The q^2 moments in inclusive $b \to c$ decays at Belle II

Joint Annual Meeting of ÖPG and SPS, Innsbruck 2021

Session: FAKT – TASK

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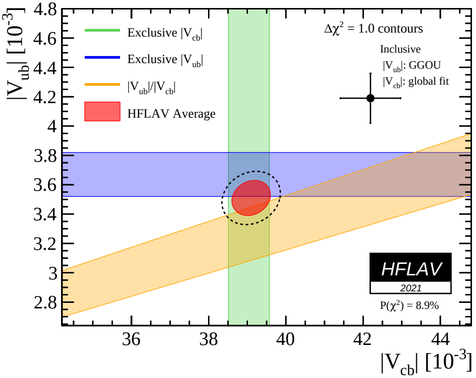
1st September 2021

Outline

- It governs the rate of the dominant weak quark transition $b \to c$
- 3.3σ tension between exclusive and inclusive determinations of $|V_{cb}|$



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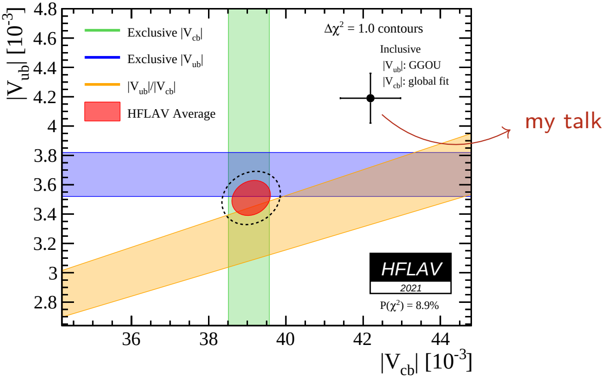

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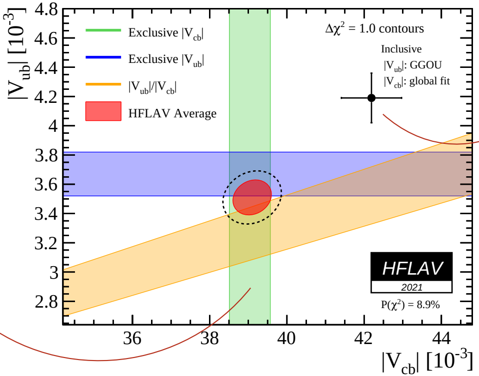

my talk

Daniel Dorner:
$B \to D^*$ [324]
Philipp Horak:
$B \to D^0$ [325]

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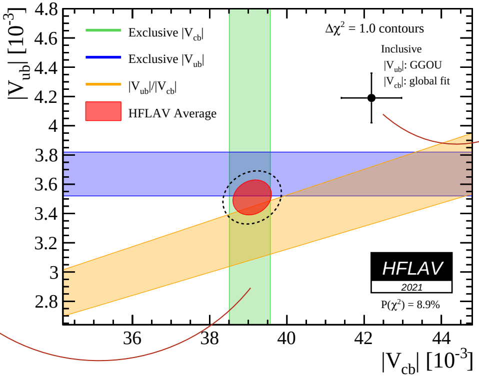

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Possible reasons for anomaly:

- An experimental issue with the exclusive or the inclusive measurement?
- A problem in the theory input?
- Physics beyond the Standard Model?

SEMILEPTONIC WIDTH

$$\Gamma = \frac{G_F^2 m_b^5}{192\pi^2} |V_{cb}|^2 \left(1 + \frac{c_5(\mu)\langle O_5\rangle(\mu)}{m_b^2} + \frac{c_6(\mu)\langle O_6\rangle(\mu)}{m_b^3} + \mathcal{O}\left(\frac{1}{m_b^4}\right)\right) \tag{1}$$

Based on the Operator Product Expansion (OPE)
$\langle O_i \rangle$: hadronic matrix elements (non-perturbative)
c_i: coefficients (perturbative)

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OTHER OBSERVABLES can be expanded with the same heavy quark parameters

→ Moments of lepton energy spectrum

$$R_n(E_{\mathrm{cut},\mu}) = \int_{E_{\mathrm{cut}}} (E_\ell - \mu)^n \frac{\mathrm{d}\Gamma}{\mathrm{d}E_\ell}\mathrm{d}E_\ell \ , \quad \langle E_\ell^n\rangle_{E_{\mathrm{cut}}} = \frac{R_n(E_{\mathrm{cut}},0)}{R_0(E_{\mathrm{cut}},0)} \quad (2)$$

→ Moments of hadron mass spectrum

$$\langle m_X^{2n}\rangle_{E_{\mathrm{cut}}} = \frac{\int_{E_{\mathrm{cut}}} (m_X^2)^n \frac{\mathrm{d}\Gamma}{\mathrm{d}m_X^2}\mathrm{d}m_X^2}{\int_{E_{\mathrm{cut}}} \frac{\mathrm{d}\Gamma}{\mathrm{d}m_X^2}\mathrm{d}m_X^2} \quad (3)$$

- Number of heavy quark parameters proliferates: 13 matrix elements up to $\mathcal{O}(1/m_b^4)$
- Reparametrization invariance links operators in the Heavy-Quark expansion, reducing the number of independent operators to **eight for the total rate at $\boldsymbol{\mathcal{O}(1/m_b^4)}$**
- This reduction does not hold for E_ℓ and M_X^2 moments...

References:
T. Mannel, S. Turczyk and N. Uraltsev, "Higher Order Power Corrections in Inclusive B Decays", JHEP 11 (2010), 109 doi:10.1007/JHEP11(2010)109 [arXiv:1009.4622 [hep-ph]]

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Going beyond: $\mathcal{O}(1/m_b^4)$

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- Reparametrization invariance links operators in the Heavy-Quark expansion, reducing the number of independent operators to **eight for the total rate at $\boldsymbol{\mathcal{O}(1/m_b^4)}$**
- This reduction does not hold for E_ℓ and M_X^2 moments...

...but: Reduction holds for moments of lepton invariant mass $\boldsymbol{q^2}$ $(\boldsymbol{q = p_\ell + p_\nu})$

$\rightarrow$ Final goal: To measure q^2 moments for determination of $|V_{cb}|$ up to $1/m_b^4$

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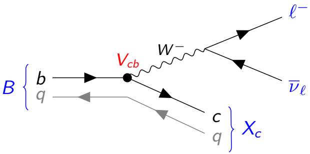
Untagged approach: the only known particle is ℓ.



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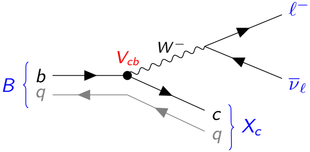

Analysis overview

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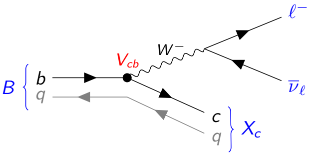

NEUTRINO MOMENTUM: $p_\nu = (|p_{\text{miss}}|, \vec{p}_{\text{miss}})$

Challenge: excluding events where other particles have (most likely) been lost:

- × Missing momentum outside of the detector acceptance
- × Large charge imbalance
- ✓ M^2_{miss} close to zero

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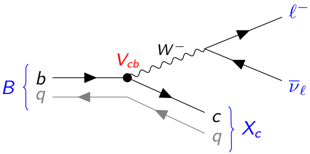

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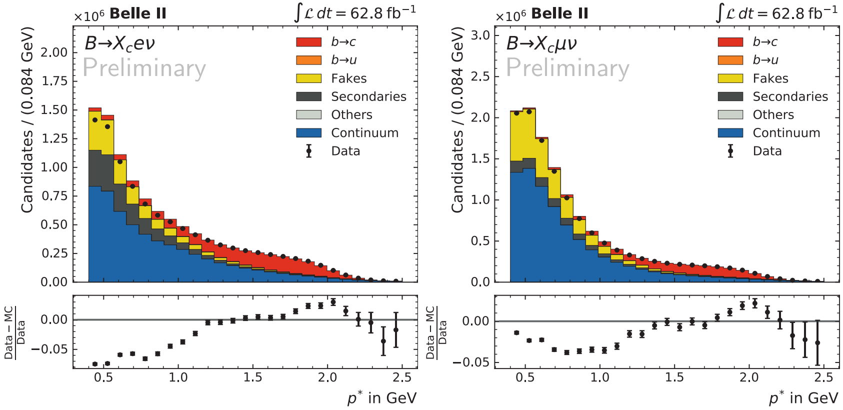
Data samples

- **Belle II data** collected in the years 2019 and 2020 equivalent to **62.8 fb^{-1}**
- Background and signal model:
  - ◇ **Continuum events** ($e^+e^- \to q\bar{q}$): replaced by *off-resonance data* of **9.2 fb^{-1}**
  - ◇ **Signal and $B\bar{B}$ backgrounds**: Monte Carlo sample of **200 fb^{-1}**


×10^6 Belle II
∫ℒ dt = 62.8 fb^−1
B→X_c eν
Preliminary
b→c
b→u
Fakes
Secondaries
Others
Continuum
Data
Candidates / (0.084 GeV)
2.00
1.75
1.50
1.25
1.00
0.75
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0.25
0.00
(Data − MC) / Data
0.00
−0.05
0.5
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p* in GeV
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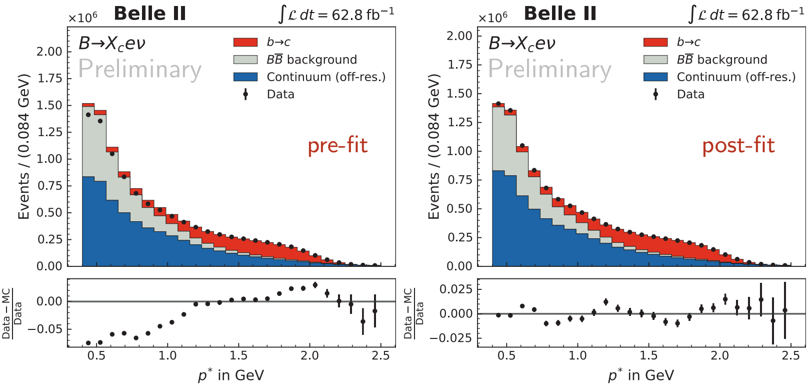
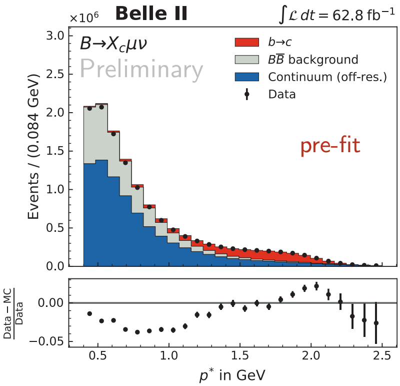
- Samples are fitted by ROOT's `TFractionFitter` (binned likelihood fit)
- Fitting range is between 0.4 GeV/c and 2.5 GeV/c
- Three templates (shapes) are used for fitting:
  - ◇ Signal (MC): events that include ℓ, ν_ℓ and c-quark containing meson
  - ◇ Continuum: shape of off-resonance data
  - ◇ $B\bar{B}$ backgrounds (MC): $b \to u$, secondary leptons, fake leptons, other lepton candidates

Electron mode: pre- and postfit results



Yields	Pre-fit	Post-fit
Signal	1849844 ± 762	1925908 ± 5834
$B\bar{B}$ background	3117962 ± 989	2641059 ± 18459
Continuum	5546871 ± 6149	5511039 ± 22370



Yields	Pre-fit	Post-fit
Signal	1436116 ± 671	1501462 ± 7277
$B\bar{B}$ background	3774497 ± 1089	4102903 ± 87668
Continuum	8033487 ± 7400	7353691 ± 89823

Two sources ($B^+ \to X_c \ell \nu_\ell$ and $B^0 \to X_c \ell \nu_\ell$) with the same semileptonic width Γ:

$$\Gamma(B \to X_c \ell \nu_\ell) = \frac{N^\ell_{\mathrm{sig}}}{2 N_{B\bar{B}} \left(f_+ \epsilon_{B^+} \tau_{B^+} + f_0 \epsilon_{B^0} \tau_{B^0} \right)} \tag{4}$$

N^ℓ_{sig} ... number of signal events

$N_{B\bar{B}}$ ... the number of $B\bar{B}$ pairs in the data sample

ϵ_{B^+/B^0} ... the total signal selection efficiency

factor 2 ... both B mesons in the $\Upsilon(4S)$ event can contribute to the signal

τ_{B^+/B^0} ... the mean life time of the B-mesons

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Branching fraction

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(Average) branching fraction, calculated using (average) lifetime τ:

$$\boldsymbol{\mathcal{B}(B \to X_c \ell \nu_\ell) = \Gamma(B \to X_c \ell \nu_\ell) \tau} \quad (5)$$

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The branching fractions for both modes are:

$$\mathcal{B}(B \to X_c e \nu_e) = (9.97 \pm 0.03_{stat})\% \tag{6}$$
$$\mathcal{B}(B \to X_c \mu \nu_\mu) = (9.47 \pm 0.05_{stat})\% \tag{7}$$

SYSTEMATIC UNCERTAINTIES

Contribution	Relative uncertainty [%] Electron mode	Muon mode
Tracking	0.69	0.69
$N_{B\bar{B}}$	1.1	1.1
Lepton ID corrections	1.64	2.33
f_0/f_+, B lifetime	1.2	1.2
$B \to X_c \ell \nu_\ell$ branching fractions	2.65	2.15
$B \to X_c \ell \nu_\ell$ form factors	1.11	1.11
$B\bar{B}$ background model	0.24	0.34
Off-resonance data model	0.34	2.91
Sum	3.77	4.79

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BRANCHING RATIOS

$$\mathcal{B}(B \to X_c e \nu_e) = (9.97 \pm 0.03_{stat} \pm 0.38_{sys})\% \quad (8)$$

$$\mathcal{B}(B \to X_c \mu \nu_\mu) = (9.47 \pm 0.05_{stat} \pm 0.45_{sys})\% \quad (9)$$

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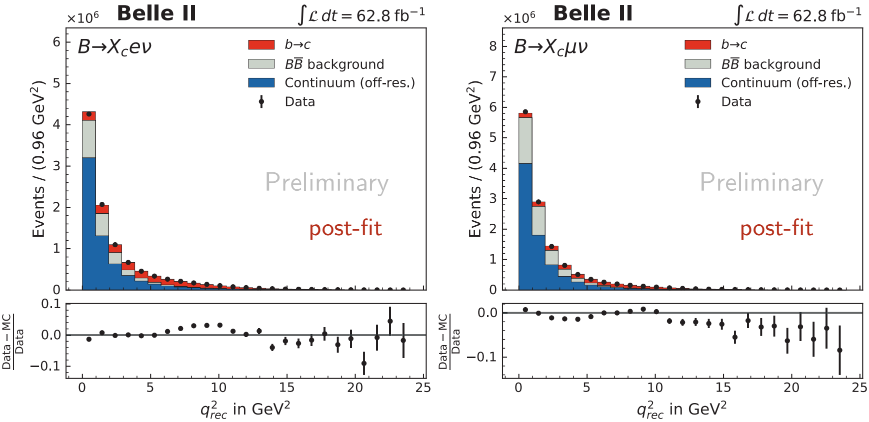
COMBINED BRANCHING RATIO

$$\boldsymbol{\mathcal{B}(B \to X_c \ell \nu_\ell) = (9.75 \pm 0.03_{stat} \pm 0.47_{sys})\%} \tag{10}$$

- Branching fraction (zeroth moment) has been determined with $B \to X_c \ell \bar{\nu}_\ell$
- Studies of q^2 and its moments are ongoing
- We are working towards the extraction of $|V_{cb}|$



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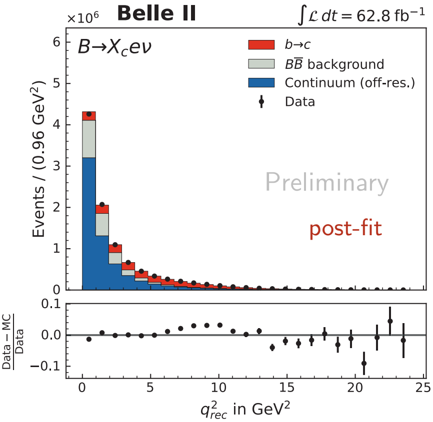
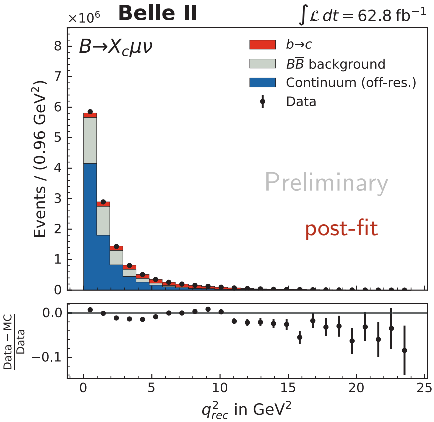
We expect to have preliminary $|V_{cb}|$ measurement next year! :)





BACKUP

Pre- and post-fit yields

Contribution	Electron mode Pre-fit	Electron mode Post-fit	Muon mode Pre-fit	Muon mode Post-fit
Signal yield	1849844 $\pm$ 1360	1932425 $\pm$ 5834	1436116 $\pm$ 1198	1501462 $\pm$ 7277
$b \rightarrow u$ background	63005 $\pm$ 251	53368 $\pm$ 373	47562 $\pm$ 218	51700 $\pm$ 1105
Fakes	1485693 $\pm$ 1219	1258451 $\pm$ 8796	2901541 $\pm$ 1703	3153994 $\pm$ 67393
Secondaries	1562863 $\pm$ 1250	1323818 $\pm$ 9252	821409 $\pm$ 906	892876 $\pm$ 19078
Other MC background	6401 $\pm$ 80	5422 $\pm$ 38	3985 $\pm$ 63	4332 $\pm$ 93
Continuum	5546871 $\pm$ 2355	5511039 $\pm$ 22370	8033487 $\pm$ 2834	7353691 $\pm$ 89823
Sum	10514678 $\pm$ 3243	10084523 $\pm$ 26412	13244100 $\pm$ 3639	12958055 $\pm$ 114140

Main source: **Signal and background modeling**

Signal model

- Signal sample was divided into 30 decays (subsamples)
- The branching ratio of each subsample was varied between $\pm 1\sigma$

$B\bar{B}$ background model

- The amounts of $B\bar{B}$ components ($b \to c$, secondaries, fakes, others) were varied one by one within $\pm 5\%$

Off-resonance data model

- Uncertainty is determined from the difference of signal yield with fully floating continuum component and component floating only within the luminosity measurement uncertainty

$B \to X_c e \nu_\ell$	Γ_i/Γ_{tot} [%]	$\sigma_{\Gamma_{i/tot},-}$ [%]	$\sigma_{\Gamma_{i/tot},+}$ [%]	$N_{sig,-}$	$N_{sig,+}$	$\sigma_{N_{sig,rel}}$ [%]
$D^- \ell^+ \nu_\ell$	2.25	0.08	0.08	1931268	1933046	0.046
$D^*(2010)^- \ell^+ \nu_\ell$	5.09	0.17	0.17	1936901	1927049	0.255
$\bar{D}^0 \pi^- \ell^+ \nu_\ell$	0.41	0.05	0.05	1931239	1933644	0.062
$D_0^*(2300)^- \ell^+ \nu_\ell$, $D_0^{*-} \to \bar{D}^0 \pi^-$	0.30	0.12	0.12	1922936	1941579	0.482
$D_2^*(2460)^- \ell^+ \nu_\ell$, $D_2^{*-} \to \bar{D}^0 \pi^-$	0.121	0.033	0.033	1927194	1937459	0.266
$D^{(*)} n\pi \ell^+ \nu_\ell (n \geq 1)$	0.248[1]	0.062	0.062	1927652	1936093	0.218
$\bar{D}^{*0} \pi^- \ell^+ \nu_\ell$	0.58	0.08	0.08	1930124	1933193	0.079
$D_1(2420)^- \ell^+ \nu_\ell$	0.31	0.12	0.12	1933015	1931679	0.035
$D_2^*(2460)^- \ell^+ \nu_\ell$, $D_2^{*-} \to \bar{D}^{*0} \pi^-$	0.068	0.012	0.012	1929479	1935758	0.162
$D^- \pi^+ \pi^- \ell^+ \nu_\ell$	5.8[2]	3.0	3.0	1931024	1933486	0.064
$D^{*-} \pi^+ \pi^- \ell^+ \nu_\ell$	2.8[3]	1.4	1.4	1926079	1937582	0.298
$\bar{D}^0 \ell^+ \nu_\ell$	2.29	0.08	0.08	1931076	1932703	0.042
$\bar{D}^*(2007)^0 \ell^+ \nu_\ell$	5.58	0.26	0.26	1943118	1922732	0.527
$D^- \pi^+ \ell^+ \nu_\ell$	0.44	0.04	0.04	1930414	1941668	0.291
$\bar{D}_0^*(2420)^0 \ell^+ \nu_\ell$, $\bar{D}_0^{*0} \to D^- \pi^+$	0.25	0.05	0.05	1913151	1947373	0.885
$\bar{D}_2^*(2460)^0 \ell^+ \nu_\ell$, $D_2^{*0} \to D^- \pi^+$	0.153	0.016	0.016	1928964	1933883	0.127
$D^{(*)} n\pi \ell^+ \nu_\ell (n \geq 1)$	0.193[1]	0.022	0.022	1929894	1932582	0.070
$D^{*-} \pi^+ \ell^+ \nu_\ell$	0.60	0.04	0.04	1931840	1933072	0.032
$\bar{D}_1(2420)^0 \ell^+ \nu_\ell$, $\bar{D}_1^0 \to D^{*-} \pi^+$	0.303	0.020	0.020	1932406	1931181	0.032
$\bar{D}_1'(2430)^0 \ell^+ \nu_\ell$, $\bar{D}_1'^0 \to D^{*-} \pi^+$	0.27	0.09	0.09	1931181	1938717	0.195
$\bar{D}_2^*(2460)^0 \ell^+ \nu_\ell$, $\bar{D}_2^{*0} \to D^{*-} \pi^+$	0.101	0.024	0.024	1926970	1937601	0.275
$\bar{D}^0 \pi^+ \pi^- \ell^+ \nu_\ell$,	7.1[2]	2.1	2.1	1931527	1933351	0.047
$\bar{D}^{*0} \pi^+ \pi^- \ell^+ \nu_\ell$	1.4[3]	1.1	1.1	1920958	1944040	0.597
$D_s^- K^+ \ell^+ \nu_\ell$	0.003	0.002	0.002	1929035	1934350	0.138
$D_s^{*-} K^+ \ell^+ \nu_\ell$	0.003	0.003	0.003	1931862	1933038	0.030
$D_1(H)^+ \ell^+ \nu_\ell$ (†)	1	1	1	1916144	1947723	0.817
$D^- \eta \ell^+ \nu_\ell$ (hypothetical*)	0.201	0.201	0.201	1916398	1947962	0.817
$D^{*-} \ell^+ \nu_\ell$ (hypothetical*)	0.201	0.201	0.201	1913463	1951018	0.972
$D^0 \eta \ell^+ \nu_\ell$ (hypothetical*)	0.201	0.201	0.201	1910202	1952695	1.099
$D^{*0} \ell^+ \nu_\ell$ (hypothetical*)	0.201	0.201	0.201	1909367	1953684	1.147

[1] Values given in Γ_i/Γ_3 instead of Γ_i/Γ_{tot} [2] Values given in Γ_i/Γ_4 instead of Γ_i/Γ_{tot} [3] Values given in Γ_i/Γ_6 instead of Γ_i/Γ_{tot}

† Branching fraction was varied within $\pm 100\%$ * Decays from MC to fill the gap between the $\mathcal{B}$ of inclusive and sum of the exclusive decays

$B \to X_c \mu \nu_\ell$	Γ_i/Γ_{tot}[%]	$\sigma_{\Gamma_{i/tot},-}$[%]	$\sigma_{\Gamma_{i/tot},+}$[%]	$N_{sig,-}$	$N_{sig,+}$	$\sigma_{N_{sig,rel}}$ [%]
$D^- \ell^+ \nu_\ell$	2.25	0.08	0.08	1504186	1504672	0.016
$D^*(2010)^- \ell^+ \nu_\ell$	5.09	0.17	0.17	1514854	1508594	0.208
$\bar{D}^0 \pi^- \ell^+ \nu_\ell$	0.41	0.05	0.05	1503605	1510606	0.233
$D_0^*(2300)^- \ell^+ \nu_\ell,\ D_0^{*-} \to \bar{D}^0 \pi^-$	0.30	0.12	0.12	1504428	1508941	0.150
$D_2^*(2460)^- \ell^+ \nu_\ell,\ D_2^{*-} \to \bar{D}^0 \pi^-$	0.121	0.033	0.033	1500621	1507473	0.228
$D^{(*)} n\pi \ell^+ \nu_\ell (n \geq 1)$	0.248[1]	0.062	0.062	1501597	1505444	0.128
$\bar{D}^{*0} \pi^- \ell^+ \nu_\ell$	0.58	0.08	0.08	1509942	1513377	0.114
$D_1(2420)^- \ell^+ \nu_\ell$	0.31	0.12	0.12	1506194	1502020	0.139
$D_2^*(2460)^- \ell^+ \nu_\ell,\ D_2^{*-} \to \bar{D}^{*0} \pi^-$	0.068	0.012	0.012	1506016	1506480	0.015
$D^- \pi^+ \pi^- \ell^+ \nu_\ell$	5.8[2]	3.0	3.0	1502369	1504398	0.068
$D^{*-} \pi^+ \pi^- \ell^+ \nu_\ell$	2.8[3]	1.4	1.4	1501022	1510699	0.322
$\bar{D}^0 \ell^+ \nu_\ell$	2.29	0.08	0.08	1503849	1505237	0.046
$\bar{D}^*(2007)^0 \ell^+ \nu_\ell$	5.58	0.26	0.26	1510872	1505381	0.183
$D^- \pi^+ \ell^+ \nu_\ell$	0.44	0.04	0.04	1503896	1513751	0.328
$\bar{D}_0^*(2420)^0 \ell^+ \nu_\ell,\ \bar{D}_0^{*0} \to D^- \pi^+$	0.25	0.05	0.05	1498581	1519215	0.687
$\bar{D}_2^*(2460)^0 \ell^+ \nu_\ell,\ \bar{D}_2^{*0} \to D^- \pi^+$	0.153	0.016	0.016	1499692	1507424	0.257
$D^{(*)} n\pi \ell^+ \nu_\ell (n \geq 1)$	0.193[1]	0.022	0.022	1503631	1501530	0.070
$D^{*-} \pi^+ \ell^+ \nu_\ell$	0.60	0.04	0.04	1502226	1505597	0.112
$\bar{D}_1(2420)^0 \ell^+ \nu_\ell,\ \bar{D}_1^0 \to D^{*-} \pi^+$	0.303	0.020	0.020	1512654	1504807	0.261
$\bar{D}_1'(2430)^0 \ell^+ \nu_\ell,\ \bar{D}_1'^0 \to D^{*-} \pi^+$	0.27	0.09	0.09	1497896	1508952	0.368
$\bar{D}_2^*(2460)^0 \ell^+ \nu_\ell,\ \bar{D}_2^{*0} \to D^{*-} \pi^+$	0.101	0.024	0.024	1500761	1513068	0.410
$\bar{D}^0 \pi^+ \pi^- \ell^+ \nu_\ell,$	7.1[2]	2.1	2.1	1500541	1506281	0.191
$\bar{D}^{*0} \pi^+ \pi^- \ell^+ \nu_\ell$	1.4[3]	1.1	1.1	1500421	1510675	0.341
$D_s^- K^+ \ell^+ \nu_\ell$	0.003	0.002	0.002	1502864	1507095	0.141
$D_s^{*-} K^+ \ell^+ \nu_\ell$	0.003	0.003	0.003	1500971	1506384	0.180
$D_1(H)^+ \ell^+ \nu_\ell (\dagger)$	1	1	1	1503126	1513857	0.357
$D^- \eta \ell^+ \nu_\ell$ (hypothetical*)	0.201	0.201	0.201	1499323	1516808	0.582
$D^{*-} \ell^+ \nu_\ell$ (hypothetical*)	0.201	0.201	0.201	1490428	1516353	0.863
$D^0 \eta \ell^+ \nu_\ell$ (hypothetical*)	0.201	0.201	0.201	1496843	1525394	0.951
$D^{*0} \ell^+ \nu_\ell$ (hypothetical*)	0.201	0.201	0.201	1493486	1521830	0.944

[1] Values given in Γ_i/Γ_3 instead of Γ_i/Γ_{tot} [2] Values given in Γ_i/Γ_4 instead of Γ_i/Γ_{tot} [3] Values given in Γ_i/Γ_6 instead of Γ_i/Γ_{tot}

† Branching fraction was varied within $\pm 100\%$ * Decays from MC to fill the gap between the $\mathcal{B}$ of inclusive and sum of the exclusive decays

To crosscheck the stability of the fit, we repeated the fit over a different fitting range, excluding up to the first 7 bins, which corresponds to almost 1 GeV.

ELECTRON MODE

Fitting region (bins)	Signal yield	$\epsilon^e(B^+)$	$\epsilon^e(B^0)$	$\Delta\mathcal{B}_e$ [%]
$[2, 25]$	1902589 ± 6407	0.155	0.122	0.01
$[3, 25]$	1863009 ± 6547	0.152	0.119	0.22
$[4, 25]$	1802833 ± 6827	0.148	0.116	-0.43
$[5, 25]$	1737155 ± 7538	0.143	0.109	0.40
$[6, 25]$	1673829 ± 7013	0.132	0.104	3.32
$[7, 25]$	1603001 ± 6624	0.125	0.099	4.32

MUON MODE

Fitting region (bins)	Signal yield	$\epsilon^e(B^+)$	$\epsilon^e(B^0)$	$\Delta\mathcal{B}_e$ [%]
$[2, 25]$	1473534 ± 4390	0.129	0.099	-0.97
$[3, 25]$	1445794 ± 4595	0.127	0.098	-1.66
$[4, 25]$	1428318 ± 4398	0.125	0.097	-1.41
$[5, 25]$	1400845 ± 4833	0.123	0.095	-1.34
$[6, 25]$	1365623 ± 5510	0.120	0.092	-1.25
$[7, 25]$	1322365 ± 4843	0.111	0.086	3.00

To exclude events, where the missing momentum is likely to misrepresent the neutrino momentum, we imposed the event-level selections to the following properties: missing mass, polar angle of momentum and absolute value of event charge.

ELECTRON MODE

Selection cuts	Signal yield	$\epsilon^e(B^+)$	$\epsilon^e(B^0)$	$\Delta\mathcal{B}_e$ [%]
$M^2_{miss} < 2$ GeV/c^2	1447215 ± 5746	0.122	0.094	-2.74
$\lvert\sum_i q_i\rvert < 2$	1595620 ± 5787	0.129	0.098	2.03
$\theta_{miss} \in [0.6, 2.6]$	1712006 ± 5824	0.139	0.109	0.83

MUON MODE

Selection cuts	Signal yield	$\epsilon^\mu(B^+)$	$\epsilon^\mu(B^0)$	$\Delta\mathcal{B}_\mu$ [%]
$M^2_{miss} < 2$ GeV/c^2	1121154 ± 3631	0.100	0.076	-2.19
$\lvert\sum_i q_i\rvert < 2$	1232379 ± 3621	0.106	0.100	1.44
$\theta_{miss} \in [0.6, 2.6]$	1315029 ± 3757	0.114	0.088	0.11

Neutrino properties