Holographic QCD and the anomalous magnetic moment of the muon

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Outline of talk

- **Pion and axial vector meson transition form factors** from (chiral) holographic QCD:
  - [JL, J. Mager, A. Rebhan, 1906.11795]
  - [JL, A. Rebhan, 1912.01596]

- Comparison with recent low-energy data (BESIII)
- Short distance constraints
- hQCD prediction for hadronic light-by-light scattering contribution to \((g - 2)_\mu\)

- Results from hQCD with **finite quark masses**
  - with infinite towers of both pseudoscalars and axials

  [JL, A. Rebhan, 2108.12345]
Anomalous TFFs from holographic QCD

Pion TFF: [Grigoryan, Radyushkin, PRD76,77,78 (2007-8)]
[Cappiello, Catà, D’Ambrosio, PRD83 (2011)]

Axial-vector TFF: [JL, A. Rebhan, PRD101 (2020) - 1912.01596]
[Cappiello, Catà, D’Ambrosio, Greynat, Iyer, PRD102 (2020) - 1912.02779]

In bottom-up hQCD models, (as in the top-down string-theoretical Sakai-Sugimoto (SS) model,)
pions & (axial) vector mesons described by 5d-YM fields $\mathcal{F}_{MN}^{L,R} = \mathcal{F}_{MN}^{V} \mp \mathcal{F}_{MN}^{A}$

$$S_{YM}^{U(N_f) \times U(N_f)} \propto \text{tr} \int d^4 x \int_0^{z_0} dz \: e^{-\Phi(z)} \sqrt{-g} \: g^{PR} g^{QS} \left( \mathcal{F}^{(L)}_{PQ} \mathcal{F}^{(L)}_{RS} + \mathcal{F}^{(R)}_{PQ} \mathcal{F}^{(R)}_{RS} \right),$$

where $P, Q, R, S = 0, \ldots, 3$, $z$ and $\mathcal{F}_{MN} = \partial_M \mathcal{B}_N - \partial_N \mathcal{B}_M - i[\mathcal{B}_M, \mathcal{B}_N]$

conformal boundary at $z = 0$, either sharp cut-off of AdS$_5$ at $z_0$ (HW) or with nontrivial dilaton $z_0 = \infty$ (SW)

(SS: finite $z_0$, corresponding to point where D8 branes join; not asymptotically AdS$_5$ ⇒ no matching to pQCD possible)

Chiral symmetry breaking either from extra bifundamental scalar field (HW1)
or through different boundary conditions for vector/axial-vector fields at $z_0$ (Hirn-Sanz (HW2), as in SS)

Anomalies uniquely from Chern-Simons term: (by hand in bottom-up models, from D8 branes in SS model)

$$S_{CS}^{L} - S_{CS}^{R}, \quad S_{CS} = \frac{N_c}{24\pi^2} \int \text{tr} \left( \mathcal{B} \mathcal{F}^2 - \frac{i}{2} \mathcal{B}^3 \mathcal{F} - \frac{1}{10} \mathcal{B}^5 \right).$$
Anomalous TFFs from holographic QCD

Vector meson dominance (VMD):
Electromagnetic background fields through non-normalizable modes for $B^V$ with $B^V_\mu|_{z=0} = e QA^{e.m.}_\mu$.

Bulk-to-boundary propagator $\mathcal{J}$ contains sum over infinite tower of vector mesons,

$$\mathcal{J}^{HW}(Q, z) = Qz \left[ K_1(Qz) + \frac{K_0(Qz)}{I_0(Qz)} I_1(Qz) \right], \quad (M_Y^V = m_\rho = 775 \text{ MeV} \Rightarrow z_0 = 3.103 \text{ GeV}^{-1}),$$

$$F_{\pi\gamma^*\gamma^*}(Q_1^2, Q_2^2) = -\frac{N_c}{12\pi^2 f_\pi} \int_0^{z_0} \mathcal{J}(Q_1, z) \mathcal{J}(Q_2, z) \partial_z \alpha(z) dz$$

$$\mathcal{M}_{A\gamma^*\gamma^*} \propto \epsilon_\mu^{(1)} \epsilon_\nu^{(2)} \epsilon_A^{*\rho} \epsilon_{\mu\nu\rho\sigma} \left[ q_\sigma^{(2)} Q_1^2 A(Q_1^2, Q_2^2) - q_\sigma^{(1)} Q_2^2 A(Q_2^2, Q_1^2) \right]$$

with asymmetric $A$:

$$A(Q_1^2, Q_2^2) = \frac{2g_5}{Q_1^2} \int_0^{z_0} \left[ \frac{d}{dz} \mathcal{J}(Q_1, z) \right] \mathcal{J}(Q_2, z) \psi^A(z) dz$$

- **Landau-Yang theorem** realized by $\mathcal{J}'(Q, z) = 0$ for $Q^2 = 0$

- Amazingly, bottom-up models with asymptotic AdS$_5$ geometry reproduce asymptotic momentum dependence of pQCD (Brodsky-Lepage) for pions and axials! (see Hoferichter & Stoffer, 2004.06127 for axials)

- **HW1**: correct asymptotic prefactor
  HW2: with correct IR-fit of $m_\rho$ and $f_\pi = 92.4$ MeV only 62% of LO pPQCD value!
Holographic pion TFF and experimental data

**Slope parameter:**

\[ F_{\pi^0\gamma^*\gamma^*}(Q_1^2, Q_2^2)/F(0, 0) = 1 + \hat{\alpha}(Q_1^2 + Q_2^2) + O(Q^4) \]

Holographic predictions: (only free parameters: \( m_\rho = 775 \) MeV, \( f_\pi = 92.4 \) MeV)

<table>
<thead>
<tr>
<th>Model</th>
<th>( \hat{\alpha}[\text{GeV}^{-2}] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sakai-Sugimoto</td>
<td>-2.043</td>
</tr>
<tr>
<td>HW1</td>
<td>-1.595</td>
</tr>
<tr>
<td>HW2 (Hirn-Sanz)</td>
<td>-1.805</td>
</tr>
<tr>
<td>SW</td>
<td>-1.665</td>
</tr>
<tr>
<td>(cp. with ( m_\rho^{-2} = 1.665\text{GeV}^{-2} ))</td>
<td></td>
</tr>
</tbody>
</table>

Experimental data (fits):

<table>
<thead>
<tr>
<th>Experiment</th>
<th>( \hat{\alpha}[\text{GeV}^{-2}] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>PDG (before NA62)</td>
<td>-1.76(22)</td>
</tr>
<tr>
<td>NA62 (Dalitz decays ( \pi^0 \rightarrow \gamma e^+ e^- ))</td>
<td>-2.02(31)</td>
</tr>
<tr>
<td>PDG (after NA62)</td>
<td>-1.84(17)</td>
</tr>
<tr>
<td>DRV4 (CELLO,CLEO,BESIII)</td>
<td>-1.74(2)</td>
</tr>
</tbody>
</table>

DRV: simple monopole fits up to \( Q^2 = 4 \) or 9 GeV\(^2\) with lowest \( Q^2 \approx 0.3 \) GeV\(^2\)

[Danilkin, Redmer, Vanderhaeghen, 1901.10346]
Holographic TFFs and experimental data

Single-virtual pion TFF:
[J. Leutgeb, J. Mager & AR, 1906.11795]
(data from Danilkin et al., Prog.Part.Nucl.Phys. 107 (2019) 20)

Single-virtual axial TFF:
[J. Leutgeb & AR, 1912.01596]
dipole fit of L3 data for $f_1(1285)$ (gray band) vs. SS, HW1, and HW2 models:

Danilkin et al. (DRV) fit below 4 GeV$^2$
bracketed by HW1 and HW2!
Axial vector contributions to SDC

Infinite tower of axial-vector mesons responsible for satisfying the longitudinal SDC

- MV-SDC \( \lim_{Q_3 \to \infty} \lim_{Q \to \infty} Q^2 Q_3^2 \Pi_1(Q, Q, Q_3) = -\frac{2}{3\pi^2} : 100\% \) for HW1 and HW2(UV-fit)

HW2 model with \( g_5^2 = 4\pi^2 \) (UV-fit) at large \( Q = 50\)GeV and increasing \( Q_3 \ll Q \):

- SDC for symmetric limit \( Q_1^2 = Q_2^2 = Q_3^2 \to \infty \) satisfied qualitatively, but quantitatively only at max. 80\% level (for HW1 and HW2(UV-fit))
Total axial-vector contributions to muon $g - 2$

$$a_{\mu}^{AV} = \int_0^\infty dQ_1 \int_0^\infty dQ_2 \int_{-1}^1 d\tau \rho_a(Q_1, Q_2, \tau)$$

E.g. at $\tau = 0$:

<table>
<thead>
<tr>
<th>$j = 1$</th>
<th>$j \leq 2$</th>
<th>$j \leq 3$</th>
<th>$j \leq 4$</th>
<th>$j \leq 5$</th>
<th>$a_{\mu}^{AV}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>HW1</td>
<td>31.4</td>
<td>36.2</td>
<td>37.9</td>
<td>39.1</td>
<td>39.6</td>
</tr>
<tr>
<td>HW2</td>
<td>23.0</td>
<td>26.2</td>
<td>27.4</td>
<td>27.9</td>
<td>28.2</td>
</tr>
</tbody>
</table>

HW axial-vector results $\approx 60\%$ longitudinal + $40\%$ transversal

(long. prop.: $q_{(3)}^\mu q_{(3)}^\nu / (M_n A Q_3)^2$)
Pseudoscalar plus axial vector contributions to $a_\mu$

Our results [J. Leutgeb & AR, 1912.01596]: (combined with $a_\mu^{PS}$ [1906.11795])

($z_0$ s.t. $m_\rho = 775$ MeV, $f_\pi = 92.4$ MeV; degenerate $a_1, f_1, f_1'$)

<table>
<thead>
<tr>
<th></th>
<th>HW1 (100% LSDC)</th>
<th>HW2 (62% LSDC)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_\mu^{PS}[\pi^0 + \eta + \eta'] \times 10^{11}$</td>
<td>92 [65.2+18.2+15.6]</td>
<td>84 [59.2+15.9+13.4]</td>
</tr>
<tr>
<td>$a_\mu^{AV}[L + T] \times 10^{11}$</td>
<td>41 [23+18]</td>
<td>29 [17+12]</td>
</tr>
<tr>
<td>$a_\mu^{PS+AV} \times 10^{11}$</td>
<td>140</td>
<td>112</td>
</tr>
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</table>

(compare with MV model: longitudinal contribution estimated $\sim 38 \times 10^{-11}$)

independently at the same time:


• agreement, but different parameters:

HW2(1): $z_0$ s.t. $m_\rho = 776$ MeV, $f_\pi = 93$ MeV, $f_\eta' = 74$ MeV
HW2(2): $z_0$ s.t. 100% UV limit (but $m_\rho = 987$ MeV !)

<table>
<thead>
<tr>
<th></th>
<th>HW2(2) (100% LSDC)</th>
<th>HW2(1) (62% LSDC)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_\mu^{PS}[\pi^0 + \eta + \eta'] \times 10^{11}$</td>
<td>112 [75+21+16]</td>
<td>81 [57+14+10]</td>
</tr>
<tr>
<td>$a_\mu^{AV}[L + T] \times 10^{11}$</td>
<td>32 [18* +14]</td>
<td>28 [14+14]</td>
</tr>
<tr>
<td>$a_\mu^{PS+AV} \times 10^{11}$</td>
<td>144</td>
<td>110</td>
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*) 15 acc.to Colangelo et al. 2106.13222
Pseudoscalar plus axial vector contributions to \( a_\mu \)

Our results [J. Leutgeb & AR, 1912.01596]: (combined with \( a_\mu^{PS} [1906.11795] \))

\[
(z_0 \text{ s.t. } m_\rho = 775 \text{ MeV}, f_\pi = 92.4 \text{ MeV}; \text{ degenerate } a_1, f_1, f_1')
\]

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<th>( a_\mu^{PS}[\pi^0 + \eta + \eta'] ) ( \times 10^{11} )</th>
<th>HW1 (100% LSDC)</th>
<th>HW2 (62% LSDC)</th>
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\( a_\mu^{PS+AV} \) \( \times 10^{11} \) \( \times 10^{11} \)

| \( \times 10^{11} \) | 140 | 112 |

(compare with MV model: longitudinal contribution estimated \( \sim 38 \times 10^{-11} \))

independently at the same time:


- agreement, but different parameters:

\( \text{HW2}^{(1)} \): \( z_0 \text{ s.t. } m_\rho = 776 \text{ MeV}, f_\pi = 93 \text{ MeV}, f_{\eta'} = 74 \text{ MeV} \)

\( \text{HW2}^{(2)} \): \( z_0 \text{ s.t. } 100\% \text{ UV limit (but } m_\rho = 987 \text{ MeV !) } \)

<table>
<thead>
<tr>
<th>( \times 10^{11} )</th>
<th>HW2(^{(2)}) (100% LSDC)</th>
<th>HW2(^{(1)}) (62% LSDC)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_\mu^{PS}[\pi^0 + \eta + \eta'] ) ( \times 10^{11} )</td>
<td>112 [75+21+16]</td>
<td>81 [57+14+10]</td>
</tr>
<tr>
<td>( a_\mu^{AV}[L + T] ) ( \times 10^{11} )</td>
<td>32 [18(^{(*)})+14]</td>
<td>28 [14+14]</td>
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\(^{*})\) \( 15 \) acc.to Colangelo et al. 2106.13222
**HW models with finite quark masses (LR 2108.12345)**

**HW1**: HW1 with finite quark mass;  
**HW3**: HW1 with HW2 b.c.;  
\(\Delta^+\): scaling dimension of operator dual to \(\Phi\)

**Analytic result:** **LSDC completely saturated by axials, no contribution from heavy PS!**  
(as long as \(\Delta^+\) in holographically allowed range \(2 \leq \Delta^+ < 4\))

<table>
<thead>
<tr>
<th>model</th>
<th>PS</th>
<th>(n = 1)</th>
<th>(n = 2)</th>
<th>(n = 3)</th>
<th>AV</th>
<th>(n = 1)</th>
<th>(n = 2)</th>
<th>(n = 3)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>HW1 chiral</strong></td>
<td>(m_\pi) [MeV]</td>
<td>0.135</td>
<td>1888</td>
<td>2879</td>
<td>(m_{a_1}) [MeV]</td>
<td>1375</td>
<td>2154</td>
<td>2995</td>
</tr>
<tr>
<td>(\Delta^+ = 3)</td>
<td>(a_\mu \cdot 10^{11})</td>
<td>65.2</td>
<td>0.7</td>
<td>0.1</td>
<td>(4a_{a_1} \cdot 10^{11})</td>
<td>31.4</td>
<td>4.7</td>
<td>1.8</td>
</tr>
<tr>
<td><strong>HW1m</strong></td>
<td>(m_\pi) [MeV]</td>
<td>135*</td>
<td>1892</td>
<td>2882</td>
<td>(m_{a_1}) [MeV]</td>
<td>1367</td>
<td>2141</td>
<td>2987</td>
</tr>
<tr>
<td>(\Delta^+ = 3)</td>
<td>(a_\mu \cdot 10^{11})</td>
<td>66.0</td>
<td>0.7</td>
<td>0.1</td>
<td>(4a_{a_1} \cdot 10^{11})</td>
<td>31.4</td>
<td>4.9</td>
<td>1.8</td>
</tr>
<tr>
<td><strong>HW1m’</strong></td>
<td>(m_\pi) [MeV]</td>
<td>135*</td>
<td>1591</td>
<td>2564</td>
<td>(m_{a_1}) [MeV]</td>
<td>1230*</td>
<td>1977</td>
<td>2901</td>
</tr>
<tr>
<td>(\Delta^+ = 2.404)</td>
<td>(a_\mu \cdot 10^{11})</td>
<td>64.3</td>
<td>1.5</td>
<td>0.3</td>
<td>(4a_{a_1} \cdot 10^{11})</td>
<td>29.8</td>
<td>8.7</td>
<td>2.0</td>
</tr>
<tr>
<td><strong>HW3m</strong></td>
<td>(m_\pi) [MeV]</td>
<td>135*</td>
<td>1715</td>
<td>2513</td>
<td>(m_{a_1}) [MeV]</td>
<td>1431</td>
<td>2421</td>
<td>3398</td>
</tr>
<tr>
<td>(\Delta^+ = 3)</td>
<td>(a_\mu \cdot 10^{11})</td>
<td>66.6</td>
<td>0.8</td>
<td>0.04</td>
<td>(4a_{a_1} \cdot 10^{11})</td>
<td>32.7</td>
<td>3.4</td>
<td>1.8</td>
</tr>
<tr>
<td><strong>HW3m’</strong></td>
<td>(m_\pi) [MeV]</td>
<td>135*</td>
<td>1300*</td>
<td>2113</td>
<td>(m_{a_1}) [MeV]</td>
<td>1380</td>
<td>2355</td>
<td>3345</td>
</tr>
<tr>
<td>(\Delta^+ = 2.399)</td>
<td>(a_\mu \cdot 10^{11})</td>
<td>66.0</td>
<td>1.5</td>
<td>0.01</td>
<td>(4a_{a_1} \cdot 10^{11})</td>
<td>33.2</td>
<td>4.1</td>
<td>1.8</td>
</tr>
</tbody>
</table>

* fitted

- Numerically: some **increase** compared to chiral model with \(m_\pi\) inserted in propagator by hand  
  \((a_\mu - a_{\mu \text{chiral}})_\pi + a_1 = + (0.8 + 0.2) \times 10^{-11}\)

- Contribution of heavy PS **smaller** than in PS Regge model of Colangelo et al., where \(\Delta a_{PS}^{\pi(n>1)} = 2.7\) prior to OPE substitutions
Conclusions/Outlook

- **hQCD is not QCD, but sophisticated toy model that can give clues on**
  - how short-distance behavior can be implemented at the hadronic level
  - important fundamental role of axial-vector mesons ↔ anomaly

- semi-quantitative estimates of the ballparks to be expected (HW1–HW2 brackets experimental results for pion TFF!)

- axial-vector contributions more important numerically than estimated previously

\[ a_{\mu}^{AV}[L + T] = 35(6) \times 10^{-11} \quad \text{for HW1–HW2} \]

vs. **WP:** \[ a_{\mu}^{SDC+axials} = 21(16) \times 10^{-11} \]

- with quark masses LSDC still completely satisfied through tower of axial-vector mesons; massive pions have subleading contribution

**Outlook:** hQCD models to be made more realisitic:

- little change with \( u, d \) quark masses, but need SU(3) breaking mass terms
- Witten-Veneziano mechanism for U(1)\(_A\) anomaly
- LO high-energy asymptotics approached perhaps too quickly, Regge trajectories unrealistic in HW models → improved hQCD models with numerically determined deviation from conformality?