a cumulative quark formation



M.A. Braun, V.V. V , Phys.Atom.Nucl. ***63*** *(1997) 432*

Cancellations in spectator contributions to a cumulative quark formation

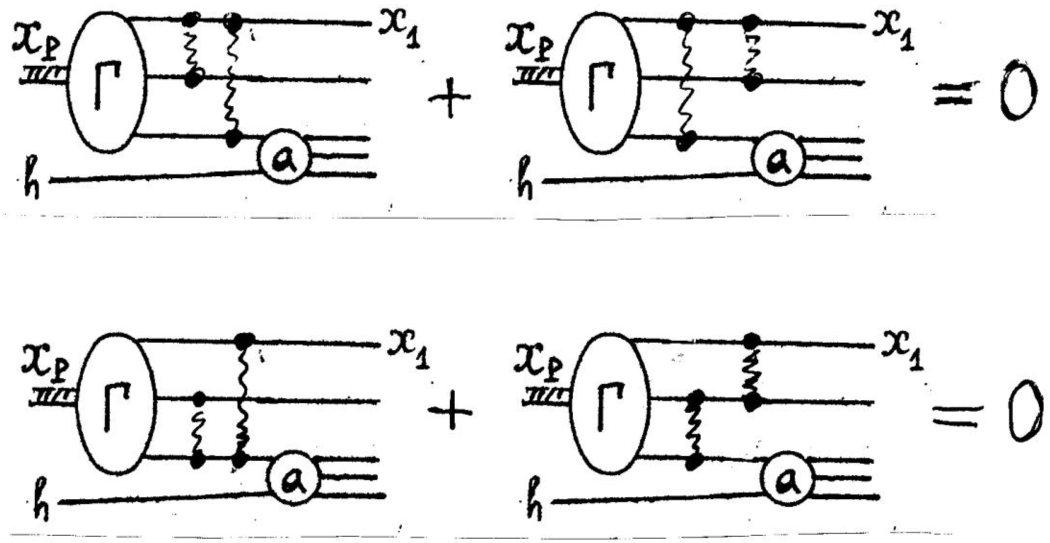
=> **all donor quarks must to interact with the projectile!**



$$\sim (x_P - x_1)^{2p-1}$$

$$x_P = 1, 2, \ldots$$

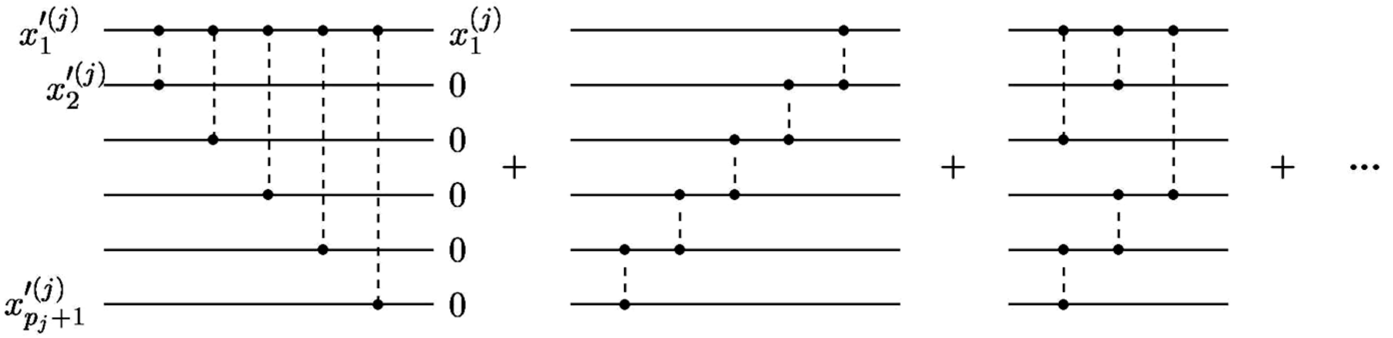
$$p = 2 \Rightarrow \sim (1 - x_1)^3$$



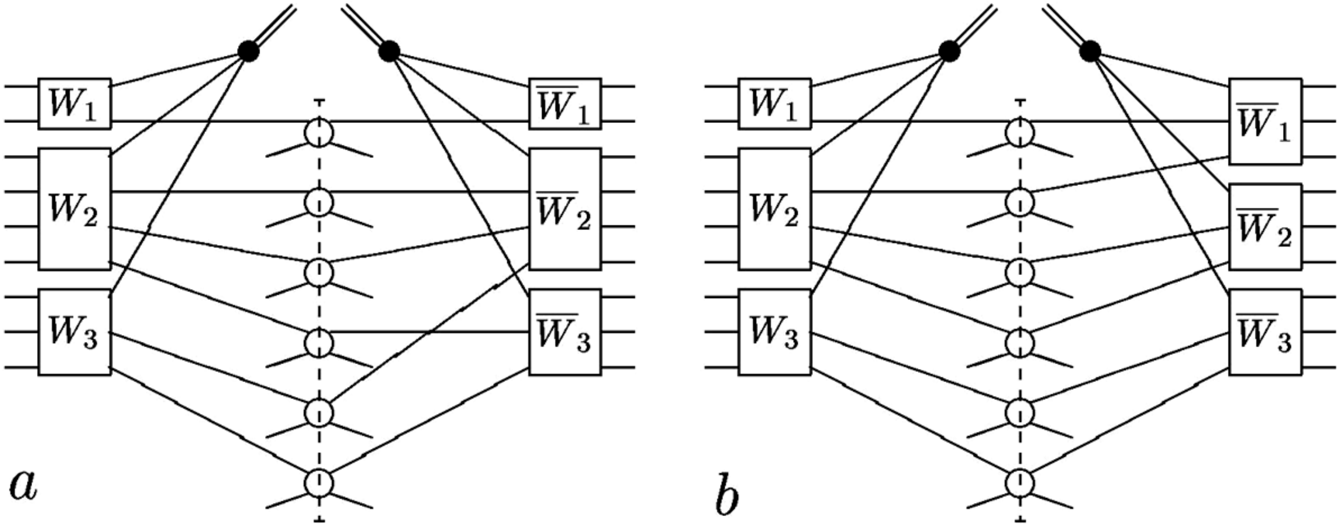
$\sim (1\text{-}x_1)^{\boldsymbol{q}}$

$q > 3$

Contributions to the blobs W_j**:**

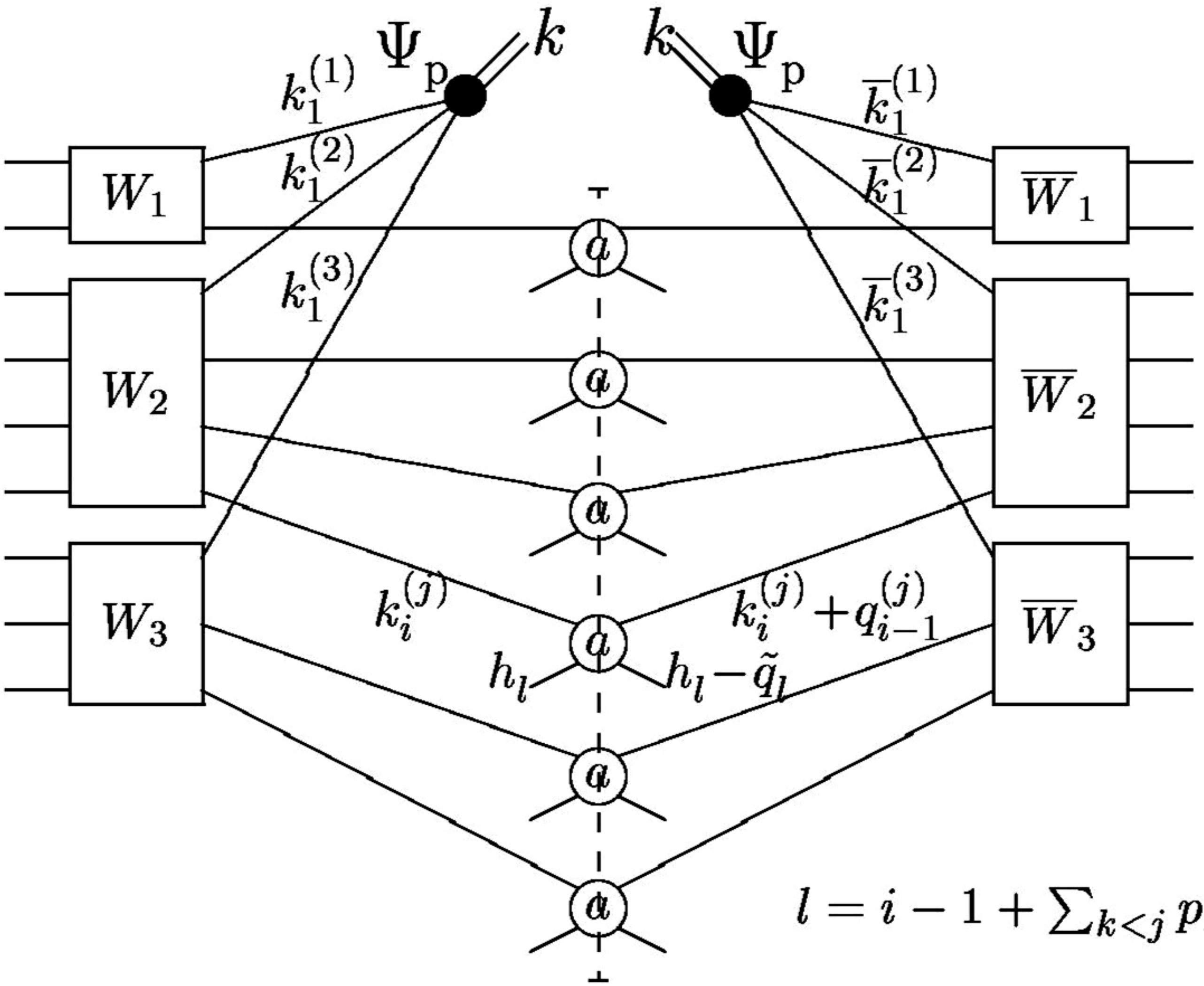


$\sim \Delta^{2p_j-1}$



The examples of two types of non-diagonal contributions to the cross section of cumulative proton production:
a – all $p_j = \overline{p}_j$, *b* – some $p_j \neq \overline{p}_j$

$$\boxed{\frac{r_q^2}{r_N^2}}$$

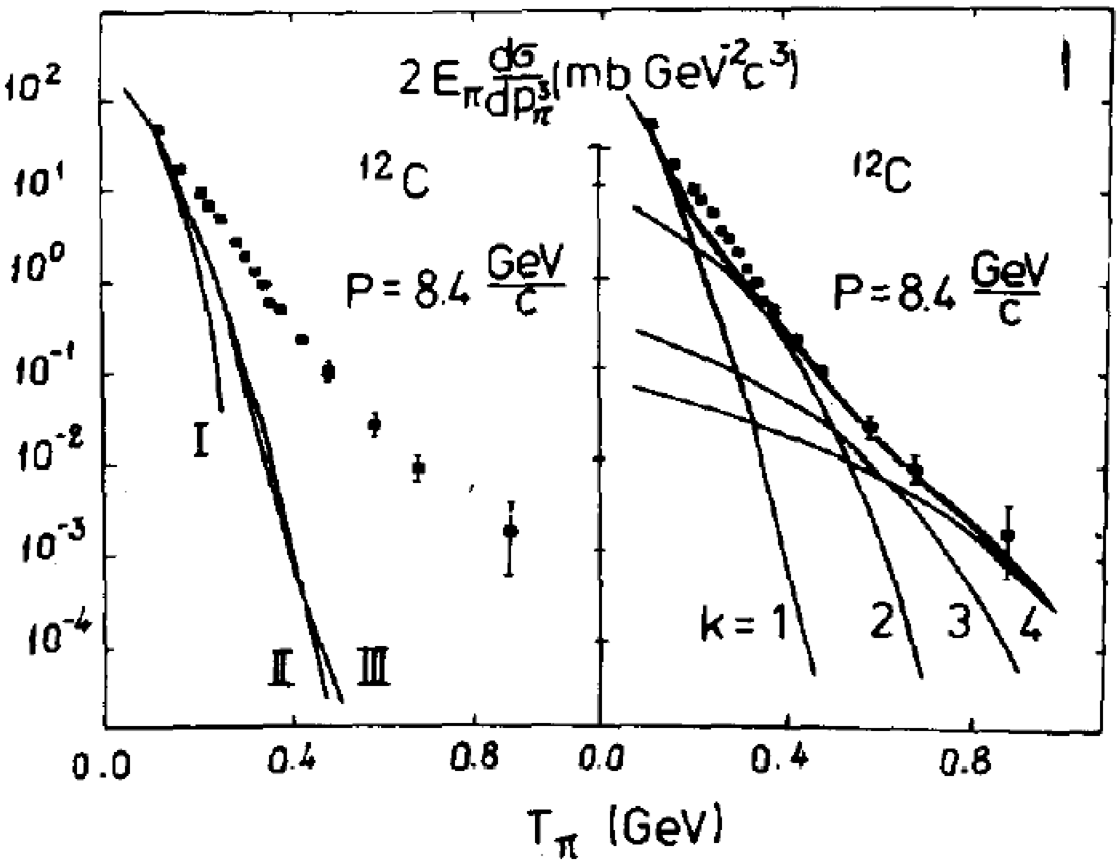


$$l = i - 1 + \sum_{k<j} p_k$$

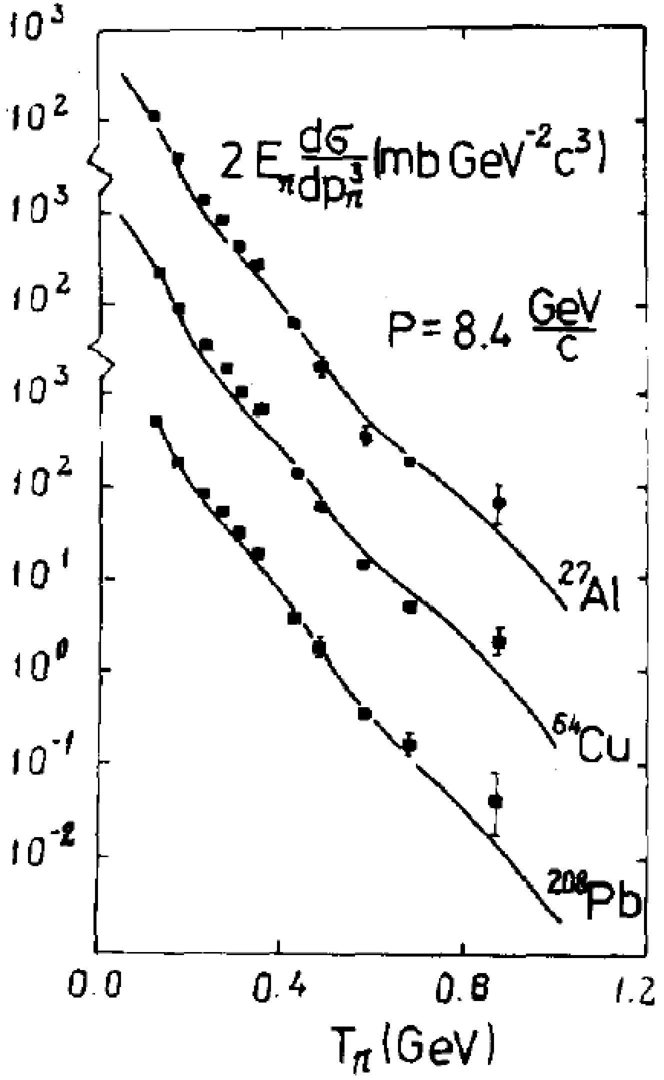
The diagonal contribution to the cross section of cumulative proton production.

Note the presence of the interference effects also in this case!

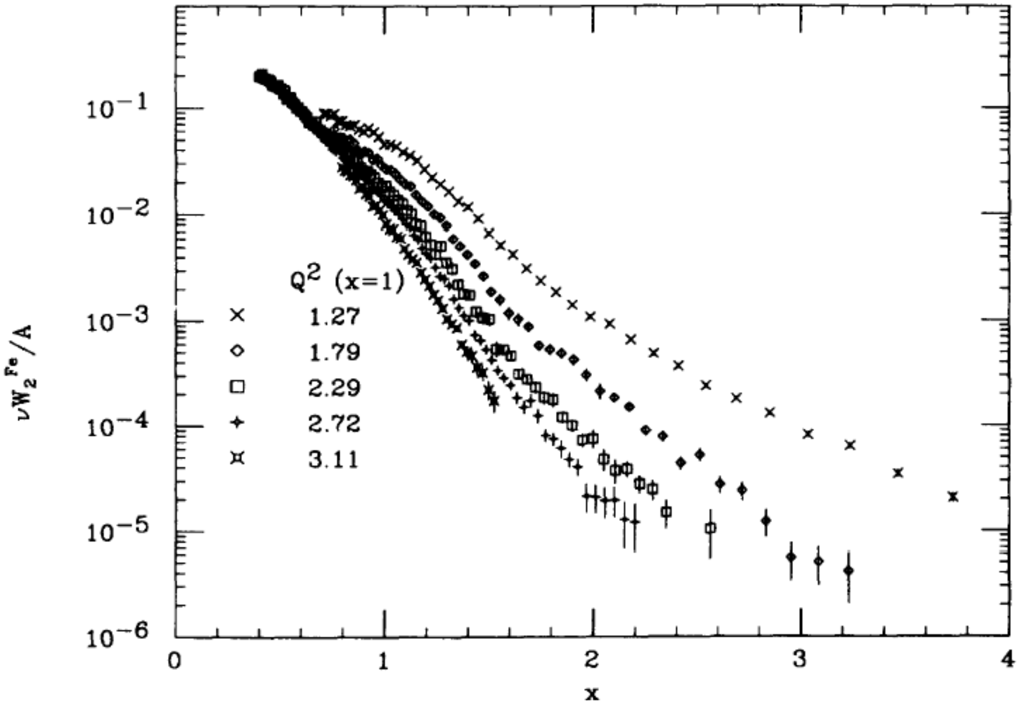
V.V. Burov, V.K. Lukyanov, A.I. Titov, Phys.Lett., 67B (1977) 46



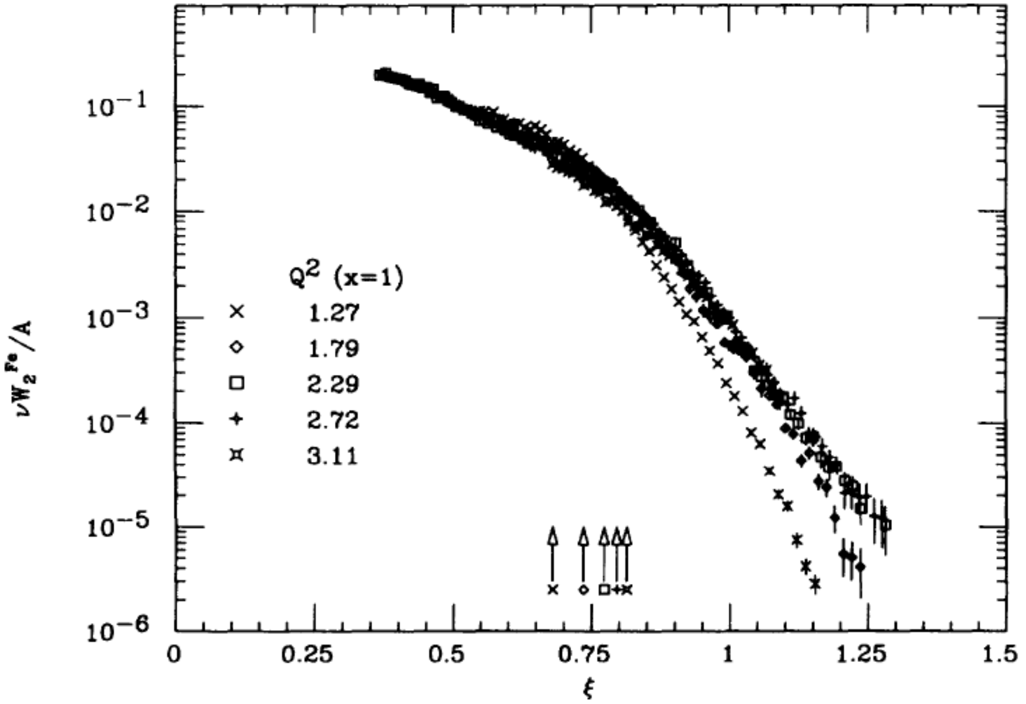
(a) Calculations of the invariant pion production cross section for ^{12}C: I – for the free proton target; II – with fermi motion; III – the relativization effect. (b) The contributions of separate fluctuons with mass $M_k = km_p$ where k is the order of cumulativity.



The experimental points from A.M. Baldin et al., Yad. Fiz. 18 (1973) 79.



Filippone B.W. et al.,
Phys.Rev.C, 45 (1992) 1582

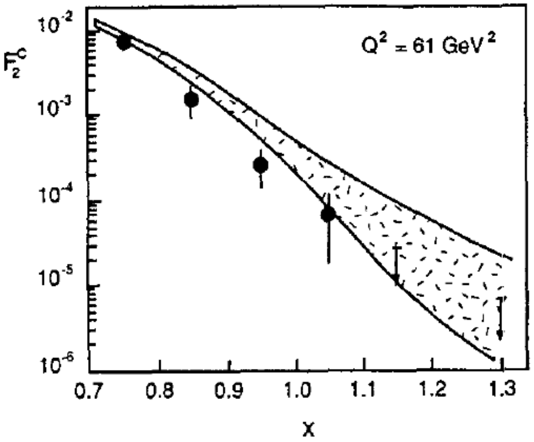


Freedom at moderate energies:
Masses in color dynamics

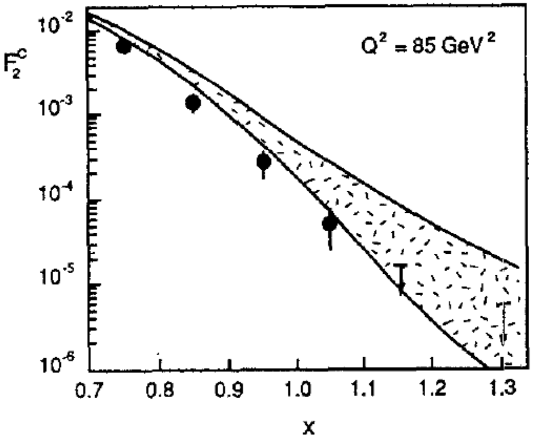
Georgi H., Politzer H.D.

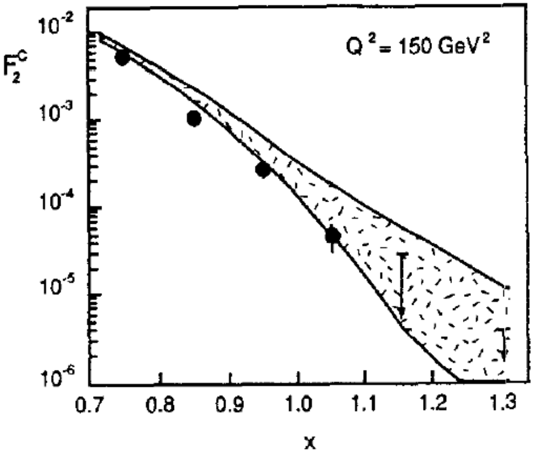
Phys. Rev. D 14, 1829 (1976)

$$\xi = 2x/[1+(1+4M^2x^2/Q^2)^{1/2}]$$

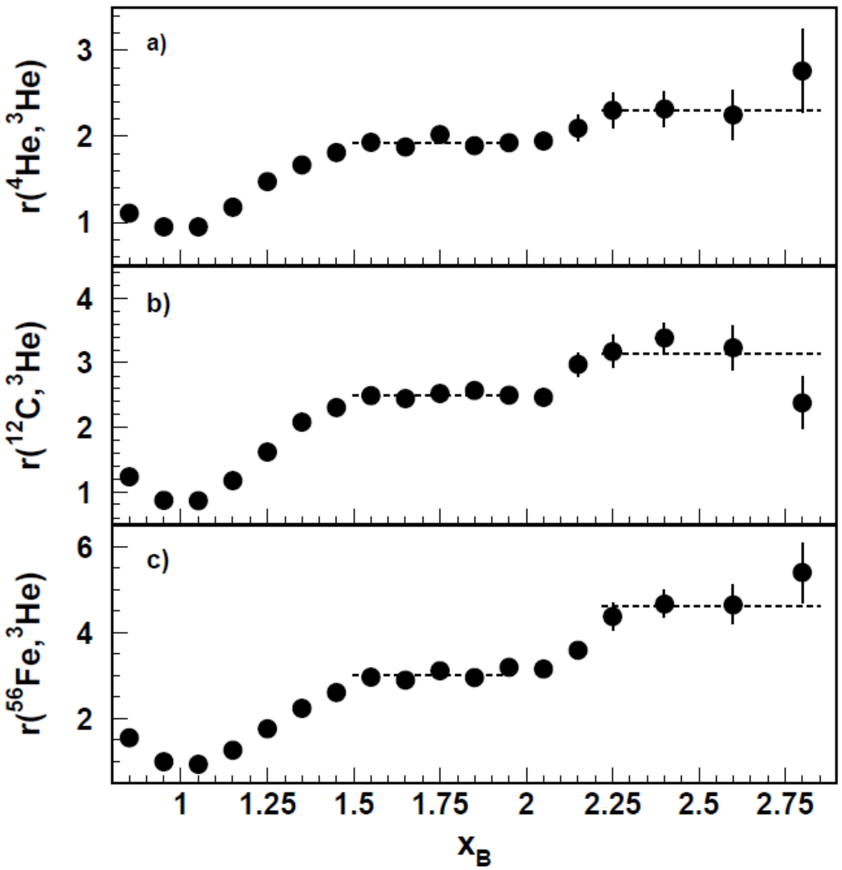


Benvenuti A.C. et al. (BCDMS collaboration) Z. Phys. C63 (1994) 29





L. Frankfurt, M. Strikman, Phys. Rep. 160 (1988) 325



K.S. Egiyan, et al.,
Phys.Rev.Lett. 96 (2006) 082501

$$r(A,{}^3\mathrm{He}) = \frac{A(2\sigma_{ep}+\sigma_{en})}{3(Z\sigma_{ep}+N\sigma_{en})}\,\frac{3\mathcal{Y}(A)}{A\mathcal{Y}({}^3\mathrm{He})}\,C^A_{\mathrm{rad}}$$

	$a_2(A/{}^3\mathrm{He})$	$a_{2N}(A)(\%)$	$a_3(A/{}^3\mathrm{He})$	$a_{3N}(A)(\%)$
^{3}He	1	8.0±1.6	1	0.18±0.06
^{4}He	1.93±0.01± 0.03	15.4±3.2	2.33±0.12±0.04	0.42±0.14
^{12}C	2.49±0.01± 0.15	19.8±4.4	3.18±0.14±0.19	0.56±0.21
^{56}Fe	2.98±0.01± 0.18	23.9±5.3	4.63±0.19±0.27	0.83±0.27

Quark counting rules for elastic and quasi elastic reactions with nuclei

Brodsky S., Farrar G. Phys.Rev.Lett. 31 (1973) 1153
Brodsky S., Chertok B.T., Phys.Rev. ***D14*** *(1976) 3003*
Matveev V.A., Muradyan R.M., Tavkhelidze A.N. Lett. Nuovo Cimento 7 (1973) 719

$s \to \infty, \quad t/s \text{ fixed}$

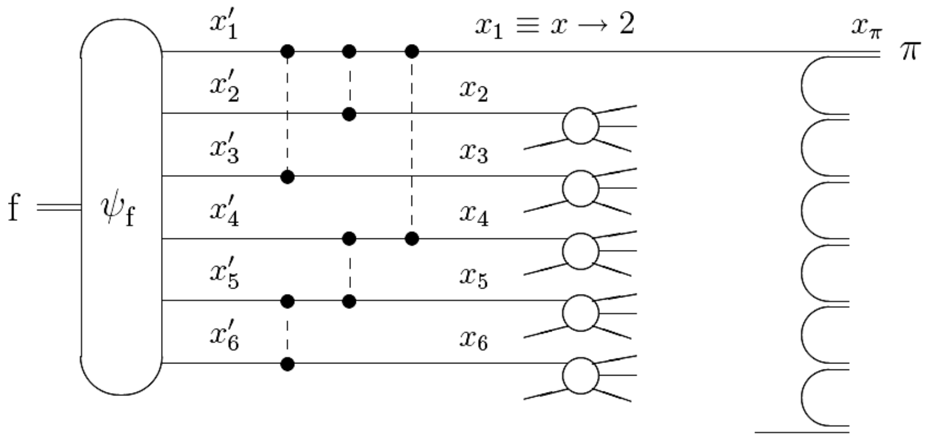
$$(d\sigma/dt)_{\pi p \to \pi p} \sim s^{-8}, \quad (d\sigma/dt)_{pp \to pp} \sim s^{-10}, \quad (d\sigma/dt)_{\gamma p \to \pi p} \sim s^{-7}, \quad (d\sigma/dt)_{\gamma p \to \gamma p} \sim s^{-6}$$

$\sim \boldsymbol{s}^{-n}$ A+B->C+D $n=n_A+n_B+n_C+n_D-2$ $n_p=3$ $n_\pi=2$ $n_\gamma=1$

$$\frac{d\sigma}{dt}(A+B \to C+D) \to \frac{1}{t^{N-2}} f(t/s)$$

$N=n_A+n_B+n_C+n_D$

Transverse momentum spectra of cumulative pions



- the cumulative pion production

k_T – dependence:
M.A. Braun, V.V. V ,
Phys.Atom.Nucl. ***63****, 1831 (2000)*

$$\sigma_{pion}(x, k_\perp; p) = C(p)\ (x_{frag} - x)^{2p-1}\ f_p\left(\frac{k_\perp}{m}\right)$$

$$x < x_{frag}(p) = 1/3 + p/3$$

p – the number of “donors”, stopped quarks
m – the constituent quark mass

$$f_p(t) = \frac{1}{\pi^p} \int \prod_{i=1}^{p} \frac{d^2 t_i}{(t_i^2+1)^2} (2\pi)^2 \delta^{(2)}\left(\sum_{i=1}^{p} t_i + t\right)$$

$$t = k_\perp/m, \quad t_i = k_{i\perp}/m$$

$$f_p(t) = 2\pi \int_0^\infty dz\, z J_0(tz)[zK_1(z)]^p$$

$$\langle |K_\perp| \rangle = pm \int_0^\infty dz K_0(z)(zK_1(z))^{p-1}$$