



Angular correlations of particle yield ratios

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NUCLEUS-2021
September 23

Intro: observables for heavy-ion collision studies

In central Pb-Pb collisions at LHC energies, ~2000 particles within $|\eta| < 0.5$.

Many “event-averaged” observables can be studied: particle yields, spectra, flow harmonics...

More differentially, one can study correlations, e.g. in emission angles

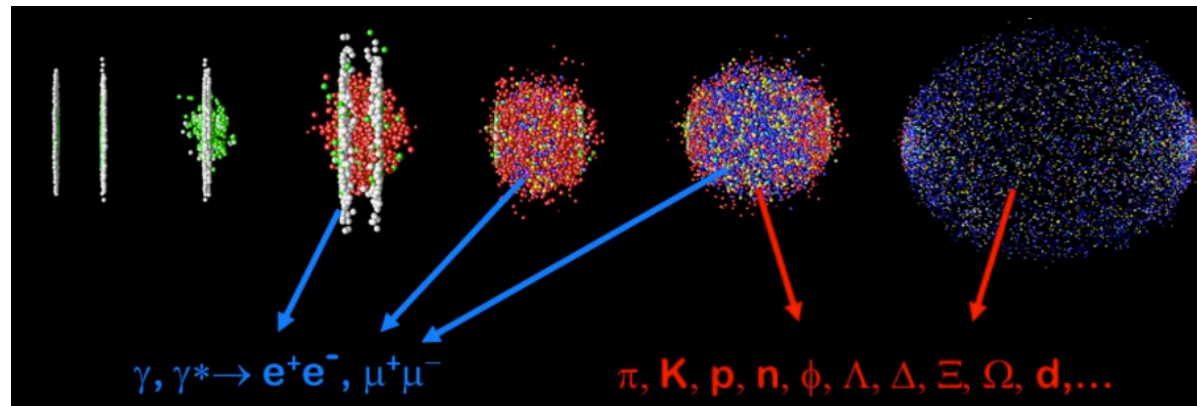
Event-by-event measurements:

the fluctuations are studied over the ensemble of the events.

- fluctuating net-charge, number of protons, mean p_T , forward-backward yields, etc.

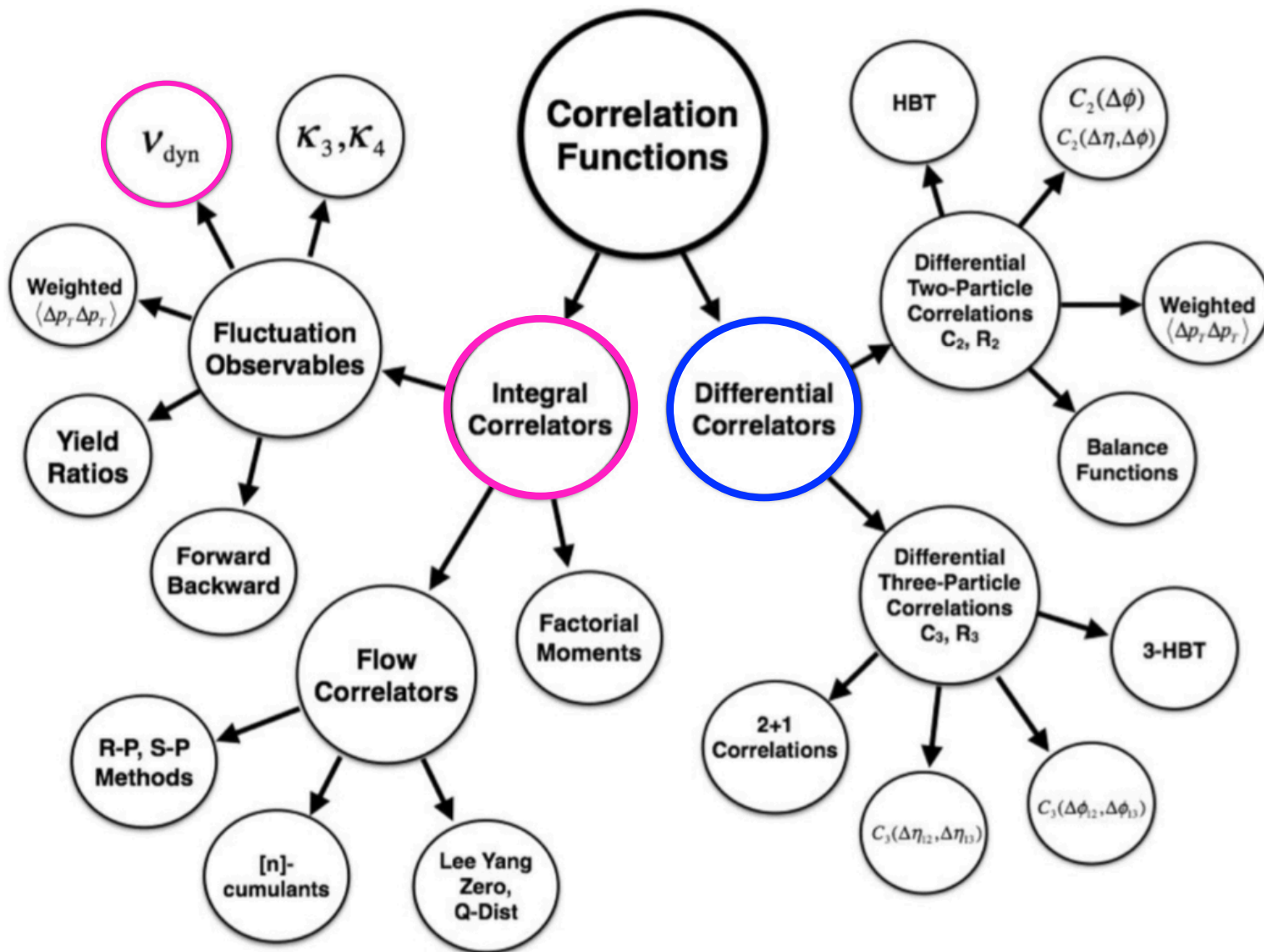
Why e-by-e fluctuations:

- they help to characterize the **properties of the “bulk” of the system**
- fluctuations also are closely related to **dynamics of the phase transitions**



Landscape of integral and differential observables

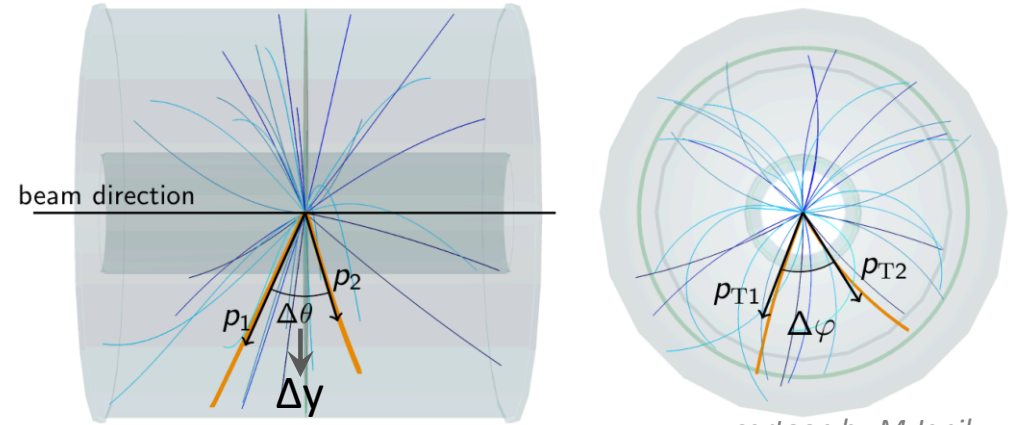
C. A. Pruneau, "Data Analysis Techniques for Physical Scientists" (2017)



Differential correlations between + – charges (Balance Functions)

$$B(\Delta\eta) = \frac{1}{2} \left(\frac{\rho_2^{(+,-)} - \rho_2^{(+,+)}}{\rho_1^{(+)}} + \frac{\rho_2^{(-,+)} - \rho_2^{(-,-)}}{\rho_1^{(-)}} \right)$$

Bass et al., Phys. Rev. Lett. 85 (2000) 2689

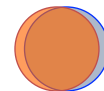
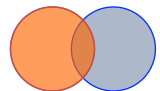
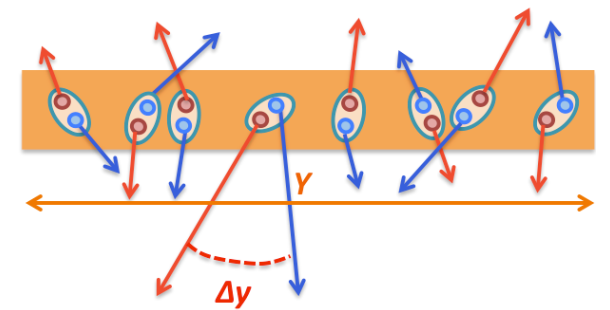
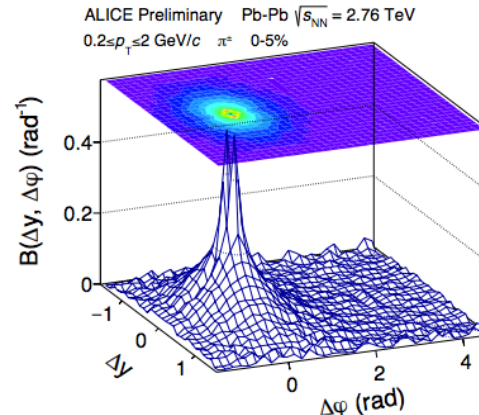
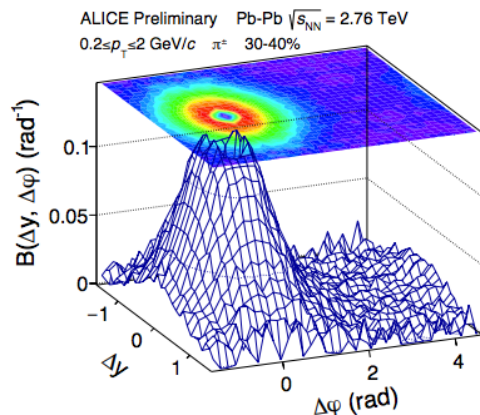
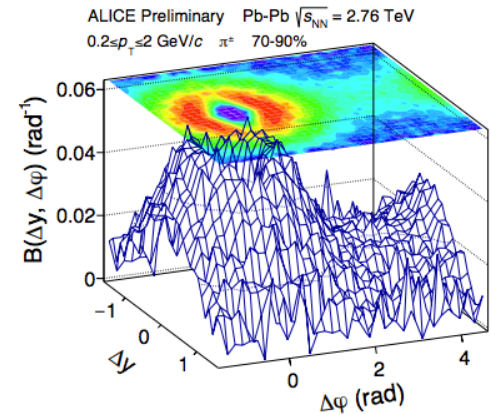


cartoon by M.Janik

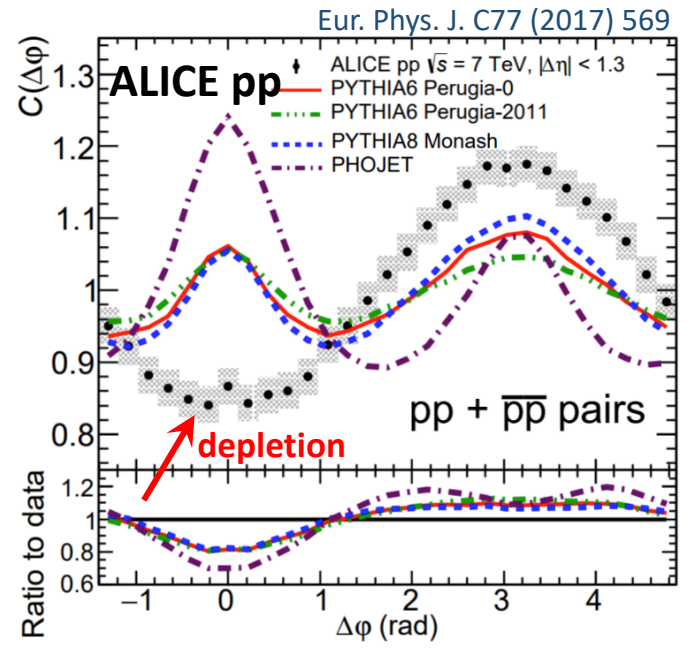
Physics picture: charge diffusion in the medium
 Affected by: radial flow, resonance decays, HBT, Coulomb effects

→ STAR and ALICE reveal narrowing of BF with centrality:

J. Pan, QM2018, arxiv:1807.10377



Differential angular correlations: typical definitions



ALICE:

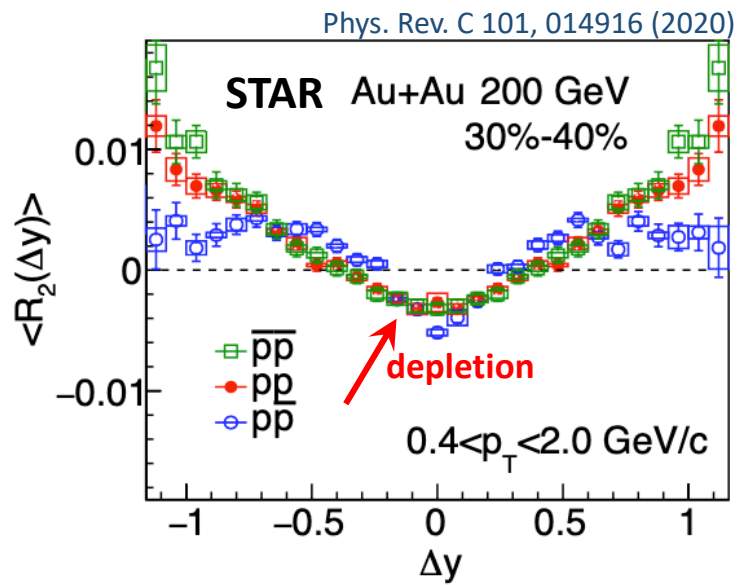
$$C(\Delta\eta, \Delta\phi) = \frac{N_{pairs}^{mixed}}{N_{pairs}^{signal}} \left(\frac{d^2 N_{pairs}^{signal}}{d\Delta\eta d\Delta\phi} / \frac{d^2 N_{pairs}^{mixed}}{d\Delta\eta d\Delta\phi} \right)$$

“angular” part

STAR:

$$R_2(\Delta y, \Delta\phi) = \frac{\langle n \rangle^2}{\langle n(n-1) \rangle} \left(\frac{\rho_2(\Delta y, \Delta\phi)}{\rho_1(y_1, \phi_1) \rho_1(y_2, \phi_2)} \right) - 1$$

This normalization removes sensitivity to E-by-E fluctuations
 → R_2 is **identically zero** in the absence of any two-particle correlations

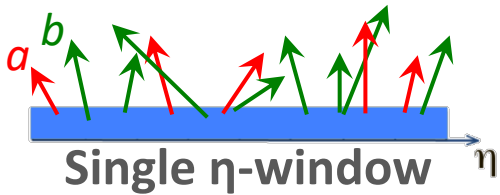


Figures: note anti-correlation of same-sign baryons at small angular separations

Integrated observable: particle number fluctuations

$$r = n_a/n_b \text{ - ratio of yields}$$

Integrated observable (approximation):



$$\nu_{dyn} \equiv \nu - \nu_{stat} = \frac{\langle n_a(n_a - 1) \rangle}{\langle n_a \rangle^2} + \frac{\langle n_b(n_b - 1) \rangle}{\langle n_b \rangle^2} - 2 \frac{\langle n_a n_b \rangle}{\langle n_a \rangle \langle n_b \rangle}$$

Pruneau, Voloshin, Gavin, Phys.Rev. C66 (2002) 044904

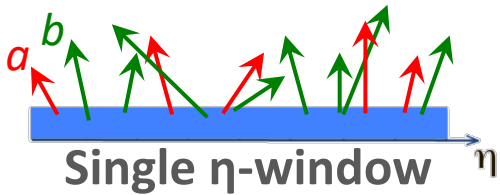
- measures deviations from Poissonian behaviour
- robust against volume fluctuations, efficiency losses
- sensitive to correlations between species *a*, *b*
- affected by resonance decays

Variance of the ratio (norm.):

$$\nu \equiv \frac{\langle \Delta r^2 \rangle}{\langle r \rangle^2}$$

Integrated observable: particle number fluctuations

$r = n_a/n_b$ – ratio of yields



Integrated observable (approximation):

$$v_{dyn} \equiv v - v_{stat} = \frac{\langle n_a(n_a - 1) \rangle}{\langle n_a \rangle^2} + \frac{\langle n_b(n_b - 1) \rangle}{\langle n_b \rangle^2} - 2 \frac{\langle n_a n_b \rangle}{\langle n_a \rangle \langle n_b \rangle}$$

Pruneau, Voloshin, Gavin, Phys.Rev. C66 (2002) 044904

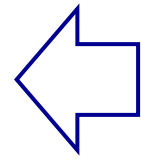
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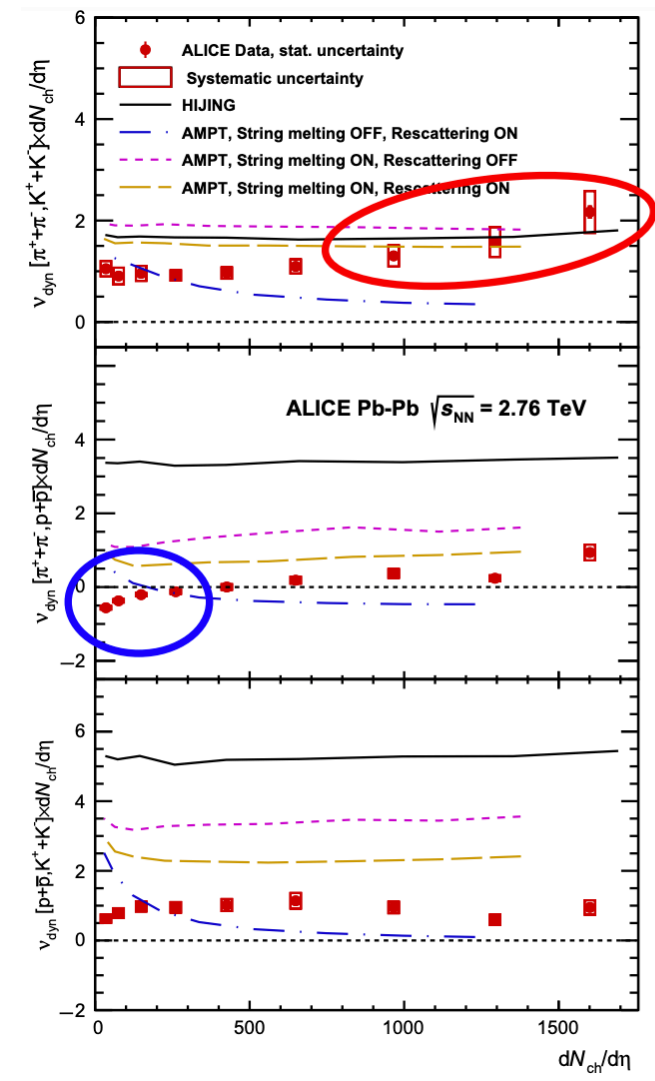
$$v \equiv \frac{\langle \Delta r^2 \rangle}{\langle r \rangle^2}$$

Observations:

- $v_{dyn}[\pi, p]$: increasing correlation with decreasing centrality
- $v_{dyn}[\pi, K]$: increasing anti-correlation between π and K or increasing dynamical fluctuations with increasing centrality
- Models fail to describe data



ALICE, EPJ C79 (2019) 236



$v_{dyn}[\pi, K]$

$v_{dyn}[\pi, p]$

$v_{dyn}[p, K]$

Differential correlations between ratios of particle yields

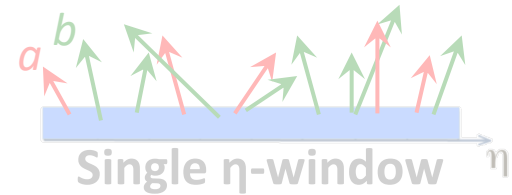
$r = n_a/n_b$ – ratio of yields

Integrated observable (approximation):

$$v_{dyn} \equiv v - v_{stat} = \frac{\langle n_a(n_a - 1) \rangle}{\langle n_a \rangle^2} + \frac{\langle n_b(n_b - 1) \rangle}{\langle n_b \rangle^2} - 2 \frac{\langle n_a n_b \rangle}{\langle n_a \rangle \langle n_b \rangle}$$

Pruneau, Voloshin, Gavin, Phys.Rev. C66 (2002) 044904

- robust against volume fluctuations, efficiency losses



$$v \equiv \frac{\langle \Delta r^2 \rangle}{\langle r \rangle^2}$$



Differential observable (approximation):

$$v_{FB} \approx \underbrace{\frac{\langle n_a^F n_a^B \rangle}{\langle n_a^F \rangle \langle n_a^B \rangle} + \frac{\langle n_b^F n_b^B \rangle}{\langle n_b^F \rangle \langle n_b^B \rangle}}_{\text{"same-species" terms}} - \underbrace{\frac{\langle n_a^F n_b^B \rangle}{\langle n_a^F \rangle \langle n_b^B \rangle} - \frac{\langle n_b^F n_a^B \rangle}{\langle n_b^F \rangle \langle n_a^B \rangle}}_{\text{"cross-species" terms}}$$

- keeps good properties of v_{dyn}
- if independent particle production $\rightarrow v_{FB} = 0$
- if only short-range effects (decays, jets) \rightarrow at large η_{gap} $v_{FB} = 0$

– not the case for the “classical” v_{dyn}

Examples:

- kaon-to-pion ratio $r = n_K / n_\pi$
- baryon-to-pion ratio $r = n_{proton} / n_\pi$

Physics cases of interest:

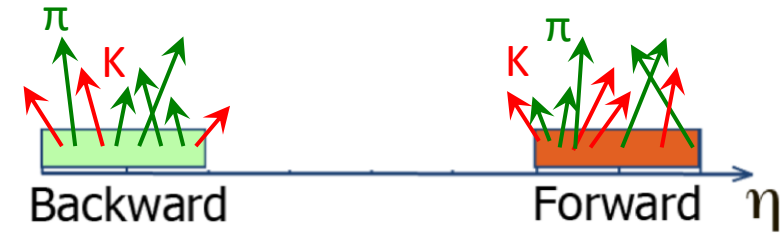
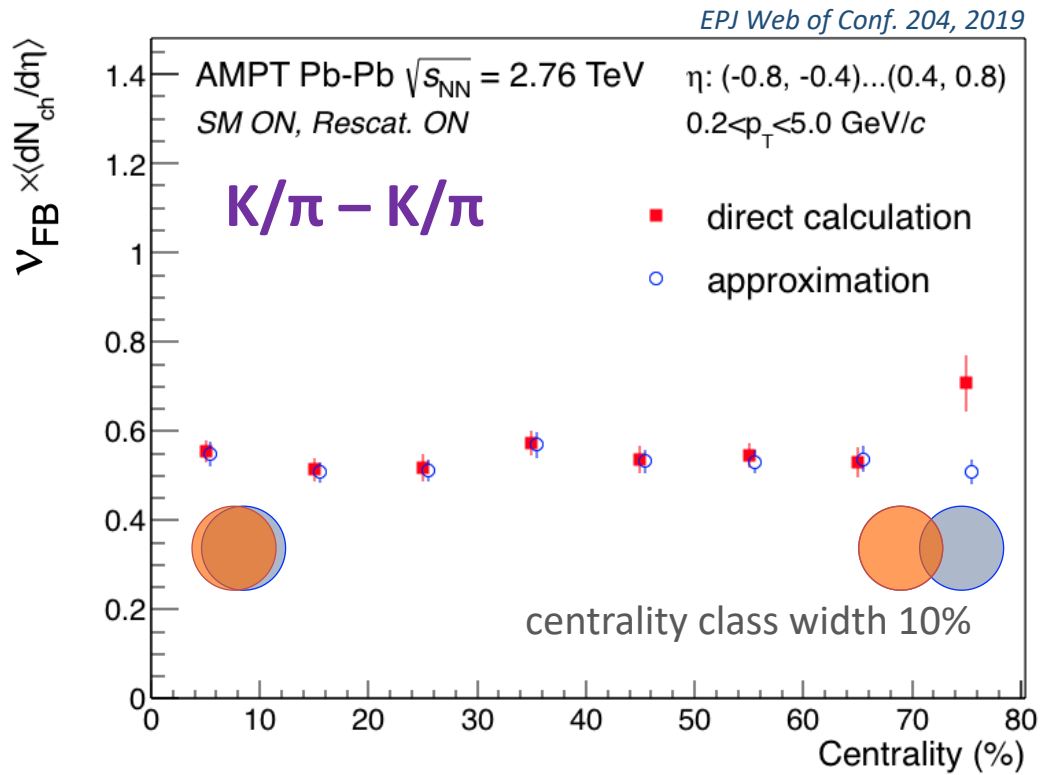
- correlations between strangeness or baryon production *at large η gaps* (string interactions, thermal models, ...)

Correlation strength:

$$v_{FB} = \frac{\langle r^F \cdot r^B \rangle}{\langle r^F \rangle \langle r^B \rangle} - 1$$

I.A., EPJ Web of Conf. 204, 2019
arXiv:1901.01635

Angular correlations between yield ratios: check approximation

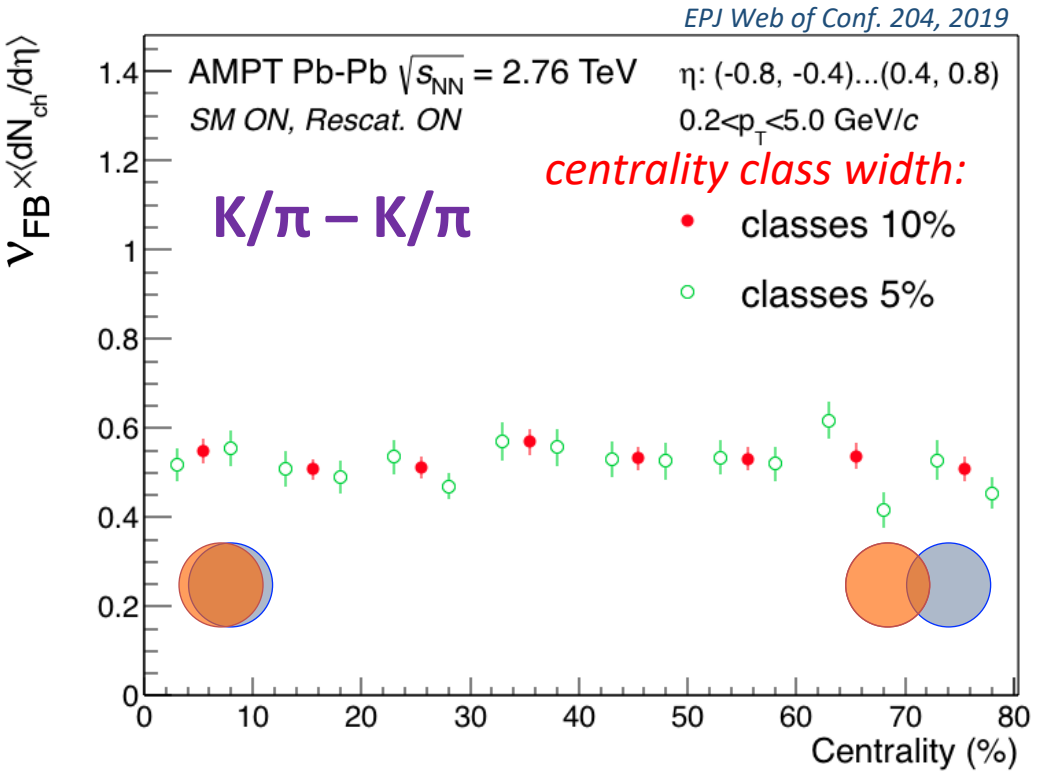


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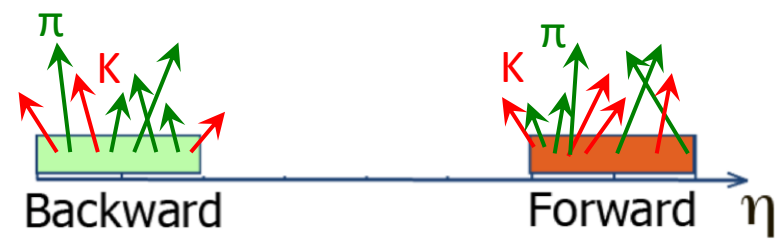
$$V_{FB} \approx \frac{\langle n_K^F n_K^B \rangle}{\langle n_K^F \rangle \langle n_K^B \rangle} + \frac{\langle n_\pi^F n_\pi^B \rangle}{\langle n_\pi^F \rangle \langle n_\pi^B \rangle} - \frac{\langle n_K^F n_\pi^B \rangle}{\langle n_K^F \rangle \langle n_\pi^B \rangle} - \frac{\langle n_\pi^F n_K^B \rangle}{\langle n_\pi^F \rangle \langle n_K^B \rangle}$$

- Good agreement between direct calculations and the approximation

Angular correlations between yield ratios: check robustness to VF



➡ Observables of this type are **robust to Volume and Volume Fluctuations**



Volume Fluctuations: when system size changes E-by-E
 (e.g. due impact parameter fluctuations)

It can be shown that in an independent sources model:

$$v_{FB} = \frac{1}{\langle N_{sources} \rangle} v_{FB}^{source}$$



A convenient to rescale:
 $v_{FB} \cdot \langle N_{sources} \rangle \sim v_{FB} \cdot \langle dN/d\eta \rangle$
 (as for v_{dyn})

Toy model for yield ratio correlations

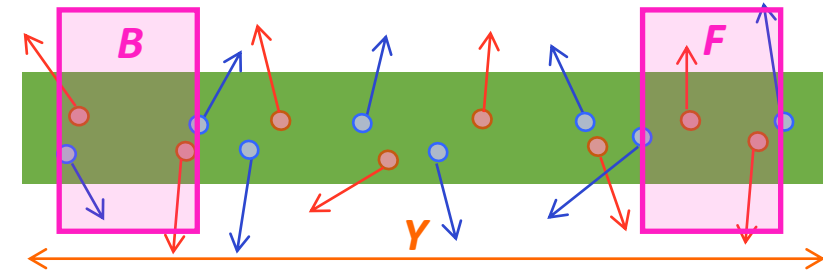
Assumptions:

- number of *positive* particles in each event is from Gauss, $\langle N \rangle = 80$, $\sigma = 4$
- particles are distributed within $|\eta| < 2$
- for each positive particle there is one *negative* (charge conservation)

Then particle ID is assigned, and we can simulate

- a binomial distribution of K^+ (others are pions) \rightarrow GCE
- ... or assign a strictly fixed fraction of K^+ \rightarrow CE

$$V_{FB} = \frac{\langle r^F \cdot r^B \rangle}{\langle r^F \rangle \langle r^B \rangle} - 1$$

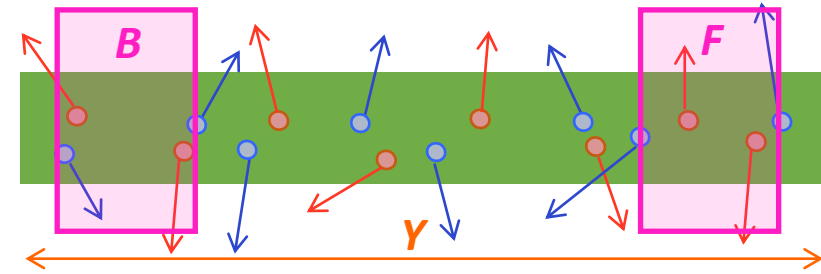


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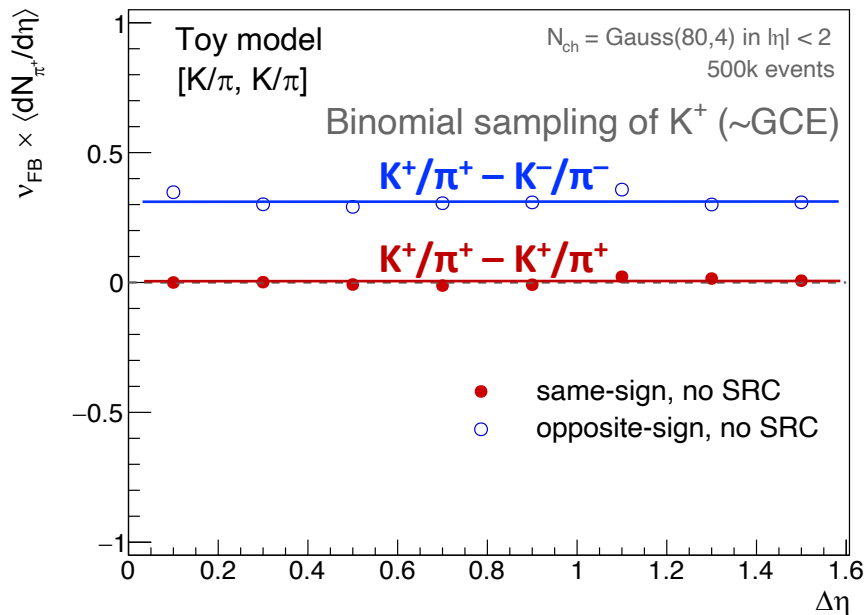
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Grand Canonical ensemble for K^+ :

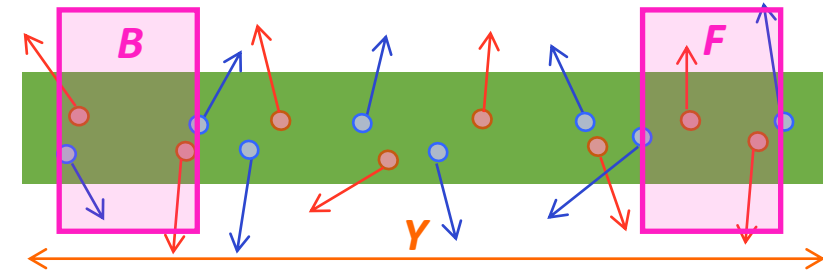


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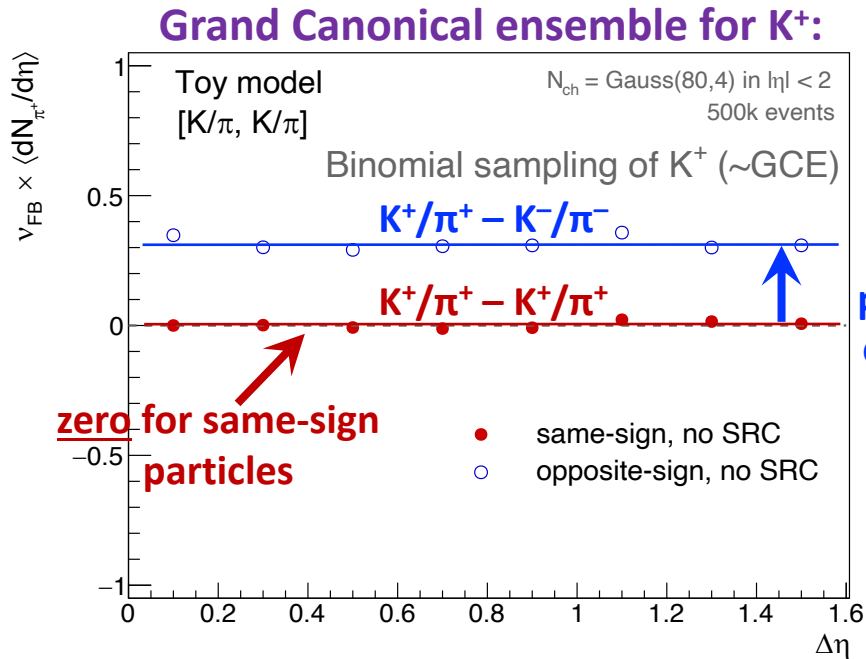
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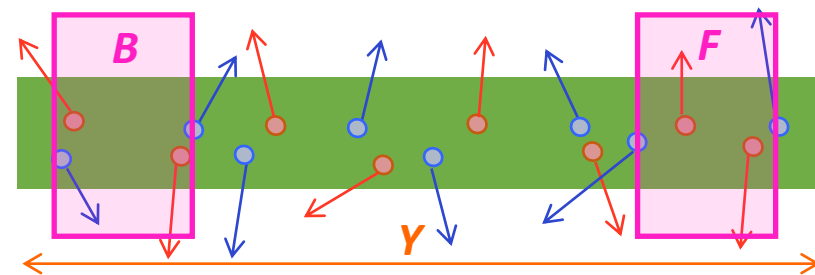
positive impact from charge conservation

Toy model for yield ratio correlations

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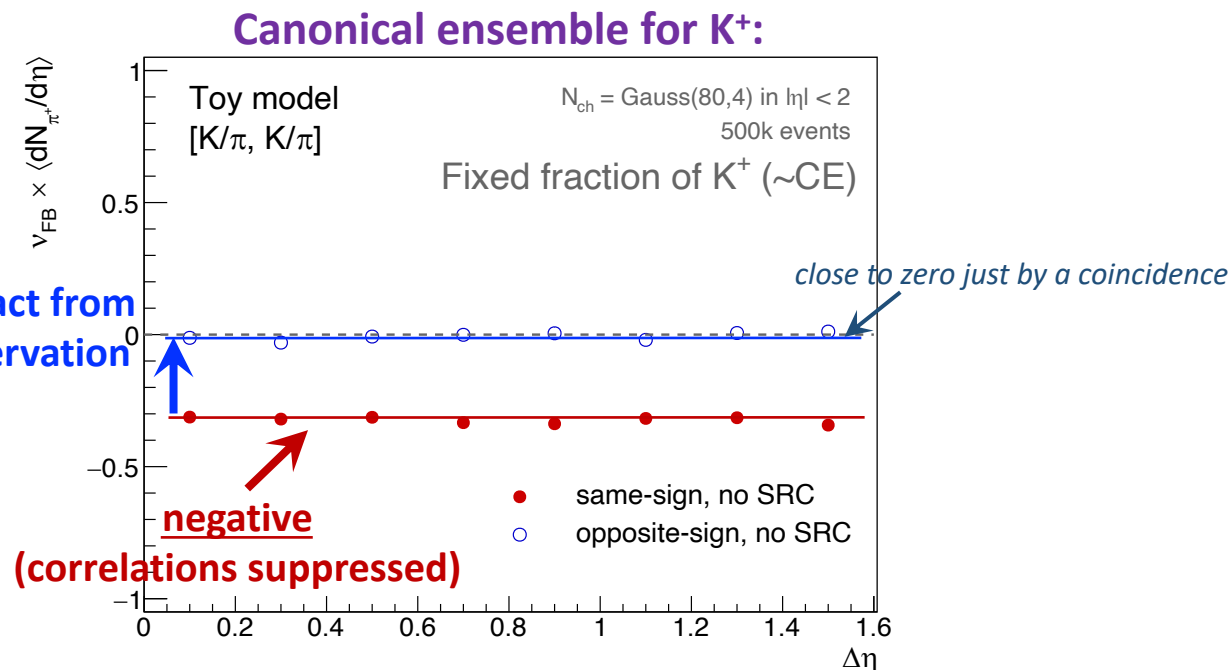
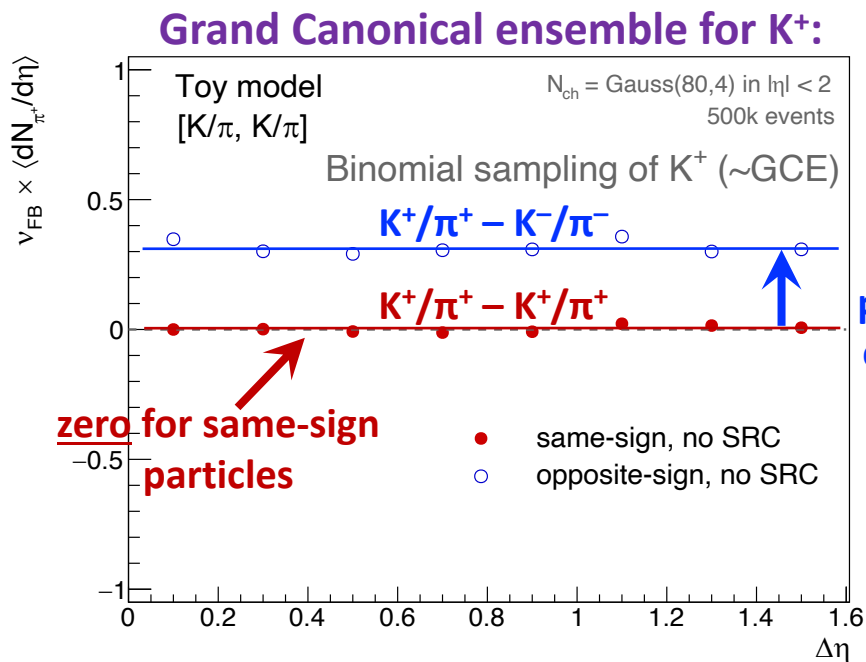
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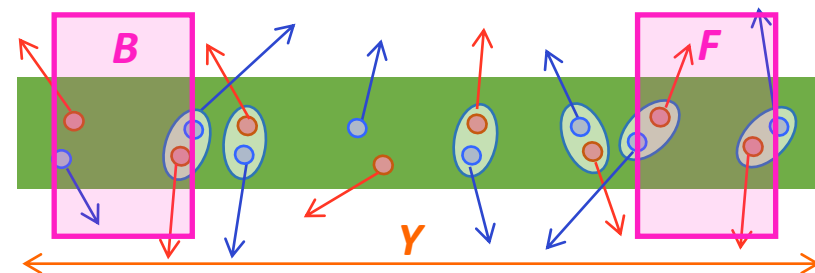


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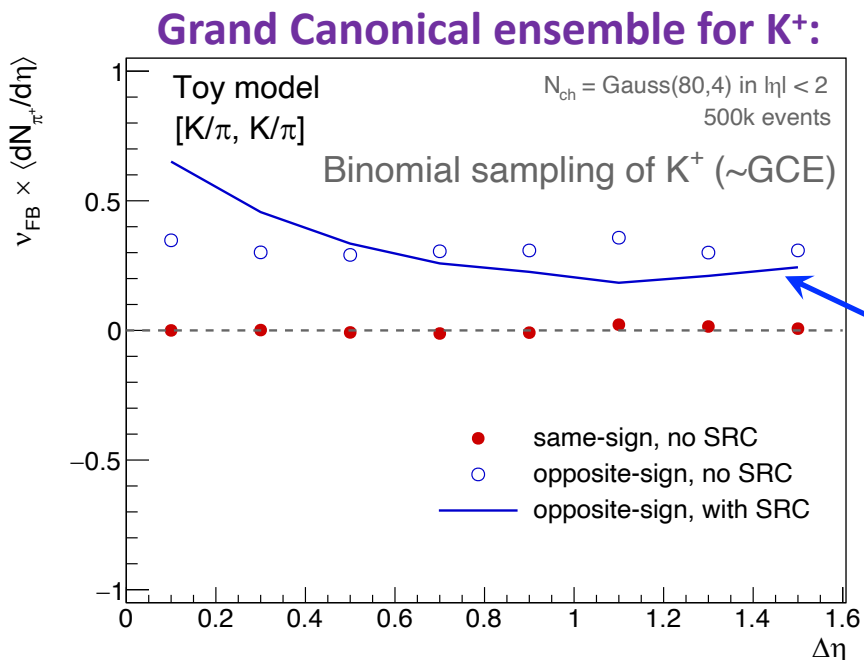
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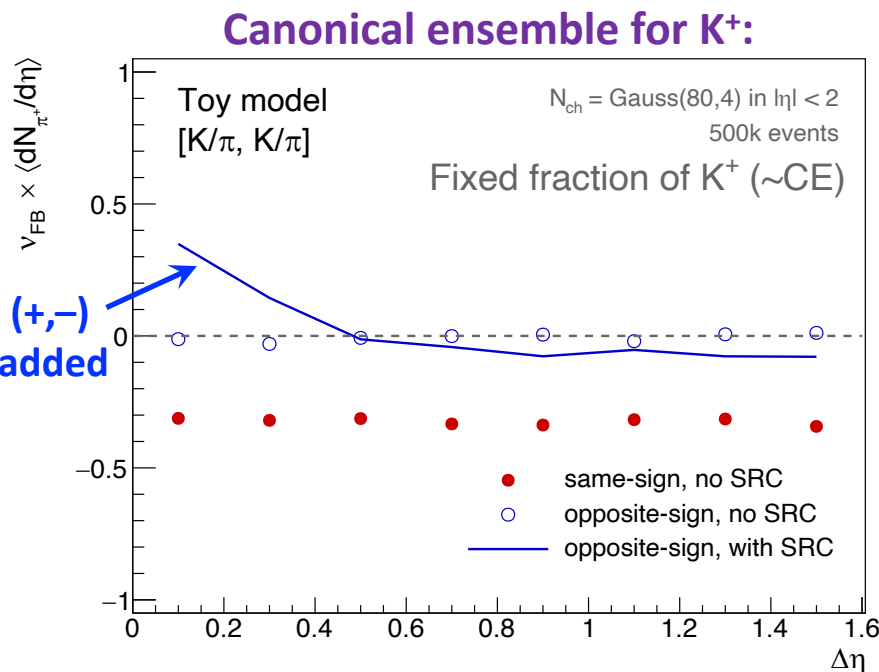


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short-range (+,-) correlations added

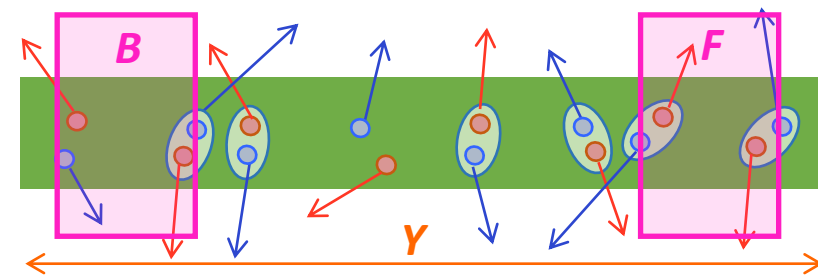


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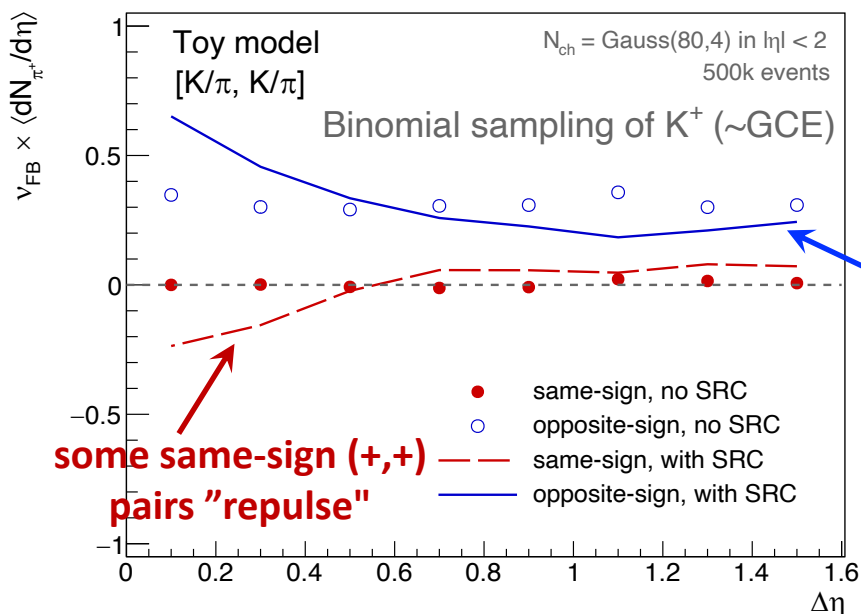
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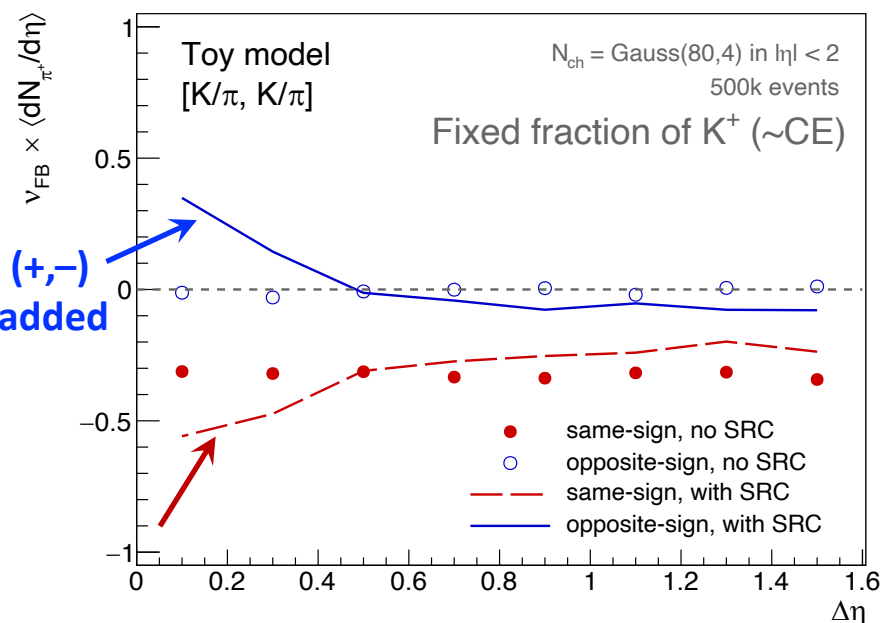
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Grand Canonical ensemble for K^+ :



Canonical ensemble for K^+ :

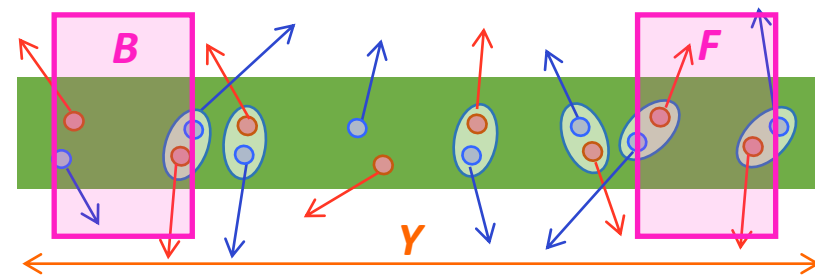


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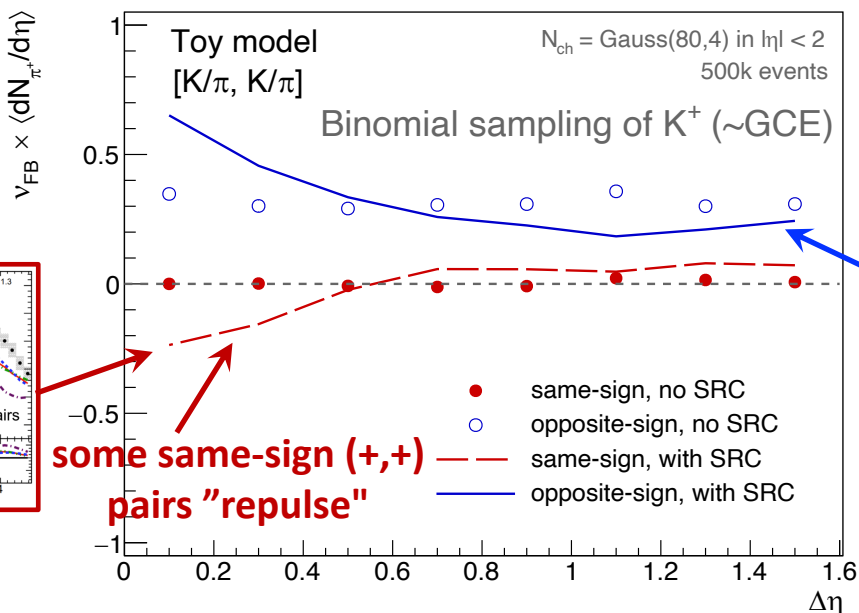
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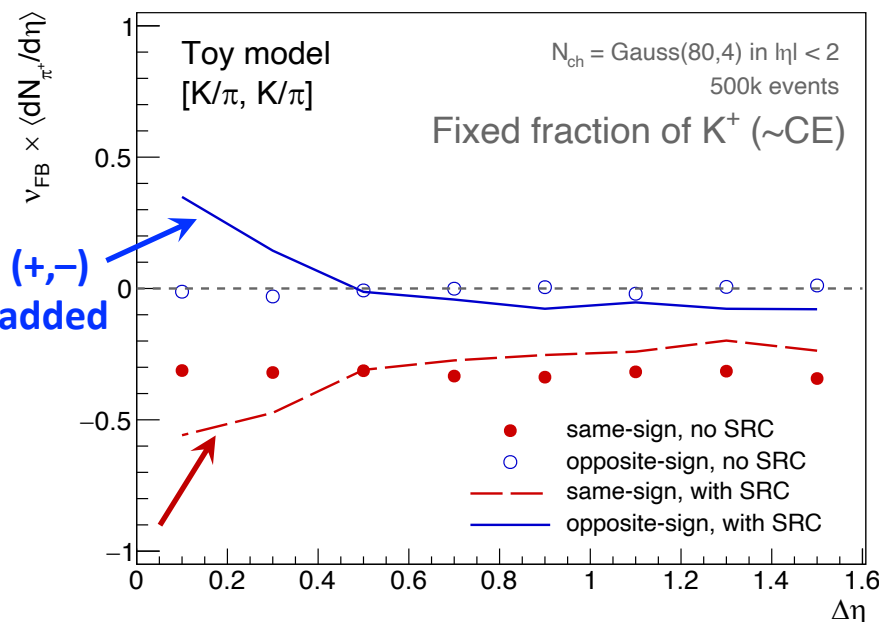
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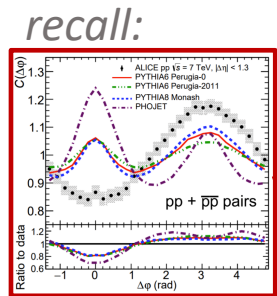


Canonical ensemble for K^+ :



short-range (+,-) correlations added

some same-sign (+,+) pairs "repulse"

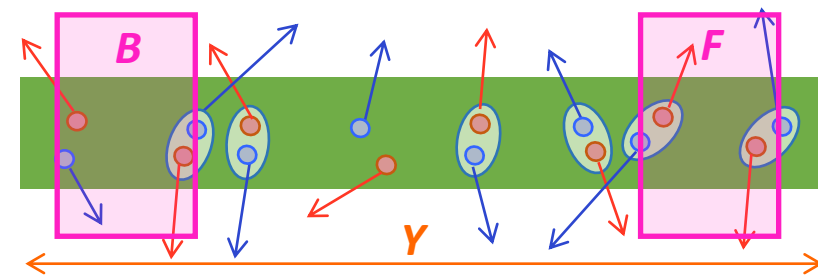


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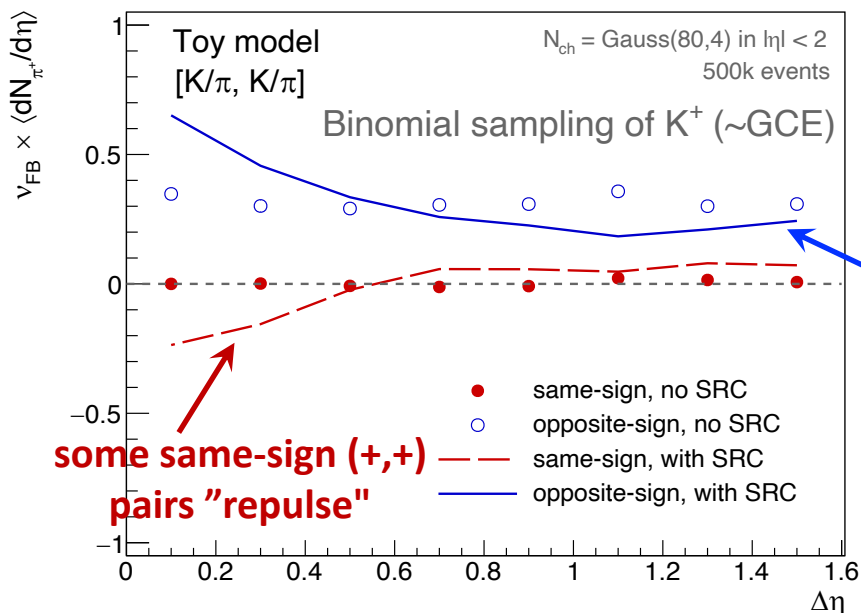


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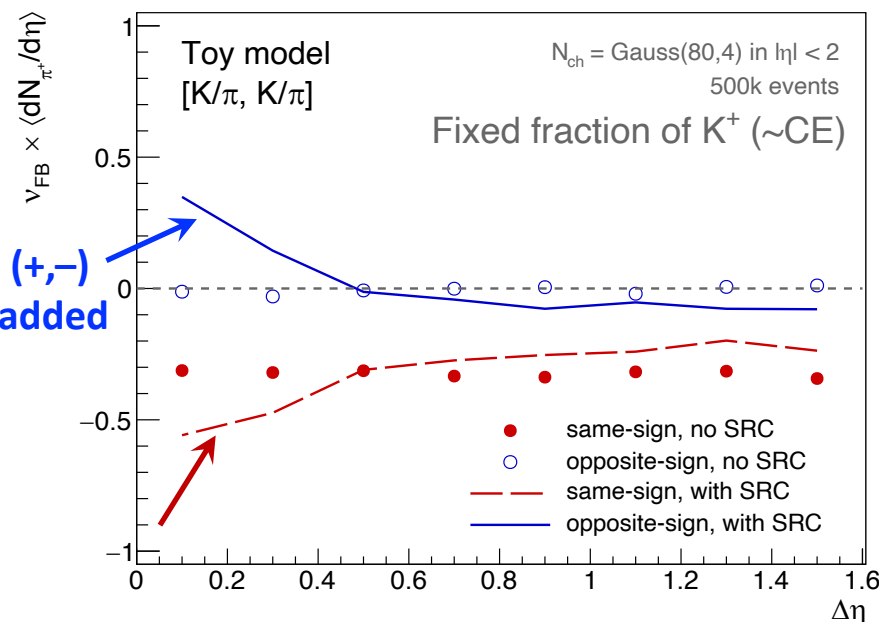
- a binomial distribution of K^+ (others are pions) \rightarrow GCE
- ... or assign a strictly fixed fraction of K^+ \rightarrow CE

$\rightarrow v_{FB}$ allows one to separate short-range effects from "global" fluctuations

Grand Canonical ensemble for K^+ :

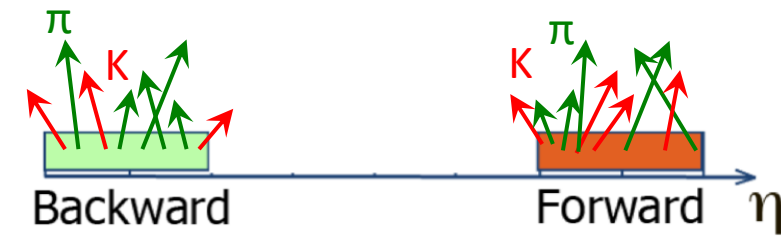


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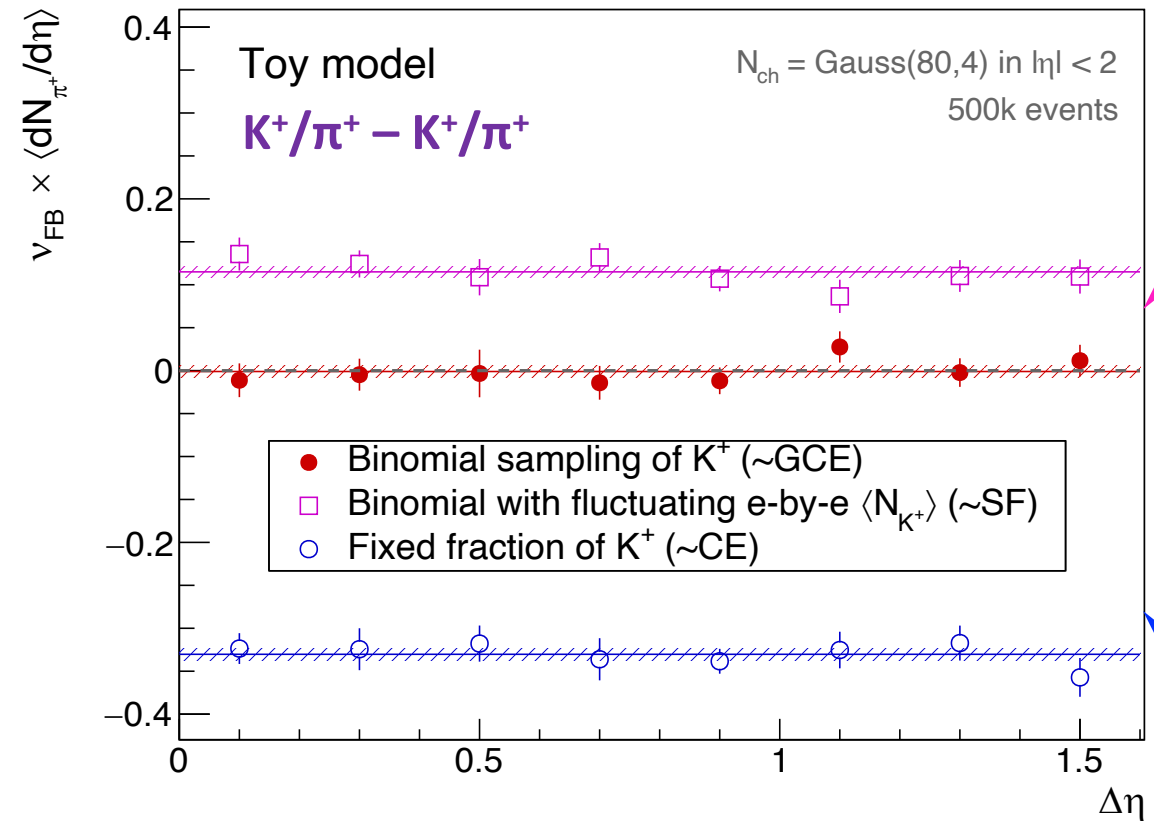


Yield ratio correlations as a measure of fluctuations

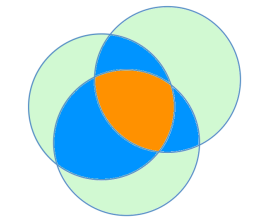
$$v_{FB} = \frac{\langle r^F \cdot r^B \rangle}{\langle r^F \rangle \langle r^B \rangle} - 1$$



- Remove short-range effects, leave only "global" scenarios:



raise in String Fusion-like models
(when strange particle yield depends on source density fluctuates e-by-e)



Nucl. Phys. B 390 542-558 (1993)

suppression for Canonical Ensemble

→ v_{FB} allows one to separate short-range effects from "global" fluctuations

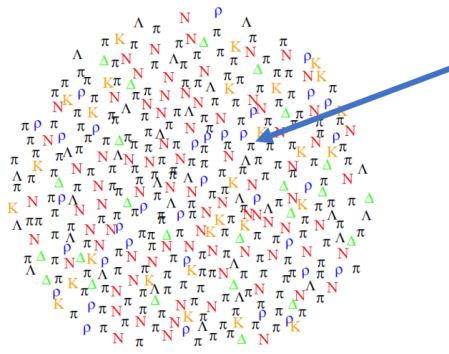
Yield ratio correlations in Hadron Resonance Gas models



Thermal-FIST package

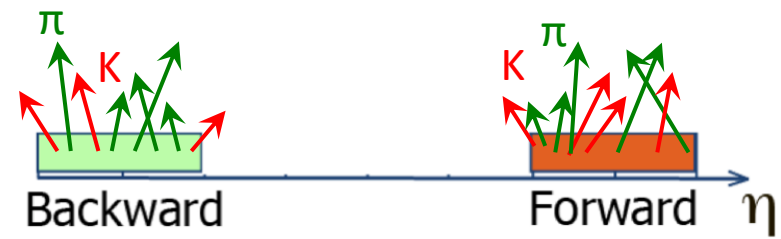
Vovchenko, Stoecker, Comput. Phys. Commun. 244, 295 (2019)

[Source code](#)



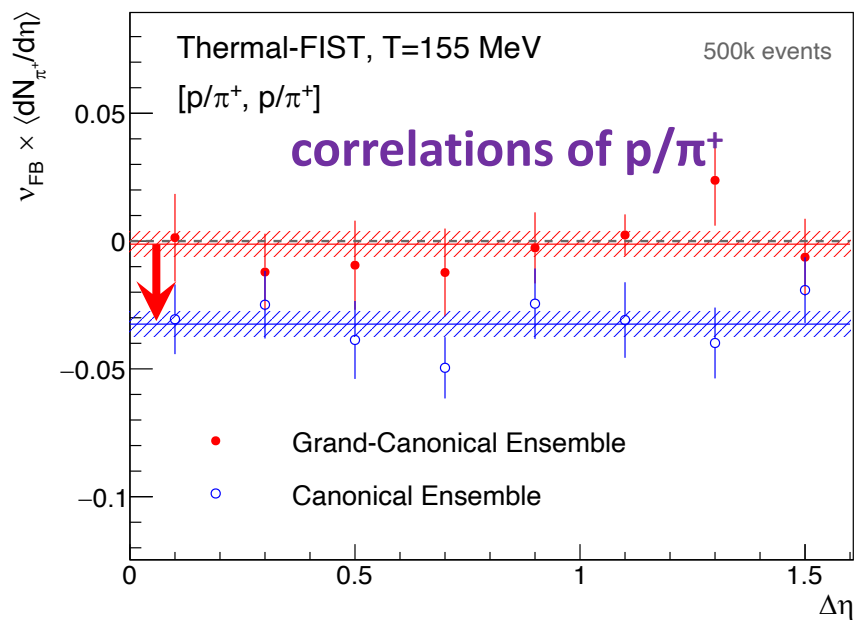
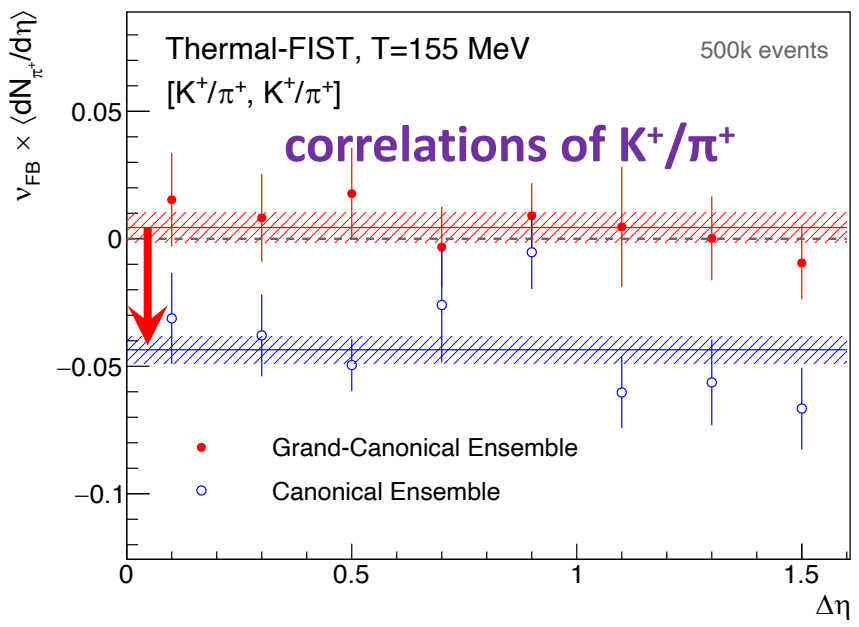
Thermal model:
equilibrated Hadron Resonance Gas at the chemical freeze-out stage

Parameters:
 T – temperature
 μ_B, μ_Q, μ_S – chemical potentials
 V – system volume



$$v_{FB} = \frac{\langle r^F \cdot r^B \rangle}{\langle r^F \rangle \langle r^B \rangle} - 1$$

→ Run in Monte Carlo mode (HRG + radial flow + decays) for *Canonical* and *Grand-Canonical* Ensembles:

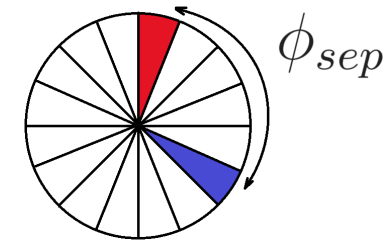


→ The pattern of suppression of v_{FB} for the Canonical Ensemble

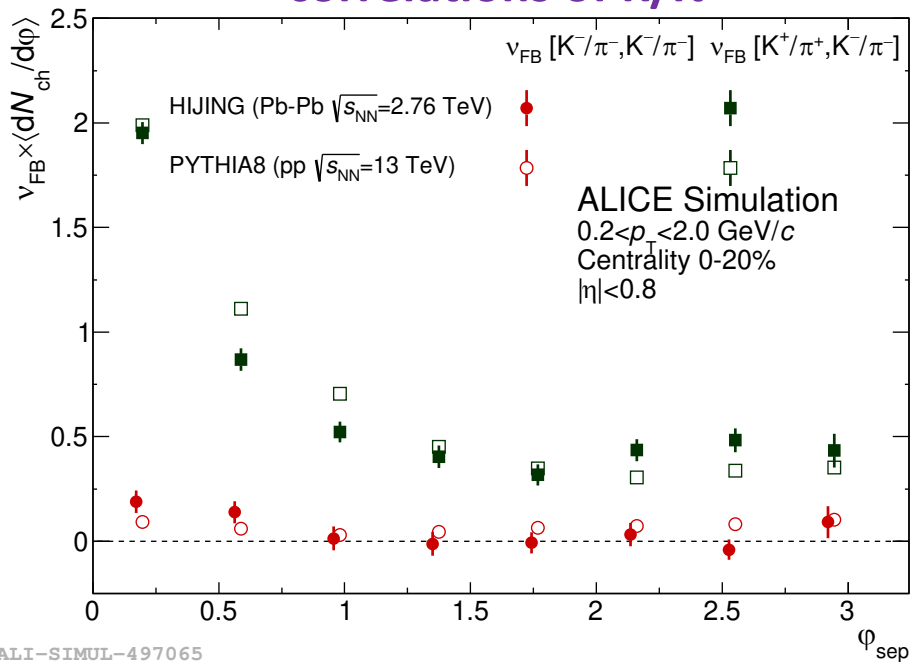
What with realistic models of pp and A-A collisions?

$$v_{FB} = \frac{\langle r^F \cdot r^B \rangle}{\langle r^F \rangle \langle r^B \rangle} - 1$$

Calculations are done in HIJING (Pb-Pb) and PYTHIA (pp) collisions at LHC energies.

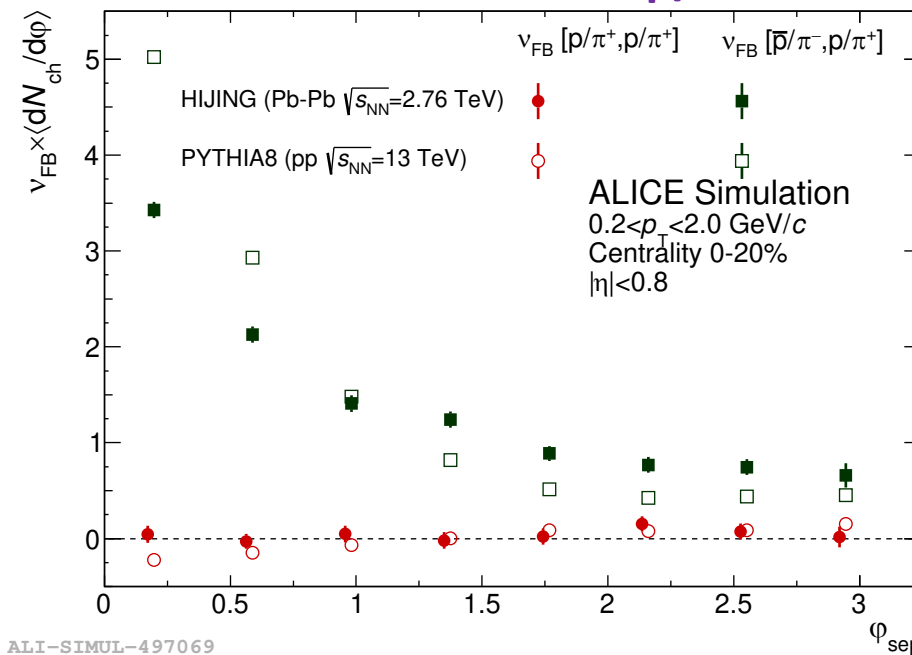


correlations of K/π



ALI-SIMUL-497065

correlations of p/π



ALI-SIMUL-497069

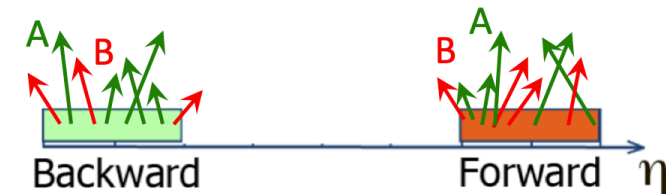
- Similar results for azimuthal intervals

*plots by V. Petrov

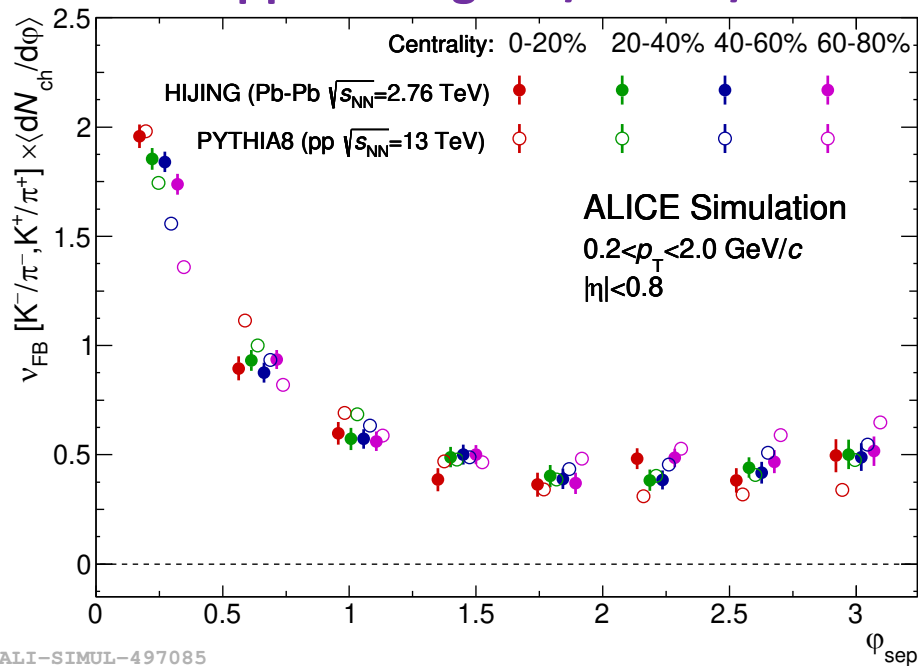
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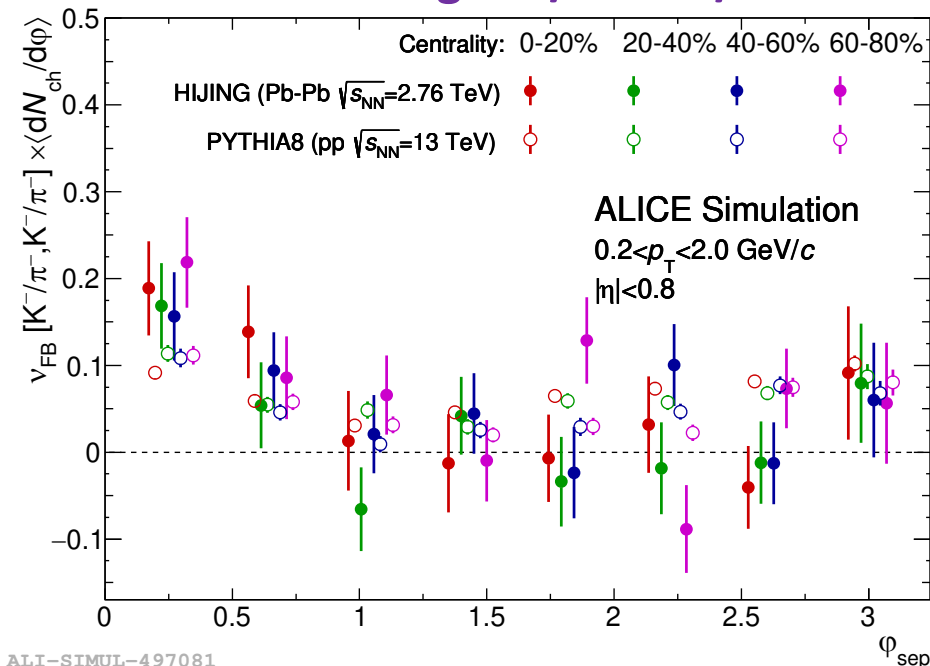


opposite-sign $K^+/\pi^+ - K^-/\pi^-$



ALI-SIMUL-497085

same-sign $K^-/\pi^- - K^+/\pi^+$



ALI-SIMUL-497081

*plots by V. Petrov

- **Centrality (multiplicity) dependence:**
 - HIJING: similar values in all classes due to absence of collective effects
 - PYTHIA: slight dependence, probably due to Color Reconnection

Summary

- **Event-by-event measurements** help to characterize the properties of the “bulk” of the system, they also are closely related to dynamics of the phase transitions.
- **Challenges from the experimental point of view:**
 - fluctuations of the volume of the created system
 - corrections on efficiency and contamination, limited acceptance
 - difficult to interpret the data due to resonance decays, conservation laws
- **Angular correlations between ratios of identified particle yields in two windows** were discussed
 - robust observable, allows to suppress contributions from SRC
 - v_{FB} allows one to separate short-range effects from global fluctuation patterns caused by canonical suppression, etc.
 - experimental studies – to be done

Thank you for your attention!

This work is supported by the Russian Science Foundation, grant 17-72-20045.