Holographic QCD and the anomalous magnetic moment of the muon

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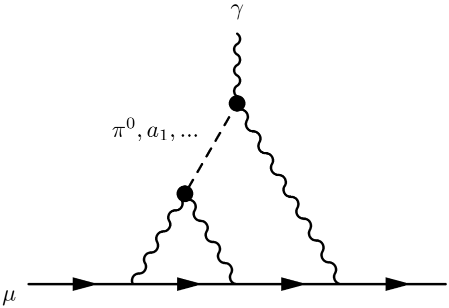
Outline of talk

- **Pion and axial vector meson transition form factors** from (chiral) holographic QCD:

  [JL, J. Mager, A. Rebhan, 1906.11795]
  [JL, A. Rebhan, 1912.01596]

  - Comparison with recent low-energy data (BESIII)
  - Short distance constraints
  - hQCD prediction for hadronic light-by-light scattering contribution to $(g-2)_\mu$
- Results from hQCD with **finite quark masses**
  with infinite towers of both pseudoscalars and axials

  [JL, A. Rebhan, 2108.12345]



Anomalous TFFs from holographic QCD

Pion TFF: [Grigoryan, Radyushkin, PRD76,77,78 (2007-8)]
[Cappiello, Catà, D'Ambrosio, PRD83 (2011)]
[JL, J. Mager, A. Rebhan, PRD100 (2019) - 1906.11795]
Axial-vector TFF: [JL, A. Rebhan, PRD101 (2020) - 1912.01596]
[Cappiello, Catà, D'Ambrosio, Greynat, Iyer, PRD102 (2020) - 1912.02779]

In bottom-up hQCD models, (as in the top-down string-theoretical Sakai-Sugimoto (SS) model,)
pions & (axial) vector mesons described by 5d-YM fields $\mathcal{F}^{L,R}_{MN} = \mathcal{F}^{V}_{MN} \mp \mathcal{F}^{A}_{MN}$

$$S_{\rm YM}^{U(N_f)\times U(N_f)} \propto \text{tr} \int d^4x \int_0^{z_0} dz\, e^{-\Phi(z)} \sqrt{-g}\, g^{PR} g^{QS} \left(\mathcal{F}^{(L)}_{PQ} \mathcal{F}^{(L)}_{RS} + \mathcal{F}^{(R)}_{PQ} \mathcal{F}^{(R)}_{RS} \right),$$

where $P, Q, R, S = 0, \ldots, 3, z$ and $\mathcal{F}_{MN} = \partial_M \mathcal{B}_N - \partial_N \mathcal{B}_M - i[\mathcal{B}_M, \mathcal{B}_N]$

conformal boundary at $z = 0$, either sharp cut-off of AdS$_5$ at z_0 (HW) or with nontrivial dilaton $z_0 = \infty$ (SW)

(SS: finite z_0, corresponding to point where D8 branes join; not asymptotically AdS$_5$ $\Rightarrow$ no matching to pQCD possible)

Chiral symmetry breaking either from extra bifundamental scalar field (HW1)
or through different boundary conditions for vector/axial-vector fields at z_0 (Hirn-Sanz (HW2), as in SS)

Anomalies uniquely from *Chern-Simons term:* (by hand in bottom-up models, from D8 branes in SS model)

$$S^L_{\rm CS} - S^R_{\rm CS}, \quad S_{\rm CS} = \frac{N_c}{24\pi^2} \int \text{tr} \left(\mathcal{B}\mathcal{F}^2 - \frac{i}{2}\mathcal{B}^3\mathcal{F} - \frac{1}{10}\mathcal{B}^5 \right).$$

Anomalous TFFs from holographic QCD

Vector meson dominance (VMD):
Electromagnetic background fields through non-normalizable modes for $\mathcal{B}^V$ with $\mathcal{B}^V_\mu|_{z=0} = e\mathcal{Q}A^{e.m.}_\mu$
Bulk-to-boundary propagator $\mathcal{J}$ contains sum over infinite tower of vector mesons,

$$\mathcal{J}^{\mathrm{HW}}(Q,z) = Qz\left[K_1(Qz) + \frac{K_0(Qz_0)}{I_0(Qz_0)} I_1(Qz)\right], \qquad (M_1^V = m_\rho = 775\text{ MeV} \Rightarrow z_0 = 3.103\text{ GeV}^{-1}),$$

$$F_{\pi\gamma^*\gamma^*}(Q_1^2,Q_2^2) = -\frac{N_c}{12\pi^2 f_\pi}\int_0^{z_0} \mathcal{J}(Q_1,z)\mathcal{J}(Q_2,z)\partial_z\alpha(z)dz$$

$$\mathcal{M}_{\mathcal{A}\gamma^*\gamma^*} \propto \epsilon^\mu_{(1)}\epsilon^\nu_{(2)}\epsilon^{*\rho}_{\mathcal{A}}\epsilon_{\mu\nu\rho\sigma}\left[q^\sigma_{(2)}Q_1^2 A(Q_1^2,Q_2^2) - q^\sigma_{(1)}Q_2^2 A(Q_2^2,Q_1^2)\right]$$

with asymmetric A:

$$A(Q_1^2,Q_2^2) = \frac{2g_5}{Q_1^2}\int_0^{z_0}\left[\frac{d}{dz}\mathcal{J}(Q_1,z)\right]\mathcal{J}(Q_2,z)\,\psi^{\mathcal{A}}(z)\,dz$$

- **Landau-Yang theorem** realized by $\mathcal{J}'(Q,z)=0$ for $Q^2=0$
- Amazingly, bottom-up models with asymptotic AdS$_5$ geometry reproduce **asymptotic momentum dependence of pQCD (Brodsky-Lepage) for pions and axials!** (see Hoferichter & Stoffer, 2004.06127 for axials)
- HW1: **correct asymptotic prefactor**
  HW2: with correct IR-fit of m_ρ and $f_\pi = 92.4$ MeV only 62% of LO pPQCD value!

Holographic pion TFF and experimental data

Slope parameter:

$$F_{\pi^0\gamma^*\gamma^*}(Q_1^2, Q_2^2)/F(0,0) = 1 + \hat{\alpha}(Q_1^2 + Q_2^2) + O(Q^4)$$

Holographic predictions: (only free parameters: $m_\rho = 775$ MeV, $f_\pi = 92.4$ MeV)

Model	$\hat{\alpha}$[GeV^{-2}]
Sakai-Sugimoto	-2.043
HW1	-1.595
HW2 (Hirn-Sanz)	-1.805
SW	-1.665

(cp. with $m_\rho^{-2} = 1.665$GeV^{-2})

Experimental data (fits):

Experiment	$\hat{\alpha}$[GeV^{-2}]
PDG (before NA62)	-1.76(22)
NA62 (Dalitz decays $\pi^0 \to \gamma e^+ e^-$)	-2.02(31)
PDG (after NA62)	-1.84(17)
DRV4 (CELLO,CLEO,BESIII)	-1.74(2)

DRV: simple monopole fits up to $Q^2 = 4$ or 9 GeV^2 with lowest $Q^2 \approx 0.3$ GeV^2

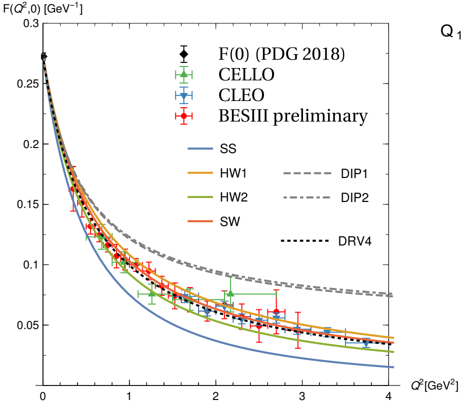
[Danilkin, Redmer, Vanderhaeghen, 1901.10346]

Holographic TFFs and experimental data

Single-virtual pion TFF:

[J. Leutgeb, J. Mager & AR, 1906.11795]

(data from Danilkin et al., Prog.Part.Nucl.Phys. 107 (2019) 20)



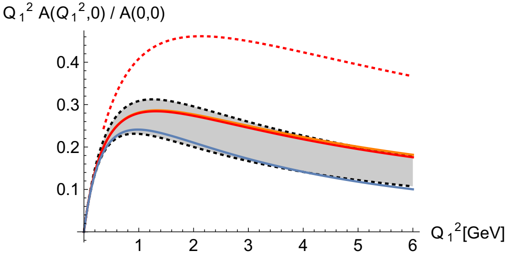
Danilkin et al. (DRV) fit below 4 GeV^2

bracketed by HW1 and HW2!

Single-virtual axial TFF:

[J. Leutgeb & AR, 1912.01596]

dipole fit of L3 data for $f_1(1285)$ (gray band) vs. SS, HW1, and HW2 models:



hQCD results:	HW1	HW2
$\|A(0,0)\|$ [GeV^{-2}]	21.04	16.63

$$A(0,0)^{\text{L3 exp.}}_{f_1(1285)} = 16.6(1.5)\,\text{GeV}^{-2}$$

Roig & Sanchez-Puertas, 1910.02881:

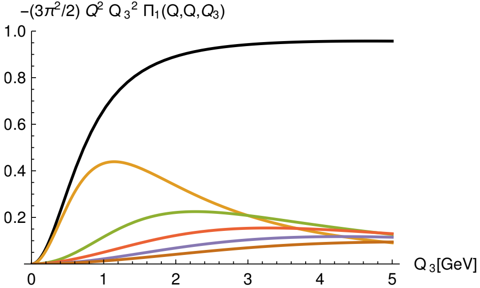
$$A(0,0)_{a_1(1230)} = 19.3(5.0)\,\text{GeV}^{-2}$$

Axial vector contributions to SDC

Infinite tower of axial-vector mesons responsible for satisfying the longitudinal SDC

- MV-SDC $\lim_{Q_3\to\infty}\lim_{Q\to\infty} Q^2 Q_3^2 \bar{\Pi}_1(Q,Q,Q_3) = -\frac{2}{3\pi^2}$: 100% for HW1 and HW2(UV-fit)

HW2 model with $g_5^2 = 4\pi^2$ (UV-fit) at large $Q = 50$GeV and increasing $Q_3 \ll Q$:



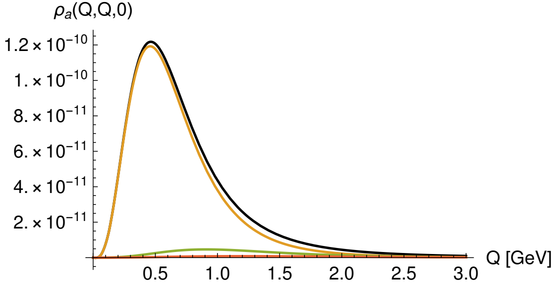
black line: infinite sum
colored lines: first 5 axial vector modes

- SDC for symmetric limit $Q_1^2 = Q_2^2 = Q_3^2 \to \infty$ satisfied qualitatively, but quantitatively only at max. 80% level (for HW1 and HW2(UV-fit))

Total axial-vector contributions to muon $g-2$

$$a_\mu^{\mathrm{AV}} = \int_0^\infty dQ_1 \int_0^\infty dQ_2 \int_{-1}^1 d\tau \, \rho_a(Q_1, Q_2, \tau)$$

E.g. at $\tau = 0$:



Strongly dominated by lowest axials, but nonnegligible contribution from higher modes:

	$j=1$	$j \le 2$	$j \le 3$	$j \le 4$	$j \le 5$	a_μ^{AV}
HW1	31.4	36.2	37.9	39.1	39.6	40.6 $\times 10^{-11}$
HW2	23.0	26.2	27.4	27.9	28.2	28.7 $\times 10^{-11}$

HW axial-vector results ≈ 60% longitudinal + 40% transversal (long. prop.: $q^\mu_{(3)} q^\nu_{(3)} / (M_n^A Q_3)^2$)

Pseudoscalar plus axial vector contributions to a_μ

Our results [J. Leutgeb & AR, 1912.01596]: (combined with $a_\mu^{\rm PS}$ [1906.11795])

(z_0 s.t. $m_\rho = 775$ MeV, $f_\pi = 92.4$ MeV; degenerate a_1, f_1, f_1')

	HW1 (100% LSDC)	HW2 (62% LSDC)
$a_\mu^{\rm PS}[\pi^0+\eta+\eta'] \times 10^{11}$	92 [65.2+18.2+15.6]	84 [59.2+15.9+13.4]
$a_\mu^{\rm AV}[L+T] \times 10^{11}$	41 [23+18]	29 [17+12]
$a_\mu^{\rm PS+AV} \times 10^{11}$	140	112

(compare with MV model: longitudinal contribution estimated $\sim$ 38 $\times 10^{-11}$)

independently at the same time:

[L. Cappiello, O. Cata, G. D'Ambrosio, D. Greynat, A. Iyer, 1912.02779]:

- **agreement**, but different parameters:

HW2$^{(1)}$: z_0 s.t. $m_\rho = 776$ MeV, $f_\pi = 93$ MeV, $f_{\eta'} = 74$ MeV
HW2$^{(2)}$: z_0 s.t. 100% UV limit (but $m_\rho = 987$ MeV !)

	HW2$^{(2)}$ (100% LSDC)	HW2$^{(1)}$ (62% LSDC)
$a_\mu^{\rm PS}[\pi^0+\eta+\eta'] \times 10^{11}$	112 [75+21+16]	81 [57+14+10]
$a_\mu^{\rm AV}[L+T] \times 10^{11}$	32 [18$^{*)}$+14]	28 [14+14]
$a_\mu^{\rm PS+AV} \times 10^{11}$	144	110

$^{*)}$ 15 acc.to Colangelo et al. 2106.13222

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HW models with finite quark masses (LR 2108.12345)

HW1m: HW1 with finite quark mass; **HW3**: HW1 with HW2 b.c.;
Δ^+: scaling dimension of operator dual to Φ

Analytic result: **LSDC completely saturated by axials, no contribution from heavy PS!**
(as long as Δ^+ in holographically allowed range $2 \leq \Delta^+ < 4$)

model	PS	$n=1$	$n=2$	$n=3$	AV	$n=1$	$n=2$	$n=3$
HW1 chiral	m_π[MeV]	0|135	1888	2879	m_{a_1}[MeV]	1375	2154	2995
$\Delta^+ = 3$	$a_\mu^\pi \cdot 10^{11}$	65.2	0.7	0.1	$4a_\mu^{a_1} \cdot 10^{11}$	31.4	4.7	1.8
HW1m	m_π[MeV]	**135***	1892	2882	m_{a_1}[MeV]	1367	2141	2987
$\Delta^+ = 3$	$a_\mu^\pi \cdot 10^{11}$	66.0	0.7	0.1	$4a_\mu^{a_1} \cdot 10^{11}$	31.4	4.9	1.8
HW1m'	m_π[MeV]	**135***	1591	2564	m_{a_1}[MeV]	**1230***	1977	2901
$\Delta^+ = 2.404$	$a_\mu^\pi \cdot 10^{11}$	64.3	1.5	0.3	$4a_\mu^{a_1} \cdot 10^{11}$	29.8	8.7	2.0
HW3m	m_π[MeV]	**135***	1715	2513	m_{a_1}[MeV]	1431	2421	3398
$\Delta^+ = 3$	$a_\mu^\pi \cdot 10^{11}$	66.6	0.8	0.04	$4a_\mu^{a_1} \cdot 10^{11}$	32.7	3.4	1.8
HW3m'	m_π[MeV]	**135***	**1300***	2113	m_{a_1}[MeV]	1380	2355	3345
$\Delta^+ = 2.399$	$a_\mu^\pi \cdot 10^{11}$	66.0	1.5	0.01	$4a_\mu^{a_1} \cdot 10^{11}$	33.2	4.1	1.8

* fitted

- Numerically: some *increase* compared to chiral model with m_π inserted in propagator by hand
  $((a_\mu - a_\mu^{\text{chiral}})_{\pi+a_1} = +(0.8+0.2) \times 10^{-11})$
- Contribution of heavy PS
  smaller than in PS Regge model of Colangelo et al., where $\Delta a^{\text{PS}}_{\pi(n>1)} = 2.7$ prior to OPE substitutions

Conclusions/Outlook

- hQCD is not QCD, but sophisticated toy model that can give clues on
  – how short-distance behavior can be implemented at the hadronic level
  - important fundamental role of axial-vector mesons $\leftrightarrow$ anomaly

  – semi-quantitative estimates of the ballparks to be expected
  (HW1–HW2 brackets experimental results for pion TFF!)
  - axial-vector contributions more important numerically than estimated previously

$$a_\mu^{\text{AV}}[L+T] = \mathbf{35(6)}\ [20(3) + 15(3)] \times 10^{-11} \qquad \text{for HW1–HW2}$$

  vs. WP: $a_\mu^{\text{SDC+axials}} = \mathbf{21(16)}\ [15(10) + 6(6)] \times 10^{-11}$

  - with quark masses LSDC still completely satisfied through tower of axial-vector mesons; massive pions have subleading contribution

- Outlook: hQCD models to be made more realisitic:
  - little change with u, d quark masses, but need SU(3) breaking mass terms
  - Witten-Veneziano mechanism for U(1)$_A$ anomaly
  - LO high-energy asymptotics approached perhaps too quickly, Regge trajectories unrealistic in HW models $\rightarrow$ improved hQCD models with numerically determined deviation from conformality?