DETERMINATION OF ASYMPTOTIC NORMALIZATION COEFFICIENTS BY ANALYTICAL CONTINUATION OF DIFFERENTIAL CROSS SECTIONS

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Asymptotic normalization coefficients (ANC) determine the asymptotics of the wave functions of bound nuclear states a in binary channels $b + c$ and are expressed in terms of the residue in energy of the partial amplitude of elastic bc scattering at the pole corresponding to the binding energy of the a system. ANCs are fundamental nuclear characteristics that are important both in the physics of nuclear reactions and in the physics of nuclear structure. ANCs are on-shell observables, in contrast to the commonly used spectroscopic factors, which are off-shell quantities and cannot be reliably extracted from experimental data.

The role of ANC is especially great in determining the cross sections for nuclear reactions with charged particles at low energies, which are inaccessible for direct measurement due to the large Coulomb barrier. The most important class of such processes is astrophysical nuclear reactions. .

From the above, it follows the importance of knowing the ANCs and including them in the list of important nuclear characteristics along with such quantities as binding energies, the probabilities of electromagnetic transitions, etc.

ANCs cannot be directly measured in an experiment; to determine them, a special analysis of experimental data is required.

ANCs can be determined from the analysis of cross sections of peripheral transfer reactions within the framework of the DWBA. This method of determining ANCs is based on the statement that the cross sections of peripheral reactions should be parameterized in terms of ANCs, rather than spectroscopic factors. ANC values can also be extracted from data on radiative capture reactions at low energies.

There are approaches specifically designed to determine ANCs without making assumptions based on specific nuclear models. One of such approaches is the method based on the analytical continuation in energy of the partial amplitudes of elastic scattering, determined from the phase-shift analysis of experimental data, to the pole point located in the unphysical region of negative energies. However, there are not so many sufficiently accurate results of phase-shift analyses in published sources, which limits the field of applicability of this method.

In this work, to determine the ANC, a method (MCP) is used, which is also based on the analytical continuation, but not the partial amplitudes in energy, but the experimental differential cross sections (DCS) of nuclear transfer reactions in the variable $z = \cos\theta$, where θ is the scattering angle in the CM system. In what follows, I will use the notation $\sigma(E, z)$ for the DCS.

The idea of the method goes back to the work of G.F. Chew (1958), in which it was indicated that the extrapolation over $\cos\theta$ of the DCS of NN scattering to the pole corresponding to the virtual exchanged pion can be used to determine the pion-nucleon coupling constant.

In our case, we consider the nuclear reaction $A + x \to B + y$ with the transfer of the particle c, the contribution to the amplitude of which is made by the pole Feynman diagram 1a.

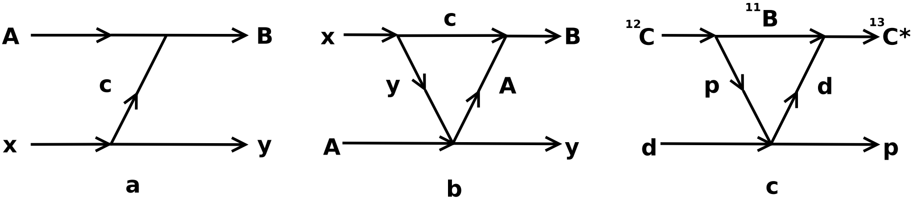

Figure:

In view of the contribution of the mechanism corresponding to diagram 1a, the DCS $\sigma(E, z)$ of the reaction has a second-order pole at $z = z_p > 1$. Introduce the quantity

$$\varphi(E, z) = (z - z_p)^2 \sigma(E, z). \quad (1)$$

$\varphi(E, z)$ has a finite limit at $z \to z_p$ and therefore can be approximated as:

$$\varphi(E, z) = \sum_{n=0}^{N} c_n(E) P_n(z), \quad (2)$$

where $P_n(z)$ are some polynomials. In particular, $P_n(z) = z^n$. The idea behind the method is that the coefficients $c_n(E)$ are fitted to the experimental values of $\varphi(E, z)$ in the physical region $-1 \leq z \leq 1$, after which the approximated value $\varphi(E, z)$ is continued analytically into the unphysical domain z up to $z = z_p > 1$.

At the point $z = z_p$, the contribution of non-pole mechanisms to $\varphi(E, z)$ vanishes due to the presence of the factor $(z - z_p)^2$ and the remaining contribution of the pole mechanism 1a is expressed in terms of ANCs corresponding to the vertices $x \to y + c$ and $A + c \to B$.

From the expression for the contribution of the pole mechanism (diagram 1a) to the DCS in the case when the spin of the transferred particle c is equal to 0 or 1/2, we can obtain the relation:

$$\sum_{L_x, S_x} C^2(L_x S_x) \cdot \sum_{L_B, S_B} C^2(L_B S_B) = 16 \frac{(2J_A + 1)(2J_c + 1)}{2J_B + 1} g^4 m_c^2 \frac{p}{p'} E E' R(E), \quad (3)$$

where $C(LS)$ are ANCs, $g^2 = m_y m_A / (m_x m_B)$ and

$$R(E) = \lim_{z \to z_p} \varphi(E, z) = \lim_{z \to z_p} [(z - z_p)^2 \sigma(E, z)]. \quad (4)$$

Having found the value of $R(E)$ on the basis of experimental data, it is possible to determine the product of ANCs squared on the left side (3). If the ANC values for one of the vertices are known (for example, if this is the $d \to n + p$ vertex), then from (3) and (4) information about the ANC for the second vertex is obtained.

ACCOUNT OF COULOMB EFFECTS

The Coulomb interaction significantly affects the analytical properties of the amplitudes of the processes. It was shown (A.M.Mukhamedzhanov) that the inclusion of the Coulomb interaction between particles in a Feynman diagram does not change the position of the singularities of the diagrams, but changes their type and power. In the case of the pole mechanism (diagram 1a), taking into account the Coulomb interaction in the initial, final, and intermediate states and/or at the vertices of the diagram leads to the fact that the pole singularity turns into a branch point. For three-ray vertices in diagram 1a, the inclusion of the Coulomb interaction between y and c and between A and c leads to the appearance of additional factors $(z - z_p)^{\eta_{xyc}}$ and $(z - z_p)^{\eta_{BAc}}$, where η_{ijk} is the Coulomb parameter for the bound state.

We consider the (d, p) reaction, in which the transferred particle c in diagram 1a is a neutron; therefore, there are no Coulomb effects at the vertices of diagram 1a. In this case, the remaining Coulomb corrections, which we consider, do not change the factor $(z - z_p)^{-2}$ in the expression for the DCS. In reality, we consider corrections due to the Coulomb interaction in the initial and final states, as well as the Coulomb interaction in the intermediate state.

The interaction in the intermediate state is determined by the sum of the diagrams of an infinite rescattering series in the three-body system A, x, c, on which the 4-ray vertices correspond to pure Coulomb scattering. The first of the diagrams of this series is shown in Fig. 1b. The amplitudes of all diagrams of such a series have a pole at the same point $z = z_p$ as diagram 1a, and in fact lead to a renormalization of the residue at this pole.

The Coulomb corrections to the DCS for the pole mechanism were estimated earlier by studying the asymptotics of the partial amplitudes $f_l(E)$ of the reaction with respect to the angular momentum variable l and using the unambiguous relationship between the form of the asymptotics of $f_l(E)$ in l and the behavior of the total amplitude of the reaction $f(E,z)$ near its nearest singularity in z (V.S.Popov). The analytical expressions obtained for these corrections are very cumbersome, and I do not present them here. These expressions are given in the review
L.B., A.M.Mukhamedzhanov, A.N.Safronov, EChAYa, **15**, 1296.

We used the MCP to determine the ANC for the vertex
$^{13}\mathrm{C}(1/2^+;\ 3.09\ \mathrm{MeV}) \rightarrow ^{12}\mathrm{C}(0^+;\ 0\ \mathrm{MeV}) + n$

by analytic continuation of the differential cross section for the reaction
$^{12}\mathrm{C}(0^+;\ 0\ \mathrm{MeV})(d,p)^{13}\mathrm{C}(1/2^+;\ 3.09\ \mathrm{MeV})$.

In diagram 1a, for this reaction $A = ^{12}\mathrm{C}$, $x = d$, $B = ^{13}\mathrm{C}$, $y = p$.

The results are presented in the table.

Table: Calculated ANCs

E_d, MeV	β	β_t	C_0, $\mathrm{fm}^{-1/2}$	C, $\mathrm{fm}^{-1/2}$
3.7	0.0822	0.0853	0.543	1.858
5.03	0.156	0.162	0.783	1.948
9.0	0.357	0.369	1.320	2.174
12.0	0.469	0.484	1.238	1.780
30.0	0.776	0.797	1.379	1.543

Coulomb correction β was calculated taking into account only the interaction in the initial and final states, and β_t additionally takes into account the Coulomb interaction in the intermediate state. β and β_t are close, from which we can conclude that the main contribution to the corrections is made by the Coulomb interaction in the initial and final states. β_t (or β) must be added as a factor to the left side of (3). In the 4th and 5th columns of the table we present the ANCs calculated without (C_0) and with (C) Coulomb corrections. ANC C is related to C_0 as $C = C_0/\sqrt{\beta_t}$.

It follows from a comparison of the values of C and C_0 that the considered Coulomb corrections have a significant effect on the ANC values obtained with the help of the MCP. This effect is especially noticeable at low energies.

From presented in table ANC values obtained from the analysis of various experimental data, it follows that the mean value of the ANC obtained is $C = 1.86 \pm 0.16$ fm$^{-1/2}$.

In the paper J.T.Huang et al, At. Data Nucl. Data Tables **96**, 824 (2010), from the analysis of data on the radiative capture $^{12}\mathrm{C}(n,\gamma)^{13}\mathrm{C}$, $C = 1.61$ fm$^{-1/2}$ was found. An analytical continuation in energy of the data of phase-shift analysis of elastic neutron scattering by $^{12}\mathrm{C}$ gives a significantly higher value
$C = 2.07 \pm 0.13$ fm$^{-1/2}$ (L.B. et al., PRC C**100**, 024627 (2019)).
Thus, the value of ANC C obtained in this work lies between the values given in the above cited papers.

CONCLUSIONS

In this work, the MCP is used to determine the ANC for the channel
$^{13}\mathrm{C}(1/2^+;\ 3.09\ \mathrm{MeV}) \to {}^{12}\mathrm{C}(0^+;\ 0\ \mathrm{MeV}) + n$
based on experimental data on the differential cross sections for the reaction
$^{12}\mathrm{C}(0^+;\ 0\ \mathrm{MeV})(d,p)^{13}\mathrm{C}(1/2^+;\ 3.09\ \mathrm{MeV})$
at different energies.

An essentially new element is the allowance for the corrections due to the Coulomb interaction in the initial, final, and intermediate states of the reaction. It is shown that these corrections strongly affect the extracted ANC values, especially at low energies.

MCP has some advantages over other methods for determining ANCs based on experimental data. It does not rely on the use of any nuclear models, as in the determination of ANCs from the analysis of transfer reactions within the framework of the DWBA. In contrast to the method based on the analytical continuation of the phase-shift analysis data in energy, the MCP uses only the directly measured quantity, the DCS of the reaction. The accuracy of determining the ANC using the MCP is mainly determined by the presence of sufficiently accurate experimental data on the differential cross sections at small scattering angles.

The MCP was used in a number of rather old papers (Borbely, Dumbrajs) to determine the nuclear vertex constants proportional to ANCs for several lightest and light nuclei. However, these works did not take into account the Coulomb corrections considered in this work, and also ignored the problem associated with nonzero orbital angular momenta at the vertices of the pole diagram. These circumstances force us to treat the results obtained in these papers with caution.

Thank You for Patience