

# The $q^2$ moments in inclusive $b \rightarrow c$ decays at Belle II

*Joint Annual Meeting of ÖPG and SPS, Innsbruck 2021*

*Session: FAKT – TASK*

Manca Mrvar

Institute of High Energy Physics (HEPHY, Vienna)

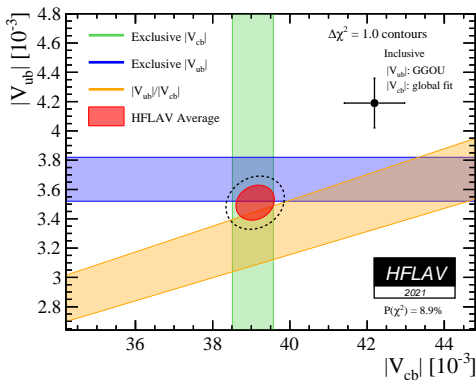
*manca.mrvar@oeaw.ac.at*

1st September 2021

- 1 Motivation
- 2 Theoretical background
- 3 Analysis
- 4 Branching ratio
- 5 Results & Outlook

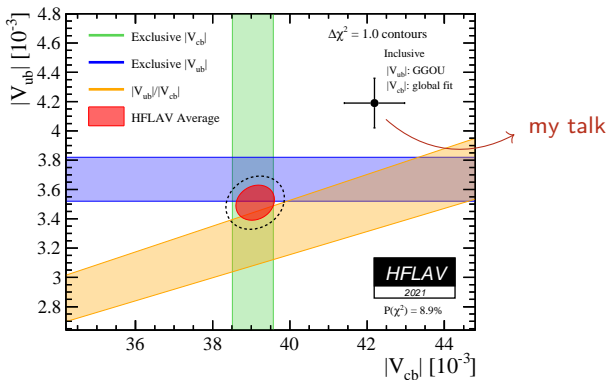
It governs the rate of the dominant weak quark transition  $b \rightarrow c$

### 3.3 tension between exclusive and inclusive determinations of $|V_{cb}|$



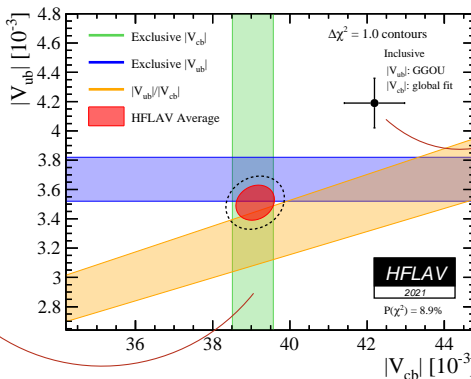
It governs the rate of the dominant weak quark transition  $b \rightarrow c$

### 3.3 tension between exclusive and inclusive determinations of $|V_{cb}|$



It governs the rate of the dominant weak quark transition  $b \rightarrow c$

### 3.3 tension between exclusive and inclusive determinations of $|V_{cb}|$

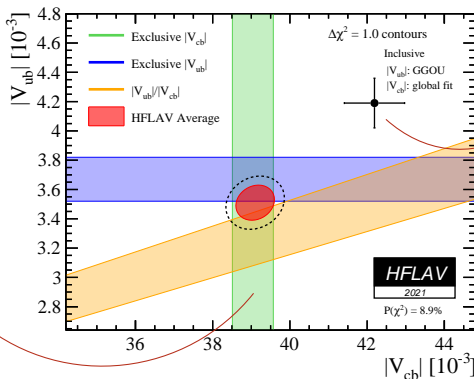


my talk

Daniel Dorner:  
 $B \rightarrow D$  [324]  
 Philipp Horak:  
 $B \rightarrow D^0$  [325]

It governs the rate of the dominant weak quark transition  $b \rightarrow c$

### 3.3 tension between exclusive and inclusive determinations of $|V_{cb}|$



my talk

Daniel Dorner:  
 $B \rightarrow D$  [324]  
 Philipp Horak:  
 $B \rightarrow D^0$  [325]

Possible reasons for anomaly:

An experimental issue with the exclusive or the inclusive measurement?

A problem in the theory input?

Physics beyond the Standard Model?

## SEMILEPTONIC WIDTH

$$\Gamma = \frac{G_F^2 m_b^5}{192} |V_{cb}|^2 \left[ 1 + \frac{c_5 \langle hO_5 \rangle}{m_b^2} + \frac{c_6 \langle hO_6 \rangle}{m_b^3} + \mathcal{O}\left(\frac{1}{m_b^4}\right) \right] \quad (1)$$

Based on the Operator Product Expansion (OPE)  
 $\langle hO_i \rangle$ : hadronic matrix elements (non-perturbative)  
 $c_i$ : coefficients (perturbative)

## SEMILEPTONIC WIDTH

$$\Gamma = \frac{G_F^2 m_b^5}{192} |V_{cb}|^2 \left[ 1 + \frac{c_5 \langle hO_5 \rangle}{m_b^2} + \frac{c_6 \langle hO_6 \rangle}{m_b^3} + \mathcal{O}\left(\frac{1}{m_b^4}\right) \right] \quad (1)$$

Based on the Operator Product Expansion (OPE)  
 $\langle hO_i \rangle$ : hadronic matrix elements (non-perturbative)  
 $c_i$ : coefficients (perturbative)

OTHER OBSERVABLES can be expanded with the same heavy quark parameters

! Moments of lepton energy spectrum

$$R_n(E_{\text{cut}}) = \int_{E_{\text{cut}}}^Z (E)^n \frac{d\Gamma}{dE} dE; \quad \langle E^n \rangle_{E_{\text{cut}}} = \frac{R_n(E_{\text{cut}}; 0)}{R_0(E_{\text{cut}}; 0)} \quad (2)$$

! Moments of hadron mass spectrum

$$\langle m_X^{2n} \rangle_{E_{\text{cut}}} = \frac{\int_{E_{\text{cut}}}^R (m_X^2)^n \frac{d\Gamma}{dm_X^2} dm_X^2}{\int_{E_{\text{cut}}}^R \frac{d\Gamma}{dm_X^2} dm_X^2} \quad (3)$$



Number of heavy quark parameters proliferates: 13 matrix elements up to  $\mathcal{O}(1/m_b^4)$   
Reparametrization invariance links operators in the Heavy-Quark expansion, reducing the number of independent operators to **eight** for the total rate at  $\mathcal{O}(1/m_b^4)$   
This reduction does not hold for  $E$  and  $M_X^2$  moments...

References:

T. Mannel, S. Turczyk and N. Uraltsev, "Higher Order Power Corrections in Inclusive B Decays", JHEP 11 (2010), 109 doi:10.1007/JHEP11(2010)109 [arXiv:1009.4622 [hep-ph]]

M. Fael, T. Mannel and K. Keri Vos, " $\int Vcbj$  determination from inclusive  $b \rightarrow c$  decays: an alternative method", JHEP 02, 177 (2019) doi:10.1007/JHEP02(2019)177 [arXiv:1812.07472 [hep-ph]].

Number of heavy quark parameters proliferates: 13 matrix elements up to  $\mathcal{O}(1/m_b^4)$   
 Reparametrization invariance links operators in the Heavy-Quark expansion, reducing the number of independent operators to **eight for the total rate at  $\mathcal{O}(1/m_b^4)$**   
 This reduction does not hold for  $E$ - and  $M_X^2$  moments...

...but: Reduction holds for moments of lepton invariant mass  $q^2$  ( $q = p_+ + p_-$ )

! **Final goal: To measure  $q^2$  moments for determination of  $JV_{cbj}$  up to  $1/m_b^4$**

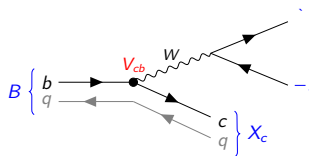
References:

T. Mannel, S. Turczyk and N. Uraltsev, "Higher Order Power Corrections in Inclusive B Decays", JHEP 11 (2010), 109 doi:10.1007/JHEP11(2010)109 [arXiv:1009.4622 [hep-ph]]

M. Fael, T. Mannel and K. Keri Vos, " $JV_{cbj}$  determination from inclusive  $b \rightarrow c$  decays: an alternative method", JHEP 02, 177 (2019) doi:10.1007/JHEP02(2019)177 [arXiv:1812.07472 [hep-ph]].

Inclusive  $B \rightarrow X_c \ell^+ \ell^-$ ,  $\ell = e, \mu$ .

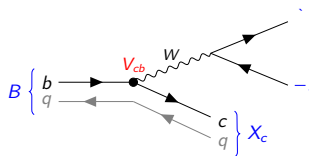
Untagged approach: the only known particle is  $\ell^-$ .



Inclusive  $B \rightarrow X_c e^+ e^-$ ,  $\ell = e, \mu$ .

Untagged approach: the only known particle is  $\ell$ .

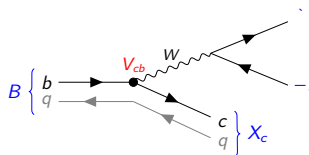
$$q^2 = M^2 = 2(E_\ell E - \vec{p}_\ell \cdot \vec{p})$$



Inclusive  $B \rightarrow X_c \ell \bar{\nu}_\ell$ ,  $\ell = e, \mu$ .

Untagged approach: the only known particle is  $\ell$ .

$$q^2 = M^2 = 2(E_\ell E - \vec{p}_\ell \cdot \vec{p})$$



**NEUTRINO MOMENTUM:**  $\vec{p} = (|\vec{p}_{\text{miss}}|; \vec{p}_{\text{miss}})$

Challenge: excluding events where other particles have (most likely) been lost:

Missing momentum outside of the detector acceptance

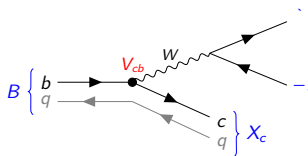
Large charge imbalance

×  $M_{\text{miss}}^2$  close to zero

Inclusive  $B \rightarrow X_c e \bar{\nu}_e$ ,  $\bar{B} \rightarrow X_c e \nu_e$ .

Untagged approach: the only known particle is  $e$ .

$$q^2 = M^2 = 2(E_e \cdot E_{X_c} - \vec{p}_e \cdot \vec{p}_{X_c})$$



## NEUTRINO MOMENTUM: $\vec{p} = (|\vec{p}_{miss}|; \vec{p}_{miss})$

Challenge: excluding events where other particles have (most likely) been lost:

- Missing momentum outside of the detector acceptance

- Large charge imbalance

- $\times M_{miss}^2$  close to zero

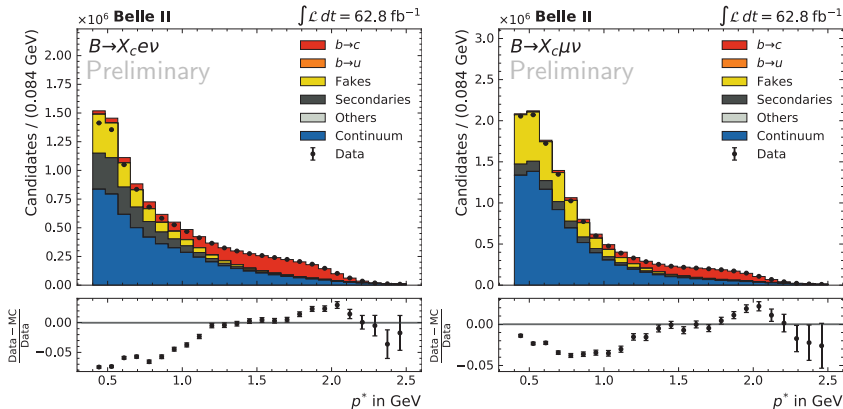
## Data samples

**Belle II data** collected in the years 2019 and 2020 equivalent to **62.8 fb<sup>-1</sup>**

Background and signal model:

- Continuum events** ( $e^+e^- \rightarrow q\bar{q}$ ): replaced by  $\rho$ -resonance data of **9.2 fb<sup>-1</sup>**

- Signal and BB backgrounds:** Monte Carlo sample of **200 fb<sup>-1</sup>**



Samples are fitted by ROOT's `TFractionFitter` (binned likelihood fit)

Fitting range is between  $0.4 \text{ GeV} = c$  and  $2.5 \text{ GeV} = c$

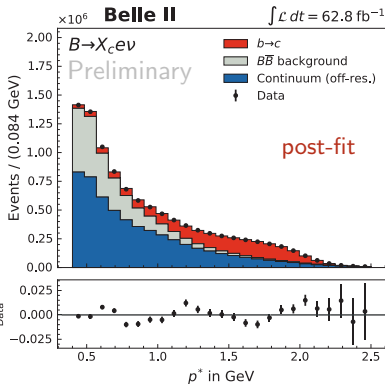
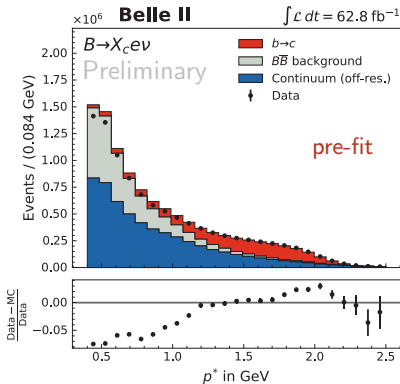
Three templates (shapes) are used for fitting:

- Signal (MC): events that include  $\bar{c}$ ,  $c$  and  $c$ -quark containing meson

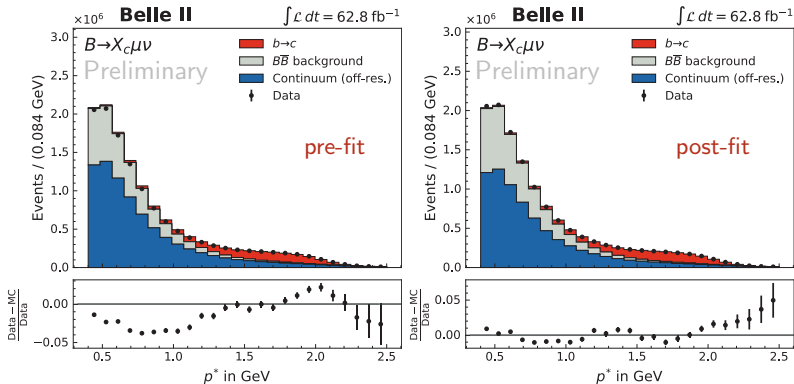
- Continuum: shape of off-resonance data

- $B\bar{B}$  backgrounds (MC):  $b \rightarrow u$ , secondary leptons, fake leptons, other lepton candidates





Yields	Pre-fit		Post-fit	
Signal	1849844	762	1925908	5834
$B\bar{B}$ background	3117962	989	2641059	18459
Continuum	5546871	6149	5511039	22370



Yields	Pre-fit		Post-fit	
Signal	1436116	671	1501462	7277
$B\bar{B}$ background	3774497	1089	4102903	87668
Continuum	8033487	7400	7353691	89823

Two sources ( $B^+ \rightarrow X_c^- \dots$  and  $B^0 \rightarrow X_c^- \dots$ ) with the same semileptonic width  $\Gamma$ :

$$(B \rightarrow X_c^- \dots) = \frac{N_{\text{sig}}}{2N_{B\bar{B}} f_{+B^+B^+} + f_{0B^0B^0}} \quad (4)$$

$N_{\text{sig}}$  ... number of signal events

$N_{B\bar{B}}$  ... the number of  $B\bar{B}$  pairs in the data sample

$f_{+B^+B^+}$  ... the total signal selection efficiency

factor 2 ... both  $B$  mesons in the  $\Upsilon(4S)$  event can contribute to the signal

$f_{0B^0B^0}$  ... the mean life time of the  $B$ -mesons

$f_{+B^+B^+}$  ... the proportions of  $\Upsilon(4S)$  decaying into  $B^+B^+$  and  $B^0\bar{B}^0$  pairs

Two sources ( $B^+ \rightarrow X_c^- \dots$  and  $B^0 \rightarrow X_c^- \dots$ ) with the same semileptonic width  $\Gamma$ :

$$(B \rightarrow X_c^- \dots) = \frac{N_{\text{sig}}}{2N_{B\bar{B}} (f_{+B^+} + f_{0B^0})} \quad (4)$$

$N_{\text{sig}}$  ... number of signal events  
 $N_{B\bar{B}}$  ... the number of  $B\bar{B}$  pairs in the data sample  
 $f_{+B^+}$  ... the total signal selection efficiency  $g_{MC}$   
 factor 2 ... both  $B$  mesons in the  $\Upsilon(4S)$  event can contribute to the signal  
 $f_{0B^0}$  ... the mean life time of the  $B$ -mesons  
 $f_{+B^+}$  ... the proportions of  $\Upsilon(4S)$  decaying into  $B^+B^+$  and  $B^0\bar{B}^0$  pairs

PDG

Two sources ( $B^+ \rightarrow X_c^- \gamma$  and  $B^0 \rightarrow X_c^- \gamma$ ) with the same semileptonic width  $\Gamma$ :

$$B(B \rightarrow X_c^- \gamma) = \frac{N_{\text{sig}}}{2N_{B\bar{B}} (f_{+B^+} + f_{0B^0})} \quad (4)$$

<p><math>N_{\text{sig}}</math> ... number of signal events</p> <p><math>N_{B\bar{B}}</math> ... the number of <math>B\bar{B}</math> pairs in the data sample</p> <p><math>f_{B^+=B^0}</math> ... the total signal selection efficiency <math>gMC</math></p> <p>factor 2 ... both <math>B</math> mesons in the <math>\Upsilon(4S)</math> event can contribute to the signal</p> <p><math>f_{B^+=B^0}</math> ... the mean life time of the <math>B</math>-mesons</p> <p><math>f_{\pm=0}</math> ... the proportions of <math>\Upsilon(4S)</math> decaying into <math>B^+B^+</math> and <math>B^0\bar{B}^0</math> pairs</p>	)
---	---

PDG

(Average) branching fraction, calculated using (average) lifetime :

$$B(B \rightarrow X_c^- \gamma) = \langle B(B \rightarrow X_c^- \gamma) \rangle \quad (5)$$

Two sources ( $B^+ \rightarrow X_c^- e^+$  and  $B^0 \rightarrow X_c^- e^+$ ) with the same semileptonic width  $\Gamma$ :

$$B(B \rightarrow X_c^- e^+) = \frac{N_{sig}}{2N_{B\bar{B}} f_{+B^+} + f_{0B^0}} \quad (4)$$

$N_{sig}$  ... number of signal events  
 $N_{B\bar{B}}$  ... the number of  $B\bar{B}$  pairs in the data sample  
 $f_{B^+=B^0}$  ... the total signal selection efficiency  $g_{MC}$   
 factor 2 ... both  $B$  mesons in the  $\Upsilon(4S)$  event can contribute to the signal  
 $\tau_{B^+=B^0}$  ... the mean life time of the  $B$ -mesons  
 $f_{\pm=0}$  ... the proportions of  $\Upsilon(4S)$  decaying into  $B^+B^+$  and  $B^0\bar{B}^0$  pairs

PDG

(Average) branching fraction, calculated using (average) lifetime :

$$B(B \rightarrow X_c^- e^+) = \dots \quad (5)$$

The branching fractions for both modes are:

$$B(B \rightarrow X_c^- e^+) = (9.97 \pm 0.03_{stat})\% \quad (6)$$

$$B(B \rightarrow X_c^- e^+) = (9.47 \pm 0.05_{stat})\% \quad (7)$$

## SYSTEMATIC UNCERTAINTIES

Contribution	Relative uncertainty [%]	
	Electron mode	Muon mode
Tracking	0.69	0.69
$N_{B\bar{B}}$	1.1	1.1
Lepton ID corrections	1.64	2.33
$f_0=f_+$ , $B$ lifetime	1.2	1.2
$B \rightarrow X_c \ell \bar{\nu}$ branching fractions	2.65	2.15
$B \rightarrow X_c \ell \bar{\nu}$ form factors	1.11	1.11
$B\bar{B}$ background model	0.24	0.34
Off-resonance data model	0.34	2.91
Sum	3.77	4.79

## SYSTEMATIC UNCERTAINTIES

Contribution	Relative uncertainty [%]	
	Electron mode	Muon mode
Tracking	0.69	0.69
$N_{B\bar{B}}$	1.1	1.1
Lepton ID corrections	1.64	2.33
$f_0=f_+$ , $B$ lifetime	1.2	1.2
$B \rightarrow X_c e e$ branching fractions	2.65	2.15
$B \rightarrow X_c e e$ form factors	1.11	1.11
$B\bar{B}$ background model	0.24	0.34
Off-resonance data model	0.34	2.91
Sum	3.77	4.79

## BRANCHING RATIOS

$$B(B \rightarrow X_c e e) = (9.97 \pm 0.03_{stat} \pm 0.38_{sys})\% \quad (8)$$

$$B(B \rightarrow X_c e e) = (9.47 \pm 0.05_{stat} \pm 0.45_{sys})\% \quad (9)$$



## SYSTEMATIC UNCERTAINTIES

Contribution	Relative uncertainty [%]	
	Electron mode	Muon mode
Tracking	0.69	0.69
$N_{B\bar{B}}$	1.1	1.1
Lepton ID corrections	1.64	2.33
$f_0=f_+$ , $B$ lifetime	1.2	1.2
$B \rightarrow X_c \ell \bar{\nu}$ branching fractions	2.65	2.15
$B \rightarrow X_c \ell \bar{\nu}$ form factors	1.11	1.11
$B\bar{B}$ background model	0.24	0.34
Off-resonance data model	0.34	2.91
Sum	3.77	4.79

## BRANCHING RATIOS

$$B(B \rightarrow X_c e \bar{\nu}) = (9.97 \pm 0.03_{stat} \pm 0.38_{sys})\% \quad (8)$$

$$B(B \rightarrow X_c \mu \bar{\nu}) = (9.47 \pm 0.05_{stat} \pm 0.45_{sys})\% \quad (9)$$

## COMBINED BRANCHING RATIO

$$B(B \rightarrow X_c \ell \bar{\nu}) = (9.75 \pm 0.03_{stat} \pm 0.47_{sys})\% \quad (10)$$

Branching fraction (zeroth moment) has been determined with  $B \rightarrow X_c \ell \bar{\nu}$   
Studies of  $q^2$  and its moments are ongoing  
We are working towards the extraction of  $\int jV_{cbj}$

Preliminary

post-fit

Preliminary

post-fit

Branching fraction (zeroth moment) has been determined with  $B \rightarrow X_c \ell \bar{\nu}$   
Studies of  $q^2$  and its moments are ongoing  
We are working towards the extraction of  $|jV_{cbj}|$

**We expect to have preliminary  $|jV_{cbj}|$  measurement next year! :)**

Preliminary

post-fit

Preliminary

post-fit

# BACKUP

Contribution	Electron mode				Muon mode			
	Pre-fit		Post-fit		Pre-fit		Post-fit	
Signal yield	1849844	1360	1932425	5834	1436116	1198	1501462	7277
$b \neq u$ background	63005	251	53368	373	47562	218	51700	1105
Fakes	1485693	1219	1258451	8796	2901541	1703	3153994	67393
Secondaries	1562863	1250	1323818	9252	821409	906	892876	19078
Other MC background	6401	80	5422	38	3985	63	4332	93
Continuum	5546871	2355	5511039	22370	8033487	2834	7353691	89823
Sum	10514678	3243	10084523	26412	13244100	3639	12958055	114140

Main source: **Signal and background modeling**

## Signal model

Signal sample was divided into 30 decays (subsamples)

The branching ratio of each subsample was varied between  $\pm 1\%$

## $B\bar{B}$ background model

The amounts of  $B\bar{B}$  components ( $b \rightarrow c$ , secondaries, fakes, others) were varied one by one within  $\pm 5\%$

## Off-resonance data model

Uncertainty is determined from the difference of signal yield with fully floating continuum component and component floating only within the luminosity measurement uncertainty

$B \rightarrow X_c e$	$\Gamma_i = \Gamma_{tot} [\%]$	$\Gamma_i = \Gamma_{tot}^- [\%]$	$\Gamma_i = \Gamma_{tot}^+ [\%]$	$N_{sig}^-$	$N_{sig}^+$	$N_{sig}^{rel} [\%]$
$D^- \rightarrow \bar{c} \ell$	2.25	0.08	0.08	1931268	1933046	0.046
$D^{*0} (2010) \rightarrow \bar{c} \ell$	5.09	0.17	0.17	1936901	1927049	0.255
$D^0 \rightarrow \bar{c} \ell$	0.41	0.05	0.05	1931239	1933644	0.062
$D_0^{*0} (2300) \rightarrow \bar{c} \ell$ ; $D_0^{*-0} \rightarrow \bar{D}^0 \rightarrow \bar{c} \ell$	0.30	0.12	0.12	1922936	1941579	0.482
$D_2^{*0} (2460) \rightarrow \bar{c} \ell$ ; $D_2^{*-0} \rightarrow \bar{D}^0 \rightarrow \bar{c} \ell$	0.121	0.033	0.033	1927194	1937459	0.266
$D^{(*)0} \rightarrow \bar{c} \ell$ ( $n \geq 1$ )	0.248 <sup>1</sup>	0.062	0.062	1927652	1936093	0.218
$D^{*0} \rightarrow \bar{c} \ell$	0.58	0.08	0.08	1930124	1933193	0.079
$D_1 (2420) \rightarrow \bar{c} \ell$	0.31	0.12	0.12	1933015	1931679	0.035
$D_2^{*0} (2460) \rightarrow \bar{c} \ell$ ; $D_2^{*-0} \rightarrow D^{*0} \rightarrow \bar{c} \ell$	0.068	0.012	0.012	1929479	1935758	0.162
$D^- \rightarrow \bar{c} \ell$	5.8 <sup>2</sup>	3.0	3.0	1931024	1933486	0.064
$D^{*-} \rightarrow \bar{c} \ell$	2.8 <sup>3</sup>	1.4	1.4	1926079	1937582	0.298
$D^0 \rightarrow \bar{c} \ell$	2.29	0.08	0.08	1931076	1932703	0.042
$D^{*-} (2007) \rightarrow \bar{c} \ell$	5.58	0.26	0.26	1943118	1922732	0.527
$D^- \rightarrow \bar{c} \ell$	0.44	0.04	0.04	1930414	1941668	0.291
$D_0^{*-} (2420) \rightarrow \bar{c} \ell$ ; $D_0^{*0} \rightarrow D^- \rightarrow \bar{c} \ell$	0.25	0.05	0.05	1913151	1947373	0.885
$D_2^{*-} (2460) \rightarrow \bar{c} \ell$ ; $D_2^{*0} \rightarrow D^- \rightarrow \bar{c} \ell$	0.153	0.016	0.016	1928964	1933883	0.127
$D^{(*)-} \rightarrow \bar{c} \ell$ ( $n \geq 1$ )	0.193 <sup>1</sup>	0.022	0.022	1929894	1932582	0.070
$D^{*-} \rightarrow \bar{c} \ell$	0.60	0.04	0.04	1931840	1933072	0.032
$\bar{D}_1 (2420) \rightarrow \bar{c} \ell$ ; $\bar{D}_1^0 \rightarrow D^{*-} \rightarrow \bar{c} \ell$	0.303	0.020	0.020	1932406	1931181	0.032
$\bar{D}_1^0 (2430) \rightarrow \bar{c} \ell$ ; $\bar{D}_1^0 \rightarrow D^{*-} \rightarrow \bar{c} \ell$	0.27	0.09	0.09	1931181	1938717	0.195
$\bar{D}_2^{*-} (2460) \rightarrow \bar{c} \ell$ ; $\bar{D}_2^{*0} \rightarrow D^{*-} \rightarrow \bar{c} \ell$	0.101	0.024	0.024	1926970	1937601	0.275
$\bar{D}^0 \rightarrow \bar{c} \ell$	7.1 <sup>2</sup>	2.1	2.1	1931527	1933351	0.047
$D^{*0} \rightarrow \bar{c} \ell$	1.4 <sup>3</sup>	1.1	1.1	1920958	1944040	0.597
$D_s^- \rightarrow K^+ \ell$	0.003	0.002	0.002	1929035	1934350	0.138
$D_s^{*-} \rightarrow K^+ \ell$	0.003	0.003	0.003	1931862	1933038	0.030
$D_1(H) \rightarrow \bar{c} \ell$ ( $\dagger$ )	1	1	1	1916144	1947723	0.817
$D^- \rightarrow \bar{c} \ell$ (hypothetical*)	0.201	0.201	0.201	1916398	1947962	0.817
$D^{*-} \rightarrow \bar{c} \ell$ (hypothetical*)	0.201	0.201	0.201	1913463	1951018	0.972
$D^0 \rightarrow \bar{c} \ell$ (hypothetical*)	0.201	0.201	0.201	1910202	1952695	1.099
$D^{*0} \rightarrow \bar{c} \ell$ (hypothetical*)	0.201	0.201	0.201	1909367	1953684	1.147

<sup>1</sup> Values given in  $\Gamma_i = \Gamma_3$  instead of  $\Gamma_i = \Gamma_{tot}$   
 $\dagger$  Branching fraction was varied within  $\pm 100\%$

<sup>2</sup> Values given in  $\Gamma_i = \Gamma_4$  instead of  $\Gamma_i = \Gamma_{tot}$

<sup>3</sup> Values given in  $\Gamma_i = \Gamma_6$  instead of  $\Gamma_i = \Gamma_{tot}$   
 \* Decays from MC to fill the gap between the  $B$  of inclusive and sum of the exclusive decays

$B \rightarrow X_c$	$\Gamma_i = \Gamma_{tot} [\%]$	$\Gamma_{i=tot}; - [\%]$	$\Gamma_{i=tot}; + [\%]$	$N_{sig}; -$	$N_{sig}; +$	$N_{sig}; rel [\%]$
$D^- \rightarrow +$	2.25	0.08	0.08	1504186	1504672	0.016
$D^*(2010) \rightarrow +$	5.09	0.17	0.17	1514854	1508594	0.208
$D^0 \rightarrow +$	0.41	0.05	0.05	1503605	1510606	0.233
$D_0^*(2300) \rightarrow +; D_0^{*-} \rightarrow \bar{D}^0 -$	0.30	0.12	0.12	1504428	1508941	0.150
$D_2^*(2460) \rightarrow +; D_2^{*-} \rightarrow \bar{D}^0 -$	0.121	0.033	0.033	1500621	1507473	0.228
$D^{(*)n} \rightarrow +, (n \geq 1)$	0.248 <sup>1</sup>	0.062	0.062	1501597	1505444	0.128
$D^{*0} \rightarrow +$	0.58	0.08	0.08	1509942	1513377	0.114
$D_1(2420) \rightarrow +$	0.31	0.12	0.12	1506194	1502020	0.139
$D_2^*(2460) \rightarrow +; D_2^{*-} \rightarrow D^{*0} -$	0.068	0.012	0.012	1506016	1506480	0.015
$D^- \rightarrow +$	5.8 <sup>2</sup>	3.0	3.0	1502369	1504398	0.068
$D^{*-} \rightarrow +$	2.8 <sup>3</sup>	1.4	1.4	1501022	1510699	0.322
$D^0 \rightarrow +$	2.29	0.08	0.08	1503849	1505237	0.046
$D^{*}(2007)^0 \rightarrow +$	5.58	0.26	0.26	1510872	1505381	0.183
$D^- \rightarrow ++$	0.44	0.04	0.04	1503896	1513751	0.328
$D_0^{*-}(2420)^0 \rightarrow +; D_0^{*0} \rightarrow D^- +$	0.25	0.05	0.05	1498581	1519215	0.687
$D_2^{*-}(2460)^0 \rightarrow +; D_2^{*0} \rightarrow D^- +$	0.153	0.016	0.016	1499692	1507424	0.257
$D^{(*)n} \rightarrow ++, (n \geq 1)$	0.193 <sup>1</sup>	0.022	0.022	1503631	1501530	0.070
$D^{*-} \rightarrow ++$	0.60	0.04	0.04	1502226	1505597	0.112
$\bar{D}_1(2420)^0 \rightarrow +; \bar{D}_1^0 \rightarrow D^{*-} +$	0.303	0.020	0.020	1512654	1504807	0.261
$\bar{D}_1(2430)^0 \rightarrow +; \bar{D}_1^0 \rightarrow D^{*-} +$	0.27	0.09	0.09	1497896	1508952	0.368
$\bar{D}_2^*(2460)^0 \rightarrow +; \bar{D}_2^{*0} \rightarrow D^{*-} +$	0.101	0.024	0.024	1500761	1513068	0.410
$\bar{D}^0 \rightarrow ++$	7.1 <sup>2</sup>	2.1	2.1	1500541	1506281	0.191
$D^{*0} \rightarrow ++$	1.4 <sup>3</sup>	1.1	1.1	1500421	1510675	0.341
$D_s^- K^+ \rightarrow ++$	0.003	0.002	0.002	1502864	1507095	0.141
$D_s^{*-} K^+ \rightarrow ++$	0.003	0.003	0.003	1500971	1506384	0.180
$D_1(H) \rightarrow ++, (\dagger)$	1	1	1	1503126	1513857	0.357
$D^- \rightarrow +, (\text{hypothetical}^*)$	0.201	0.201	0.201	1499323	1516808	0.582
$D^{*-} \rightarrow +, (\text{hypothetical}^*)$	0.201	0.201	0.201	1490428	1516353	0.863
$D^0 \rightarrow +, (\text{hypothetical}^*)$	0.201	0.201	0.201	1496843	1525394	0.951
$D^{*0} \rightarrow +, (\text{hypothetical}^*)$	0.201	0.201	0.201	1493486	1521830	0.944

<sup>1</sup> Values given in  $\Gamma_i = \Gamma_3$  instead of  $\Gamma_i = \Gamma_{tot}$   
 $\dagger$  Branching fraction was varied within  $\pm 100\%$

<sup>2</sup> Values given in  $\Gamma_i = \Gamma_4$  instead of  $\Gamma_i = \Gamma_{tot}$

<sup>3</sup> Values given in  $\Gamma_i = \Gamma_6$  instead of  $\Gamma_i = \Gamma_{tot}$   
 $*$  Decays from MC to fill the gap between the  $B$  of inclusive and sum of the exclusive decays



To crosscheck the stability of the fit, we repeated the fit over a different fitting range, excluding up to the first 7 bins, which corresponds to almost 1 GeV.

## ELECTRON MODE

Fitting region (bins)	Signal yield		$\epsilon(B^+)$	$\epsilon(B^0)$	$\Delta B_e$ [%]
[2; 25]	1902589	6407	0.155	0.122	0.01
[3; 25]	1863009	6547	0.152	0.119	0.22
[4; 25]	1802833	6827	0.148	0.116	-0.43
[5; 25]	1737155	7538	0.143	0.109	0.40
[6; 25]	1673829	7013	0.132	0.104	3.32
[7; 25]	1603001	6624	0.125	0.099	4.32

## MUON MODE

Fitting region (bins)	Signal yield		$\epsilon(B^+)$	$\epsilon(B^0)$	$\Delta B_e$ [%]
[2; 25]	1473534	4390	0.129	0.099	-0.97
[3; 25]	1445794	4595	0.127	0.098	-1.66
[4; 25]	1428318	4398	0.125	0.097	-1.41
[5; 25]	1400845	4833	0.123	0.095	-1.34
[6; 25]	1365623	5510	0.120	0.092	-1.25
[7; 25]	1322365	4843	0.111	0.086	3.00

To exclude events, where the missing momentum is likely to misrepresent the neutrino momentum, we imposed the event-level selections to the following properties: missing mass, polar angle of momentum and absolute value of event charge.

## ELECTRON MODE

Selection cuts	Signal yield		$e(B^+)$	$e(B^0)$	$\Delta B_e$ [%]
$M_{miss}^2 < 2 \text{ GeV}/c^2$	1447215	5746	0.122	0.094	-2.74
$j_i; q_i < 2$	1595620	5787	0.129	0.098	2.03
$miss \in [0.6; 2.6]$	1712006	5824	0.139	0.109	0.83

## MUON MODE

Selection cuts	Signal yield		$(B^+)$	$(B^0)$	$\Delta B$ [%]
$M_{miss}^2 < 2 \text{ GeV}/c^2$	1121154	3631	0.100	0.076	-2.19
$j_i; q_i < 2$	1232379	3621	0.106	0.100	1.44
$miss \in [0.6; 2.6]$	1315029	3757	0.114	0.088	0.11

