From Quark and Nucleon Correlations to Nuclear Drip-line

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- Motivation
- The model: Strongly Correlated Quark Model (SCQM)
  - Nucleon Structure
  - Nucleus Structure
- Face Centered Cubic (FCC) Nuclear lattice
- Nuclear Drip line
- Summary

What is Nuclear Structure?

- **Nuclear Models** in terms of nucleons and mesons

  Conventional models
  - Independent Particle Models (Shell Model, …)
  - Collective models (Liquid Drop Model, …)
  - Cluster models
  - Modifications of above models
- Non-conventional model
  - Lattice models
  - many others …
- There are more than 40 models … (*W.Greiner et al.)*

Do nuclei possess lattice-like structure?

- **Nuclear Models** in terms of nucleons and mesons

  Conventional models

  – Independent Particle Models (Shell Model.

  – Collective models (Liquid Drop Model, …)

  – Cluster models

  – Modifications of above models

Lattice Model based on
Quark structure of Nucleon

- Non-conventional model
  – Lattice models
  – many others …
- There are more than 40 models … (*W.Greiner et al.)*

QCD – fundamental theory of strong interactions

- **Constituents of hadrons** – **quarks** of different flavors carrying spin, charge, color.
  - **flavors: u, d, s, c, b, t**
  - **spin: ½**
  - **charge: ⅓ , ⅔**
  - **color: $SU(3)_{Color}$** - *R, G, B*
- **Fields – gluons** $(R\bar{R}, G\bar{G}, B\bar{B}, RG, RB, GR, GB, BR, BG)$ perform interactions between quarks.
- **Nucleons –** 3–quark (**u/d**), color-singlet systems
- **Mesons –** quark-antiquark systems

The Model:

Strongly Correlated Quark Model of Hadron Structure

↓

Strongly Correlated Quark Model of Nucleus Structure

Quarks – Solitons

SCQM $\equiv$ Breather Solution of Sine-Gordon equation

$$\partial_\mu \partial^\mu \phi(x,t) + \sin\phi(x,t) = 0$$

Breather – oscillating soliton-antisoliton pair, the periodic solution of SG:

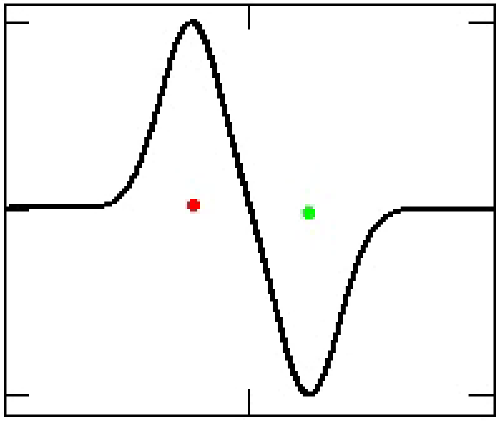
$$\phi(x,t)_{s-as} = 4\tan^{-1}\left[\frac{\sinh\left(ut/\sqrt{1-u^2}\right)}{u\cosh\left(x/\sqrt{1-u^2}\right)}\right]$$

$$\varphi(x,t)_{s-as} = \frac{\partial\phi(x,t)_{s-as}}{\partial x}$$

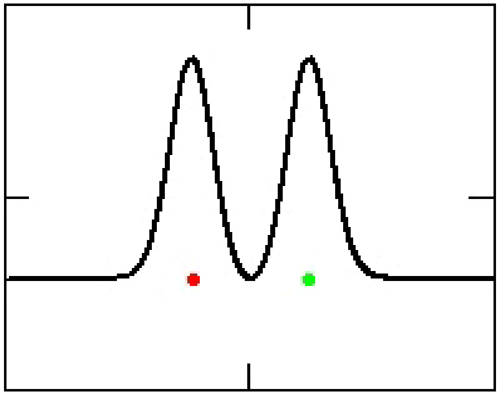
is identical to our quark-antiquark system.

quark – antiquark pair
soliton – antisoliton pair

$\varphi(x,t)$



$\varphi^2(x,t)$



The Strongly Correlated Quark Model (SCQM)

Hamiltonian of the quark – antiquark System

$$H = \frac{m_{\bar{q}}}{(1-\beta_{\bar{q}}^{\;2})^{1/2}} + \frac{m_q}{(1-\beta_q^{\;2})^{1/2}} + V_{\bar{q}q}(2x)$$

$m_{\bar{q}}, m_q$ - current masses of quarks,

$\beta = \beta(x)$ – velocity of the quark (antiquark),

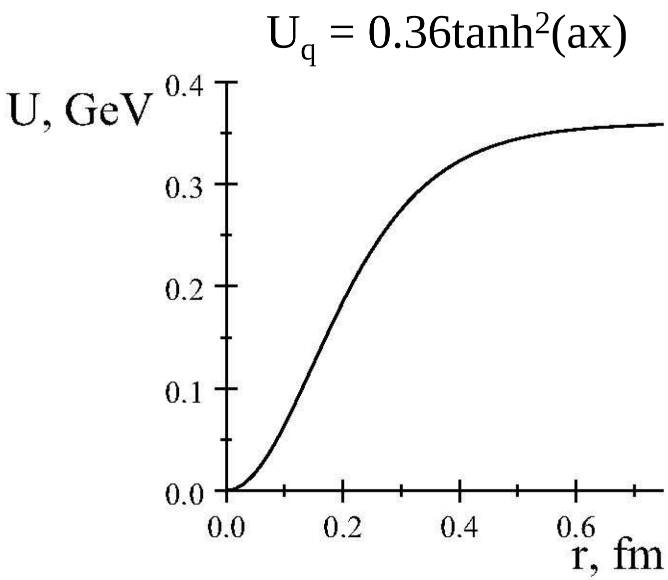
$V_{\bar{q}q}$ - quark–antiquark potential.

$$H = \left[\frac{m_{\bar{q}}}{(1-\beta_{\bar{q}}^{\;2})^{1/2}} + U(x)\right] + \left[\frac{m_q}{(1-\beta_q^{\;2})^{1/2}} + U(x)\right] = H_{\bar{q}} + H_q$$

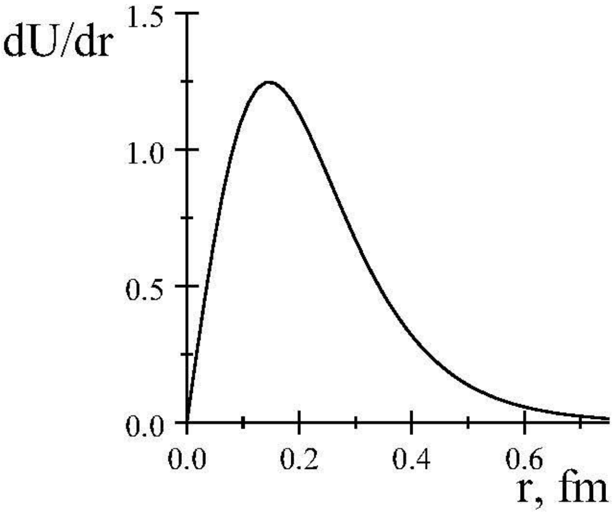
$U(x) = \frac{1}{2} V_{\bar{q}q}(2x)$ is the potential energy of a single quark/antiquark.

$$U(x) = \frac{1}{2} V_{\bar{q}q}(2x) = m \tanh^2(ax)$$

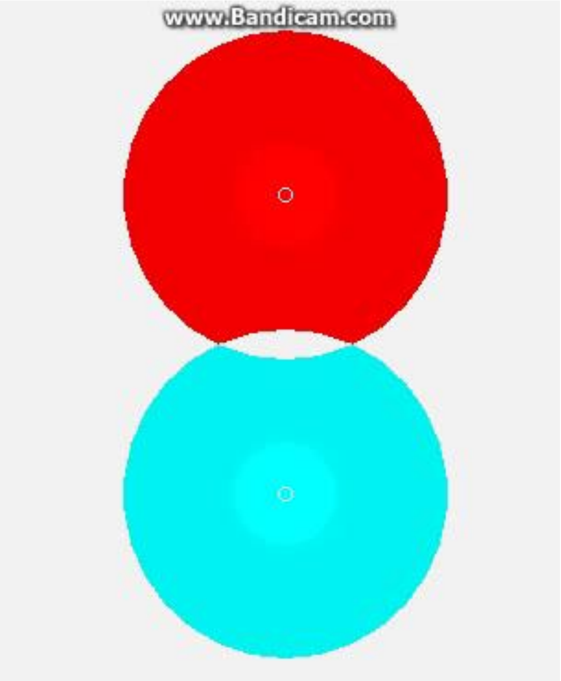
Quark Potential



Force
of quark-antiquark interaction



quark–antiquark pair
meson



QCD: Exchange by gluons $\sqrt{\frac{1}{2}}(R\bar{R}+B\bar{B})$

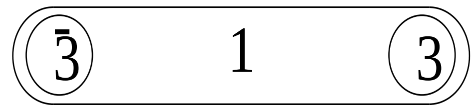
SCQM: Overlap of color fields

Generalization to the 3 – quark system (baryons)

$$SU(3)_{Color}$$

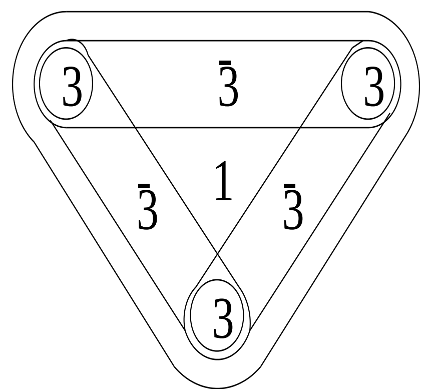
$$q \Rightarrow SU(3) \Leftrightarrow RGB \qquad \bar{q} \Rightarrow SU(\bar{3}) \Leftrightarrow CMY$$

$$\bar{q}q \Rightarrow$$

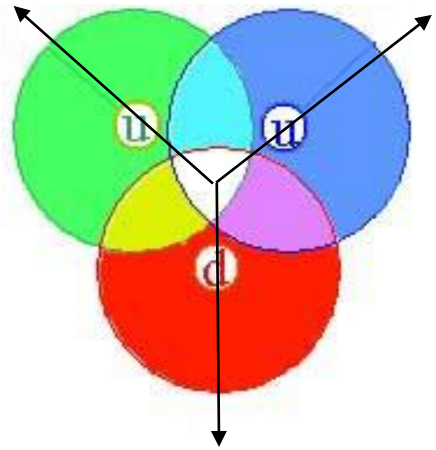


$$qq \rightarrow 3 \times 3 = 6 \oplus \bar{3} \qquad \Rightarrow \qquad \bar{q} \rightarrow qq$$

$$qqq \Rightarrow$$



Nucleon

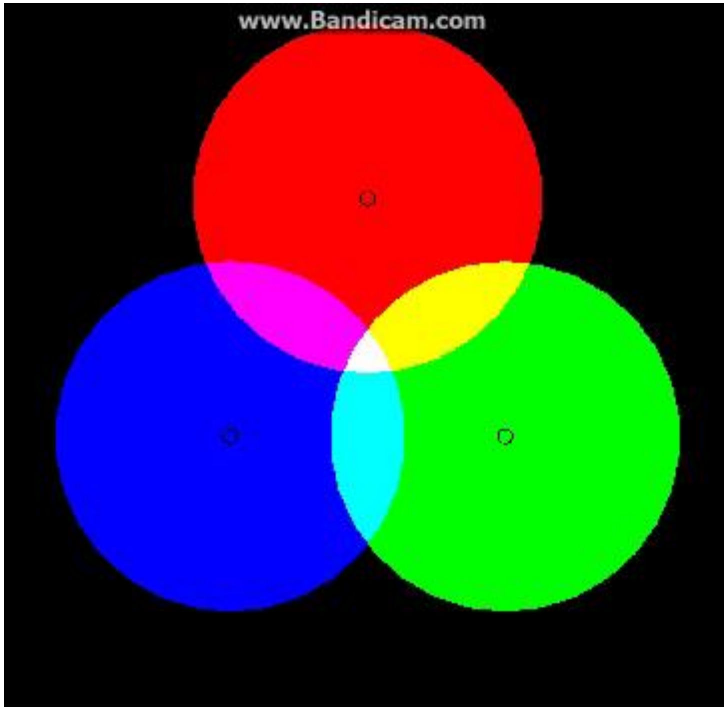


Nucleon wave function composed of color quarks

$$\psi(x) \rightarrow \frac{1}{\sqrt{6}} \sum_{ijk} e_{ijk} \left|c_i\right\rangle \left|c_j\right\rangle \left|c_k\right\rangle$$

Where $\left|c_i\right\rangle$ are orthonormal states with i,j,k → R,G,B

Nucleon

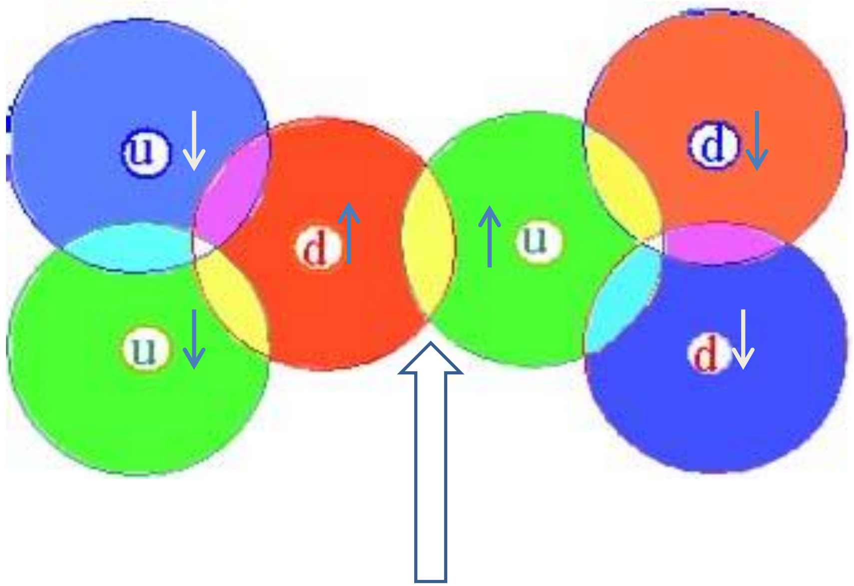


Quark Arrangement inside Nuclei

Strongly Correlated Quark Model

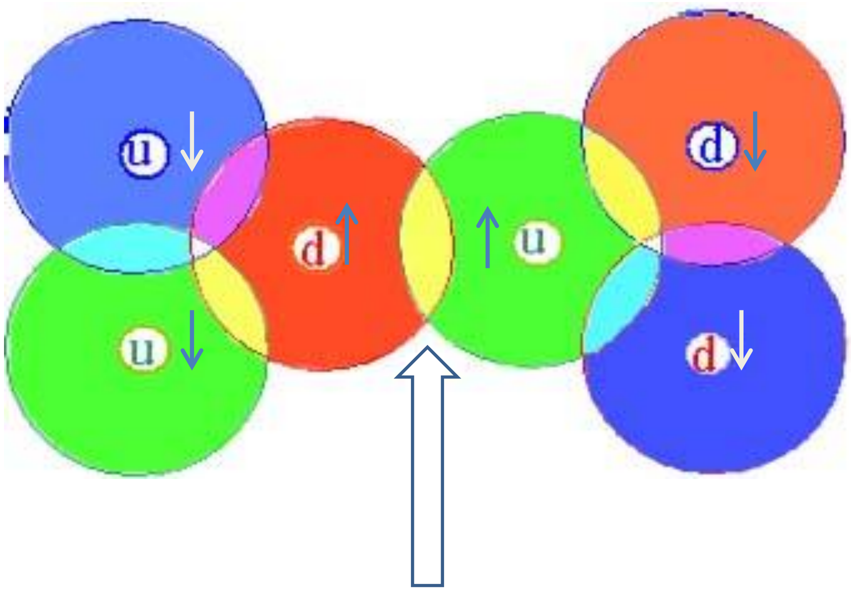
Lattice-like arrangement of Nuclear Structure

Two Nucleon System in SCQM



Interaction between nucleons is due to **overlap** of their quark color fields

Two Nucleon System in SCQM

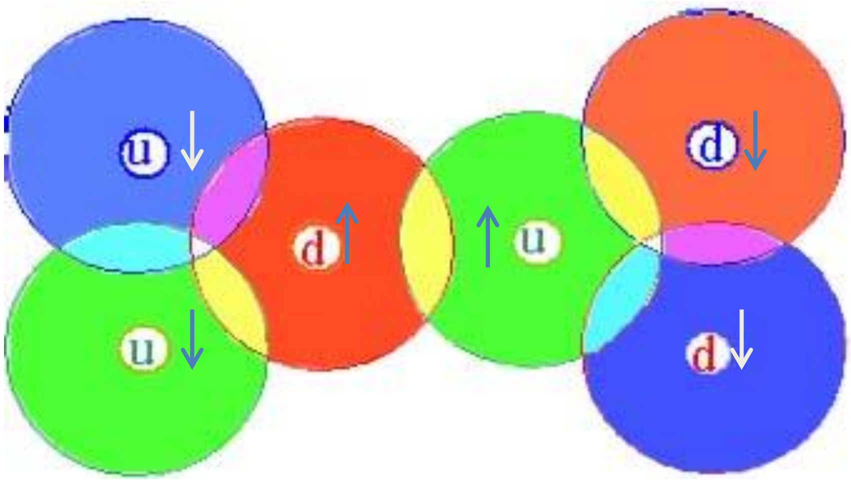


Antisymmetrization

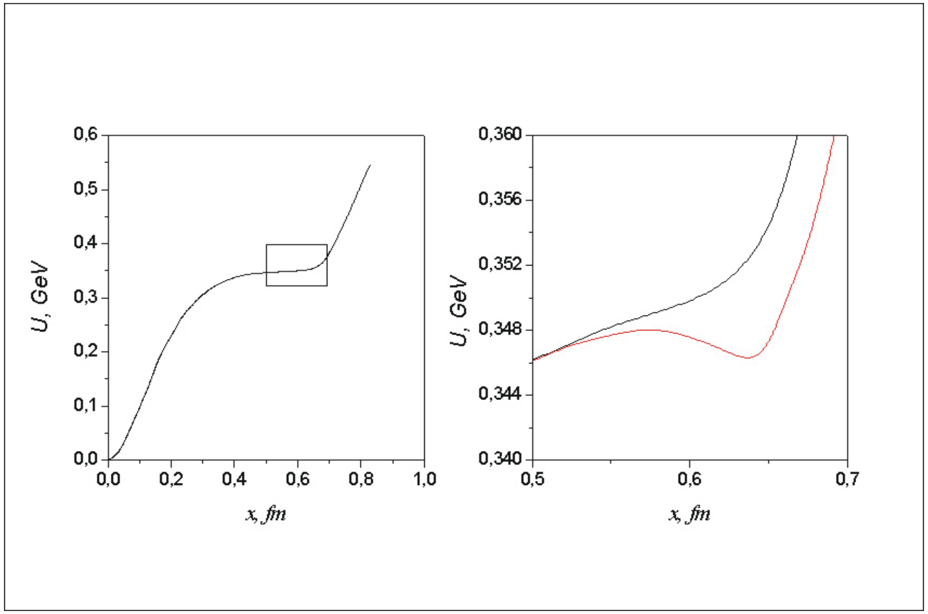
Selection rules for binding two quarks of neighboring nucleons at a junction:

- $SU(3)_{Color}$ – of different colors
- $SU(2)_{Flavor}$ – of different flavors
- $SU(2)_{Spin}$ – of parallel spins

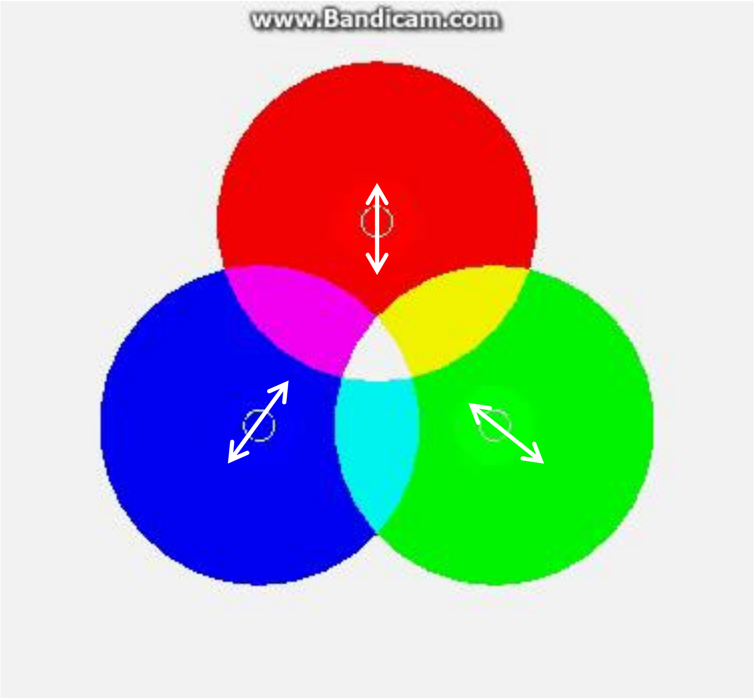
Two Nucleon System in SCQM



Quark Potential Inside Nuclei

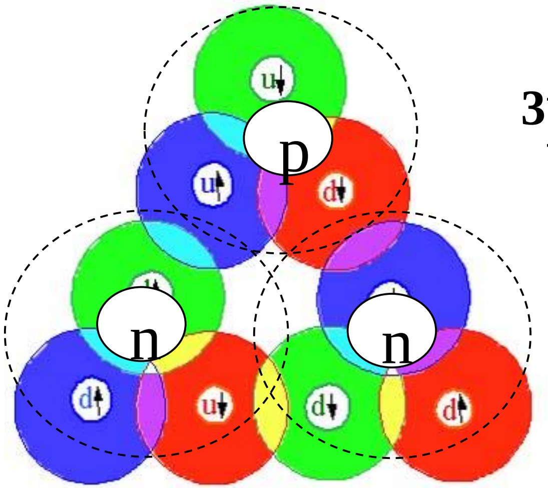


Quarks inside nucleus

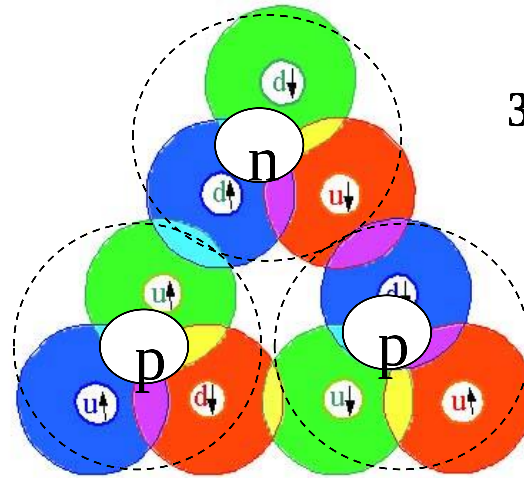


Quarks oscillate with small amplitudes near maximal displacements

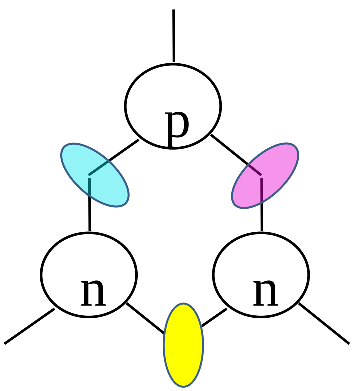
Three Nucleon Systems in SCQM



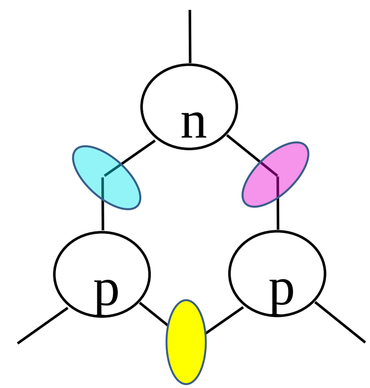
^3H



^3He



Summary color of 3 junctions is white, total color charge = zero!

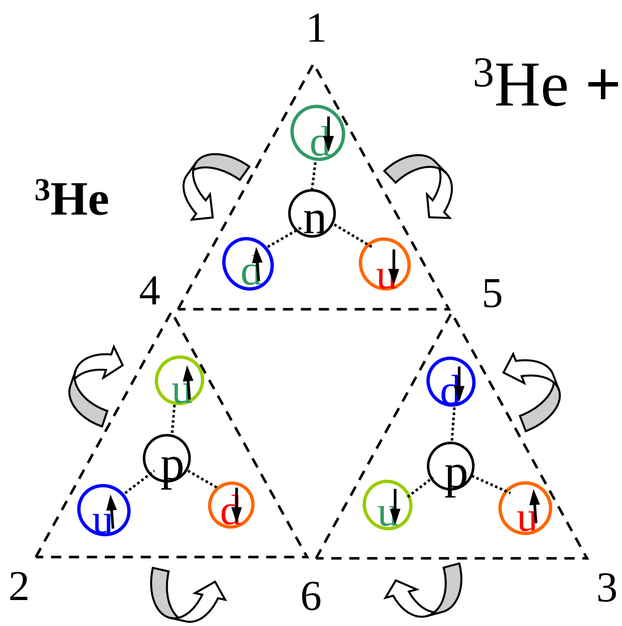


Quark loop formed by 3 nucleons → 3–body force

4-nucleon system: ^{4}He

n

^{3}He + neutron or ^{3}H + proton

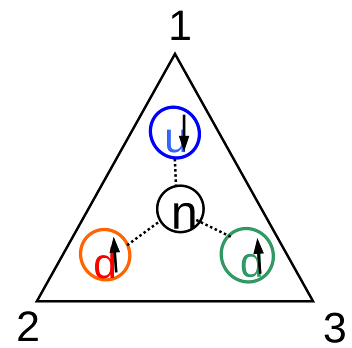


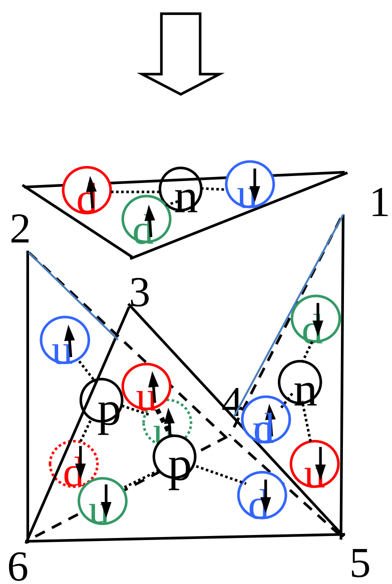
Junctures

1 ←→ 1

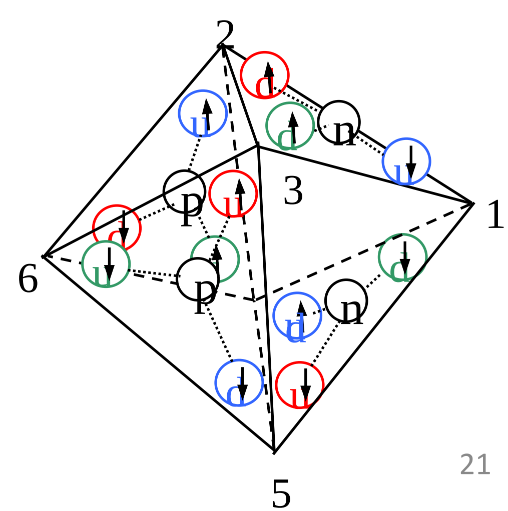
2 ←→ 2

3 ←→ 3





4 quark loops



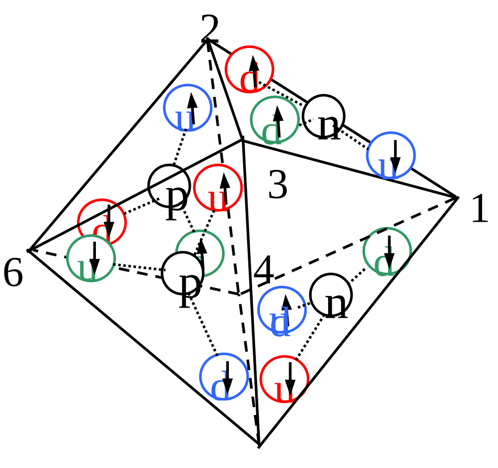
The closed shell n = 0, nucleus ^{4}He

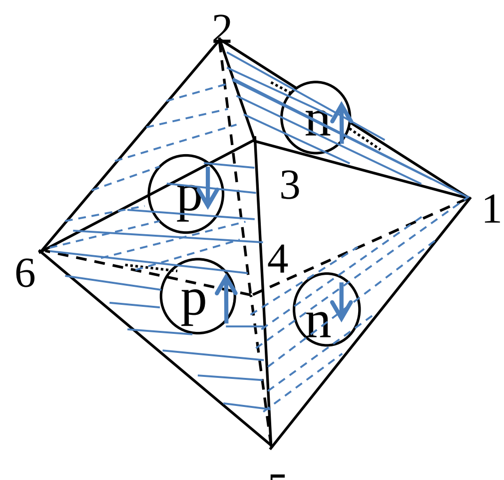
Antisymmetisation of
12 quarks in SU(12) state
$SU(2)_F \times SU(2)_S \times SU(2)_C$

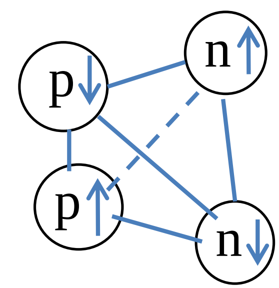
⇒

Totally antisymmetrized
4 nucleons in **s**-state

Shell Closure







Selection rules for binding two quarks of neighboring nucleons at a junction:

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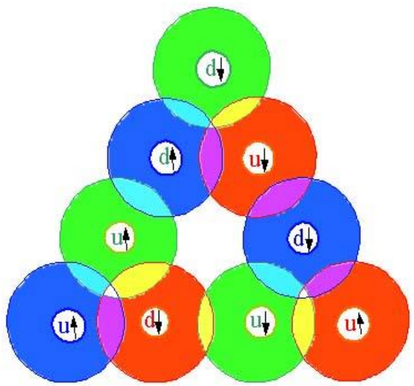
Experimental Binding Energy of Stable Nuclei and Quark Loops

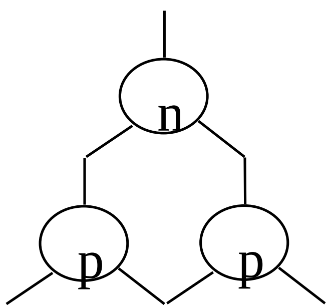
Nucleus	E_B, MeV per nucleon	Number of quark loops	Free quark ends	Nuclear forces
d	1.1	0	4	2-body
^3H	2.83	1	3	3-body
^3He	2.57	1	3	3-body
^4He	7.07	4	0	4-body

The more quark loops, the more a binding energy!

Building blocks in Shell Structure

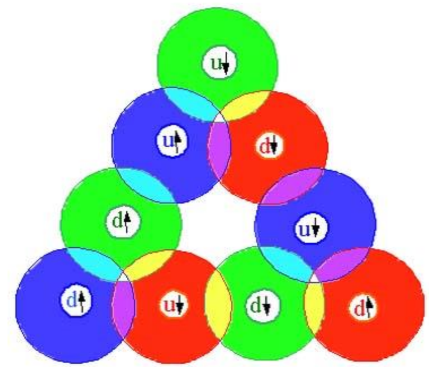
^{3}He

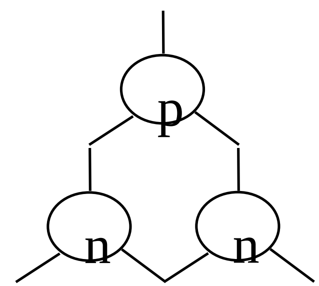




^{3}He – block

^{3}H

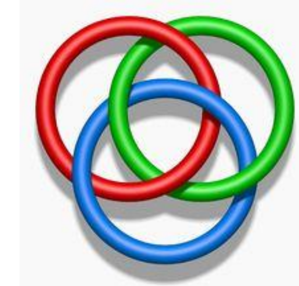


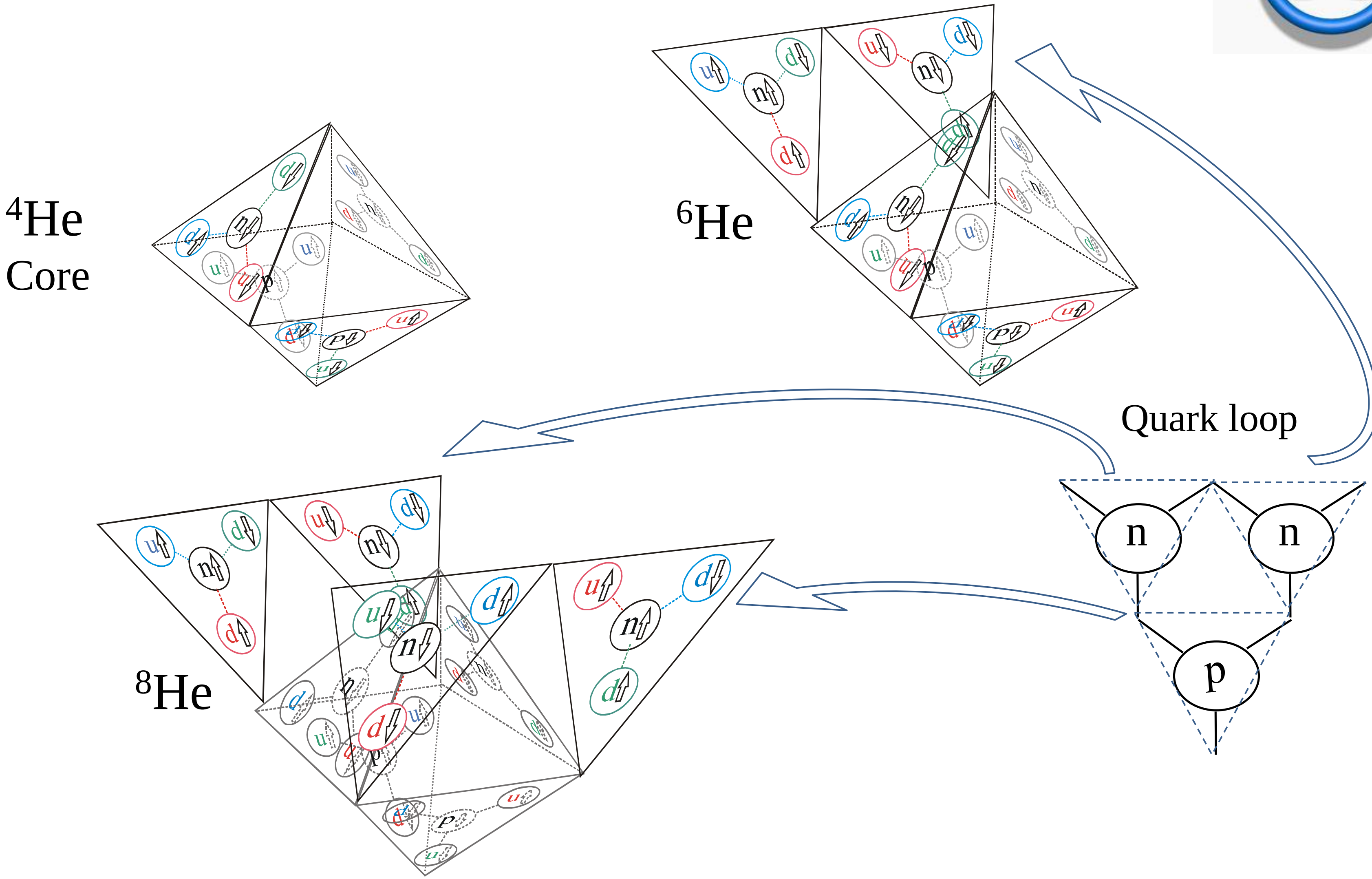


^{3}H – block

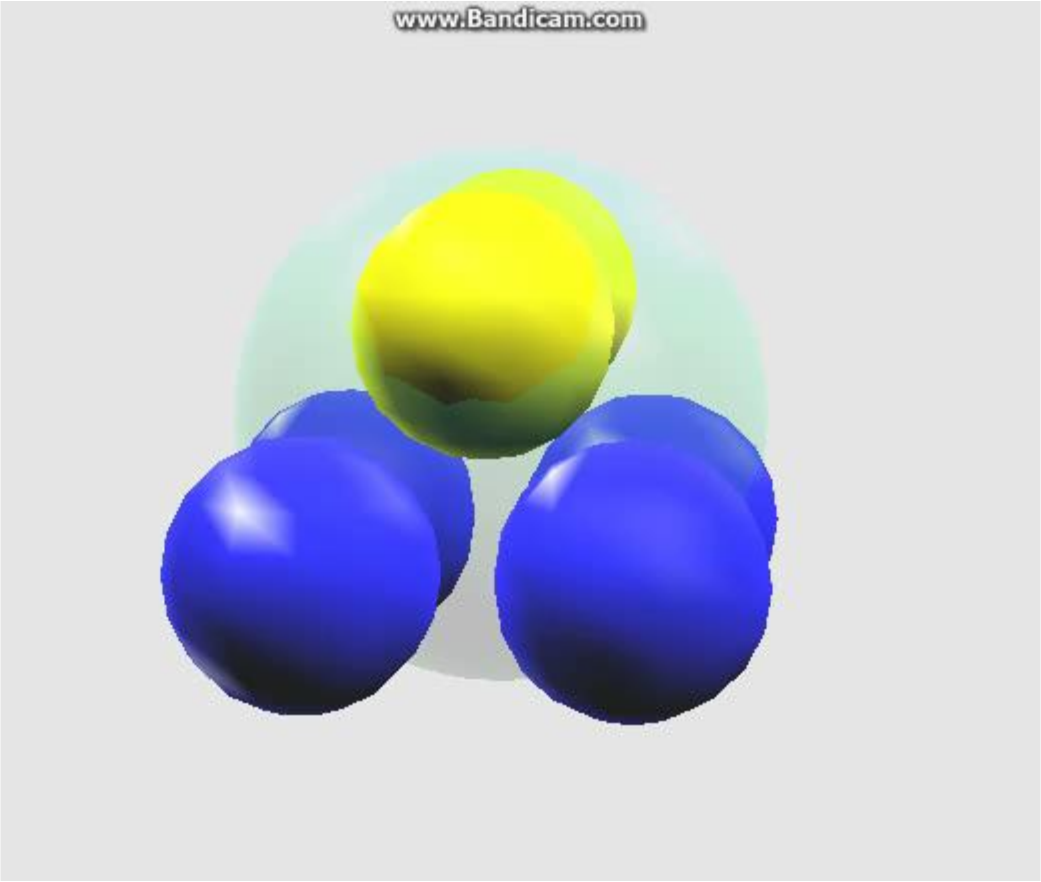
Forms Neutron Halo

Helium Isotopes
Borromean Nuclei

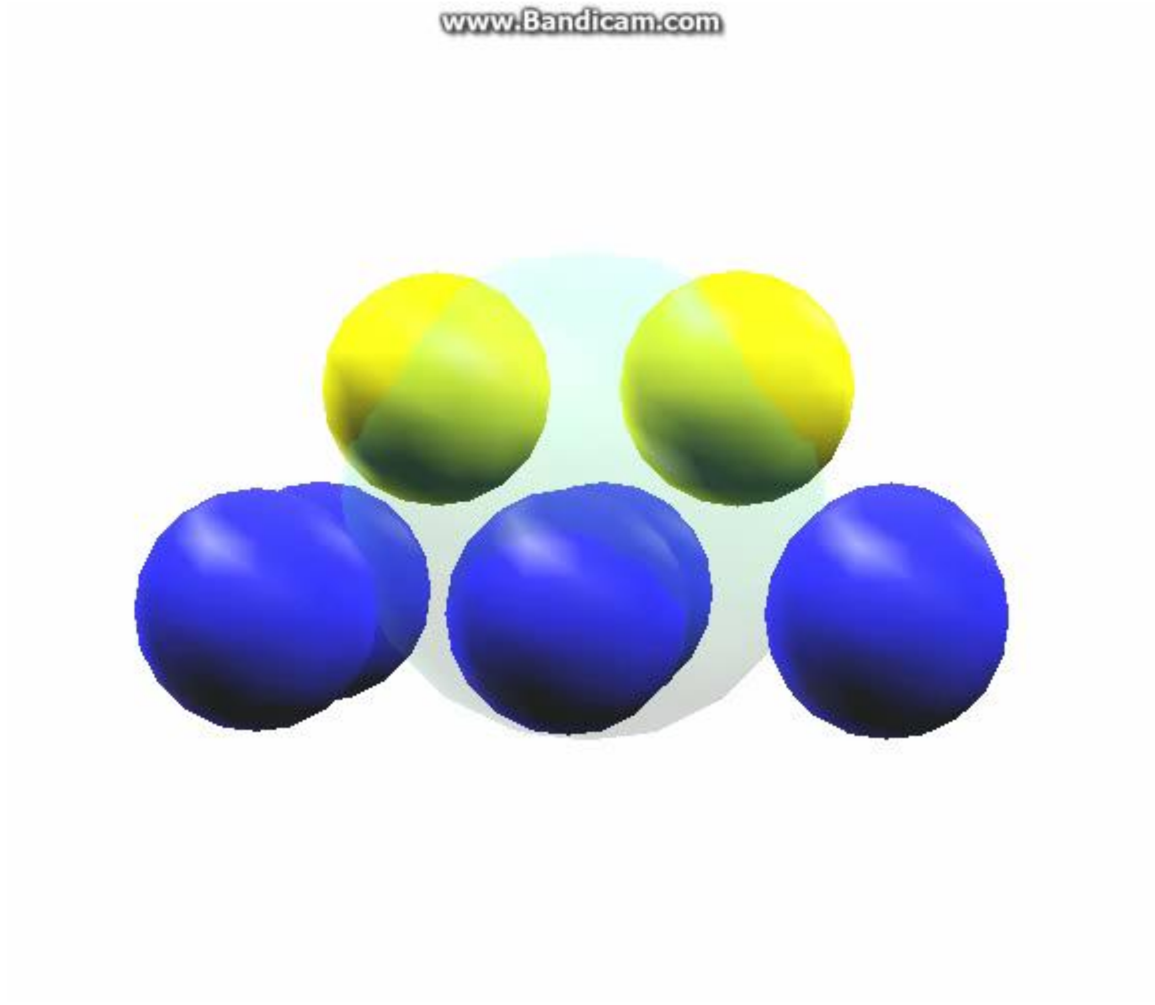




^{6}He, borromean



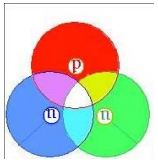
^{8}He, borromean

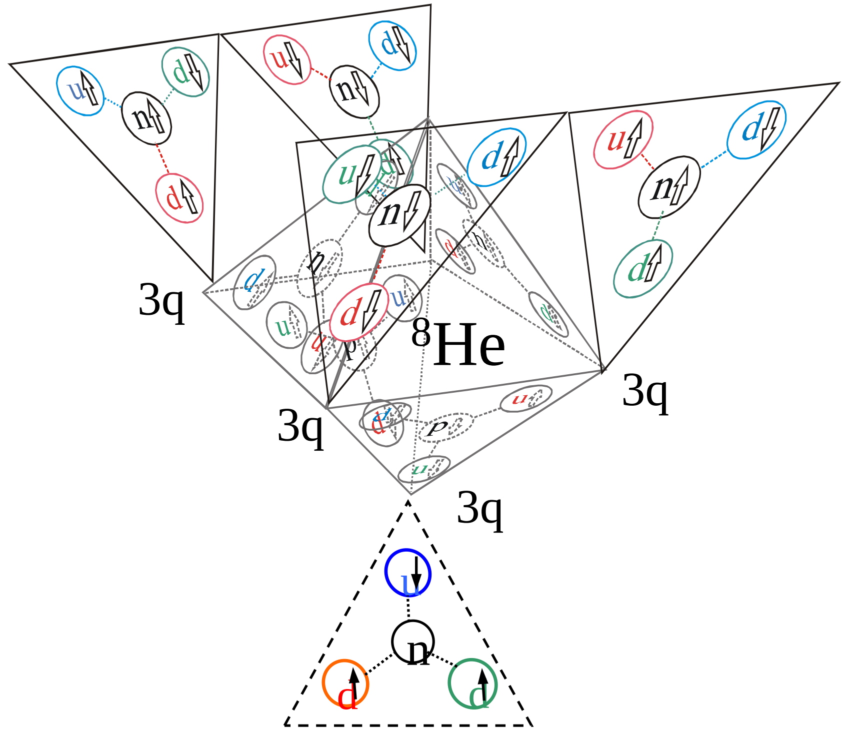


^{10}He – as a bound system does not exist!

Why? Only up to 3 quarks (RGB) can be linked at a junction

3q

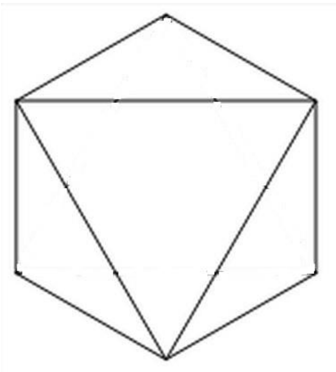


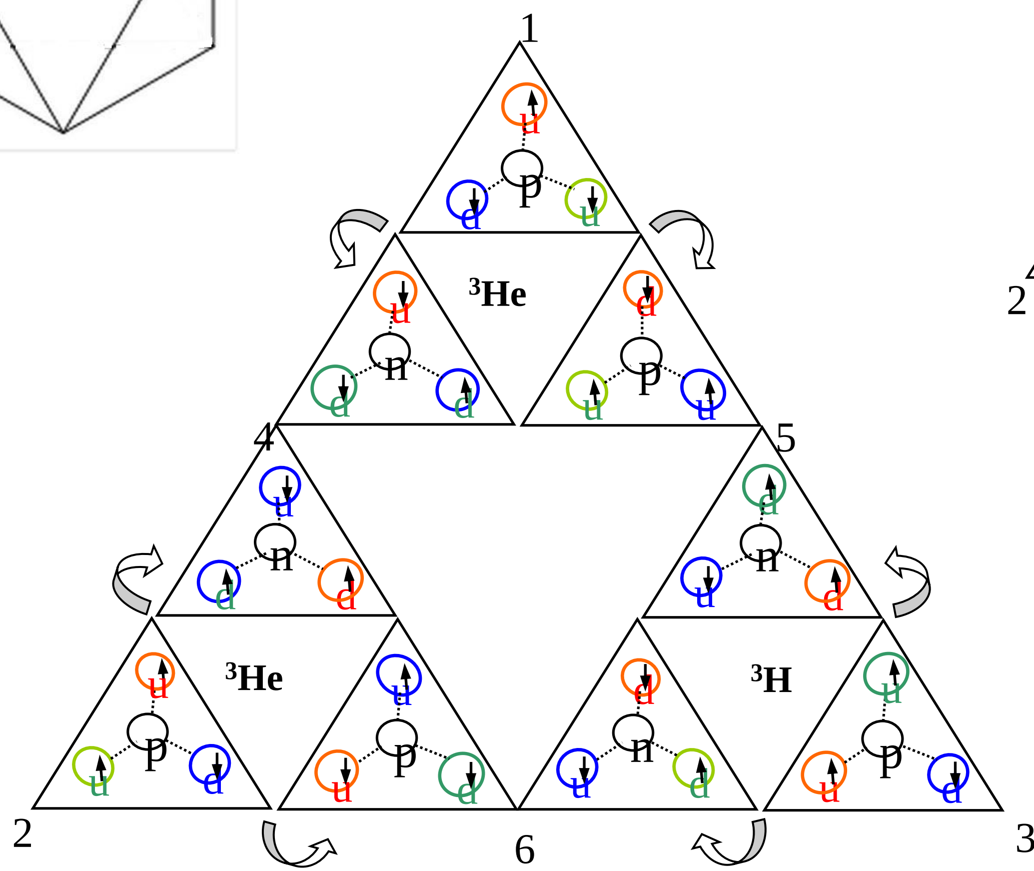


^{9}He- unbound, resonant state

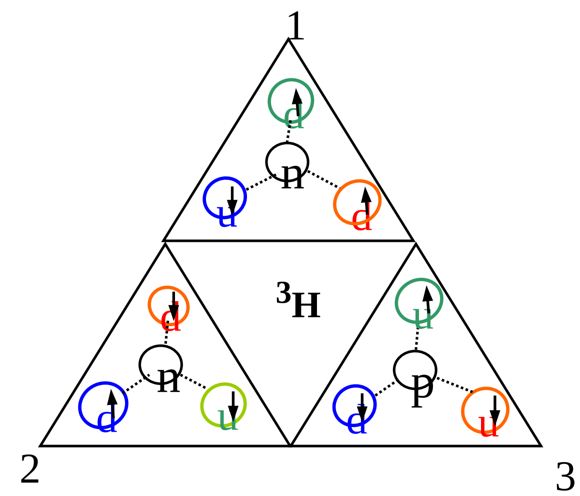
The closed shell n = 1, ^{16}O

Shell Closure





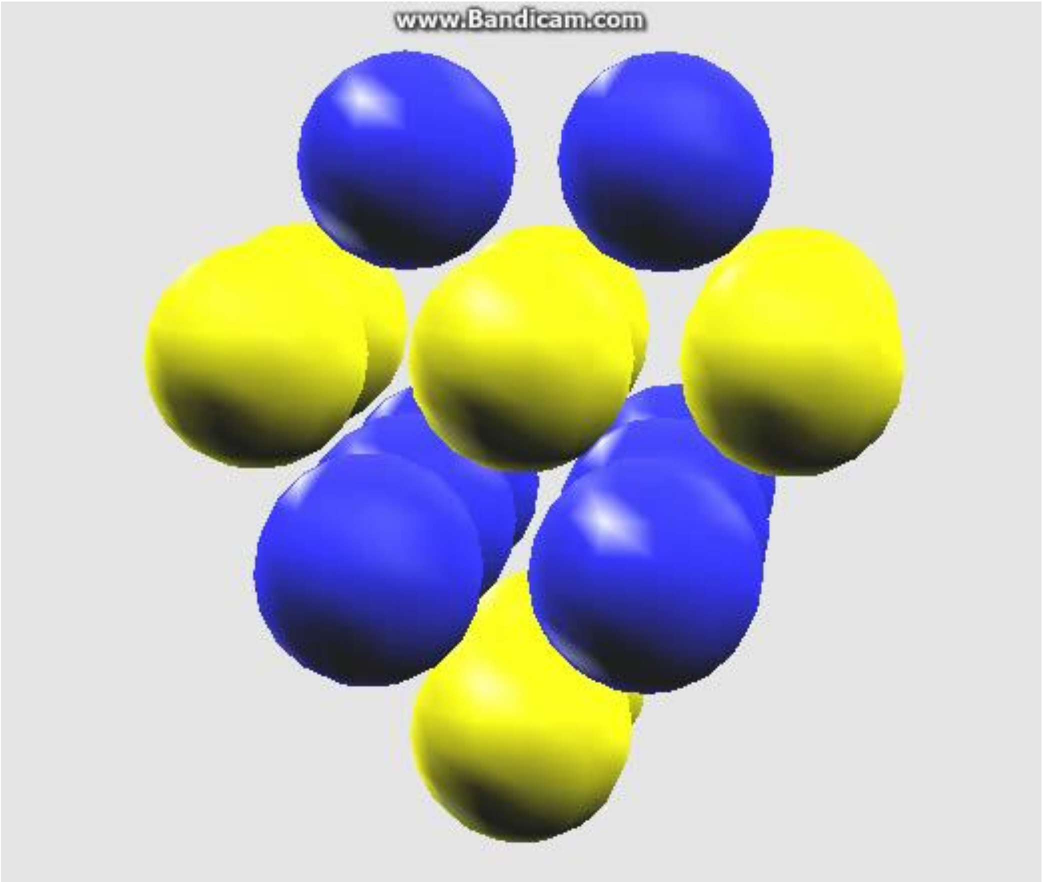
Face of ^{16}O octahedron



In analogy with ^{4}He

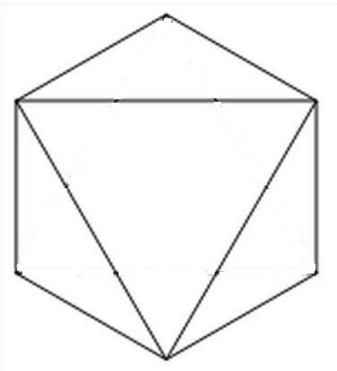
^{3}H and ^{3}He as proton and neutron in ^{4}He

^{16}O

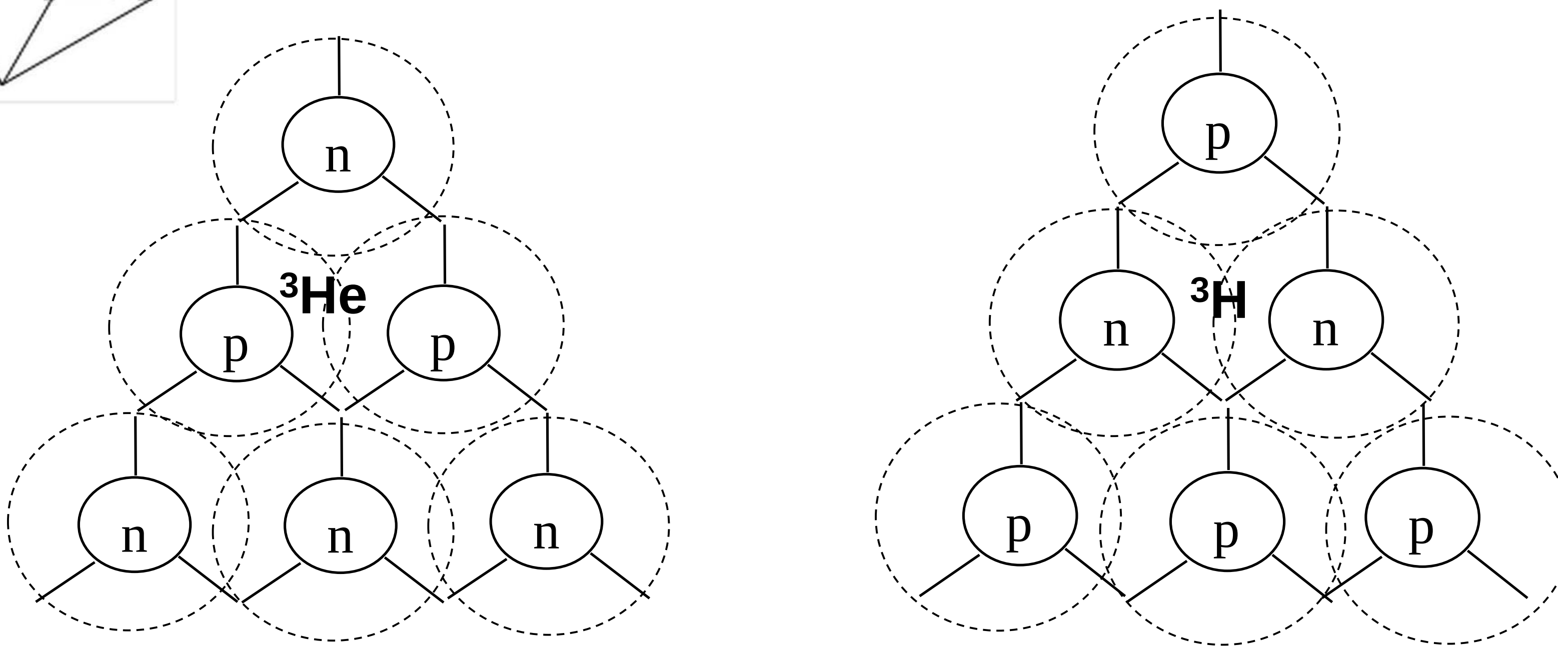


The closed shell n = 2, ^{40}Ca

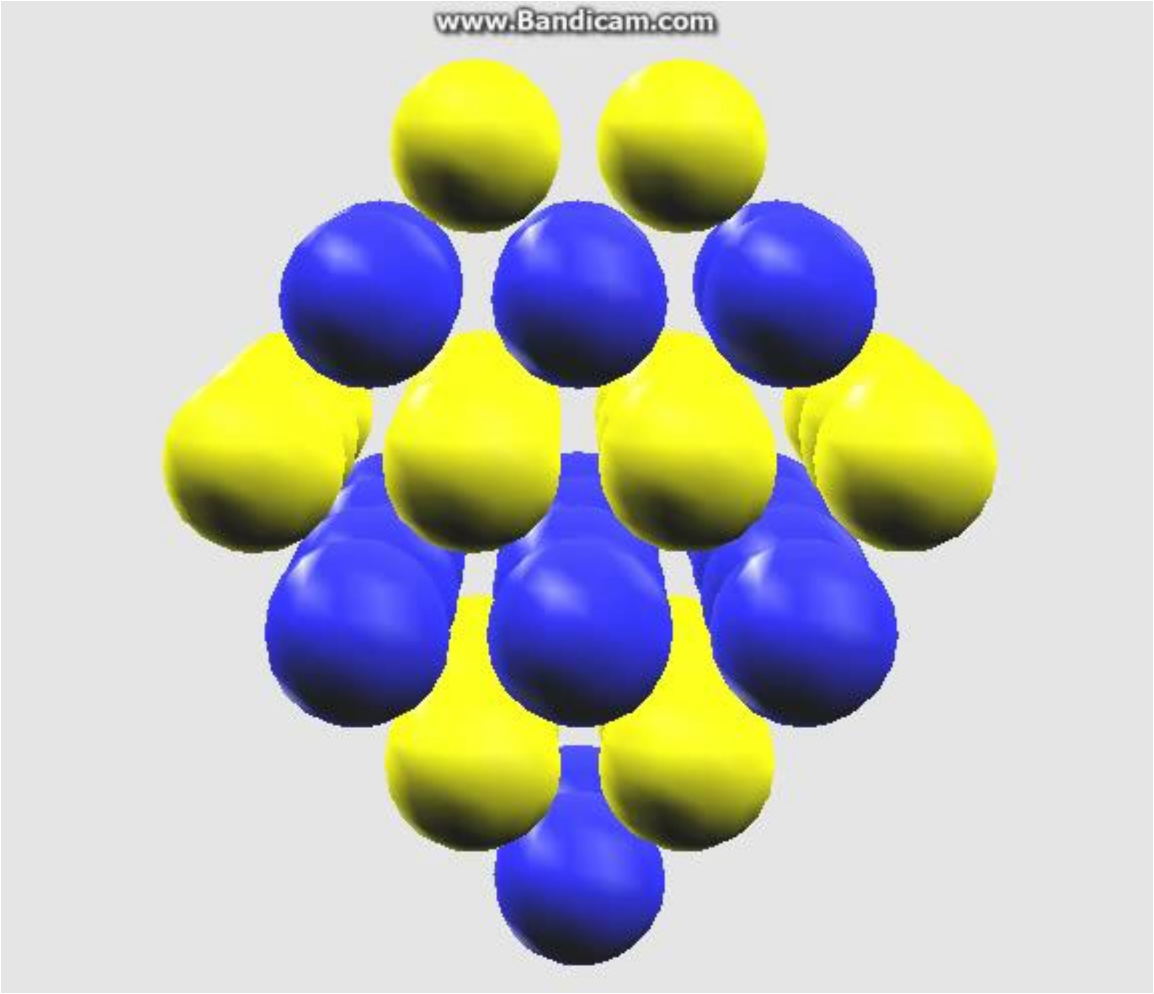
Shell Closure



Faces of ^{40}Ca octahedron

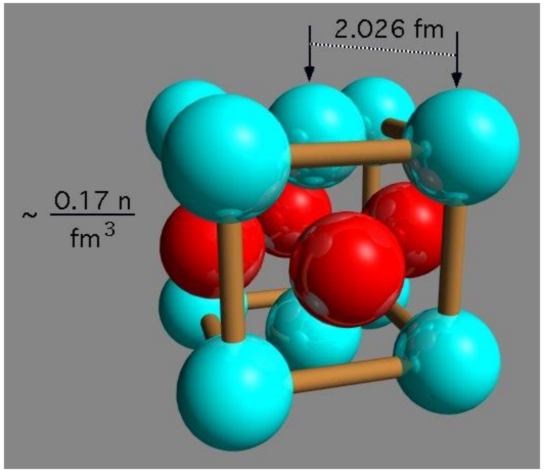


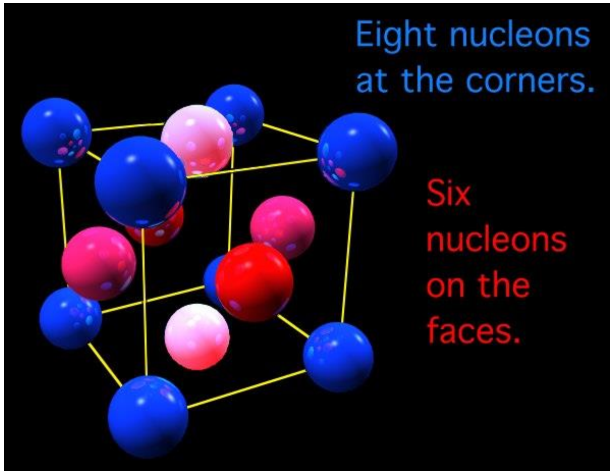
^{40}Ca

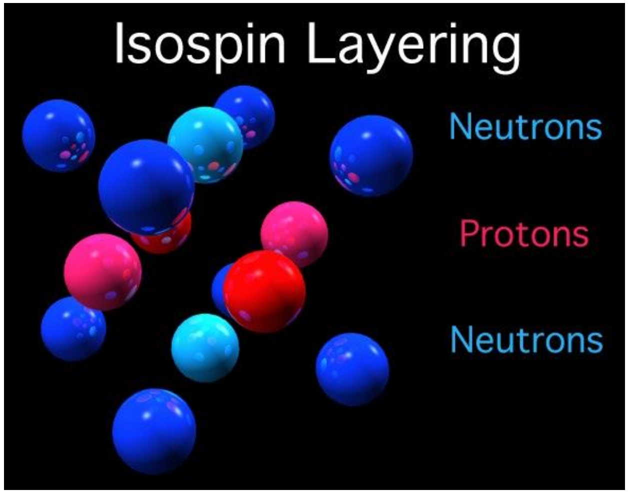


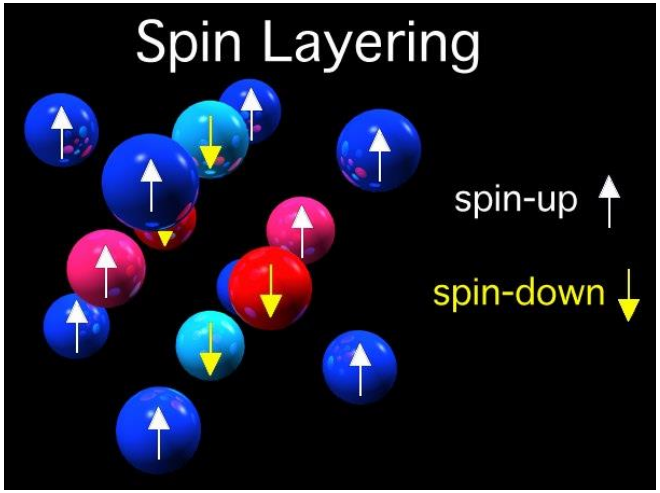
SCQM → Face-Centered Cubic Lattice

Nucleons are arranged in face-centered cubic lattce









Particle in 3D box

FCC vs Shell model (SM)

$$-(\hbar/2m\, d^2\Psi/dr^2) + V(r)\, \Psi(r) = E\Psi(r)$$

For harmonic oscillator

$$E = \hbar\omega_0(n_x + n_y + n_z + 3/2) = \hbar\omega_0(N + 3/2)$$

$$N = 0, 1, 2, 3, ,,,$$

Different combinations of $\mathbf{n_x}$, $\mathbf{n_y}$ and $\mathbf{n_z}$ that give the same total **N** – value denote spatially distinct "degenerate" states, with the same energy.
Wigner 1935, Cook 1987

If the origin of the coordinate system is taken as the center of the central tetrahedron, then the closure of each consecutive, symmetrical (x=y=z) geometrical shell in the lattice composes precisely the numbers of nucleons in the shells derived from the three-dimensional Schrodinger equation.

FCC Lattice Model

- Principal quantum number, **n**

Assuming x, y and z coordinates of nucleons are odd – integers,

$$\mathbf{n_{nucleon} = (|x_{nucleon}| + |y_{nucleon}| + |z_{nucleon}| - 3)/2}$$

The first shell (**s**-shell, **n** = 0) contains 4 nucleons with coordinates 111, -1-11, 1-1-1, -11-1.

The second shell (**p**-shell, **n** = 1): 12 nucleons
31-1, 3-11, -311, -3-1-1, 1-31, -131, 13-1, -1-3-1, -113,
11-3, 1-13, -1-1-3
and so on…

- Total angular momentum, **j**

$$\mathbf{j = (|x_{nucleon}| + |y_{nucleon}| - 1)/2}$$

- Magnetic quantum number, **m**

$$\boldsymbol{m = |x|/2}$$

Resume on nuclear symmetry

- It turns out that nucleon centers are located on the sites of face-centered cubic lattice.
- Nucleons are arranged in alternating (antiferromagnetic) spin, isospin layers.
- nucleons are strongly correlated
- If one connects the nucleons positions by bonds the nuclei with closure shells have a shape of tetrahedron (s-shell) and truncated tetrahadrons (p, d, f, …-shells).
- All nuclei are non-spherical, deformed

Summary

- Quarks play an explicit role in formation of the nuclear structure.
- Quark loops are building blocks of nuclear binding.
- Quarks and nucleons (protons and neutrons) inside nuclei are strongly correlated.
- ‘Halo’ nuclei **– fruits of quark-loop bindings**
- Effect of quark looping: $E_{sep} < E_{bound}/A$

Summary (cont.)

Nuclei possess crystal-like structure:

- Nucleon centers are arranged according to FCC lattice
- All Bound nuclei are composed of **virtual α-clusters**
- Closed Shells = Octahedral Faces
- All nuclei are deformed
- **Symmetry energy** is a consequence of strong quark correlations → strong correlations of protons and neutrons.
- The **pairing effect** is a consequence lattice structure

$$E_{cв} = a_1 A - a_2 A^{2/3} - a_3 \frac{Z^2}{A^{1/3}} - a_4 \frac{(A-2Z)^2}{A} + \delta A^{-3/4}$$

Nuclear Isotopes. Neutron Dripline

Building nuclei with $A \geq 12$

- A_{bound} – all nucleons are arranged in **virtual** α-clusters
- Node occupation in stable nuclei

  $j_A = j_A^{min}$ for neutrons and protons

  $E^{coul} = (E^{coul})_{min}$ for protons
- $A > A_{bound}$ – last excess neutrons are arranged in **triton-like** clusters (halo neutrons)

Comparison with nuclear data

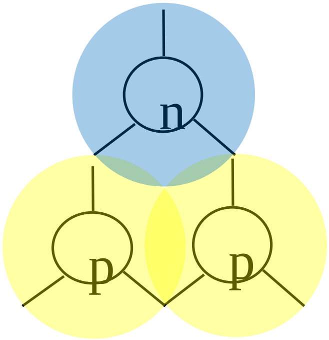
A_{bound} – nuclei, $(t_{1/2})_{bound} \geq$ *microseconds*

$A \rightarrow$ drip-line, $(t_{1/2}) <$ *microsecond*

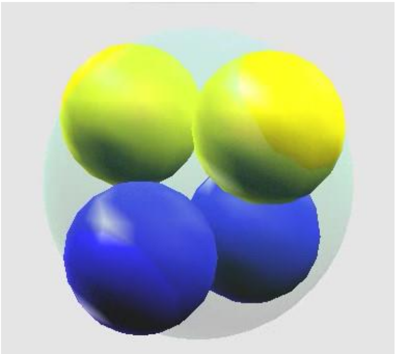
Helium Isotopes

$A_{bound} \leq 8$, $A_{DL} = 8$

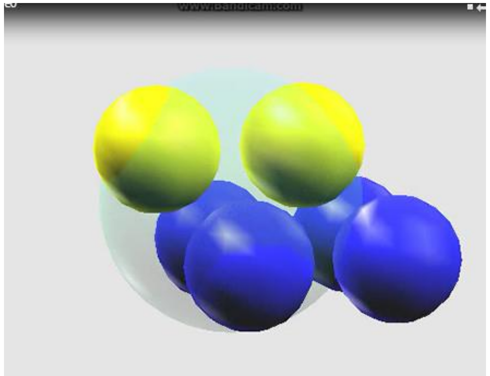
^{3}He $1/2^{+}$



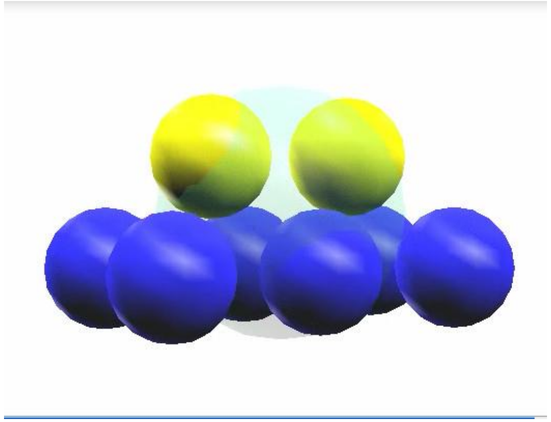
^{4}He 0^{+}



^{6}He 0^{+}



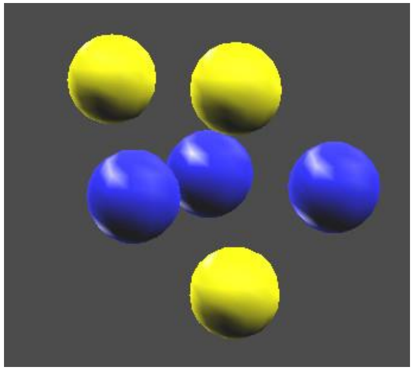
^{8}He 0^{+}



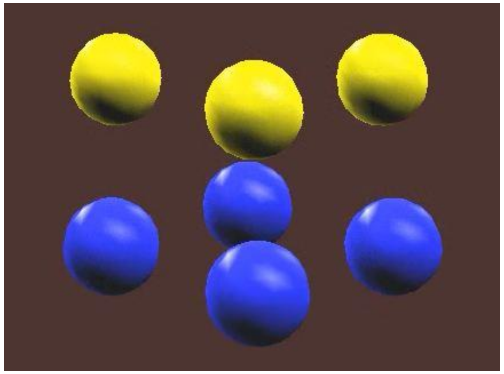
Lithium Isotopes

$A_{bound} \le 9, A_{DL} = 11$

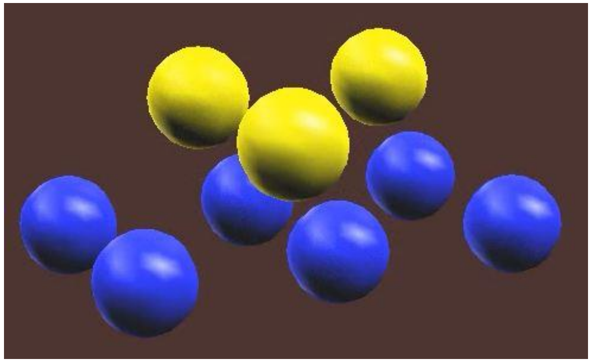
^{6}Li 1^{+}



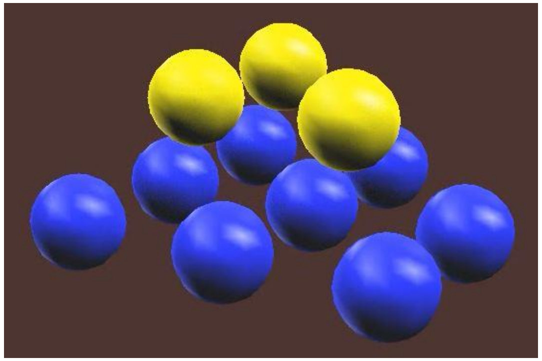
^{7}Li $3/2^{-}$



^{9}Li $3/2^{-}$



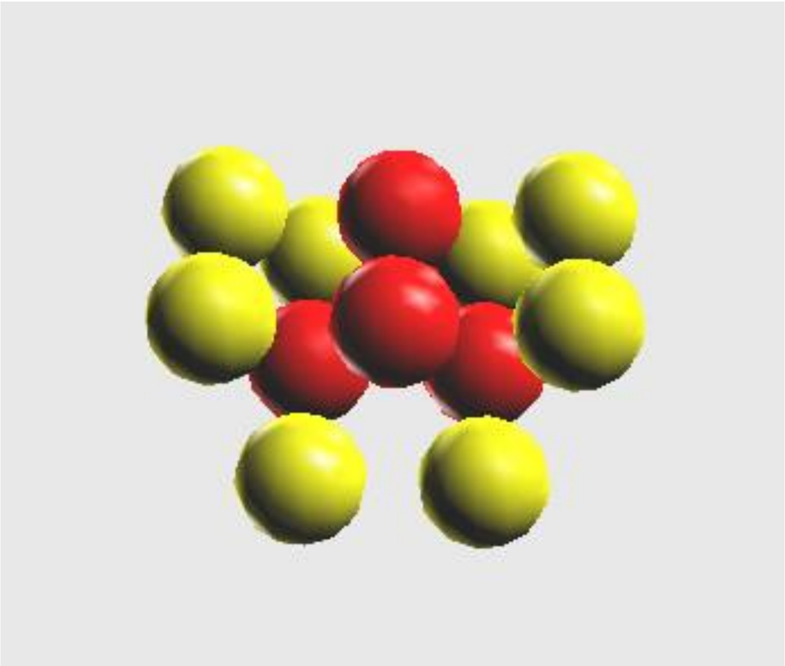
^{11}Li $3/2^{-}$



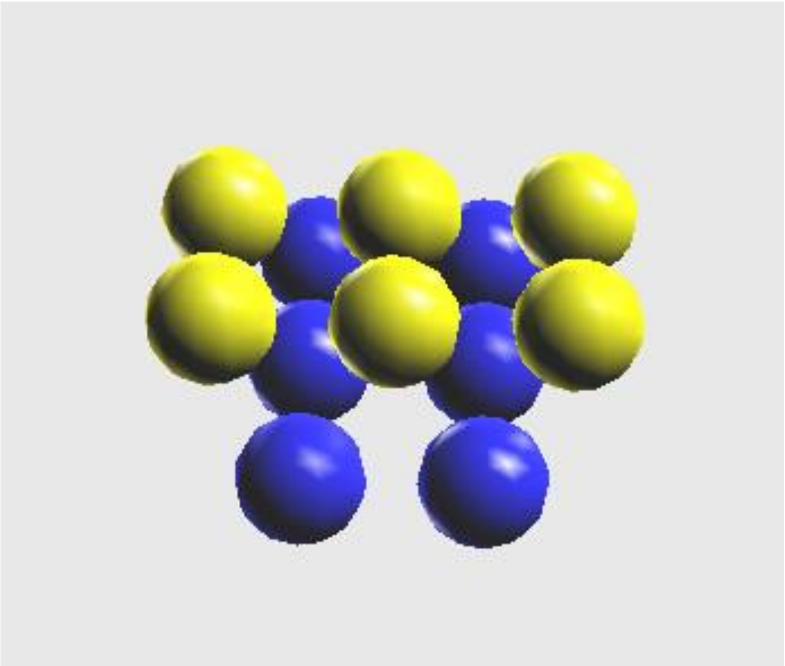
^{12}C

6 protons, 6 neutrons

n, principal number
n=0, red; n=1, yellow

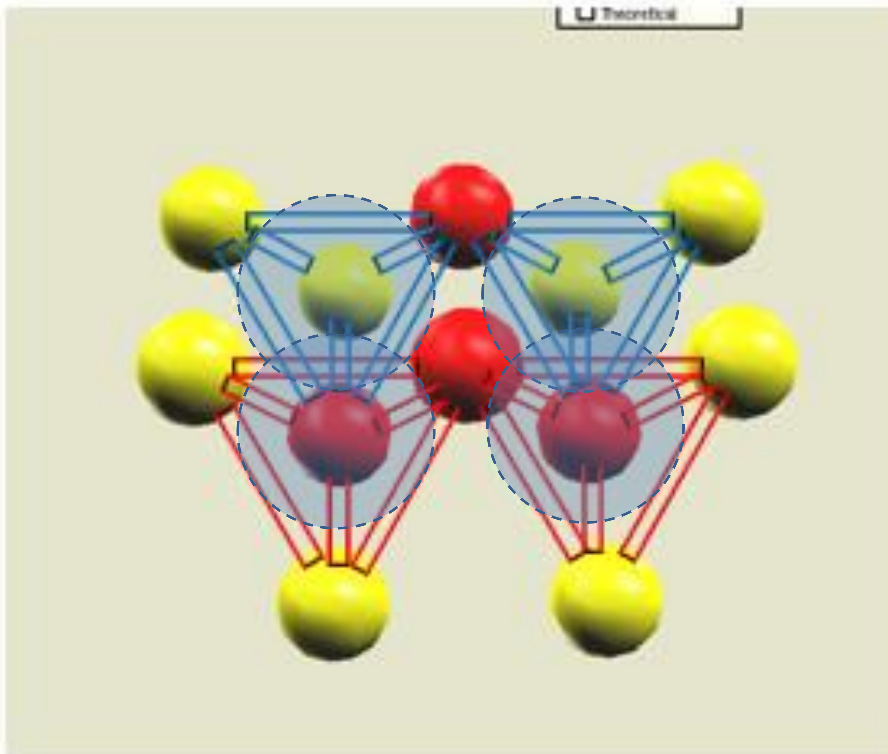


i, isospin
yellow – protons
blue - neutrons



^{12}C - 4 virtual α-clusters

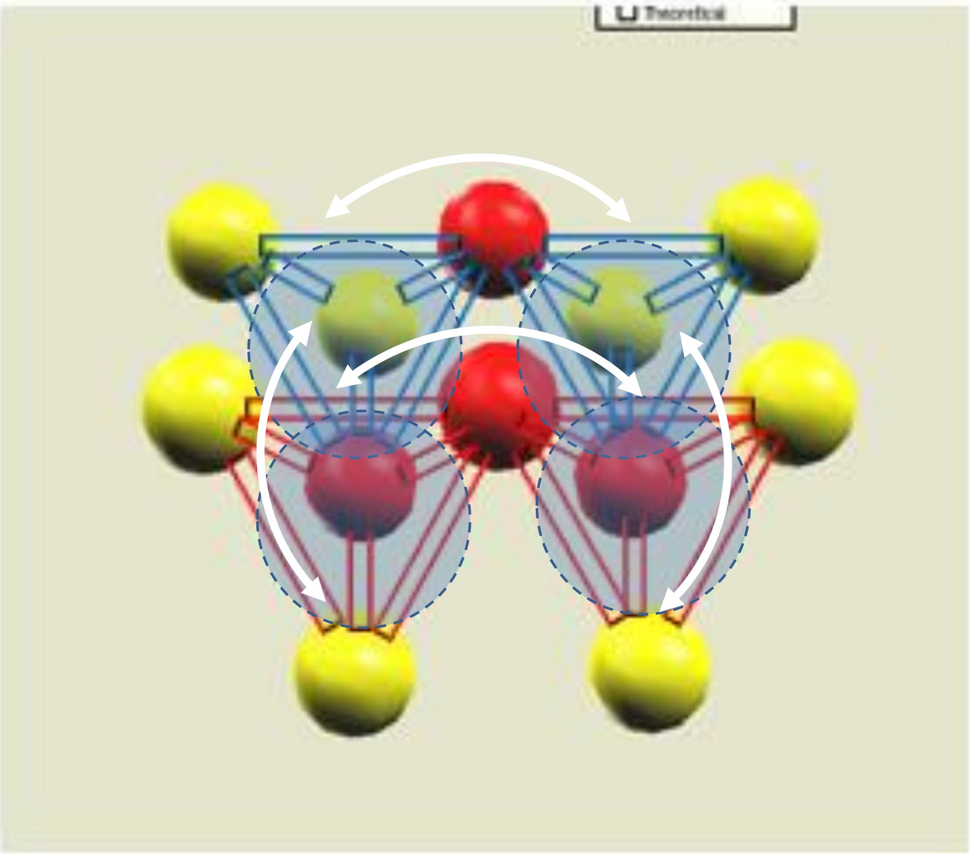
Rearrangement (disappearance) of s-shell



4 nucleons of ***s***-shell (red) form with
6 nucleons of ***p***-shell (yellow) 4 virtual α-clusters
s-shell nucleons are exchange particles

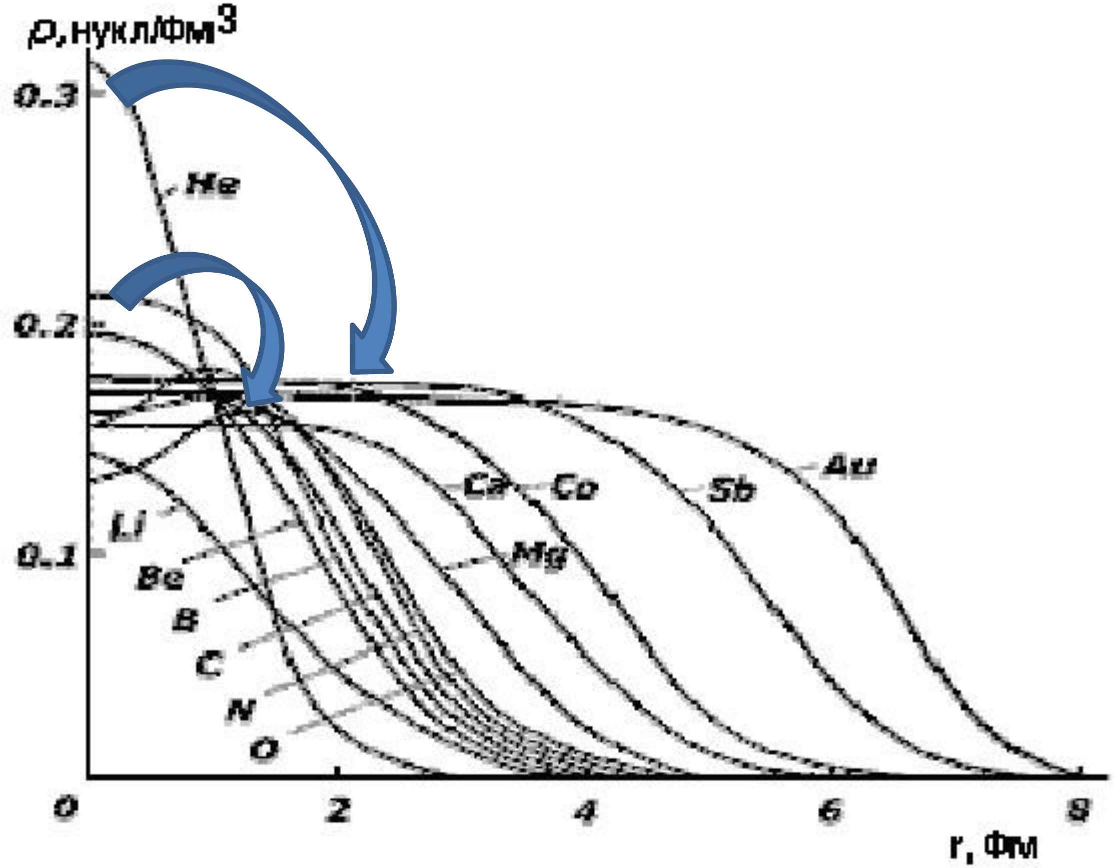
^{12}C

Crosswise bindings of 4 virtual α-clusters
by exchange (red) nucleons of s-shell

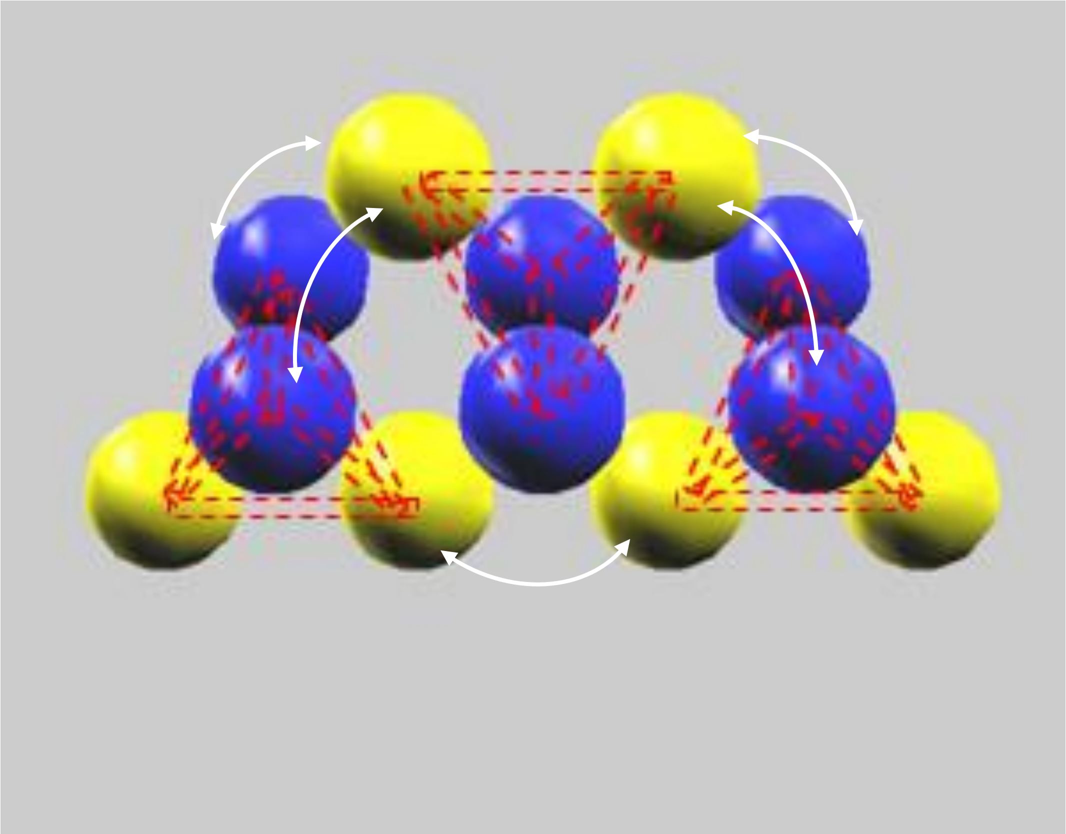


Nuclear density

Result of rearrangement – of ***s***-shell
No core structure for A ≥ 12



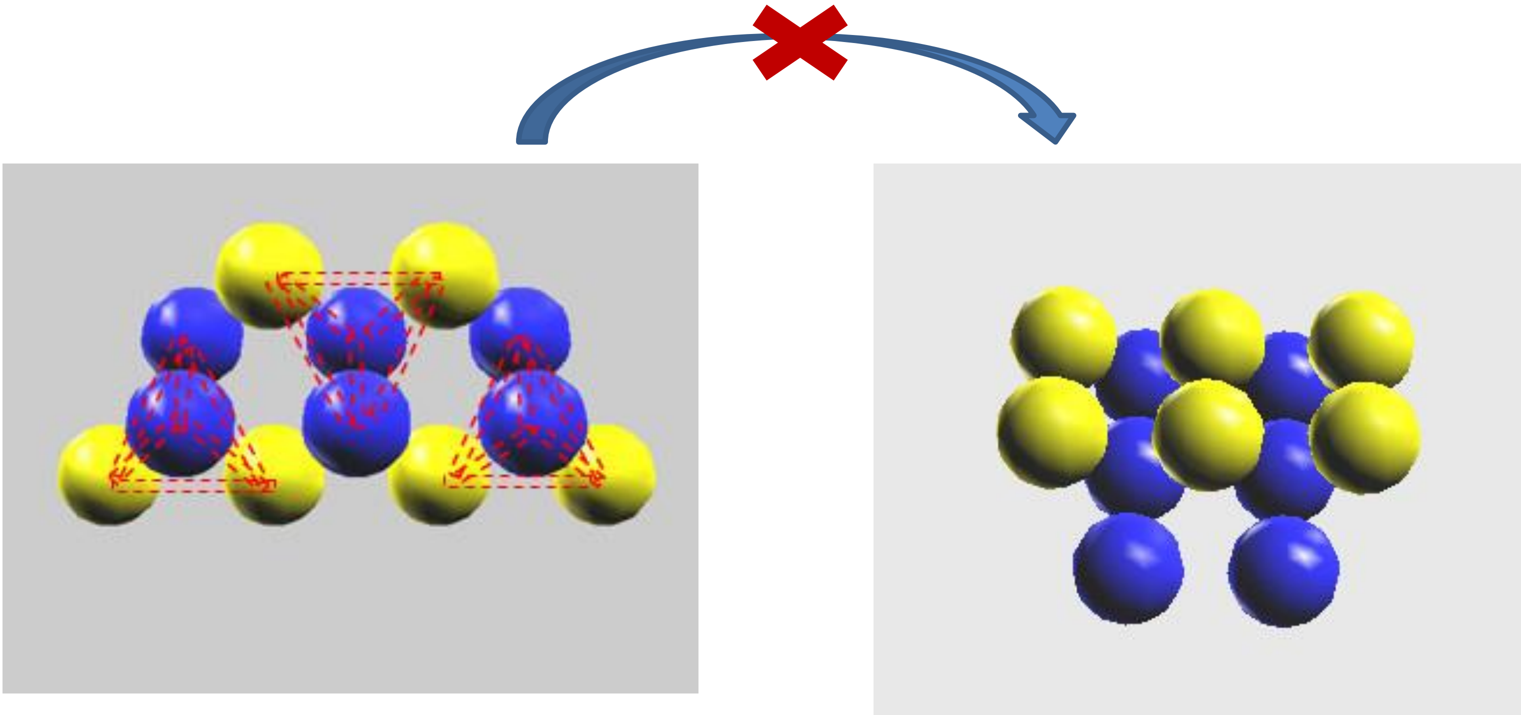
^{12}C Hoyle state
Borromean nucleus
Loosely bound 3 real α- cluster nucleus



Frames of α-clusters are depicted as tethrahadrons.
Neutrons of left and right α-clusters are bound with protons of central α-cluster (like in ^{8}He), and their 2 nearest protons are bound together.

Nucleosinthesis

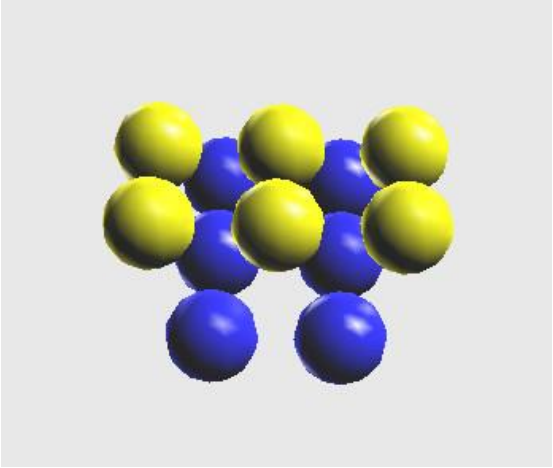
Transition from Hoyle state to ^{12}C g.s. is
IMPOSSIBLE



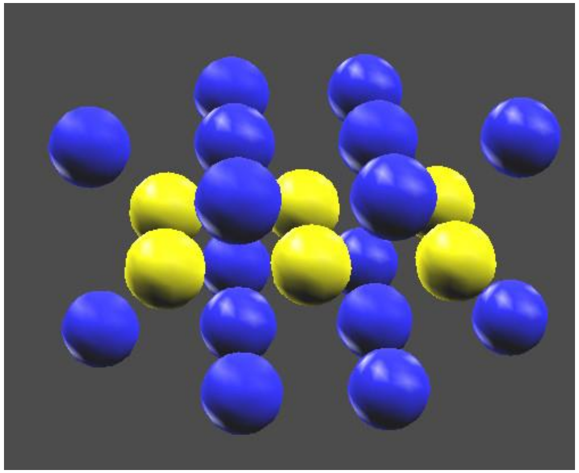
^{12}C

$A_{bound} \leq 22$, $A_{DL} = 30$

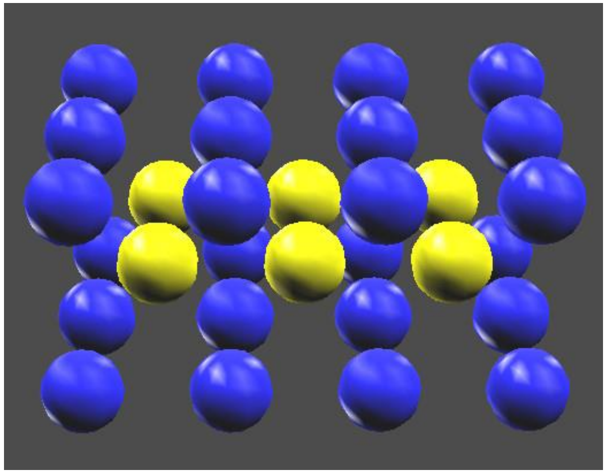
^{12}C



^{22}C



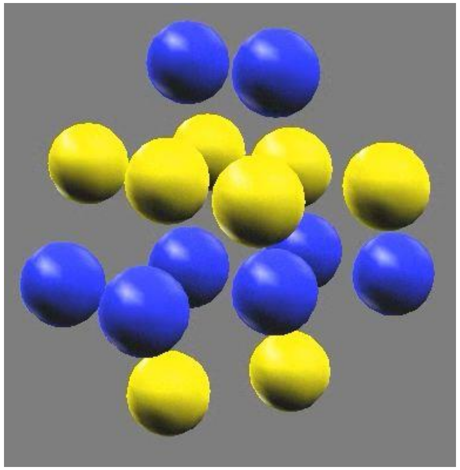
^{30}C



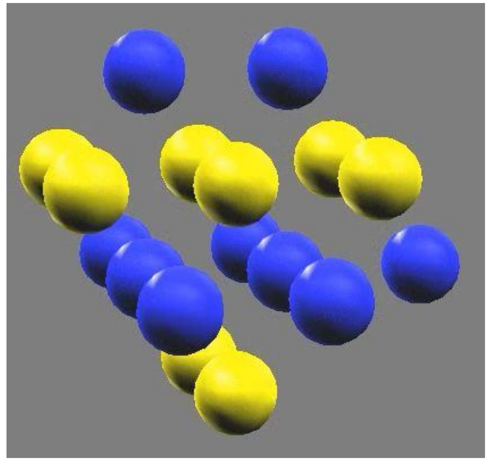
protons – yellow
neutrons - blue

Oxygen Isotopes
$A_{bound} \leq 24$, $A_{DL} = 32$

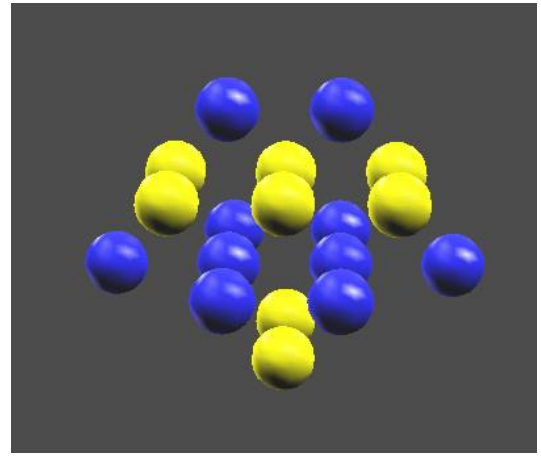
^{16}O



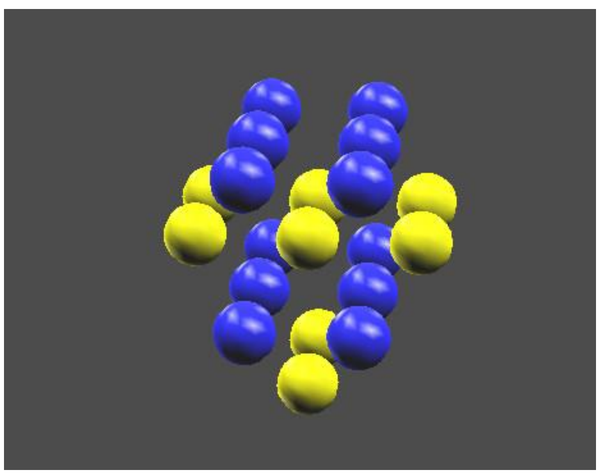
^{17}O



^{18}O

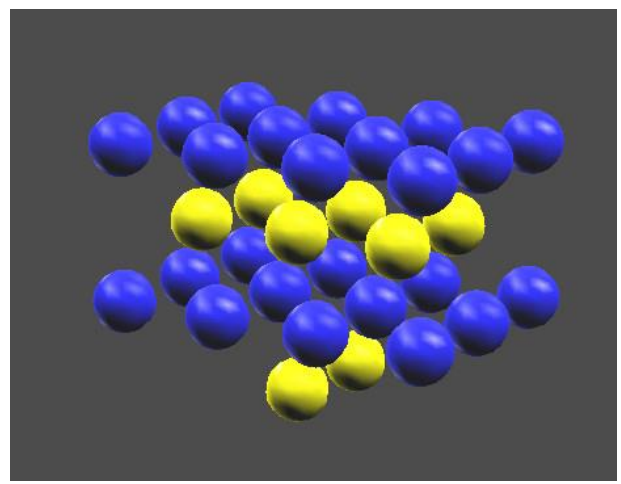


^{24}O



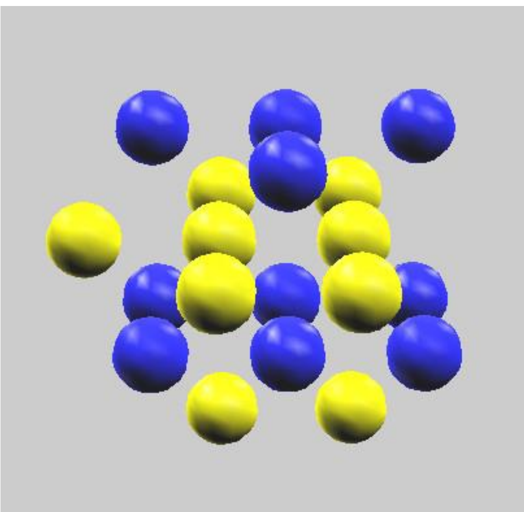
protons – yellow
neutrons - blue

^{32}O

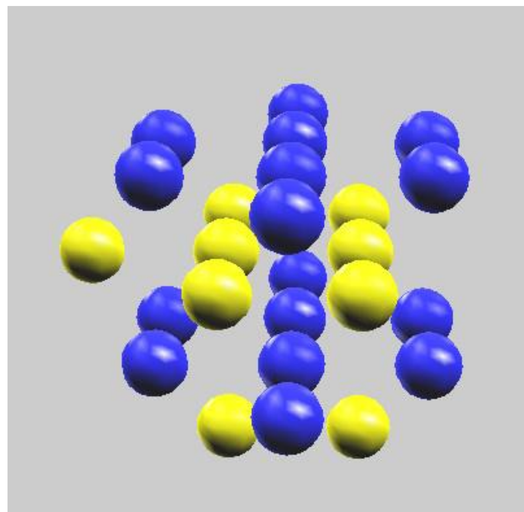


Fluorine Isotopes
$A_{bound} \leq 25$, $A_{DL} = 37$

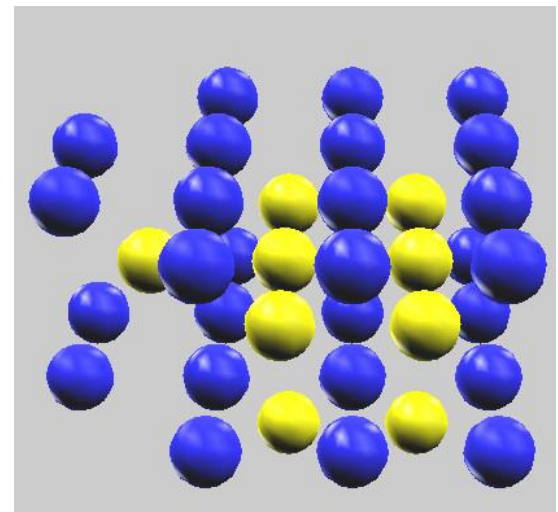
^{19}F



^{25}F



^{37}F

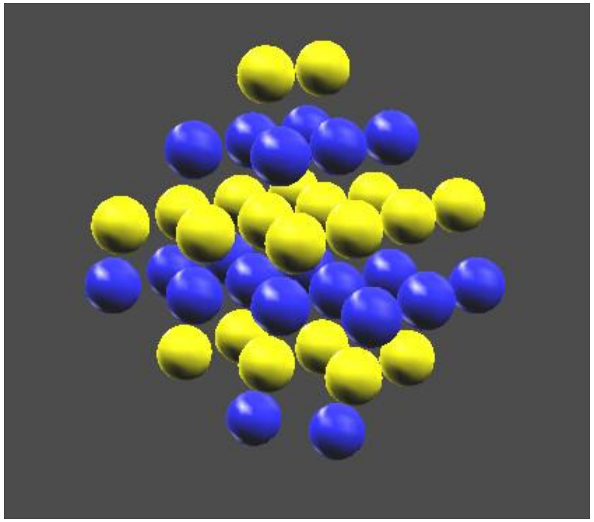


protons – yellow
neutrons - blue

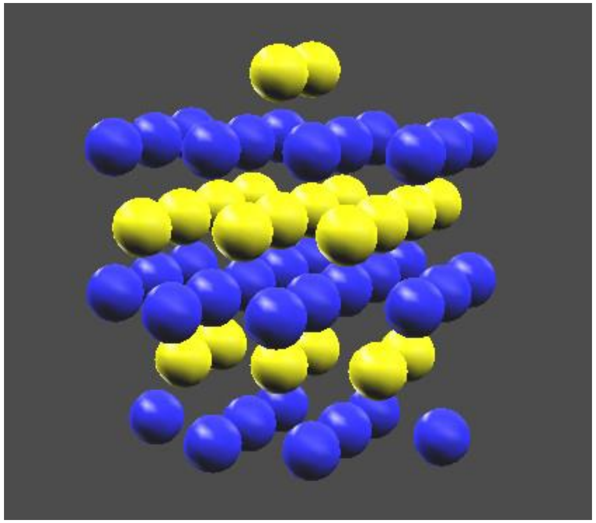
Calcium Isotopes

$A_{bound} \leq 56, A_{DL} = 72$

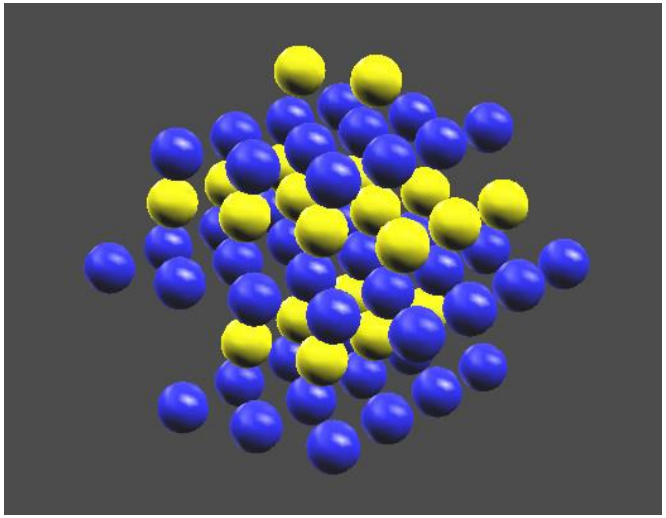
^{40}Ca



^{56}Ca



^{72}Ca

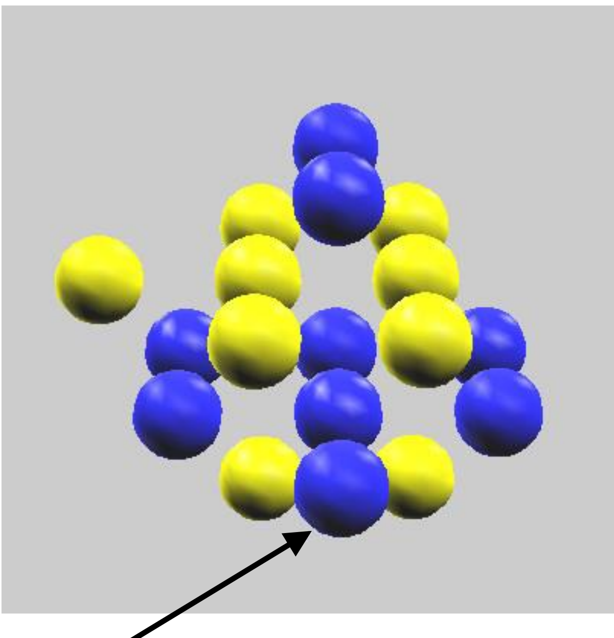


protons – yellow
neutrons - blue

Fluorine Isomers

$^{18}F_m$

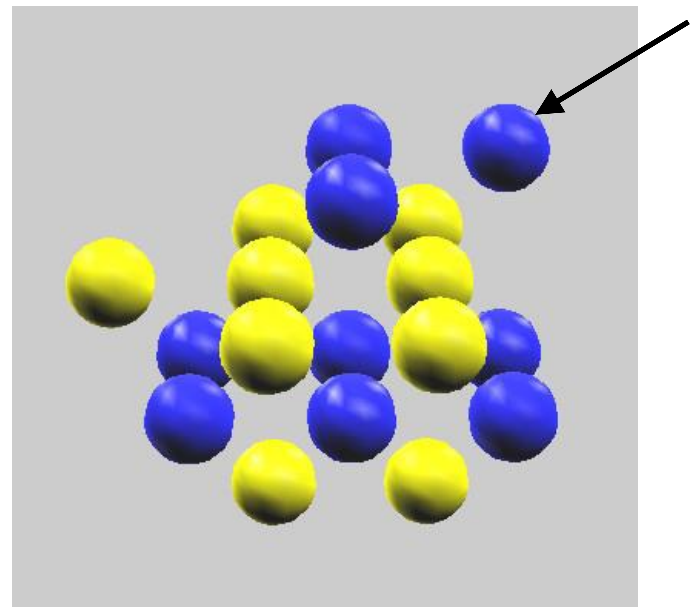
$t_{1/2}$ ~ 200 nc



5^+

$^{18}F_n$

$t_{1/2}$ ~ 110 min



1^+

protons – yellow
neutrons - blue

Thank you

Back Slides

Introduction (cont.)

QCD is non-abelian theory → hard to derive the features of hadrons and nuclei from the first principles of QCD.

Hadronic processes with high Q^2

pQCD: $\alpha_S < 1$, $m_q \to 0$, chiral symmetry

Low energy hadron and nuclear physics

non-pQCD: $\alpha_S > 1$, $m_q \neq 0$, chiral symmetry breaking

- Low energy approx. of QCD
- QCD–inspired phenomenology
  - NR constituent quark models
  - Bag models
  - Chiral quark models
  - Soliton models

pQCD →

What is Chiral Symmetry and its Breaking?

- $m_q = 0$
- Chiral Symmetry

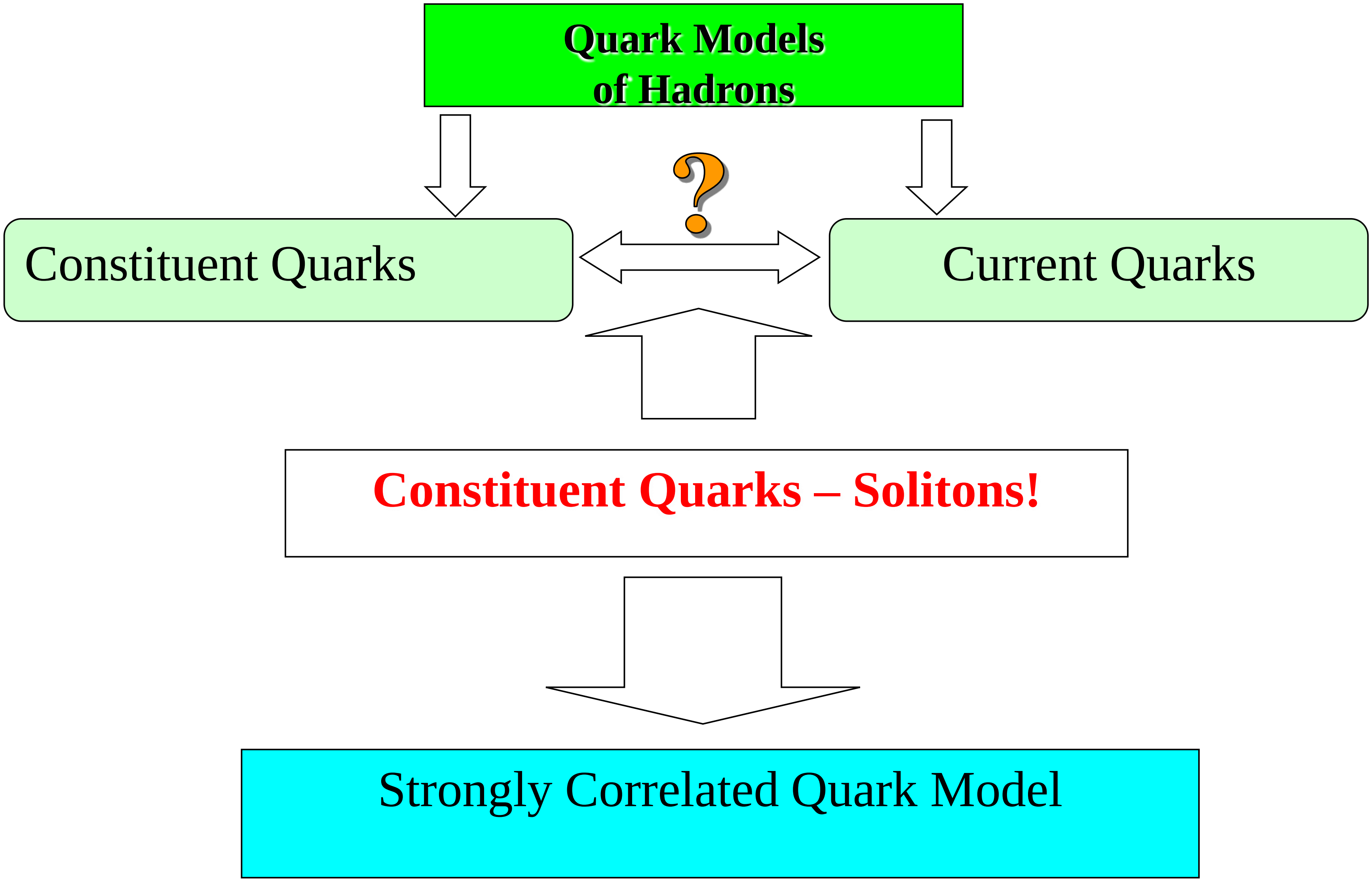
  $SU(2)_L \times SU(2)_R$ for $\psi_{L,R} = u, d$ – current quarks

- Chiral symmetry breaking ≡ quark or *chiral* condensate:

  $\langle \bar{\psi}\psi \rangle \simeq -(250 \text{ MeV})^3, \quad \psi = u,d$

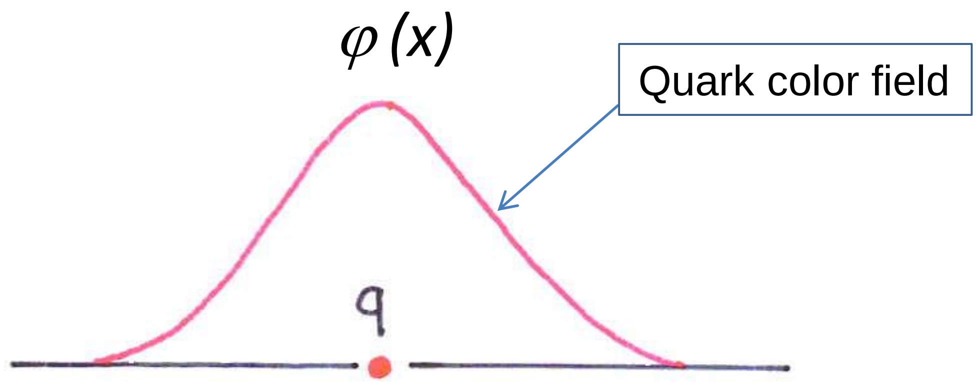
- As a consequence massless valence quarks (u, d) acquire dynamical masses which we call constituent quarks

  $M_C \approx 350 - 400$ MeV


Quark Models
of Hadrons
?
Constituent Quarks
Current Quarks
Constituent Quarks – Solitons!
Strongly Correlated Quark Model

Strongly Correlated Quark Model

(SCQM)



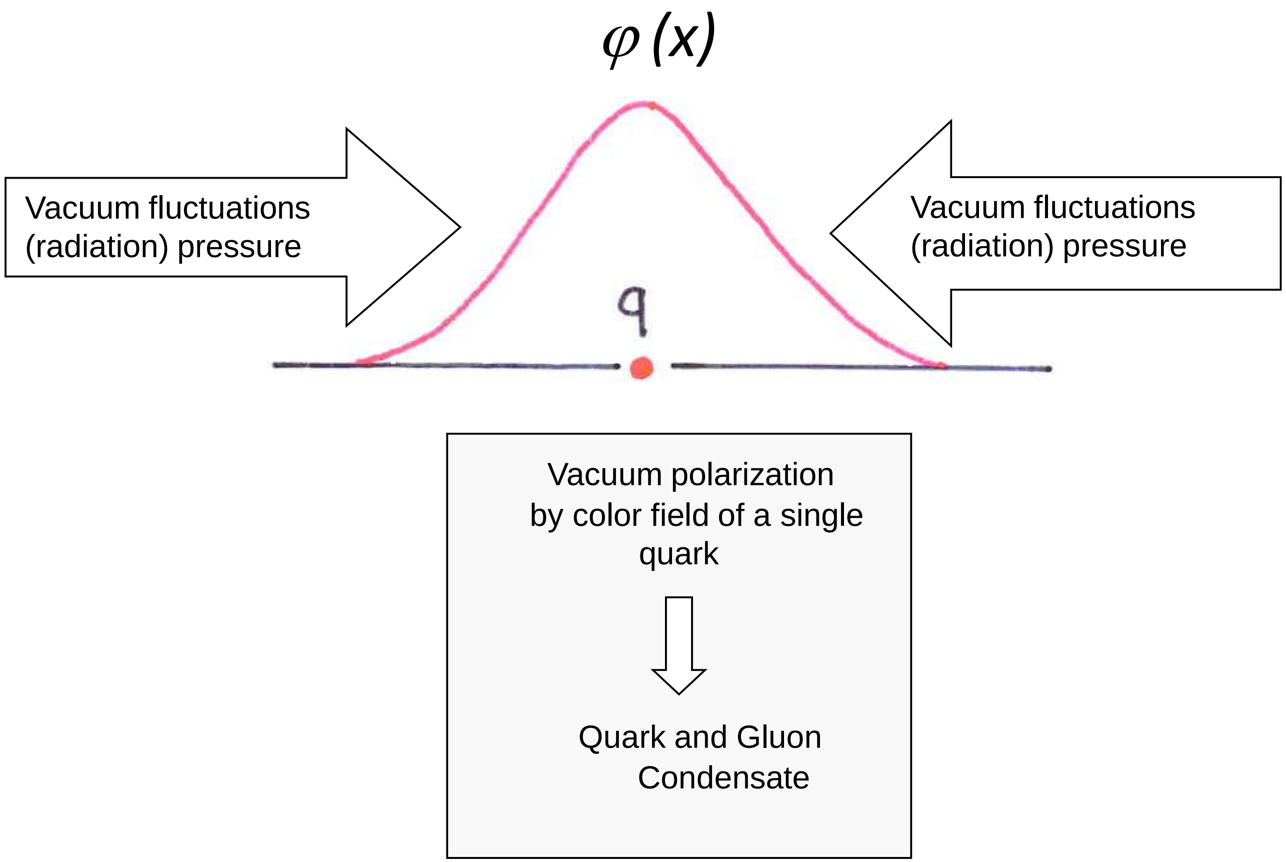
Vacuum polarization by color field of a single quark

⇩

Quark and Gluon Condensate

Strongly Correlated Quark Model

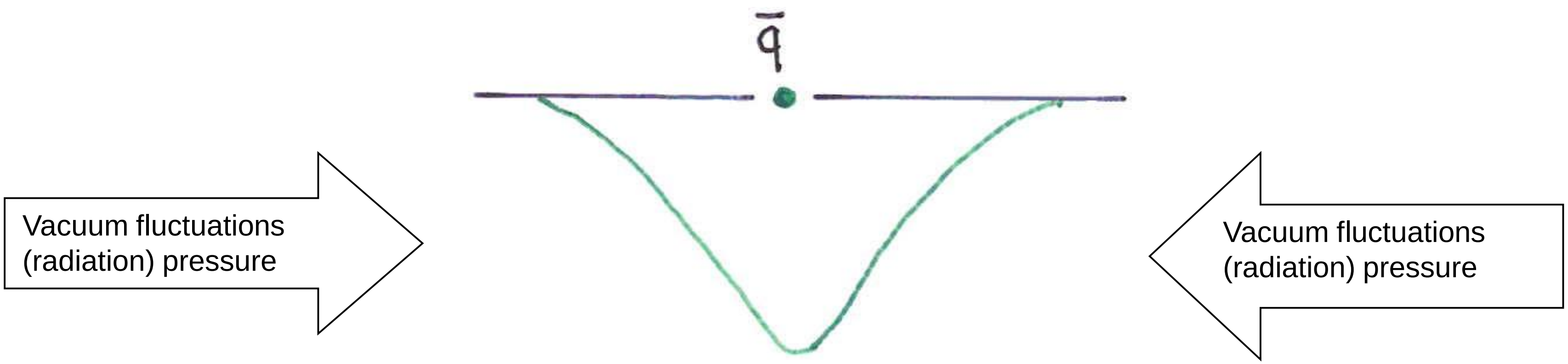
(SCQM)



Strongly Correlated Quark Model

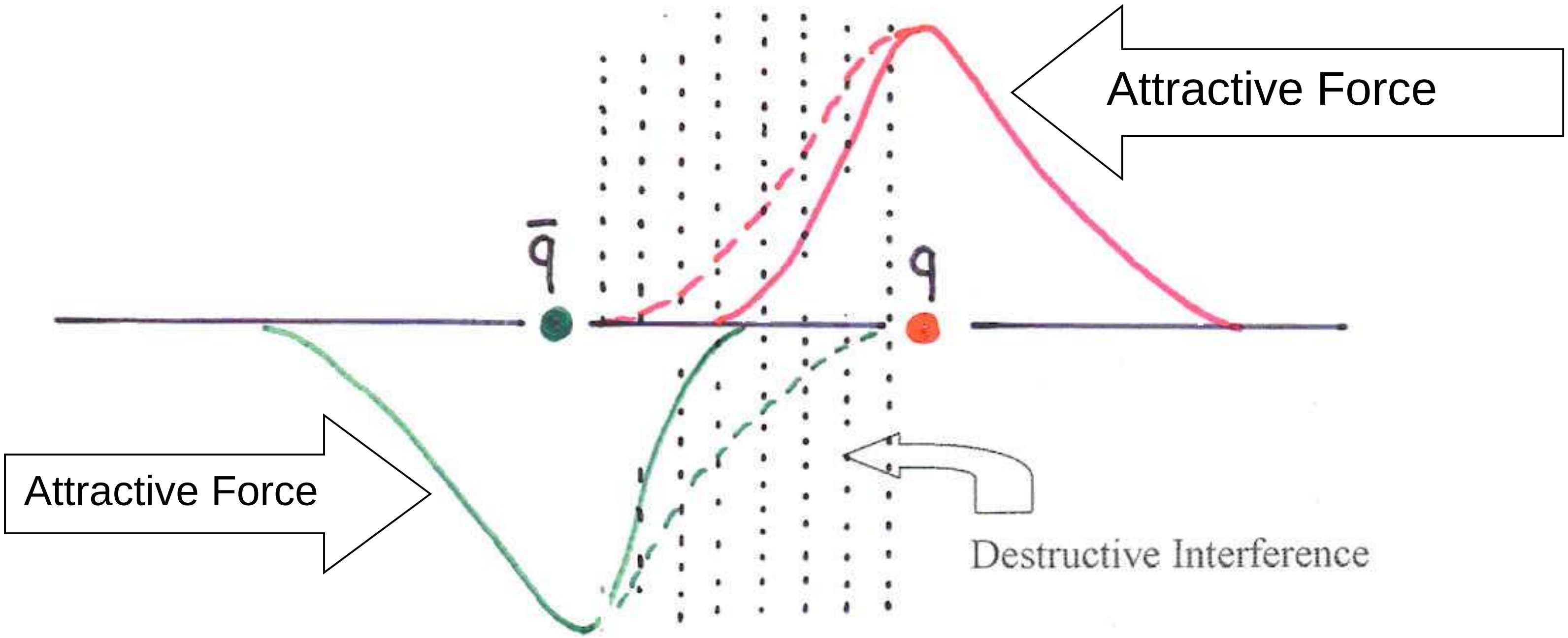
(SCQM)

$\varphi\,(x)$



Strongly Correlated Quark Model

(SCQM)



Overlap of opposite color fields → attraction force between quark and antiquark

Parameters of SCQM for the Nucleon

1.Mass of Consituent Quark

$$M_{Q(\overline{Q})}(x_{\max}) = \frac{1}{3}\left(\frac{m_{\Delta} + m_N}{2}\right) \approx 360 MeV,$$

2.Amplitude of VQs oscillations : x_{max}*=0.64 fm,*

3.Constituent quark dimensions (parameters of gaussian distribution): $\sigma_{x,y}$*=0.24 fm,* σ_z *=0.12 fm*

Parameters 2 and 3 are derived from the calculations of Inelastic Overlap Function (IOF) and σ_{tot} in $\bar{p}\,p$ and pp – collisions.

Do nuclei possess lattice-like structure?

Lattice Models (in terms of nucleons)

- Simple Cubic Lattice – *not relevant*
- Body Centered Lattice – *not relevant*
- **Face Centered Cubic Lattice (FCC)**

E.Wigner, Phys. Rev. 51(1937)106

Cook N. and V. Dallacasa, Phys. Rev. C35(1987)1883

FCC Nuclear Lattice

Skyrme model of baryon

- T. H. R. Skyrme, *Proc. R. Soc. London, Ser. A* ***260*** *(1961)127*
  Nucleons as topological solitons (skyrmions) in a nonlinear theory of
  π mesons.

- G. S. Adkins, C. R. Nappi and E. Witten, *Nucl. Phys. B228 (1983) 552*
  Protons and neutrons are spin-half quantum states
  of the basic Skyrmion, and they are combined into an
  isospin-half doublet of nucleons

FCC Nuclear Lattice

Skyrme models of nuclei

FCC Lattice of modified Skyrmions

M. Gillard et al., *Skyrmions with low binding energies*, Nucl. Phys. B895 (2015) 272;

M. Gillard, et al., *A point particle model of lightly bound Skyrmions*, Nucl. Phys. B917 (2017) 286;

- Skyrmionic nuclear ground state (g.s.) turns out to be a crystal.
- The symmetry is described by a group that includes elements of space groups and spin-isospin transformations.
- Skyrmions as particles are located precisely on the FCC vertices.
- Binding energies are compatible with data for $A \leq 28$

Summary (cont.)

Quantization

Rigid body quantization

As a rigid body Nuclei can possess:

- particle – hole excitations
- collective modes of excitations
  - Shape vibrations and fluctuations
  - Rotations
  - Isospin vibrations
  - Sissor fluctuations

Point-nucleon charge distributions of ^{3}**He** and ^{4}**He**
Hole inside ^{3}**He** and ^{4}**He**

I. Sick, PRC, vol. 15, No.4; LNP, vol. 87, p.236

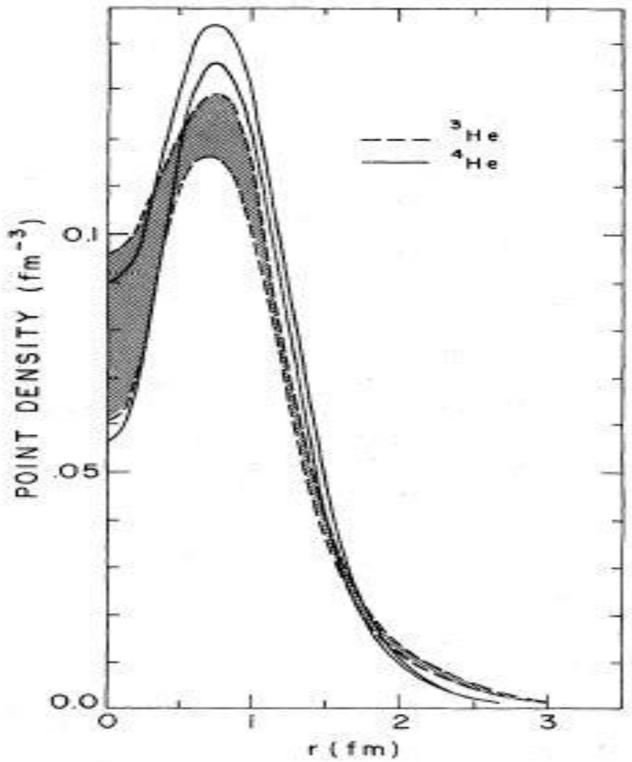


FIG. 15. Model-independent densities of pointlike protons in ^{3,4}He.

The hole is due to non-spherical, oblate shape of nucleons

Charge distributions ^{3}**He** and ^{4}**He**

I. Sick, PRC, vol. 15, No.4; LNP, vol. 87, p.236

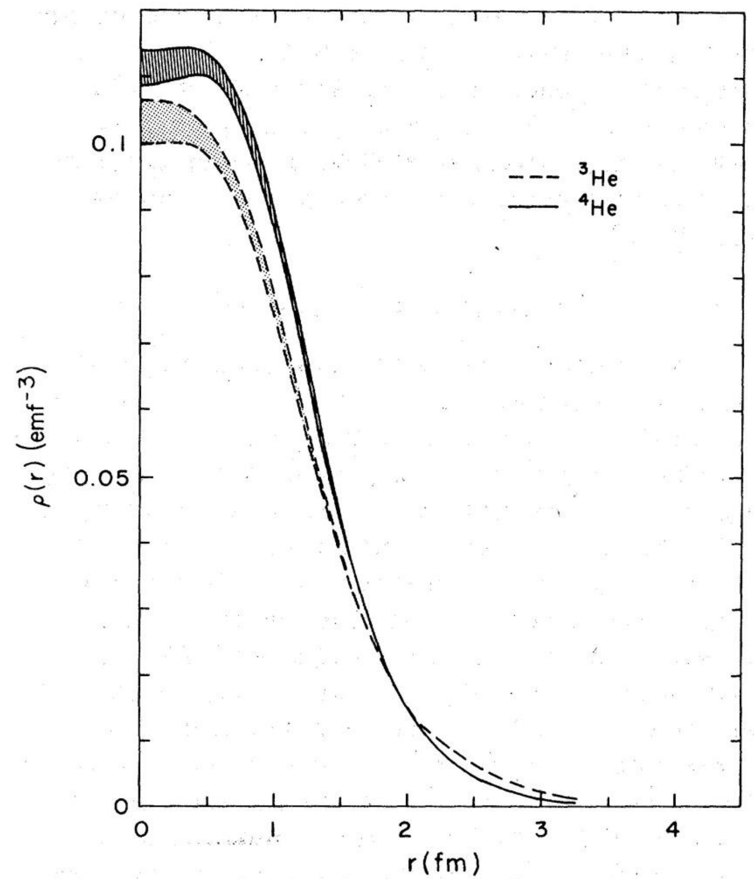
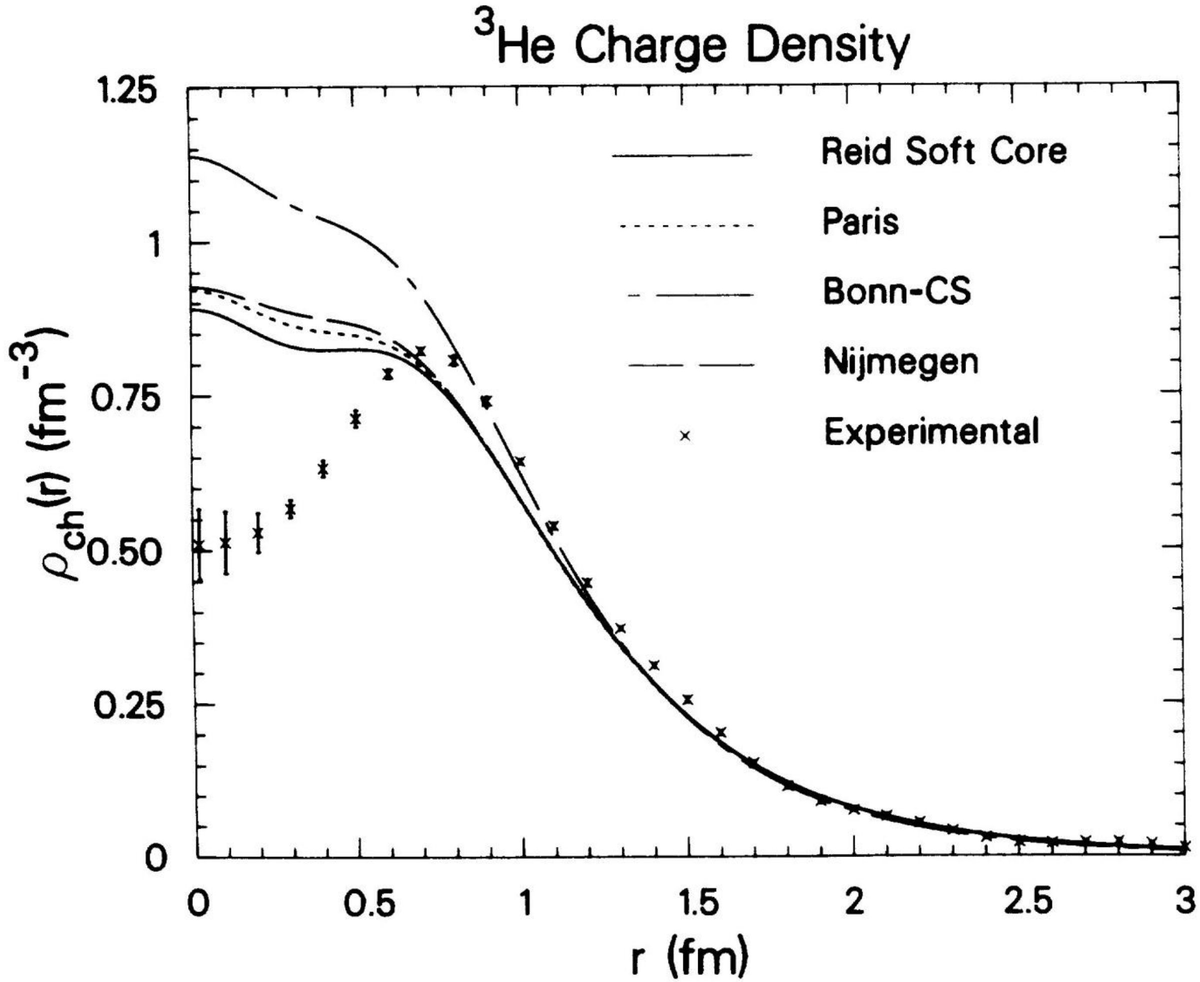


FIG. 11. ^{3,4}He model-independent charge densities.

Point-nucleon charge distributions
Hole inside ^{3}H and ^{3}He

I. Sick, PRC, vol. 15, No.4; LNP, vol. 87, p.236



Bound Hydrogen Isotopes