Black Holes, Entropy and Holography

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Overview

1. Motivation
2. Gravity in (2+1) Dimensions
3. BTZ Black Hole
4. Asymptotically AdS_3 Spacetimes
5. Entropy of BTZ Black Hole
6. Current Research

1.) Motivation

Known since the 70s after seminal works of Hawking, Bekenstein, Bardeen and Carter that black holes behave like thermodynamic systems

- they have a temperature
- they have a large entropy
  - for black hole in center of the milky way $S \sim 10^{88} k_B$

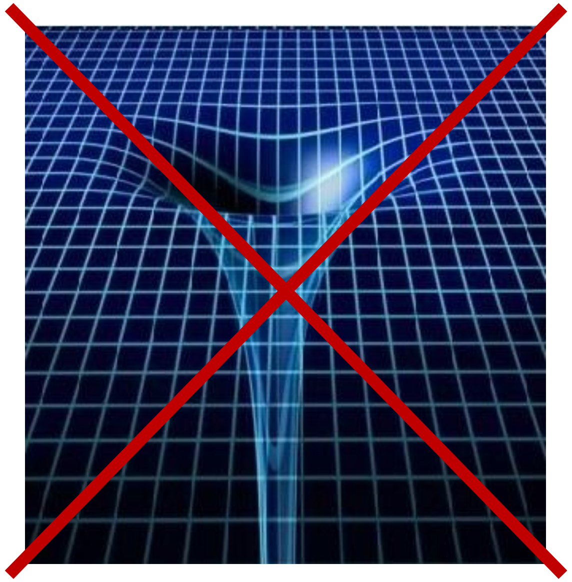
Q: Can this entropy be derived through a microscopic derivation?

2.) Gravity in (2+1) Dimensions

- Useful to consider toy models to address this question
- One such toy model: 3d Einstein gravity
  - No gravitational waves in 3d Einstein gravity
  - Curvature is constant for all solutions of Einstein's equations

$$R_{\mu\nu\rho\sigma} = \Lambda\left(g_{\mu\rho}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\rho}\right)$$

Λ ... Cosmological constant



2.) Gravity in (2+1) Dimensions

- Depending on cosmological constant, every solution looks locally like

  - Anti-de-Sitter space (AdS_3) for $\Lambda < 0$
  - de-Sitter space for $\Lambda > 0$
  - Minkowski for $\Lambda = 0$

- Locally 3d Einstein gravity seems trivial

  ⇒ Global properties allow for interesting solutions

3.) BTZ Black Hole

- 1992 Bañados, Teitelboim, Zanelli found 2-parameter family of solutions to Einstein's equations with $\Lambda = -\frac{1}{l^2} < 0$

$$ds^2 = -N(r)^2 dt^2 + \frac{1}{N(r)^2} dr^2 + r^2 (d\varphi + N_\varphi(r) dt)^2$$

$$N(r) = -8\,G\,M + \frac{r^2}{l^2} + \frac{16 G^2 J^2}{r^2} \qquad N_\varphi(r) = -\frac{4\,G\,J}{r^2}$$

for $M > 0$ and $|J| \leq M\,l$ Black Hole

3.) BTZ Black Hole

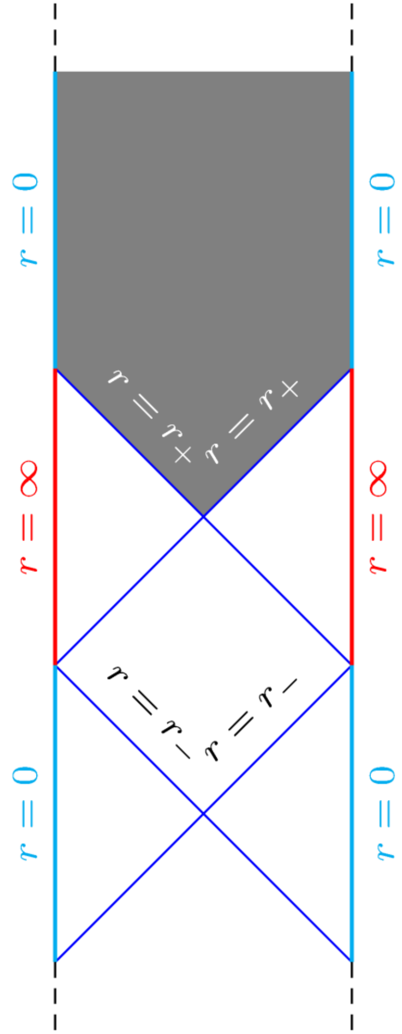
- BTZ black hole and AdS_3 locally the same
- BTZ black hole can be obtained as a quotient of AdS_3

$$M = \frac{r_+^2 + r_-^2}{8\,G l^2} \qquad J = \frac{r_+\, r_-}{4\,G\,l}$$

$$S = \frac{A}{4\,G} = \frac{2\,\pi\, r_+}{4\,G} \qquad T = \frac{r_+^2 - r_-^2}{2\,\pi\, l^2 r_+}$$

- Can we reproduce the entropy of BTZ black hole through a microscopic calculation?

Units are chosen such that $k_B = \hbar = c = 1$



4.) Asymptotically AdS3 Spacetimes

- asymptotically AdS_3 spacetimes: metric approaches AdS_3 space when approaching infinity

$$ds^2 \xrightarrow{r \to \infty} {ds^2}_{AdS}$$

  - Amongst asymptotically AdS_3 spacetimes: AdS_3 space, BTZ black hole

- Brown and Henneaux (1986) studied asymptotic symmetries of these asymptotically AdS_3 spacetimes
  - asymptotic symmetries = symmetries that leave asymptotic form of metric invariant

4.) Asymptotically AdS3 Spacetimes

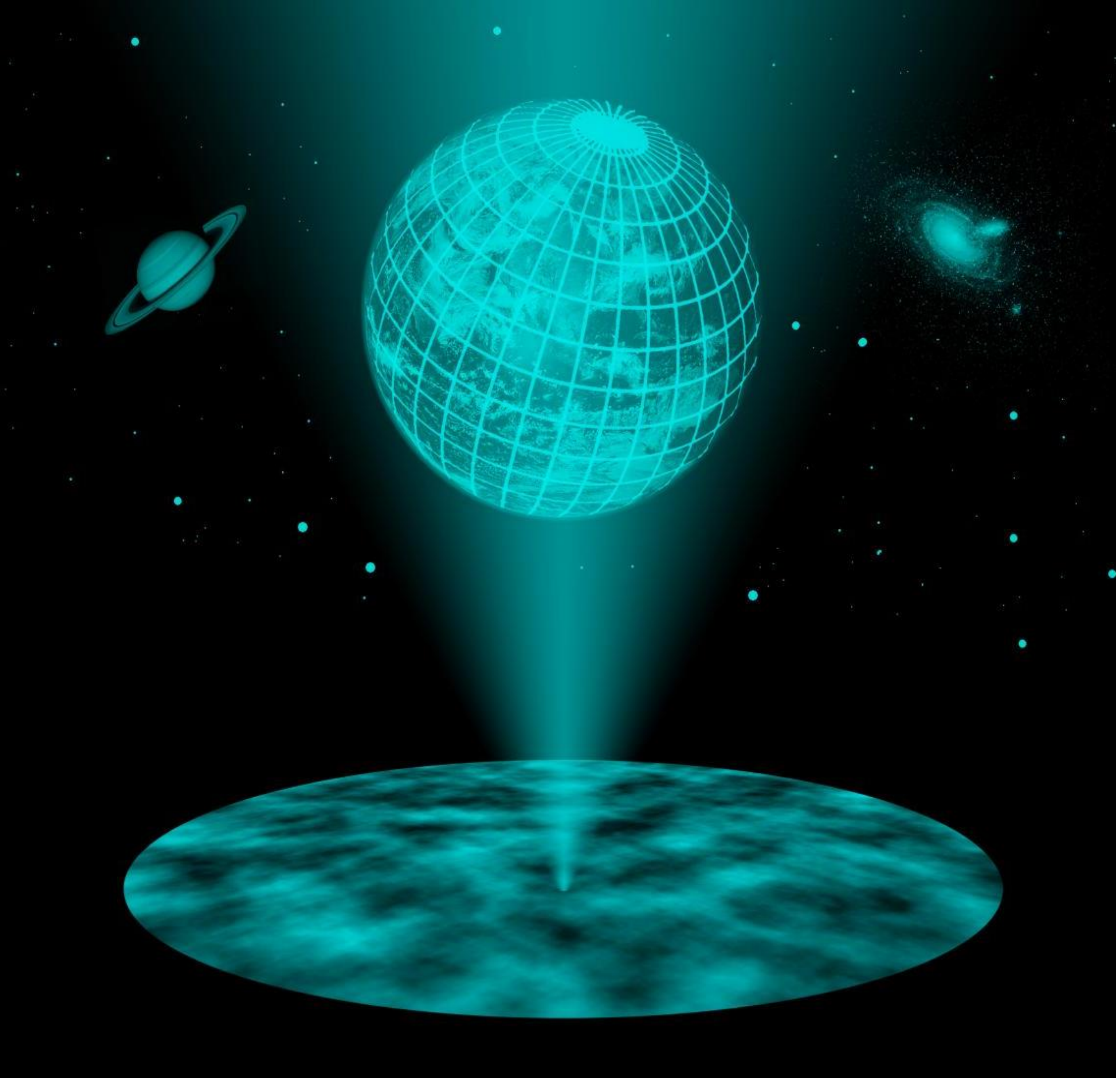
- Brown and Henneaux (1986)
  - infinitely many asymptotic symmetries and associated charges

$$\mathrm{i}\,\{L_n^{\pm}\,, L_m^{\pm}\} = (n-m)\,L_{n+m}^{\pm} + \frac{c}{12}\,(n^3-n)\,\delta_{n+m,0} \qquad \text{with } \mathrm{c} = \frac{3\,l}{2\,G}$$

$$\mathrm{i}\,\{L_n^{+}, L_m^{-}\} = 0$$

$$\mathrm{M} = \frac{L_0^{+} + L_0^{-}}{l} \qquad\qquad \mathrm{J} = L_0^{+} - L_0^{-}$$

  - symmetries of a two-dimensional conformal field theory (CFT_2)
  - precursor of the AdS_3 / CFT_2 correspondence



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4.) AdS_3 / CFT_2 Correspondence

- AdS_3 / CFT_2 Correspondence: quantum gravity on asymptotically AdS_3 spacetimes is dual to a two-dimensional conformal field theory
- General conjecture= Holographic Principle:

  Quantum gravity in (d+1)-dimensions $\leftrightarrow$ d-dimensional non-gravitational quantum field theory

5.) Entropy of BTZ Black Hole

- Use symmetries to derive entropy in 2d conformal field theory (Cardy 1986)

$$[L_n^{\pm}, L_m^{\pm}] = (n-m)\, L_{n+m}^{\pm} + \frac{c}{12}\,(n^3 - n)\,\delta_{n+m,0} \quad \text{with } c = \frac{3\,l}{2\,G}$$
$$[L_n^{+}, L_m^{-}] = 0$$

$$S_{CFT} = 2\pi\sqrt{\frac{c}{6}\,\Delta^{+}} + 2\pi\sqrt{\frac{c}{6}\,\Delta^{-}}$$

- $\Delta^{\pm}$ ... Eigenvalues of $L_0^{\pm}$

- Cardy formula counts states in a 2d conformal field theory

5.) Entropy of BTZ Black Hole

- Strominger (1997): entropy of BTZ black hole matches Cardy formula

$$S_{BTZ} = \frac{2\,\pi\, r_+}{4\,G} = 2\,\pi\,\sqrt{\frac{c}{6}\,\Delta^+} + 2\,\pi\,\sqrt{\frac{c}{6}\Delta^-} = S_{CFT}$$

$$\mathrm{M} = \frac{\Delta^+ + \Delta^-}{l} \qquad\qquad \mathrm{J} = \Delta^+ - \Delta^-$$

- Entropy of BTZ black hole can be reproduced from counting in CFT_2

6.) Current Research

- How general is holography?
- Can we move away from AdS spacetimes
- JHEP11 (2020) with S. Detournay, W. Merbis, G. S. Ng
  - considered particular solutions to topologically massive gravity with positive cosmological constant in 3d
  - Reproduced their entropy through Cardy-like counting
- Ongoing work with Daniel Grumiller in 4d
  - Goal: reproduce entropy of cylindrical Kerr-AdS black holes through Cardy-like counting

Thank you!

One more thing …

Strings 22

- Main Organizers: Stefan Fredenhagen, Daniel Grumiller
- ~ 500 participants expected
- Takes place since 1989
- Outreach Activities: January – July 2022
- More information: www.strings22.at

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