The Lipatov's effective action and Balitsky hierarchy

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Effective action

The Lipatov's effective action is gauge invariant and written in the covariant form in terms of gluon field v as

$$S_{eff} \,=\, -\int d^4 x \left(\frac{1}{4}\, G^{a}_{\mu\nu}\, G^{\mu\nu}_{a} \right. \\ \left. +\, \frac{1}{N} tr \left[\left(A_{+}(v_{+}) \,-\, A_{+} \right) \partial_i^2\, A^{+}_{a} \,+\, \left(A_{-}(v_{-}) \,-\, A_{-} \right) \partial_i^2\, A^{-}_{a} \right] \right),$$

where

$$A_{\pm}(v_{\pm}) \,=\, \frac{1}{g}\, \partial_{\pm}\, O(x^{\pm}, v_{\pm});\;\; O(x^{\pm}, v_{\pm}) \,=\, P\, e^{g \int_{-\infty}^{x^{\pm}} dx'^{\pm}\, v_{\pm}}\,.$$

There are additional kinematical constraints for the reggeon fields

$$\partial_{-}\, A_{+} \,=\, \partial_{+}\, A_{-} \,=\, 0\,,$$

[1] L. N. Lipatov, Nucl. Phys. B **452**, 369 (1995); Phys. Rept. **286**, (1997) 131;

The Lipatov's effective action formulated as RFT

$$e^{\imath\,\Gamma[A]} \,=\, \int\, Dv\; e^{\imath\, S_{eff}[v,\,A\,]}$$

The result of this action is that the effective action Γ can be expanded in terms of reggeon fields A_- and A_+ as

$$\begin{aligned}
\Gamma \;&=\; \sum_{n,m=1} \left(\mathcal{A}_{+}^{a_1} \,\cdots\, \mathcal{A}_{+}^{a_n} \, \left(K_{-\,\cdots\,-}^{+\,\cdots\,+}\right)_{b_1\cdots\, b_m}^{a_1\cdots\, a_n} \, \mathcal{A}_{-}^{b_1} \,\cdots\, \mathcal{A}_{-}^{b_m} \right) \\
&=\; -\mathcal{A}_{+\,x}^{a}\, \partial_i^2\, \mathcal{A}_{-\,x}^{a} \,+\, \mathcal{A}_{+\,x}^{a}\, \left(K_{xy}^{a\,b}\right)_{-}^{+}\, \mathcal{A}_{-\,y}^{b} \,+\, \cdots\,,
\end{aligned}$$

that determines this expression as functional of reggeon fields and provides effective vertices of the interactions of the reggeized gluons in the RFT calculus.

[2] S. Bondarenko, L. Lipatov, S. Pozdnyakov, A. Prygarin, Eur. Phys. J. C **77** (2017) no.9, 630.

The Lipatov's effective action formulated as RFT

1. From the Lipatov's formulation of the high energy scattering we know that the action can be written in terms of the physical degrees of freedom which are longitudinal reggeized gluons. Therefore, our local in rapidity action can be fully written in terms of the Reggeon, i.e. the action can reformulated as RFT Regge Field Theory (RFT) action.
2. In the formalism of Lipatov's effective action, formulated as RFT, there is an additional source of perturbative and unitarity corrections to high energy QCD amplitudes based on the diagrams constructed entirely in terms of Reggeon fields and their vertices of interactions
3. Classical solutions of equations of motion in the effective action are written with precision NNLO
4. Classical solutions of all orders have a common simple structure
5. We calculated the A+A+A vertex with all loop LLA precision and simpler structures, demonstrating their reggeization

The Balitsky equation

We consider a derivation of the hierarchy of correlators of orderedexponentials directly from the Lipatov's effective action [1] formulated interms of interacting ordered exponentials [3]. We have demonstrated the derivation of the Balitsky equation [4] from the hierarchy as well as the way the subleading eikonal corrections to the Balitsky equation arise from the transverse field contribution and sub-leading eikonal corrections to thequark propagator.

[3] S. Bondarenko and M. A. Zubkov, Eur. Phys. J. C 78 (2018) no.8, 617; S. Bondarenko, S. Pozdnyakov and M. A. Zubkov, Eur. Phys. J. C 81 (2021) no.7, 613.
[4] I. Balitsky, Nucl. Phys. B463 (1996) 99; Phys. Rev. D 60, 014020 (1999); In *Shifman, M. (ed.): At the frontier of particle physics, vol. 2* 1237-1342; Nucl. Phys. B 629, 290 (2002); Phys. Rev. D 72, (2005) 074027.

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The central result of the proposed framework in [5] is that it provides a useful instrument for the further perturbative calculations related to the BK equation.
First of all, the effective action allows to formulate the problem for the different QCD gauges directly in the action and, in turn, allows to determine corresponding redefinition of the correlators of interest.
Additionally, it clarifies the translation vocabulary between the different approaches. The proposed framework allows to calculate also the subleading corrections to the BK equation discussed in [5].

[5] S. Bondarenko, S. Pozdnyakov and A. Prygarin, Eur.Phys.J.C 81 (2021) 9, 793