Dualities of two-color QCD phase diagram

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RSF | Russian Science Foundation

БАЗИС
Фонд развития теоретической физики и математики

K.G. Klimenko, IHEP
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in the broad sense our group stems from
Department of Theoretical Physics, Moscow State University
Prof. V. Ch. Zhukovsky

details can be found in

Eur.Phys.J.C 80 (2020) 10, 995 arXiv:2005.05488 [hep-ph]
JHEP 06 (2020) 148 arXiv:2003.10562 [hep-ph]
Phys.Rev. D100 (2019) no.3, 034009 arXiv: 1904.07151 [hep-ph]
JHEP 1906 (2019) 006 arXiv:1901.02855 [hep-ph]
Eur.Phys.J. C79 (2019) no.2, 151, arXiv:1812.00772 [hep-ph],
Phys.Rev. D98 (2018) no.5, 054030 arXiv:1804.01014 [hep-ph],
Phys.Rev. D97 (2018) no.5, 054036 arXiv:1710.09706 [hep-ph]
Phys.Rev. D95 (2017) no.10, 105010 arXiv:1704.01477 [hep-ph]

Acknowlegements

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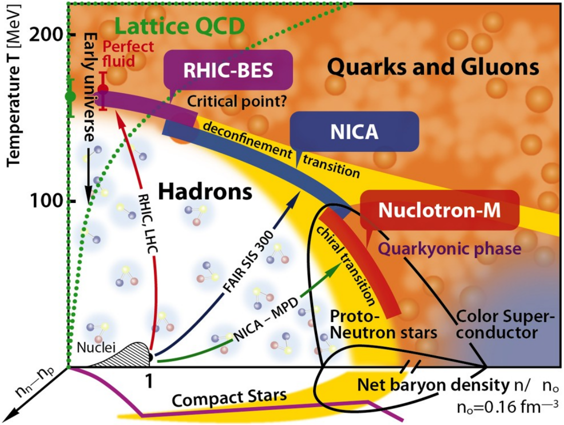
- Foundation for the Advancement of Theoretical Physics and Mathematics

БАЗИС

Фонд развития теоретической физики и математики

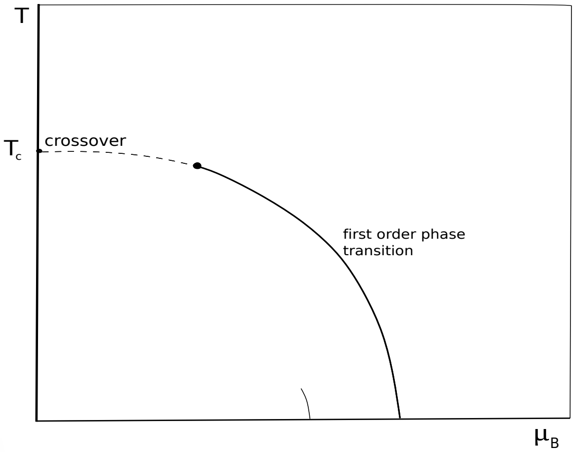
QCD Phase Diagram



Two main phase transitions

- confinement-deconfinement
- chiral symmetry breaking phase—chriral symmetric phase

QCD Phase Diagram



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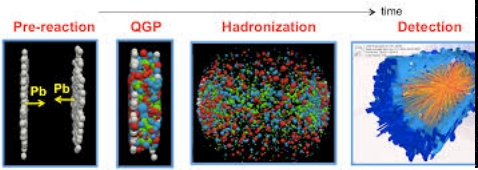
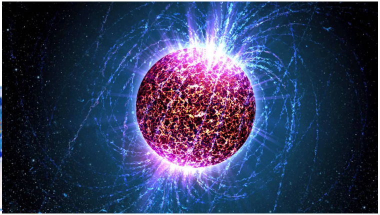
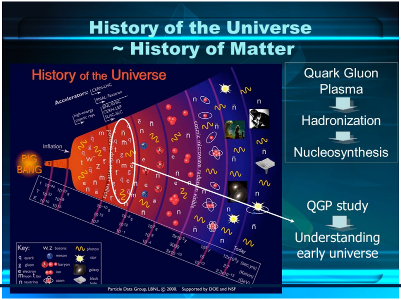


time
Pre-reaction
QGP
Hadronization
Detection
Pb
Pb



Neutron Star
Vancouver

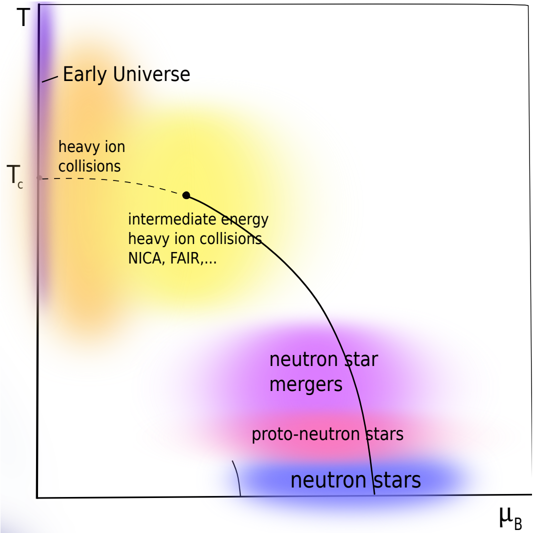


History of the Universe
~ History of Matter
History of the Universe
Accelerators:
Inflation
BIG BANG
Quark Gluon Plasma
Hadronization
Nucleosynthesis
QGP study
Understanding early universe
Key:
Particle Data Group, LBNL, © 2000. Supported by DOE and NSF

QCD Dhase Diagram

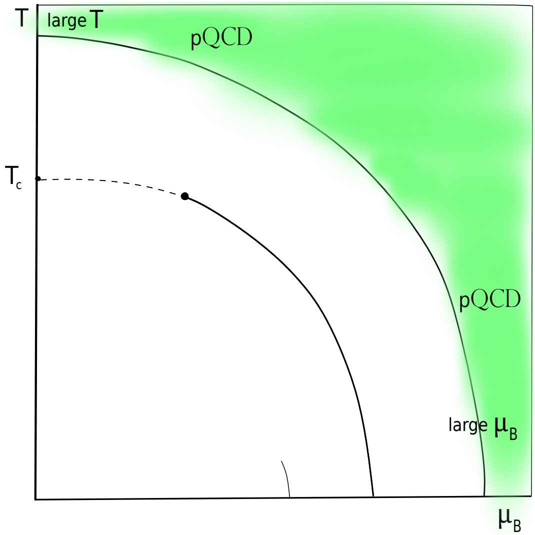
QCD at T and μ
(QCD at extreme conditions)

- Early Universe
- heavy ion collisions
- neutron stars
- proto- neutron stars
- neutron star mergers



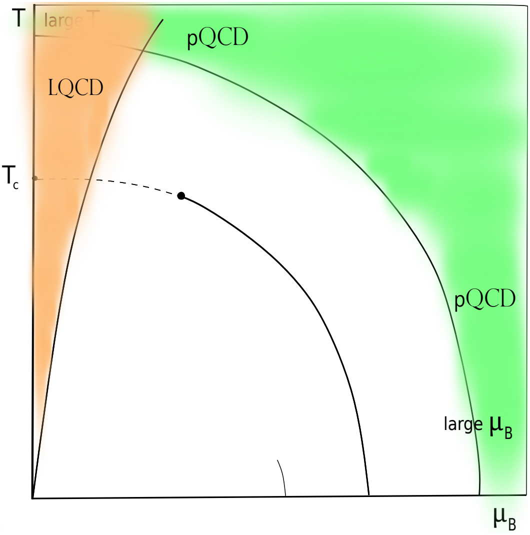
Methods of dealing with QCD

- Perturbative QCD



Methods of dealing with QCD

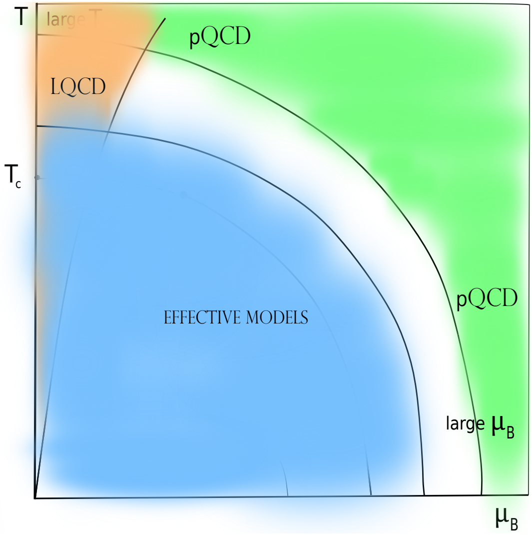
- Perturbative QCD
- First principle calculation – lattice QCD



QCD Dhase Diagram and Methods

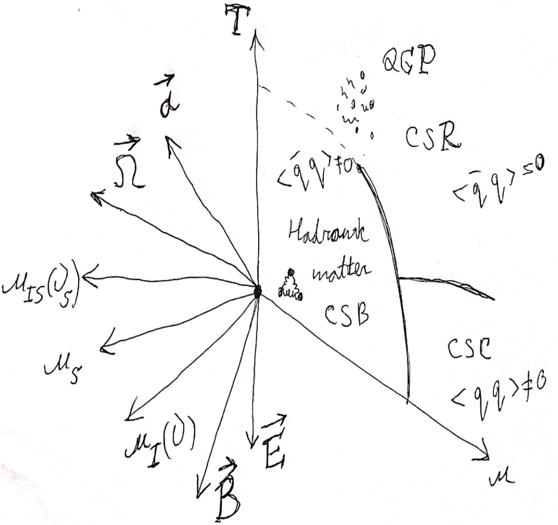
Methods of dealing with QCD

- Perturbative QCD
- First principle calculation – lattice QCD
- Effective models
- DSE, FRG
- .....



More than just QCD at (μ, T)

- more chemical potentials μ_i
- magnetic fields
- rotation of the system $\vec{\Omega}$
- acceleration $\vec{a}$
- finite size effects (finite volume and boundary conditions)



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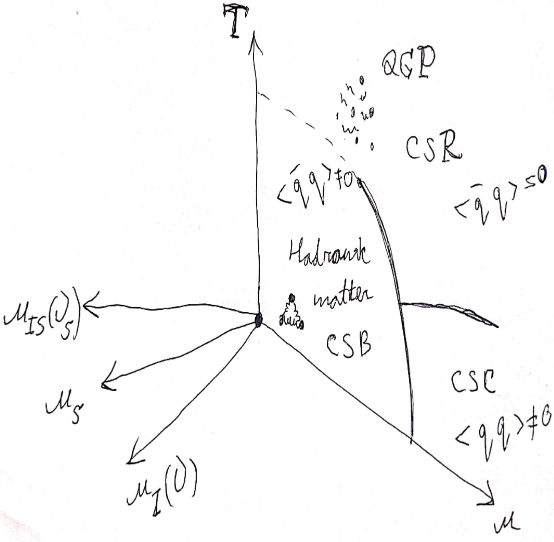

Baryon chemical potential μ_B

Allow to consider systems with non-zero baryon densities.

$$\frac{\mu_B}{3}\bar{q}\gamma^0 q = \mu\bar{q}\gamma^0 q, \qquad n_B = \frac{1}{3}(n_u + n_d)$$

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$$\frac{\mu_B}{3}\bar{q}\gamma^0 q = \mu\bar{q}\gamma^0 q, \qquad n_B = \frac{1}{3}(n_u + n_d)$$

Isotopic chemical potential μ_I

Allow to consider systems with isospin imbalance ($n_n \neq n_p$).

$$\frac{\mu_I}{2}\bar{q}\gamma^0\tau_3 q = \nu\left(\bar{q}\gamma^0\tau_3 q\right)$$

$$n_I = n_u - n_d \quad \longleftrightarrow \quad \mu_I = \mu_u - \mu_d$$

chiral (axial) chemical potential

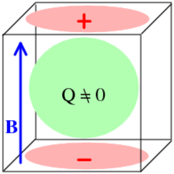
Allow to consider systems with chiral imbalance (difference between densities of left-handed and right-handed quarks).

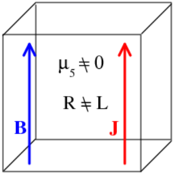
$$n_5 = n_R - n_L \quad \longleftrightarrow \quad \mu_5 = \mu_R - \mu_L$$

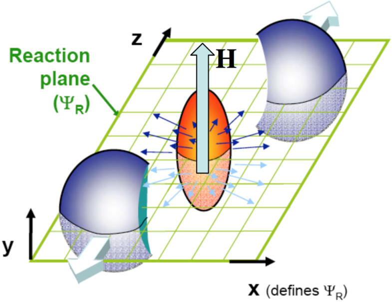
The corresponding term in the Lagrangian is

$$\mu_5 \bar{q} \gamma^0 \gamma^5 q$$

Chiral magnetic effect



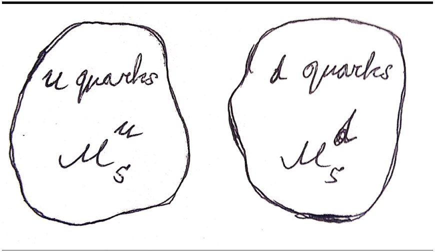




$$\vec{J} \sim \mu_5 \vec{B},$$

A. Vilenkin, PhysRevD.22.3080,
K. Fukushima, D. E. Kharzeev and H. J. Warringa, Phys. Rev. D **78** (2008) 074033

Chiral isospin imbalance



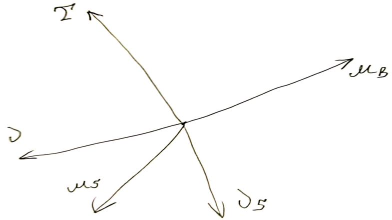
$$\mu_5^u \neq \mu_5^d \qquad \text{and} \qquad \mu_{I5} = \mu_5^u - \mu_5^d$$

Term in the Lagrangian — $\frac{\mu_{I5}}{2}\bar{q}\tau_3\gamma^0\gamma^5 q = \nu_5(\bar{q}\tau_3\gamma^0\gamma^5 q)$

$$n_{I5} = n_{u5} - n_{d5}, \qquad n_{I5} \quad \longleftrightarrow \quad \nu_5$$

Different chemical potentials and matter content

$$\mu = \frac{\mu_B}{3}, \qquad \nu = \frac{\mu_I}{2}, \qquad \mu_5, \qquad \nu_5 = \frac{\mu_{I5}}{2}$$



Dualities

It is not related to holography or gauge/gravity duality

it is the dualities of the phase structures of different systems

Dualities

Chiral symmetry breaking $\Longleftrightarrow$ pion condensation

Isospin imbalance $\Longleftrightarrow$ Chiral imbalance

The TDP

$$\Omega(T, \mu, \mu_i, ..., \langle\bar{q}q\rangle, ...)$$

The TDP

$$\Omega(T, \mu, \mu_i, ..., \langle \bar{q}q \rangle, ...) \qquad \Omega(T, \mu, \nu, \nu_5, ..., M, \pi, ...)$$

The TDP

$$\Omega(T, \mu, \mu_i, ..., \langle \bar{q} q \rangle, ...) \qquad \Omega(T, \mu, \nu, \nu_5, ..., M, \pi, ...)$$

The TDP (phase daigram) is invariant under

Interchange of - condensates - matter content

$$\Omega(M, \pi, \nu, \nu_5)$$

$$M \longleftrightarrow \pi, \qquad \nu \longleftrightarrow \nu_5$$

$$\Omega(M, \pi, \nu, \nu_5) = \Omega(\pi, M, \nu_5, \nu)$$

$\mu_B \neq 0$ impossible on lattice but if $\mu_B = 0$

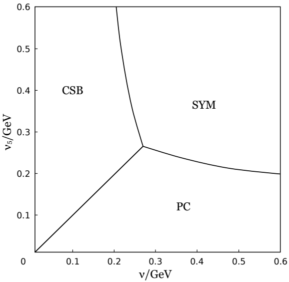
Duality is shown to take place in particular cases in lattice QCD

- **QCD at** μ_5 —- (μ_5, T)

  V. Braguta, A. Kotov et al, ITEP lattice group

- **QCD at** μ_I —- (μ_I, T)

  G. Endrodi, B. Brandt et al, Emmy Noether junior research group, Goethe-University Frankfurt, Institute for Theoretical Physics ()



Two colour QCD case

QC$_2$D

There are a lot similarities:

- similar phase transitions:

  confinement/deconfinement, chiral symmetry breaking/restoration at large T and μ

- A lot of physical quantities coincide up to few dozens percent

  Critical temperature $T_c/\sqrt{\sigma}$, topological susceptibility $\chi^{\frac{1}{4}}/\sqrt{\sigma}$ shear viscosity η/s

There are **no sign problem** in SU(2) case

$$(Det(D(\mu)))^{\dagger} = Det(D(\mu))$$

and lattice simulations at non-zero baryon density are possible

It is a great playground for studying dense matter

Phase diagram of QC_2D

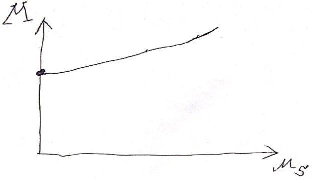
Condensates and phases

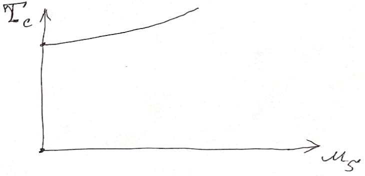
$M = \langle \sigma(x) \rangle \sim \langle \bar{q} q \rangle,$ CSB phase: $M \neq 0,$

$\pi_1 = \langle \pi_1(x) \rangle = \langle \bar{q} \gamma^5 \tau_1 q \rangle,$ PC phase: $\pi_1 \neq 0,$

$\Delta = \langle \Delta(x) \rangle = \langle qq \rangle = \langle q^T C \gamma^5 \sigma_2 \tau_2 q \rangle,$ BSF phase: $\Delta \neq 0.$

QCD at non-zero μ_5

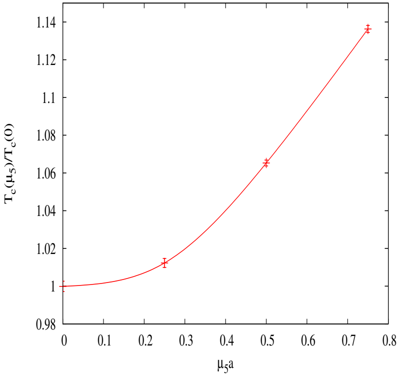




catalysis of CSB by chiral imbalance:

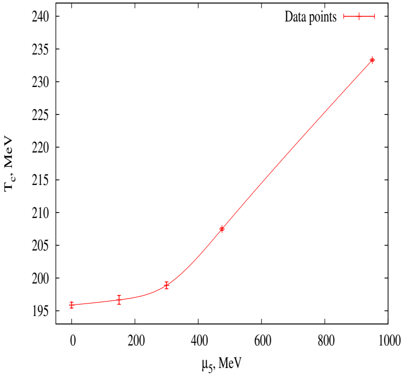
- increase of $\langle \bar{q}q \rangle$ as μ_5 increases
- increase of critical temperature T_c of chiral phase transition (crossover) as μ_5 increases

SU(2)



V. Braguta, A. Kotov et al, JHEP 1506, 094 (2015),PoS LATTICE 2014, 235 (2015)

SU(3)



V. Braguta, A. Kotov et al, Phys. Rev. D 93, 034509 (2016), arXiv:1512.05873 [hep-lat]

Instead of chiral symmetry

$$SU_L(2) \times SU_R(2)$$

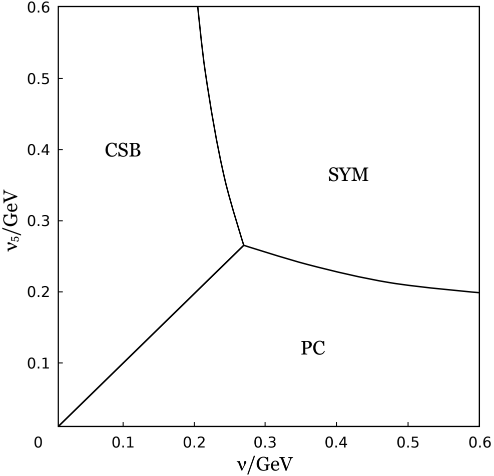
one has Pauli-Gursey flavor symmetry

$$SU(4)$$

Two colour NJL model

$$L = \bar{q}\Big[i\hat{\partial} - m_0\Big]q + H\Big[(\bar{q}q)^2 + (\bar{q}i\gamma^5\vec{\tau}q)^2 + \left(\bar{q}i\gamma^5\sigma_2\tau_2 q^c\right)\left(\overline{q^c}i\gamma^5\sigma_2\tau_2 q\right)\Big]$$

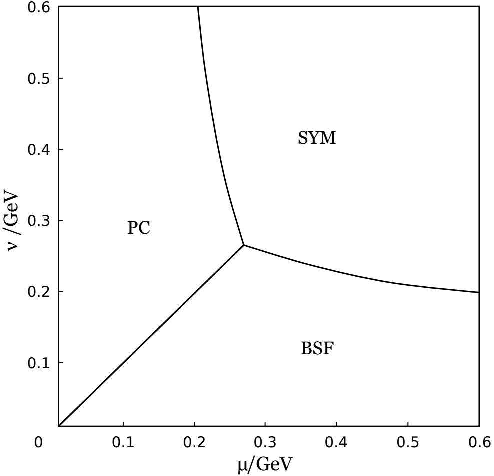
Phase structure: (ν, ν_5)-phase diagram



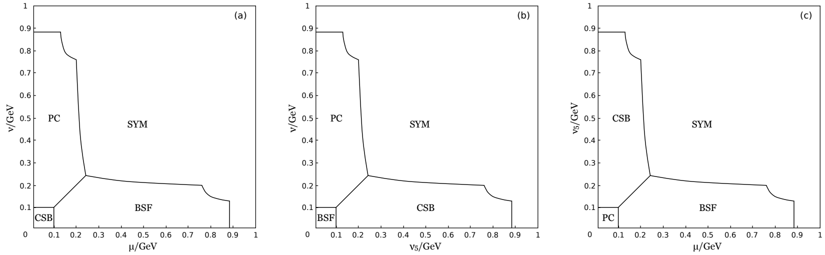
Isospin imbalance ν generates pion condensation

Chiral imbalance ν_5 generates chiral symmetry breaking

Phase structure: (μ, ν)-phase diagram



Baryon density μ generates diquark condensation



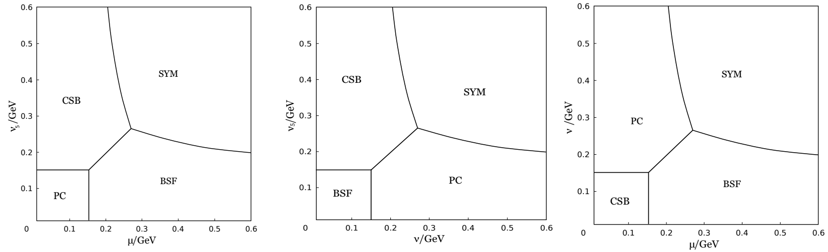
$$(a) \qquad \mathcal{D}_1 : \qquad \mu \longleftrightarrow \nu, \quad \pi_1 \longleftrightarrow |\Delta|, \qquad \text{PC} \longleftrightarrow \text{BSF}$$

J. Andersen, T. Brauner, D. T. Son, M. Stephanov, J. Kogut, ...

$$(b) \qquad \mathcal{D}_3 : \qquad \nu \longleftrightarrow \nu_5, \quad M \longleftrightarrow \pi_1, \qquad \text{PC} \longleftrightarrow \text{CSB}$$

$$(c) \qquad \mathcal{D}_2 : \qquad \mu \longleftrightarrow \nu_5, \quad M \longleftrightarrow |\Delta|, \qquad \text{CSB} \longleftrightarrow \text{BSF}$$

Uses of Dualities in two colour case



- Phase diagram is **highly symmetric** due to **dualities**

  The **whole phase diagram**, including diquark condensation, **in two color case** can be obtained from the results of behaviour of CSB and PC phases .

(μ, ν, ν_5) **phase diagram is highly symmetric due to dualities**

and intermingled by dualities

$$(\mu,\ \nu,\ \nu_5 \,|\, \mu_5)$$

μ_5 **stands alone** from other μ, ν, ν_5 in QC_2D

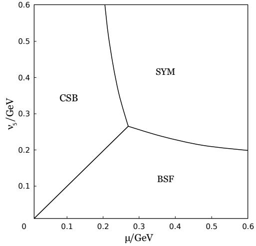
μ_5 respects the dualities but it does not transform under them

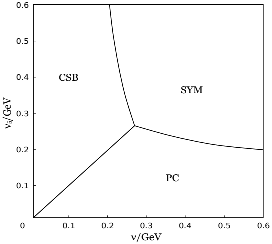
μ_5 deforms the (μ, ν, ν_5) phase diagram

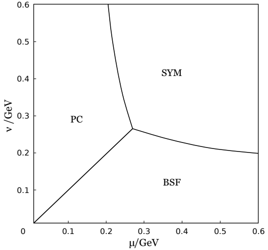
The influence of μ_5 is constrained by dual properties

- Chameleon nature of chiral imbalance μ_5 is also a consequence of dualities
- The feature of μ_5 of being universal catalizer is a consequence of dualities as well

Phase diagram in the regime of moderate values





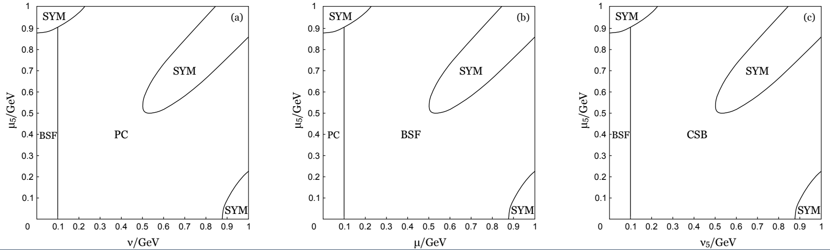


For the moderate values the phase diagram is quite concise

- Baryon density μ $\iff$ diquark condensation
- Isospin imbalance ν $\iff$ pion condensation
- Chiral imbalance ν_5 $\iff$ chiral symmetry breaking

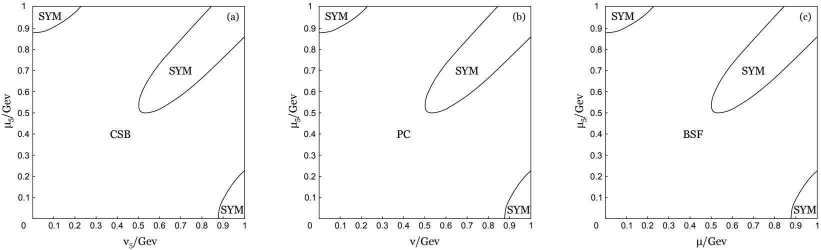
Chiral imbalance μ_5 catalyzes all the phenomena

Catalysis of CSB phenomenon was known as in $SU(3)$ as well as in $SU(2)$ cases



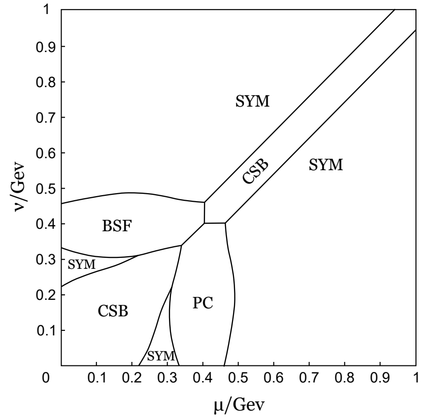
Chameleon nature of chiral imbalance μ_5

μ_5 mimics other chemical potentials μ, ν, ν_5

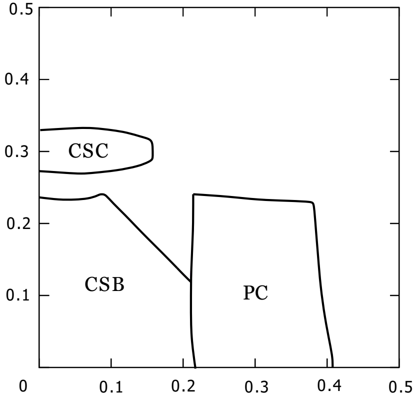


Diquark condensation at $\mu = 0$

Chiral imbalance μ_5 leads to several rather peculiar phases in the system, e. g. the **diquark condensation** in the region of the phase diagram at $\mu = 0$



Chiral imbalance μ_5 leads to the **diquark condensation** in the region of the phase diagram at $\mu = 0$ in three color case



Dualities $\mathcal{D}_1$, $\mathcal{D}_2$ and $\mathcal{D}_3$ were found in

- In the framework of NJL model

- In the mean field approximation

Dualities are connected with Pauli-Gursey group

Dualities were found in

- In the framework of NJL model beyond mean field

- In QC_2D non-pertubartively (at the level of Lagrangian)

Duality $\mathcal{D}$ is a remnant of chiral symmetry

Duality was found in

- In the framework of NJL model beyond mean field or at all orders of N_c approximation
- In QCD non-pertubartively (at the level of Lagrangian)

Conclusions and Thanks for the Attention

- $(\mu_B, \mu_I, \nu_5, \mu_5)$ phase diagram was studied in two color color case
- It was shown that there exist dualities in QCD and QC_2D

  Richer structure of Dualities in the two colour case
- There have been shown ideas how dualities can be used

  Duality is not just entertaining mathematical property but an instrument with very high predictivity power
- Dualities have been shown non-perturbetively in the two colour case
- Duality has been shown non-perturbarively in QCD