Combining Constraints on the Hadronic Light-by-Light Contribution to the Muon $g-2$

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Introduction

The magnetic moment of the muon

- muon has **magnetic moment** $\boldsymbol{\mu} = \frac{e}{2m_\mu}(\boldsymbol{L} + g_\mu \boldsymbol{S})$
- Dirac equation: $g_\mu = 2$

  $\rightarrow$ corrections parameterized by $a_\mu = \frac{1}{2}(g_\mu - 2)$
- field theory definition:

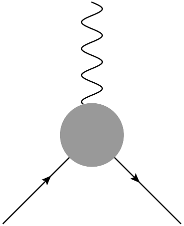
  $\langle \mu(p_2) | j^\nu(0) | \mu(p_1) \rangle = \bar{u}(p_2) \left[F_E(q^2)\gamma^\nu + F_M(q^2) \frac{i\sigma^{\nu\rho} q_\rho}{2m_\mu} \right] u(p_1)$

  with $q = p_2 - p_1$ and $a_\mu = F_M(0)$.

- most recent values BNL 2004, FNL 2021, Aoyama et al. 2020

$$a_\mu^{\text{exp}} = (116\,592\,061 \pm 41) \times 10^{-11}$$
$$a_\mu^{\text{SM}} = (116\,591\,810 \pm 43) \times 10^{-11}$$

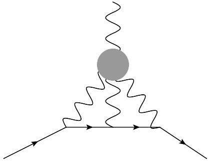
  $\rightarrow$ 4.2σ discrepancy $\rightarrow$ **new physics?**



Introduction

Master formula for HLbL

- theoretical uncertainty dominated by **hadronic effects**
  - $\rightarrow$ focus on hadronic light-by-light scattering (HLbL)
- Lorentz- and gauge invariance:
  interaction of four electromagnetic currents
  described by scalar functions $\bar{\Pi}_i$
- contribution to a_μ given by master formula



Colangelo et al. 2015

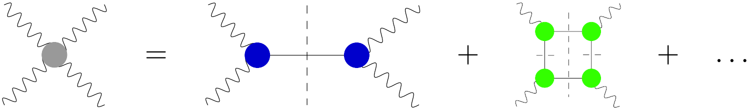
$$a_\mu^{\mathrm{HLbL}} = \frac{\alpha^3}{432\pi^2} \int_0^\infty \mathrm{d}\,\Sigma\,\Sigma^3 \int_0^1 \mathrm{d}\,r\,r\sqrt{1-r^2} \int_0^{2\pi} \mathrm{d}\,\phi \sum_{i=1}^{12} T_i(\Sigma, r, \phi)\bar{\Pi}_i(\Sigma, r, \phi)$$

- integration over space-like virtualities of internal photons with $\Sigma = Q_1^2 + Q_2^2 + Q_3^2$ and r and ϕ parameterizing the ratios
  - $\rightarrow$ $\bar{\Pi}_i$ **analytic** in integration region
- integration region takes form of a cone with tip at $Q_1^2 = Q_2^2 = Q_3^2 = 0$

Constraints

Dispersion relations

- low-energy region ($Q_1^2, Q_2^2, Q_3^2 \lesssim 1\,\text{GeV}$) can be reconstructed from **lightest** intermediate states using analyticity, unitarity and crossing symmetry Colangelo et al. 2015, 2017



- pseudoscalar poles (π^0, η, η') numerically **dominant** and only contribute to one scalar function

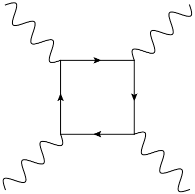
$$\bar{\Pi}_1^{P\text{-pole}} = \frac{F_{P\gamma^*\gamma^*}(-Q_1^2, -Q_2^2) F_{P\gamma^*\gamma^*}(-Q_3^2, 0)}{Q_3^2 + m_P^2}$$

$\rightarrow$ focus on $\bar{\Pi}_1$ from now on

Constraints

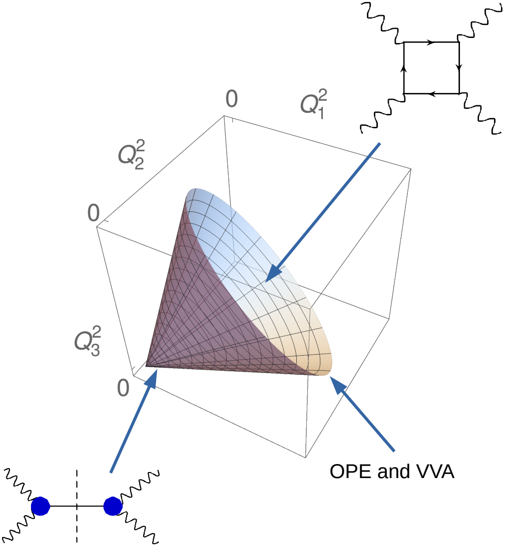
Operator product expansion and perturbative QCD

- for $Q_3^2, \Lambda_{\text{QCD}}^2 \ll Q_1^2, Q_2^2$ OPE relates HLbL to **VVA correlator**
  Melnikov and Vainshtein 2004
- for massless quarks: **VVA correlator** given by axial anomaly
  (up to gluon anomaly)
- for $Q_1^2 \sim Q_2^2 \sim Q_3^2 \gg \Lambda_{\text{QCD}}^2$ HLbL given by OPE with massless **pQCD quark loop** as leading term Bijnens et al. 2019, 2021
- perturbative and non-perturbative (OPE) corrections small (for not too small Q_i^2)



Constraints

Compilation of constraints



Strategy:

- construct functions **interpolating** between constrained regions
- calculate effect on a_μ via master formula
- **conservatively** estimate uncertainties

Interpolation

Asymptotic interpolation

- OPE **and** pertubative QCD (at $Q_1^2 = Q_2^2 = Q_3^2$) results compatible with

$$\bar{\Pi}_1^{(a),\text{asymp}} = -\frac{4N_c C_P^2}{\pi^2(Q_3^2 + m_P^2)(Q_1^2 + Q_2^2 + Q_3^2)}$$

- make use of perturbative QCD away from $Q_1^2 = Q_2^2 = Q_3^2$ by adding general polynomial and fitting the coefficients

$\rightarrow$ large Σ region **precisely** determined

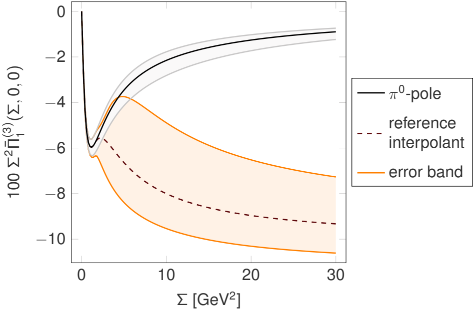
Interpolation

Connection with low-energy region

- multiply asymptotic expressions by general **analytic** function (for positive Σ) such that large-Σ behavior unchanged

$$\bar{\Pi}_1^{\text{int}} = \bar{\Pi}_1^{\text{asymp}} \left(1 + \sum_{i=1}^{N} b_i(r,\phi)\Sigma^{-i}\right)^{\pm 1}$$

- two choices and different N to estimate uncertainty
- fix coefficients by demanding **smooth matching** to dispersive result at some matching surface $\Sigma^{\text{match}}(r,\phi)$



Numerical results

- effect of longitudinal short-distance constraints on HLbL is $(9.1 \pm 5.0) \times 10^{-11}$
- compatible with recent results from Regge model and holographic QCD, but far smaller than often-used model by Melnikov and Vainshtein (2004), discrepancy well understood

Colangelo et al. 2020, Cappiello et al. 2019, Leutgeb & Rebhan 2019

$\rightarrow$ talk by Josef Leutgeb

- uncertainties **well below** near-term experimental goal of $\pm 16 \times 10^{-11}$
- uncertainties almost exclusively due to **1–2 GeV** region
- improved knowledge of **resonances** in that mass-range needed to considerably improve

Conclusions

- SM prediction for muon anomalous magnetic moment limited by **hadronic** uncertainties
- **connection** between dispersive representation and asymptotic constraints in HLbL among most important open questions (conceptually and numerically)
- presented method how to **model-independently** combine QCD constraints on HLbL
- provided estimate for effect of longitudinal constraints $(9.1 \pm 5.0) \times 10^{-11}$
- uncertainty dominated by lacking knowledge about effects of resonances in **1–2 GeV** region

Outlook

- update results with **new input** for axial transition form factors and updated Lorentz basis Zanke et al. 2021, Colangelo et al. 2021

  $\rightarrow$ talk by Josef Leutgeb

- extend analysis to **other scalar functions** $\bar{\Pi}_i$

- set up a **novel dispersive formalism** (in collab. with Peter Stoffer)
  - directly work with one **soft** photon as needed for the $g-2$ integral
  - leads to re-shuffling of terms compared to the current formalism
  - will allow to describe ***D*-wave** pion rescattering and tensor-meson poles
  - comparison with current formalism will help to estimate **truncation uncertainties**
  - apply to much simpler VVA correlator first

Thank you for your attention!