Decoding the QCD critical behaviour in A+A collisions

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Intermittency and the Order Parameter Fluctuations (OPF) - the basic idea

Order parameters

The condensate $\langle \bar{q}q \rangle$ (σ-field)

Pro: Statistics

Contra: not directly measurable

$\Downarrow$

form (π^+, π^-)-pairs (combinatorial background)

Contra: fast component

$\Downarrow$

fluctuations wash-out quickly

The net-baryon (proton) density n_B (n_p)

Pro: direct measurable

Pro: Slow component

$\Downarrow$

fluctuations sustain (N_B conservation)

Contra: Statistics

Intermittency and the Order Parameter Fluctuations (OPF) - basic idea

Infinite system

- Self-similar OPF in d-dim. space $\Rightarrow$
- Self-similar OPF in d-dim. momentum space
- Power-laws in space: $\langle n_p^\sigma(\mathbf{r}_1) n_p^\sigma(\mathbf{r}_2)\rangle \sim |\mathbf{r}_{12}|^{-\Delta_p^\sigma}$ with $\Delta_p^\sigma = d - d_{F,p}^\sigma$, $d_{F,p}^\sigma$ = fractal dimension
- Power-laws in mom. space: $\langle n_p^\sigma(\mathbf{k}_1) n_p^\sigma(\mathbf{k}_2)\rangle \sim |\mathbf{k}_{12}|^{-\tilde{\Delta}_p^\sigma}$ with $\tilde{\Delta}_p^\sigma = d_{F,p}^\sigma$

Finite system (size L)

Finite-Size Scaling (FSS) regime:

- Power-law OPF for **large** distances: $|\mathbf{r}_{12}| \approx \mathrm{O}(L)$ $\Rightarrow$
- Power-law OPF for **small** momentum differences $|\mathbf{k}_{12}| \approx \mathrm{O}(\frac{1}{L})$ $\Rightarrow$
- **Intermittency**

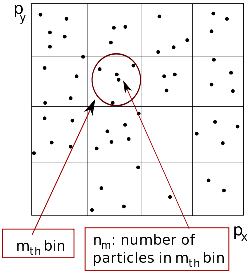
$\Downarrow$

Critical Opalescence in particle physics

Searching for the QCD critical point - basic steps

1. Determine the **order parameter** $\Rightarrow$ **proton** density here
2. Determine the **space** of analysis $\Rightarrow$ **transverse momenta at midrapidity** ($|y| < 0.75$, $\mathbf{p} \approx \mathbf{p_T} \otimes y$)
3. **Range** of analysis: $\Delta p_{\substack{x\\y}} \approx \mathrm{O}(1/L) \Rightarrow$ good choice $\frac{1}{L} < \Delta p_{\substack{x\\y}} < \frac{5}{L}$ leading to $20\ MeV \leq \Delta p_{\substack{x\\y}} \leq 100\ MeV$ for $L \approx 10\ fm$.
4. **Observable: Scaled second factorial moment**

$$F_2(M) = \frac{\sum_{m=1}^{M^2} \langle n_m(n_m-1)\rangle}{\sum_{m=1}^{M^2} \langle n_m\rangle^2}$$



Intermittency analysis and the QCD critical point (CP)

Expected behaviour

For a pure critical system (for $M \gg 1$)

$$F_{2,p}(M) \sim M^{2\phi_{2,cr}^{(p)}} \quad \left(F_{2,\sigma}(M) \sim M^{2\phi_{2,cr}^{(\sigma)}}\right)$$

with $\phi_{2,cr}^{(p)} = \frac{5}{6}$ $(\phi_{2,cr}^{(\sigma)} = \frac{2}{3})$ $\Leftrightarrow$ CP in the **3d Ising** universality class (UC)

Problems to be confronted with

- The **shape** of the function $F_{2,p}(M)$ is important $\Rightarrow$ **high statistics** needed, especially when the proton multiplicity/event is small.
- **Background** contributions must be removed (use mixed events):

$$\Delta F_{2,p}(M) = F_{2,p}^{(data)}(M) - F_{2,p}^{(mix.ev.)}(M) \quad ; \quad \Delta F_{2,p}(M) \sim M^{2\phi_{2,cr}^{(p)}}$$

- Measurements of $F_{2,p}(M)$ $(\Delta F_{2,p}(M))$ for different M-values are in general **correlated** $\Rightarrow$ fitting may lead to **wrong** values for ϕ_2.

Published experimental results - NA49 experiment

- Power-law behaviour with $\phi_2^{(p)} = 0.96^{+0.36}_{-0.25}$ and $\phi_2^{(\sigma)} \approx 0.35$ in transverse momentum space of protons and dipion pairs (π^+, π^-) observed in central Si+Si collisions at 158A GeV.

(NA49 experiment - SPS (CERN))

T. Anticic *et. al.*, Phys. Rev. C 81, 064907 (2010); Eur. Phys. J. C 75, 587 (2015).

- No such power-law behaviour observed in central C+C and Pb+Pb collisions at the same energy.
- Estimation using Critical Monte Carlo (CMC) events: the noise (background) level in Si+Si is very high $\approx 99.3\%$!
- It is likely that the Si+Si system freezes out close to the QCD critical endpoint $\Rightarrow$ Verification with much higher statistics is very important.
- A better (than usual fitting) technique for estimating the intermittency index ϕ_2 is needed.

AMIAS - a new, model independent, analysis scheme

The basics of $\underbrace{\text{A M I A S}}$ **(C. N. Papanicolas and E. Stiliaris)**

Athens Model Independent Analysis Scheme

1. Select the **physical model** for the description of the experimental data. In our case: $\Delta F_2(M) = 10^{a_0}\left(\frac{M^2}{10^4}\right)^{\phi_2}$
2. Choose randomly (uniform distribution) **any possible value** for the parameters of the physical model (here a_0 and ϕ_2) as an eventual description of the data.
3. Each choice (here (a_0, ϕ_2) pair) is **weighted with a probability** value retrieved out of the data set by a **cost function**.
4. As a result, from a given data set, we obtain a **PDF** (probability density function) for **each of the parameters** of the physical model.
5. The central value of a parameter and its uncertainty is the **expectation value** and the **standard deviation** of its **PDF**.

A test: applying AMIAS to CMC data

The AMIAS method has been successfully applied to:

- Pion photoproduction data for the extraction of the multipole excitation amplitudes

  L. Markou, E. Stiliaris, C. N. Papanicolas, Eur. Phys. J. A 54, 115 (2018).

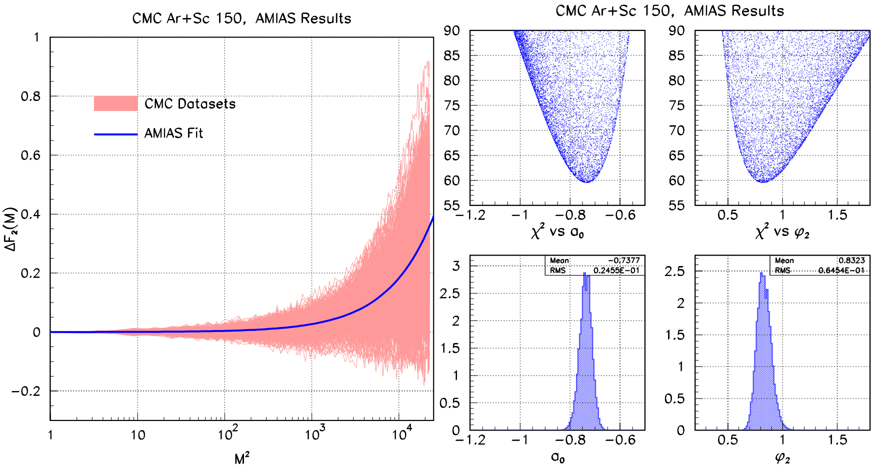
- Lattice QCD data for the detection of Tetraquark interpolating fields

  C. Alexandrou, J. Berlin, J. Finkenrath, Th. Leontiou and M. Wagner, Phys. Rev. D 101, 034502 (2020).

Testing AMIAS with contaminated CMC data:

- 400 data sets, each with 400k events with 0.7% critical protons and 99.3% noise.
- Multiplicity $\langle n_p \rangle$: 2.5p/event with standard deviation ~ 1.4 (identical to $\langle n_p \rangle$ in Ar+Sc at 150A GeV with centrality $10 - 20\%$).

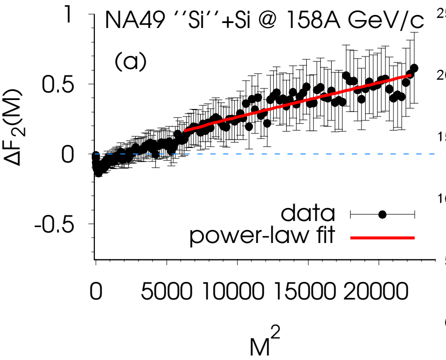
Retrieve critical protons in noisy CMC data with AMIAS

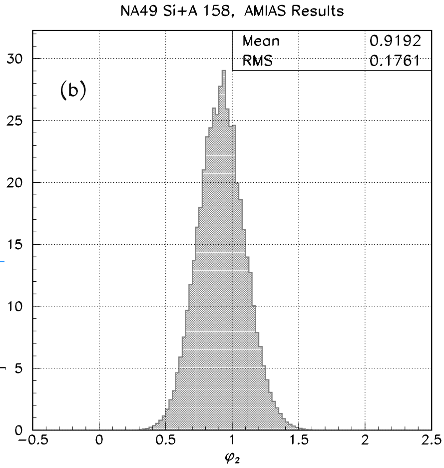


Estimated $\phi_2^{(p)}$-value: $0.83(06)$ very close to $\phi_{2,cr}^{(p)} = \frac{5}{6}$

Analyzing Si+Si NA49 data with AMIAS

For $|\mathbf{p_T}| \leq 1.5$ GeV $\Rightarrow$
scales of interest $30 \leq M \leq 150$





AMIAS result for $\phi_2^{(p)}$ in Si+Si central collisions at 158A GeV:

$$\phi_2^{(p)} = 0.92 \pm 0.18$$

Intermittency analysis in NA61/SHINE experiment

System of particular interest: protons produced in Ar+Sc at 150A GeV

- Freeze-out state of Ar+Sc should be close to that of Si+Si at 158A GeV.
- Centrality can be used to slightly shift the freeze-out state along the temperature direction.
- Intermittency analysis in Ar+Sc at 150A GeV could provide important information concerning the size of the critical region.
- Recently released data on intermittency analysis lead to inconclusive results concerning the measured ϕ_2-values.

$$\Downarrow$$

Employ AMIAS to obtain better estimates of ϕ_2

Proton intermittency analysis in Ar+Sc at 150A GeV

Analysis in **different centrality bins**

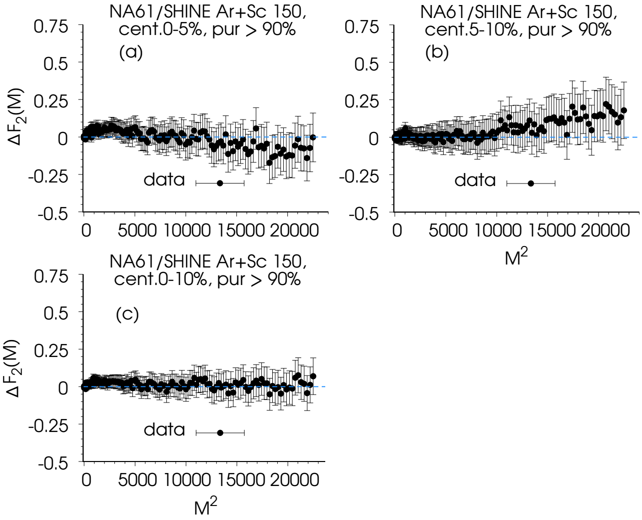
- High resolution in centrality, lower statistics
  centrality zones: $0-5\%$, $5-10\%$, $10-15\%$, $15-20\%$
- Low resolution in centrality, higher statistics
  centrality zones: $0-10\%$, $10-20\%$
- Maximize statistics $\Rightarrow$ centrality zone $0-20\%$

Lowering centrality $\Rightarrow$ expected slight freeze-out temperature increase.

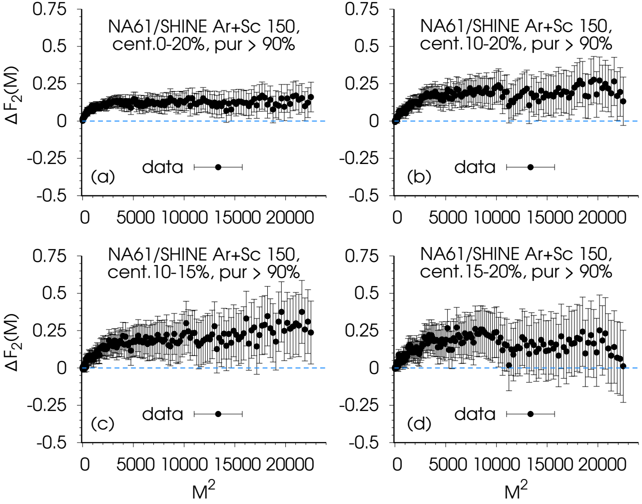
F. Becattini, M. Bleicher, E. Grossi, J. Steinheimer and R. Stock, Phys. Rev. C 90, 054907 (2014)

Suitable cuts applied to **avoid split tracks** and **proton misidentification**.

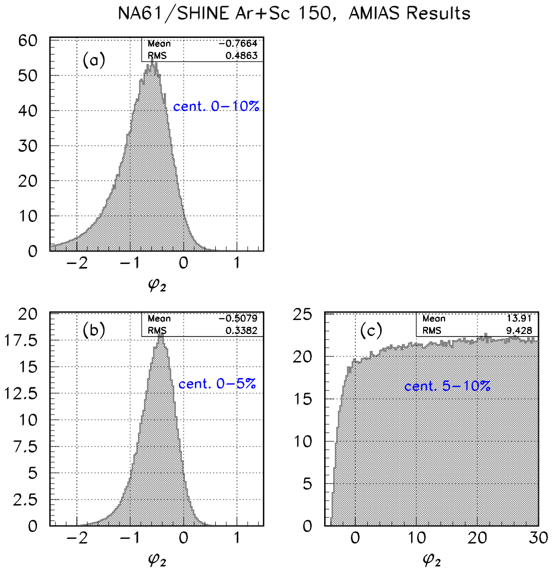
$\Delta F_2(M)$ for protons in Ar+Sc at 150A GeV



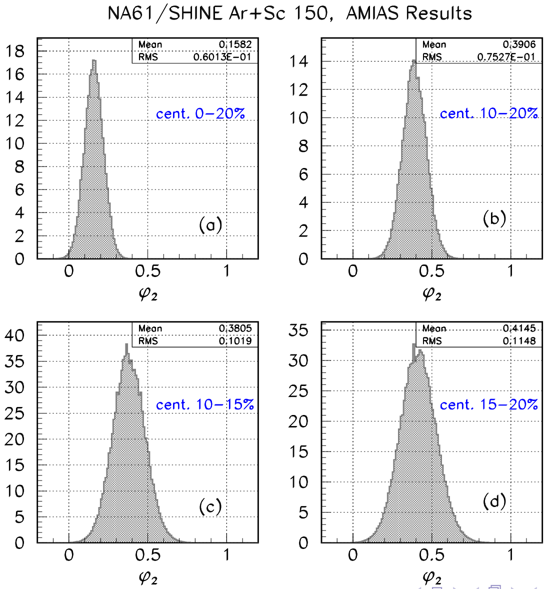
$\Delta F_2(M)$ for protons in Ar+Sc at 150A GeV



AMIAS estimates for ϕ_2 in Ar+Sc at 150A GeV

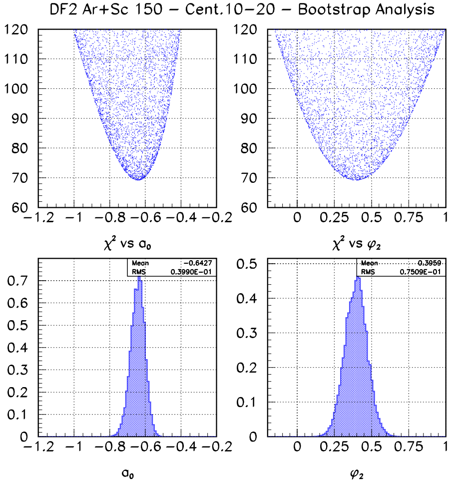


AMIAS estimates for ϕ_2 in Ar+Sc at 150A GeV



AMIAS estimates for ϕ_2 in Ar+Sc at 150A GeV

We use 1000 bootsrap samples generated from the $10 - 20\%$ centrality data set to estimate ϕ_2:



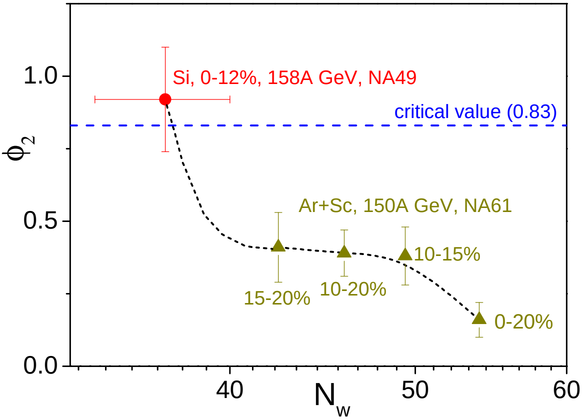
AMIAS estimates for ϕ_2 in Ar+Sc at 150A GeV

Number of wounded nucleons N_w calculated by a geometrical Glauber simulation

Reaction	centrality (%)	N_w	$\langle\phi_2\rangle$ $(\delta\phi_2)$
Ar+Sc	0-10	62(0.6)	-0.77(49)
Ar+Sc	10-20	45.9(0.5)	0.39(08)
Ar+Sc	0-5	66.6(0.9)	-0.51(34)
Ar+Sc	5-10	57.3(0.4)	–
Ar+Sc	10-15	49.4(0.4)	0.38(10)
Ar+Sc	15-20	42.4(0.5)	0.41(12)
Ar+Sc	0-20	54(0.6)	0.16(06)
Si+A	0-12	37(3)	0.92(18)

AMIAS $\langle\phi_2\rangle$ and corresponding error $\delta\phi_2$ results vs the estimated mean number of wounded nucleons N_w for central Si+Si and different Ar+Sc peripherality ranges.

Intermittency index ϕ_2 vs. N_w in Ar+Sc at 150A GeV



Inspired by: S. Pulawski [NA61/SHINE Collaboration], "Status and plans of the NA61 Experiment," News from the Experiments at CERN (131st Meeting of the SPSC), 16-17 October 2018, https://indico.cern.ch/event/758114/.

Representing the critical region in terms of N_w and ϕ_2

Ising-QCD partition function for the thermodynamic description of **proton density fluctuations** close to the **critical point**:

N. G. Antoniou, F. K. Diakonos, X. N. Maintas and C. E. Tsagkarakis, Phys. Rev. D 97, 034015 (2018)

$$\mathcal{Z} = \sum_{N=0}^{L} \zeta^N \exp\left[-\frac{1}{2}\hat{m}^2\frac{N^2}{L} - g_4\hat{m}\frac{N^4}{L^3} - g_6\frac{N^6}{L^5}\right]$$

where:

- N is the proton number in volume $V = L\beta_c^3$ ($\beta_c = \frac{1}{k_B T_c}$),
- $g_4 \approx 1$, $g_6 \approx 2$ are universal dimensionless couplings

  M. M. Tsypin, Phys. Rev. Lett. 73, 2015 (1994)
- $\hat{m} = \beta_c m$, $m = \xi^{-1}$ and $\xi = \xi_{0,\pm}|t|^{-\nu}$, $t = \frac{T-T_c}{T_c}$, ($\nu = \frac{2}{3}$, $\frac{\xi_{0,+}}{\xi_{0,-}} \approx 2$ for 3d Ising UC),
- $\zeta = e^{(\mu_B - \mu_{B,c})\beta_c}$ (fugacity)

Representing the critical region in terms of N_w and ϕ_2

The Ising-QCD partition function reproduces **all** 3d Ising scaling laws

N. G. Antoniou, F. K. Diakonos, X. N. Maintas and C. E. Tsagkarakis, unpublished

Intermittency effect contained in the size-dependence of the moments:

$$\langle N^k \rangle \sim L^{kq}, k = 1,\ 2, \ldots$$

expressing the FSS behaviour of the critical fluid

It holds: $q = \phi_2$ with $q_{cr} = \frac{5}{6}$ for the 3d Ising UC.

The power-law behaviour of $\langle N^k \rangle$ is gradually deconstructed departing from the critical point $t = 0$, $\ln \zeta = 0 \Rightarrow$ change of ϕ_2, decrease of power-law quality (R^2-criterion).

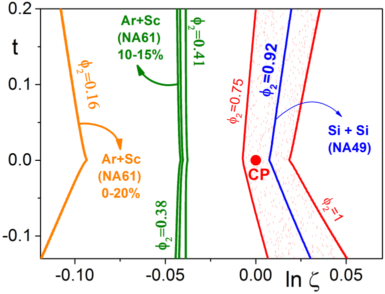
Proposal

Determine the location of the freeze-out states close to the critical point using (N_w, ϕ_2) instead of $(\ln \zeta, t)$.

Representing the critical region in terms of N_w and ϕ_2

Piecewise linear form of $\phi_2(\ln\zeta, t) = constant$

(Ising-QCD partition function with $R^2 > 0.7$ and $\underbrace{30 < L < 700}_{3\ \mathrm{fm}-9\ \mathrm{fm}}$)



upper blue line, leads to : $\begin{cases} t = a_{Si}\ln\zeta + b_{Si}\ ;\ a_{Si} = 47.52;\ b_{Si} = -0.36 \\ \mu_{B,c} = \mu_{B,Si} + \dfrac{(1+b_{Si})\,T_c - T_{Si}}{a_{Si}} \end{cases}$

Representing the critical region in terms of N_w and ϕ_2

We can also expand the function $N_w(\ln\zeta, t)$ around $N_{w,Si} \equiv N_w(\ln\zeta_{Si}, t_{Si})$ to lowest order:

$$N_w(\ln\zeta, t) = N_{w,Si} + \gamma_t(t - t_{Si}) + \gamma_\zeta(\ln\zeta - \ln\zeta_{Si})$$

use $\underbrace{N_{w,C} \equiv N_w(\ln\zeta_C, t_C)}_{(\ln\zeta_C, t_C) \text{ lies close to } (\ln\zeta_{Si}, t_{Si})}$, $N_{w,cr} \equiv N_w(0,0)$ to find γ_t, γ_ζ as:

$$\gamma_t = \frac{N_{w,Si} - N_{w,cr} - \gamma_\zeta \ln\zeta_{Si}}{t_{Si}}$$

$$\gamma_\zeta = \frac{t_{Si}(N_{w,C} - N_{w,Si}) + (t_C - t_{Si})(N_{w,cr} - N_{w,Si})}{t_{Si}(\ln\zeta_C - \ln\zeta_{Si}) - \ln\zeta_{Si}(t_C - t_{Si})}$$

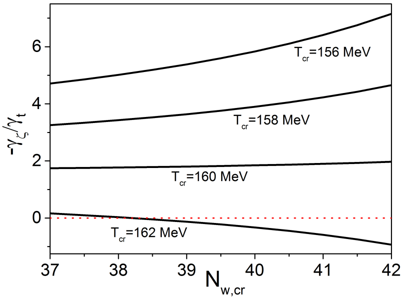
Unknown: T_c, $N_{w,cr}$ (from AMIAS analysis $37 < N_{w,cr} < 43$)

T_{Si}, $\mu_{B,Si}$, T_C, $\mu_{B,C}$ known from literature

F. Becattini, J. Manninen, M. Gazdzicki, Phys. Rev. C 73, 044905 (2006)

Representing the critical region in terms of N_w and ϕ_2

Dependence of the ratio $-\frac{\gamma_\zeta}{\gamma_t}$ on $N_{w,cr}$ for various T_c:

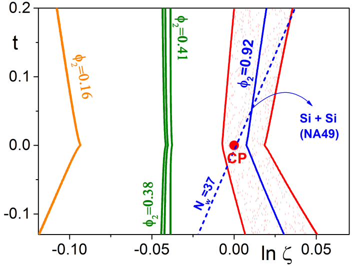


Interesting choice: $T_c = T_{Si} - \frac{(N_{w,cr} - N_{w,Si})(T_C - T_{Si})}{N_{w,Si} - N_{w,C}} \Rightarrow \gamma_\zeta = 0$

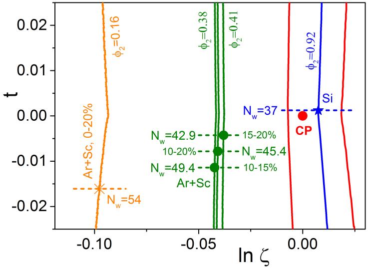
F. Becattini, J. Manninen, M. Gazdzicki, Phys. Rev. C 73, 044905 (2006);

F. Becattini, M. Bleicher, E. Grossi, J. Steinheimer and R. Stock, Phys. Rev. C 90, 054907 (2014)

Representing the critical region in terms of N_w and ϕ_2



$T = 156$ MeV, $N_{w,cr} = 39.5$



$T = 162$ MeV, $\underbrace{\gamma_\zeta = 0}_{N_{w,cr}=38.2}$

Conclusions

- The AMIAS method is used to extract distributions of ϕ_2 (intermittency index) from the preliminary NA61/SHINE results for the transverse momentum proton correlator $\Delta F_2(M)$ of the Ar+Sc system (150A GeV/c beam momentum) at different peripheralities.

- The AMIAS analysis supports the presence of a non-vanishing intermittency effect in the $10 - 20\%$ peripherality interval (Ar+Sc) and confirms the presence of critical fluctuations in central Si+Si collisions at 158A GeV/c (NA49 experiment).

Conclusions - continued

- The AMIAS results for ϕ_2, combined with an estimation of the corresponding number of wounded nucleons N_w for each considered freeze-out state, allow the mapping of the critical region and its neighbourhood in the reduced baryochemical-temperature plane, in terms of the quantities ϕ_2 and N_w.
- The derived mapping indicates that the preliminary NA61/SHINE intermittency results for Ar+Sc are fully compatible with the corresponding NA49 measurements (Si+Si) reflecting the approach of the Ar+Sc freeze-out states towards the critical region with increasing peripherality.
- The performed analysis leads to a rough estimation for the location of the critical point $T_c \approx 160$ MeV, $\mu_{B,c} \approx 258$ MeV, $N_{w,cr} \approx 39$.