Glueballs and Gauge/Gravity Duality

Unraveling the Mysteries of the (Strong) Force

• Frederic Brünner •

Goal:

Study properties of glueballs, bound states of gluons, using gauge/gravity duality.

Dissertation based on research in collaboration with
Josef Leutgeb, **Denis Parganlija**, and **Anton Rebhan**

Problem:

QCD confining and asymptotically free
$\Rightarrow$ low energy degrees of freedom: **hadrons**

Rich spectrum of hadrons observed in experiment

QCD strongly coupled, properties of hadrons not derivable

$\rightarrow$ constituent quark model, effective field theory, lattice QCD, ...

Lattice gauge theory predicts existence of **glueballs**:

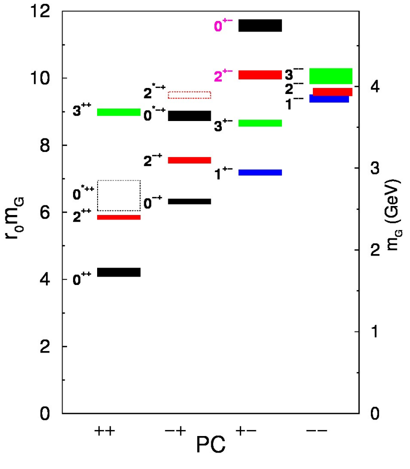
- Variety of quantum numbers:

  $J^{PC} = 0^{++}, 2^{++}, 0^{-+}, 1^{+-}, \ldots$

  scalar, tensor, pseudoscalar, pseudovector

- Mass spectrum: lightest glueball (scalar) below 2 GeV

- Rich spectrum above 2 GeV



Morningstar, Peardon, ‘99

Glueballs not unambiguously identified among the hadrons observed in experiment: **more than one candidate state**, not enough experimental data

Theoretical prediction required for properties of glueballs:

- Quantum numbers
- Mass
- Interaction Lagrangian involving other hadrons
- Decay channels and rates

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Theoretical prediction required for properties of glueballs:

- Quantum numbers ✓
- Mass ✓
- Interaction Lagrangian involving other hadrons ✗
- Decay channels and rates ✗

How to proceed? Idea: use gauge/gravity duality!

Gauge

- Gauge theory without gravity
- d dimensions
- boundary
- Large-N approximation
- Strong coupling

Gravity

- Theory including gravity
- At least $d+1$ dimensions
- bulk
- Low curvature
- Weak coupling

Duality: perfect agreement on all physical observables

$\rightarrow$ Realisation of holographic principle

$\rightarrow$ First concrete example: AdS/CFT correspondence

Holographic dual of QCD is not known, try to get as close as possible: **Witten-Sakai-Sugimoto** model (Witten, ‘98, Sakai & Sugimoto, ‘04)

Gravity side based on solution of type IIA string theory: stack of $\mathrm{N_c}$ D4-branes (glue part) intersected by $\mathrm{N_f}$ pairs of $\mathrm{D8} - \overline{\mathrm{D8}}$ flavour branes in the probe approximation $\mathrm{N_f} \ll \mathrm{N_c}$.

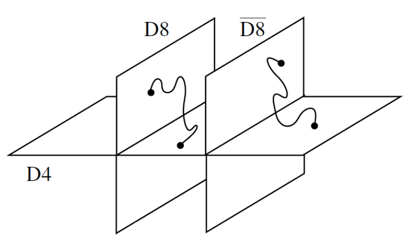


Figure by A. Schmitt

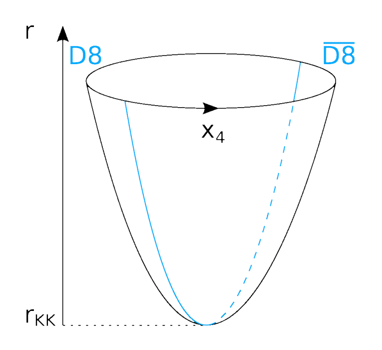
At large $\mathrm{N_c}$ and small string coupling $g_s \Rightarrow$ supergravity approximation

Non-conformal and confining theory is obtained from a compactified $\mathrm{AdS_7} \times \mathrm{S^4}$ geometry, Witten, '98:

$$ds^2 = \frac{r^2}{L^2}\left[f(r)\,dx_4^2 + \eta_{\mu\nu}dx^\mu dx^\nu + dx_{11}^2\right] + \frac{L^2}{r^2}\frac{dr^2}{f(r)} + \frac{L^2}{4}d\Omega_4^2,$$

$$f(r) = 1 - \frac{r_{\mathrm{KK}}^6}{r^6}$$

L: AdS radius, $r_{\mathrm{KK}} = M_{\mathrm{KK}} L^2/3$



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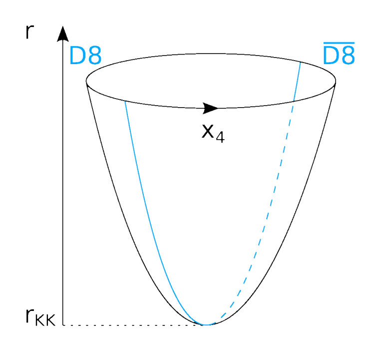
five boundary dimensions



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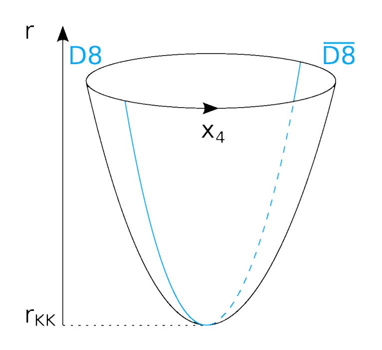
additional compact direction



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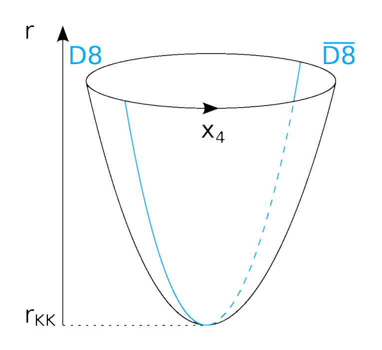
holographic coordinate



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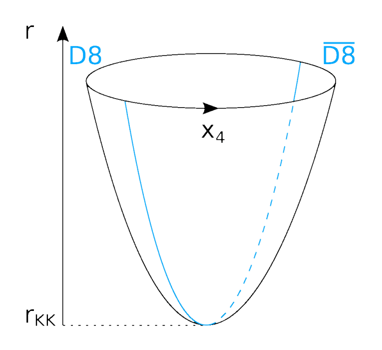
four-sphere



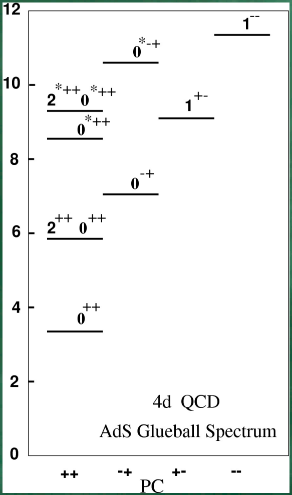
Field theory dual: five-dimensional confining gauge theory that looks four-dimensional below the compactification scale M_{KK}, at large but fixed ‘t Hooft coupling $\lambda = N_c g_{YM}^2$

Degrees of freedom are glueballs (dual to metric and form field fluctuations) as well as quark-antiquark mesons (dual to gauge fields on the probe branes)

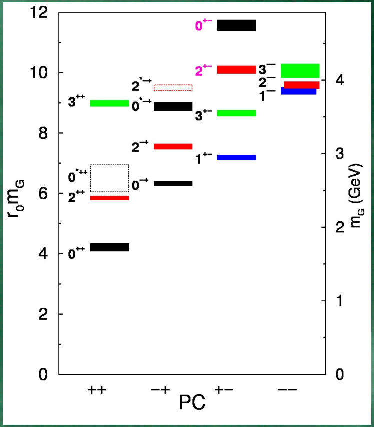
Geometric realisation of chiral symmetry breaking:

$$\mathrm{SU(N_f)_L \times SU(N_f)_R \to SU(N_f)_V}$$

Holographic spectrum of glueballs vs lattice result:



Brower, Mathur, Tan, '00



Morningstar, Peardon, '99

Two free parameters: mass scale $M_{\rm KK}$ and 't Hooft coupling λ, fixed by experimental value of m_ρ and either pion decay constant or ratio $m_\rho/\sqrt{\sigma}$:

$$M_{\rm KK} \approx 949 {\rm MeV} \qquad \lambda = 16.63 \ldots 12.55$$

Note: The Witten-Sakai-Sugimoto in its original form is chiral: quarks are massless. Quark mass is introduced by a modification of the model involving world-sheet instantons (Hashimoto et. al., '08)

Strategy to obtain decay rates:

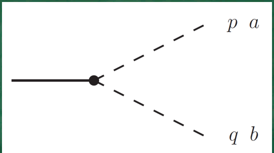
1. Solve linearized equations of motion for all fields, e.g.

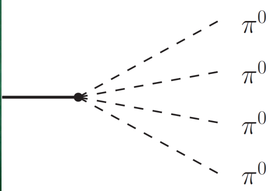
$$\delta g_{MN} = h_{MN} = q_{MN} G(x) H(r) \qquad DH(r) = M^2 H(r)$$

2. Insert fluctuation into low energy effective actions

$$h_{MN} \longrightarrow S_{\mathrm{DBI}}, S_{\mathrm{CS}}, S_{\mathrm{SUGRA}} \longrightarrow \int \qquad \Rightarrow \mathcal{L}_{\mathrm{int}}$$

3. Use Feynman rules to calculate decay rates





Plausibility check: $\rho \to 2\pi$ and $\omega \to 3\pi$ decay

$$\mathcal{L}_{\rho\pi\pi} = -g_{\rho\pi\pi}\epsilon_{abc}(\partial_\mu \pi^a)\rho^{b\mu}\pi^c, \qquad g_{\rho\pi\pi} \approx 33.98\,\lambda^{-1/2} N_c^{-1/2}$$

With $\lambda \approx 16.63 \ldots 12.55$ and $N_c = 3$ we obtain

$$\Gamma_\rho / m_\rho \approx 0.1535 \ldots 0.2034, \qquad \Gamma_\rho / m_\rho{}^{(exp)} \approx 0.191$$

$$\Gamma_\omega / m_\omega \approx 0.0033 \ldots 0.0103, \qquad \Gamma_\omega / m_\omega{}^{(exp)} \approx 0.0097$$

Results I. - Scalar Glueball, 0^{++}

FB, D. Parganlija, A. Rebhan, Phys. Rev. D **91** (2015)
FB, A. Rebhan, Phys. Rev. Lett. **115** (2015)

Interactions between glueballs and two pseudoscalars:

$$\mathcal{L}_{G\pi\pi} = \frac{1}{2} A * \partial_\mu \pi^a \partial_\nu \pi^a \left(\eta^{\mu\nu} - \frac{\partial^\mu \partial^\nu}{M_G^2} \right) G + \frac{1}{2} B (\pi^a)^2 G$$

Agreement with data for the glueball candidate $f_0(1710)$:

$f_0(1710)$	exp.	prediction
$\frac{4}{3} \cdot \Gamma(\pi\pi)/\Gamma(K\bar{K})$	$0.55^{+0.15}_{-0.23}$	0.463
$4 \cdot \Gamma(\eta\eta)/\Gamma(K\bar{K})$	1.92 ± 0.60	1.12

Results II. - Scalar Glueball, 0^{++}

FB, D. Parganlija, A. Rebhan, Phys. Rev. D **91** (2015)

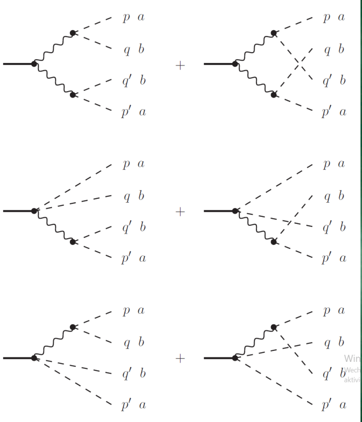
Prediction of decay into 4π:

$$\Gamma/M = 0.024 \ldots 0.030,$$

through intermediate ρ mesons as

$$f_0(1710) \to 2\rho, \rho\pi\pi \to 4\pi.$$

To be confirmed by experiment!



Results III. - Tensor Glueball, 2^{++}

FB, D. Parganlija, A. Rebhan, Phys. Rev. D **91** (2015)

$$\mathcal{L}_T = \mathrm{Tr}\left[\frac{1}{2}t_1\partial_\mu\pi\partial_\nu\pi G_T^{\mu\nu} + \frac{1}{2}t_2 M_{\mathrm{KK}}^2\rho_\mu\rho_\nu G_T^{\mu\nu} + \frac{1}{2}t_3\tilde{F}_{\mu\rho}\tilde{F}_\nu^\rho G_T^{\mu\nu} + it_5\partial_\mu\pi[\pi,\rho_\nu]G_T^{\mu\nu}\right],$$

For masses larger than 2 GeV (consistent with lattice predictions), the tensor glueball is a broad state:

$M_T = 2$ GeV: $\Gamma/M_T \approx 0.32\ldots0.45$
$M_T = 2.4$ GeV: $\Gamma/M_T \approx 0.45\ldots0.62$

Results IV. - Pseudovector Glueball, 1^{+-}

FB, J. Leutgeb, A. Rebhan, Phys. Lett. B **788** (2019)

Corresponds to Kalb-Ramond field B_2

The pseudovector glueball is a very broad state:

$\Gamma/M \approx 0.9225 \ldots 1.3685$

Summary

Witten-Sakai-Sugimoto model: interesting tool for studying glueball decay, many nontrivial results

Word of caution: uncontrolled approximation to QCD due to large-N and λ, as well as five-dimensional nature of the theory and probe limit of flavour branes

Outlook

Incorporate corrections to the approximations in order to make results more reliable