Amplitude and colour evolution: From first principles to simulations.

Simon Plätzer
Institute of Physics — NAWI, University of Graz
Particle Physics — University of Vienna

At the
51st ISMD Conference
Pitlochry/Online | 2 August 2022

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Motivation — Accurate event generators

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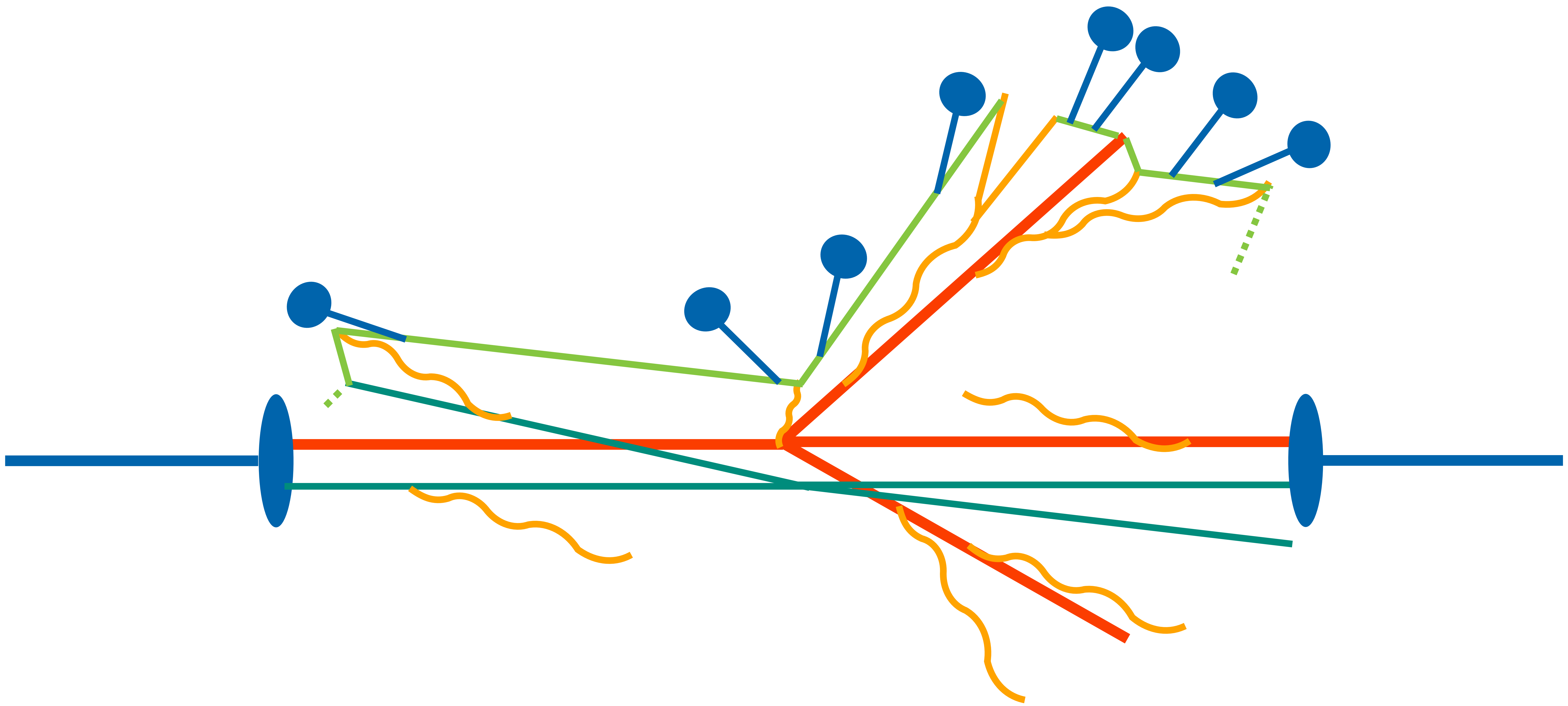

$$\mathrm{d}\sigma \sim \mathrm{L} \times \mathrm{d}\sigma_H(Q) \times \mathrm{PS}(Q \to \mu) \times \mathrm{MPI} \times \mathrm{Had}(\mu \to \Lambda) \times \ldots$$

Shower & parton branching paradigms

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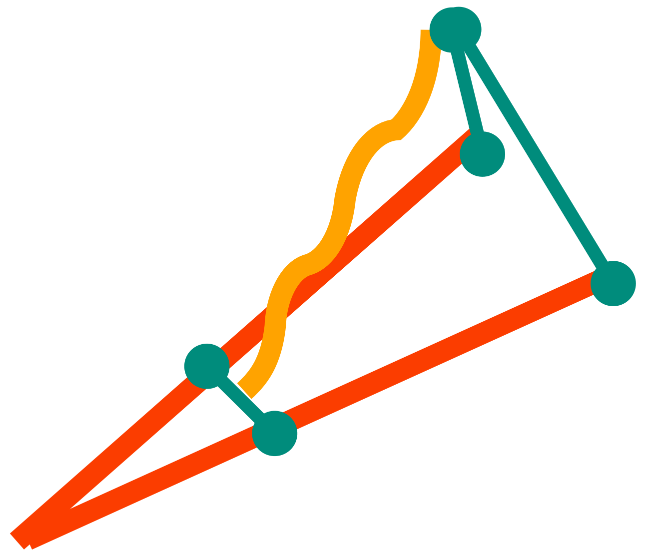
Parton branchings order in angle.

- Driven by QCD coherence
- Recoil global
- Links to analytic use of coherent branching

Herwig 7

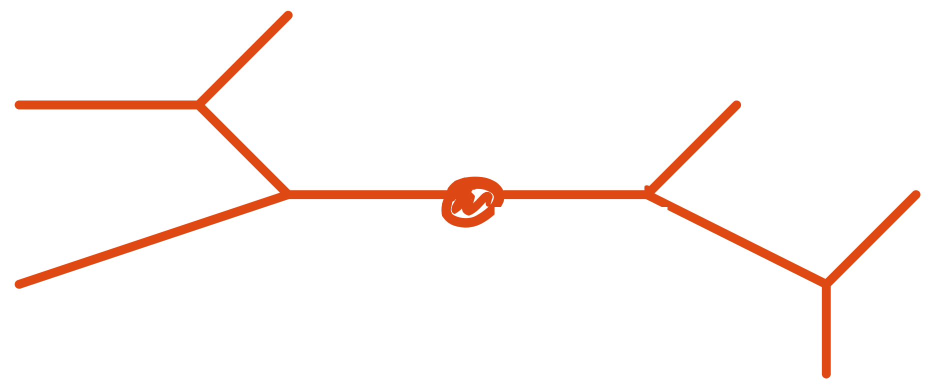


Dipole branchings order in transverse momentum.

- Driven by large-N dipole pattern and colour flows
- Momentum conservation for each emission
- Advantageous for matching & merging

Herwig 7, Pythia 8, Sherpa, PanScales, Deductor

Sequences of emission scales and momentum fractions as Markov process.
Restore momentum conservation per emissions or at end of evolution.



$$\mathrm{d}S = \frac{\alpha_s}{2\pi}\frac{\mathrm{d}\tilde{q}_i^2}{\tilde{q}_i^2}\mathrm{d}z P(z_i) \exp\left(-\int_{\tilde{q}_i^2}^{Q^2}\frac{\mathrm{d}q^2}{q^2}\int_{z_-(k^2)}^{z_+(k^2)}\mathrm{d}\xi\frac{\alpha_s}{2\pi}P(z)\right)$$

emission rate

no emission probability

$$\sigma(n \text{ jets}, \tau) \sim \sum_k \sum_{l\leq 2k} c_{nkl}\ \alpha_s^k(Q)\ \ln^l\frac{1}{\tau}$$

QCD coherence & shower accuracy

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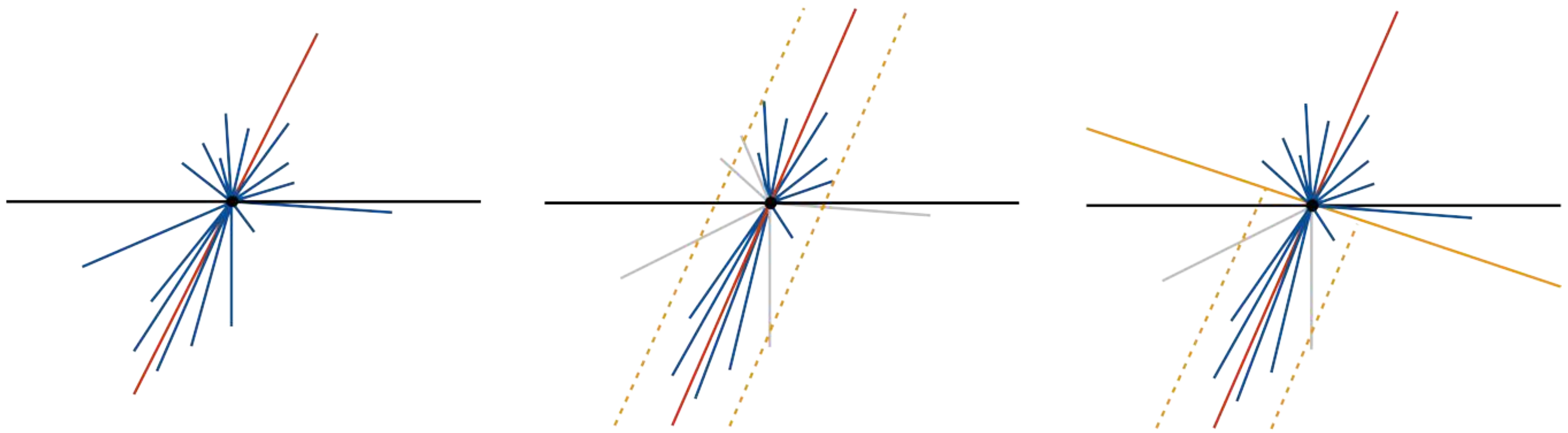

QCD coherence & shower accuracy

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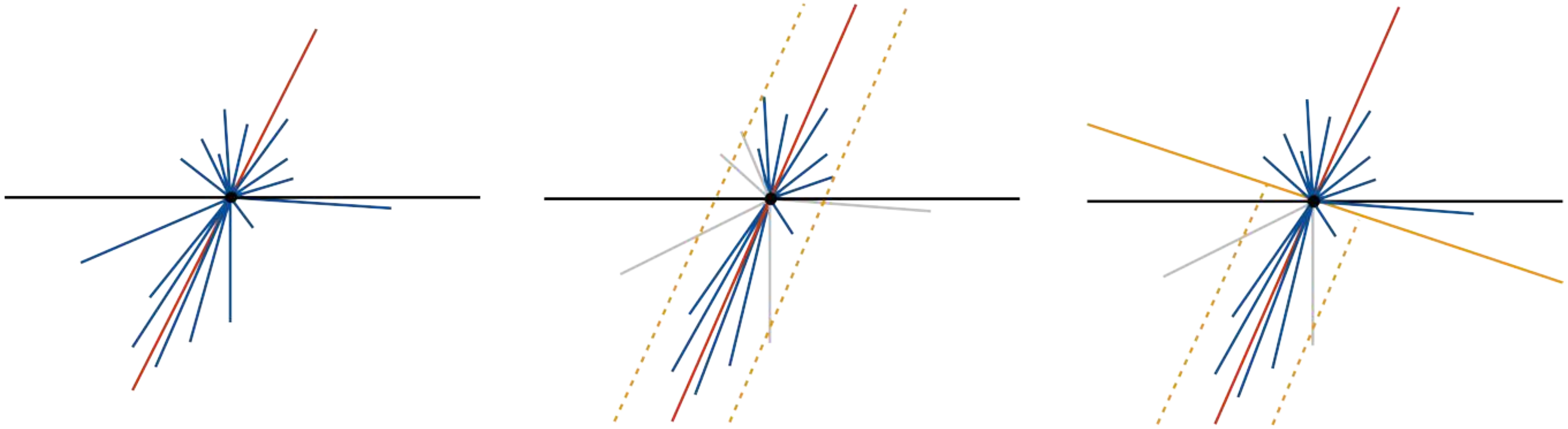

NLO with matching

NLL with coherent branching
Issues in dipole showers

Issues in coherent branching
LL with dipole showers

Understand and decide on accuracy of (existing) parton shower algorithms,
take as a starting point for incremental improvements.

$$H(\alpha_s) \times \exp\left(L g_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \ldots\right)$$

$\alpha_s L \sim 1$ LL NLL NNLL

[Dasgupta, Dreyer, Hamilton, Monni, Salam et al. — JHEP 09 (2018) 033, …]
[Hoang, Plätzer, Samitz — JHEP 1810 (2018) 200]
[Bewick, Ferrario, Richardson, Seymour — JHEP 04 (2020) 019]

QCD coherence & shower accuracy

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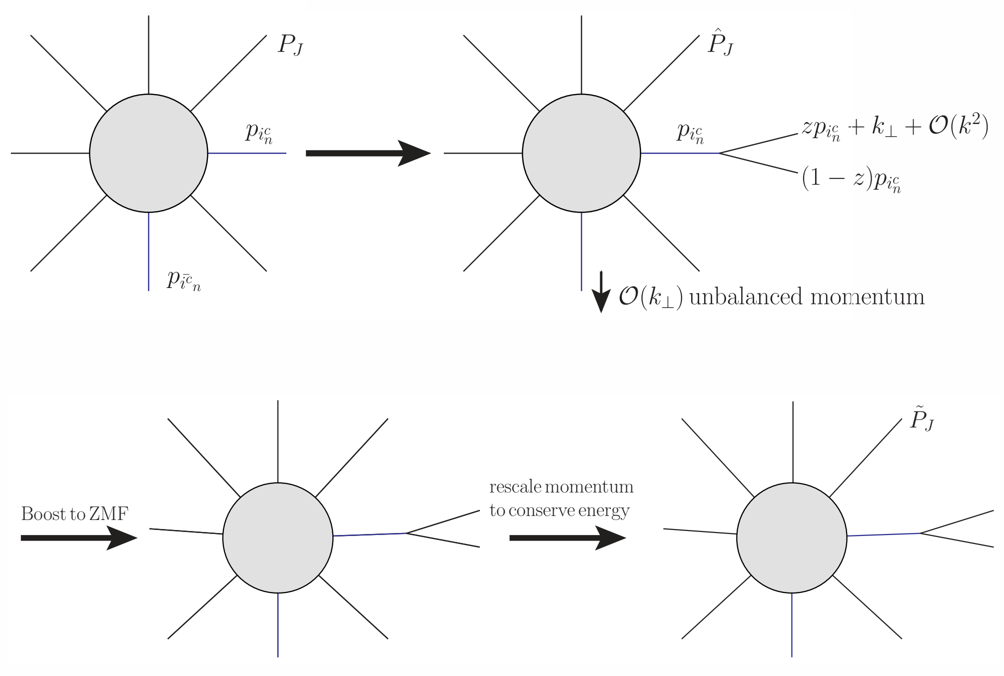
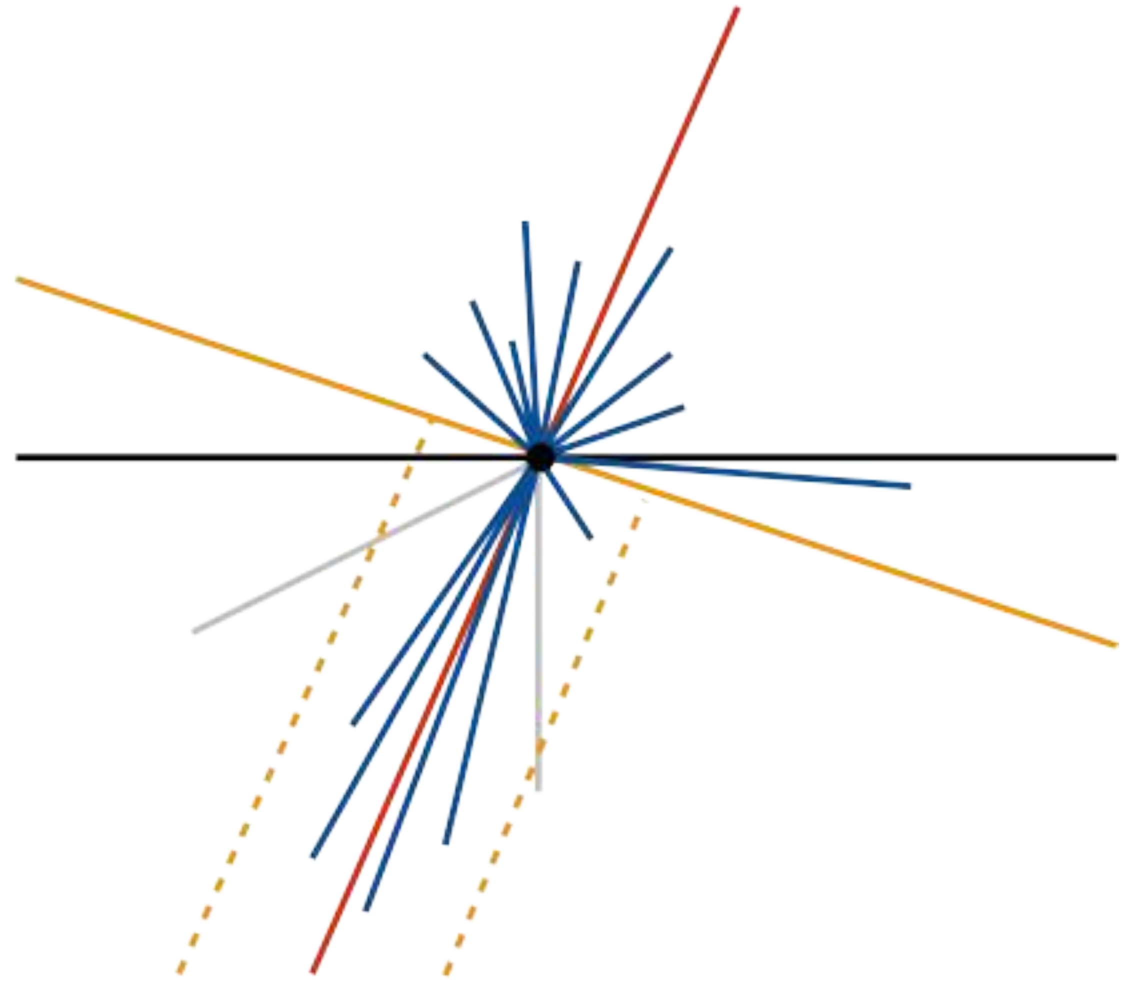
$$\frac{p_{i_n}\cdot p_{j_n}}{p_{i_n}\cdot q_n\; p_{j_n}\cdot q_n} \longrightarrow \frac{p_{i_n}\cdot p_{j_n}}{p_{i_n}\cdot q_n\; p_{j_n}\cdot q_n} - \frac{T\cdot p_{j_n}}{T\cdot q_n}\frac{1}{p_{j_n}\cdot q_n} + \frac{T\cdot p_{i_n}}{T\cdot q_n}\frac{1}{p_{i_n}\cdot q_n}$$

Partition of soft radiation



Recoil

[Dasgupta, Dreyer, Hamilton, Monni, Salam —PRL 125 (2020) 5]
[Forshaw, Holguin, Plätzer – JHEP 09 (2020) 014 & EPC C81 (2021) 4]



Dipole showers reproducing coherent branching:
NLL & NLC global, LL & LC non-global

Understand and decide on accuracy of (existing) parton shower algorithms,
take as a starting point for incremental improvements.

$$H(\alpha_s) \times \exp\left(Lg_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \ldots\right)$$

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QCD coherence & shower accuracy

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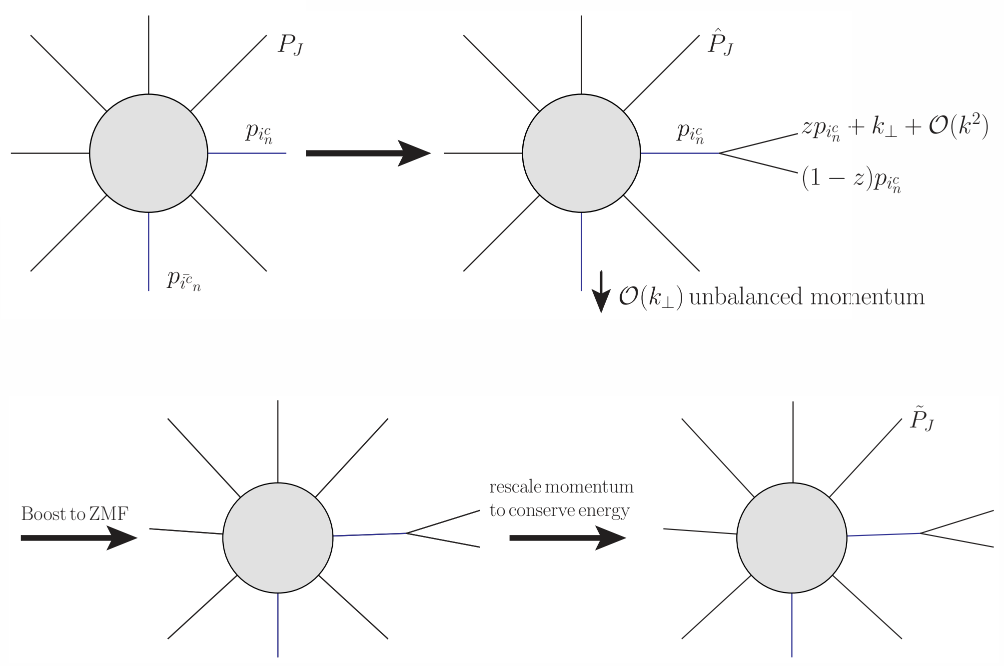
$$\frac{p_{i_n}\cdot p_{j_n}}{p_{i_n}\cdot q_n\; p_{j_n}\cdot q_n} \longrightarrow$$

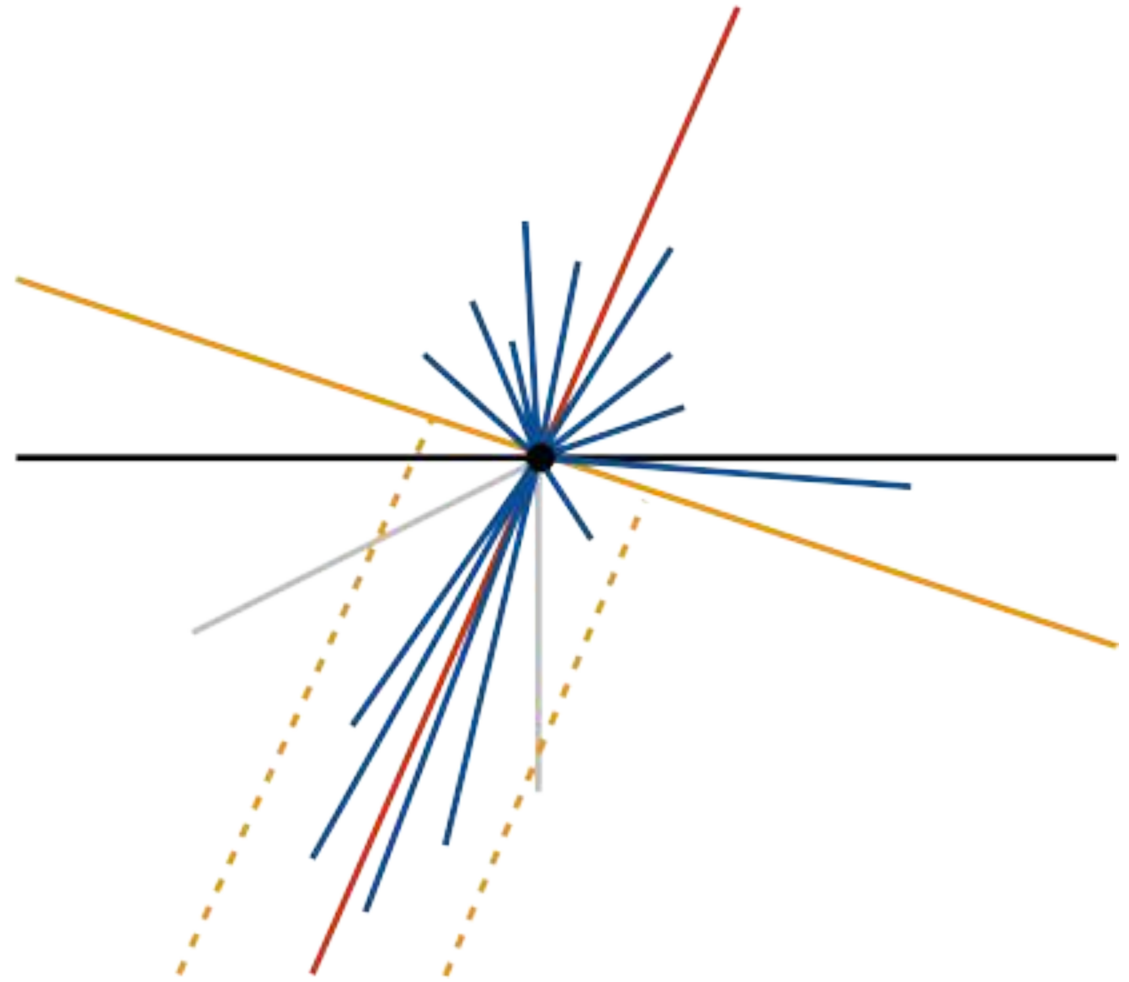
$$\frac{p_{i_n}\cdot p_{j_n}}{p_{i_n}\cdot q_n\; p_{j_n}\cdot q_n} - \frac{T\cdot p_{j_n}}{T\cdot q_n}\frac{1}{p_{j_n}\cdot q_n} + \frac{T\cdot p_{i_n}}{T\cdot q_n}\frac{1}{p_{i_n}\cdot q_n}$$

Partition of soft radiation



Recoil

[Dasgupta, Dreyer, Hamilton, Monni, Salam —PRL 125 (2020) 5]
[Forshaw, Holguin, Plätzer – JHEP 09 (2020) 014 & EPC C81 (2021) 4]



Dipole showers reproducing coherent branching:
NLL & NLC global, LL & LC non-global

Understand and decide on accuracy of (existing) pa

take as a starting point for incremental improvements.

Another issue is the large-N limit: $\alpha_s N^2 \sim 1$

$$H(\alpha_s)\times \exp\left(Lg_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \ldots\right)$$

$\alpha_s L \sim 1$ LL NLL NNLL

[Dasgupta, Dreyer, Hamilton, Monni, Salam et al. — JHEP 09 (2018) 033, …]
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A general starting point

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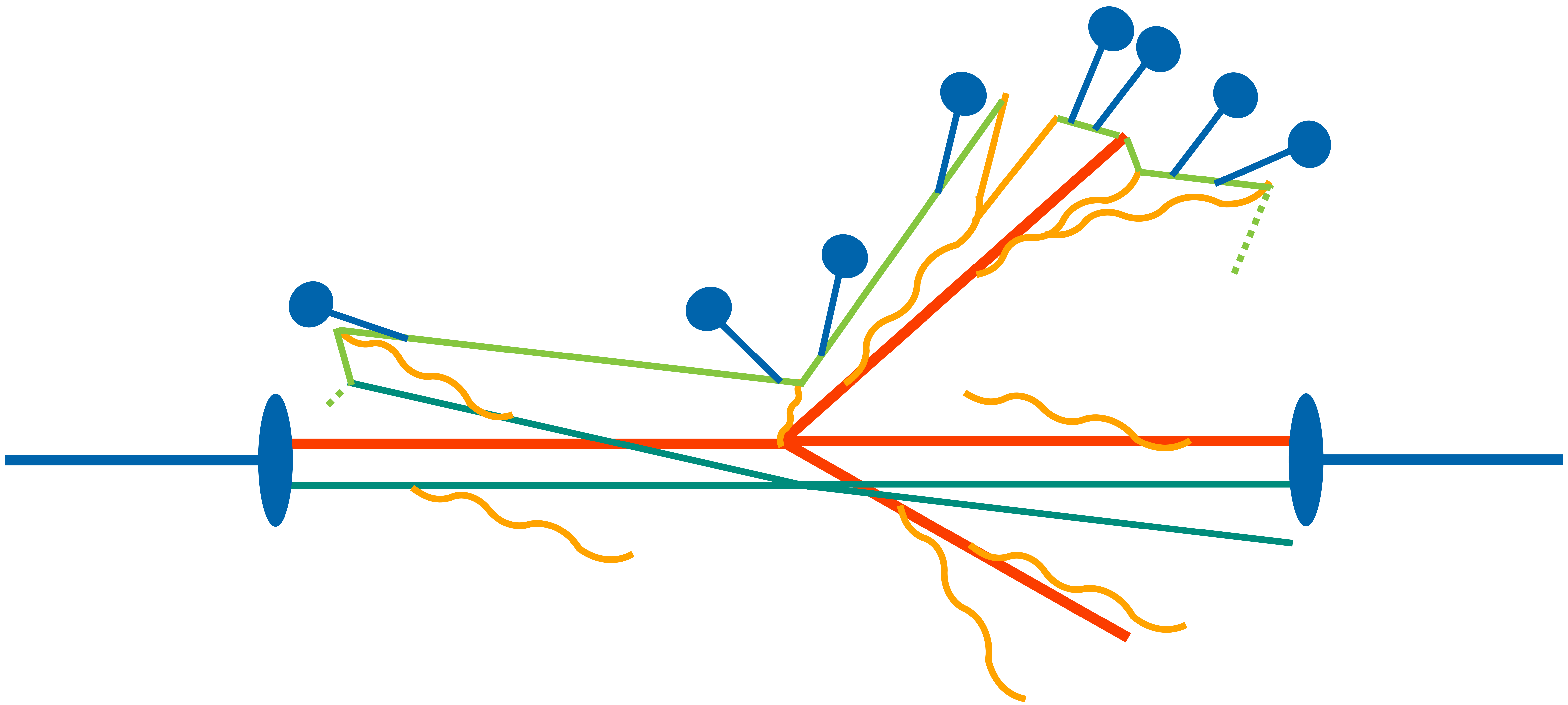

$$d\sigma \sim \mathrm{Tr}\left[\mathbf{PS}(Q \to \mu)\,d\mathbf{H}(Q)\,\mathbf{PS}^\dagger(Q \to \mu)\,\mathbf{Had}(\mu \to \Lambda)\right]$$

Amplitude evolution basics

Amplitude evolution basics

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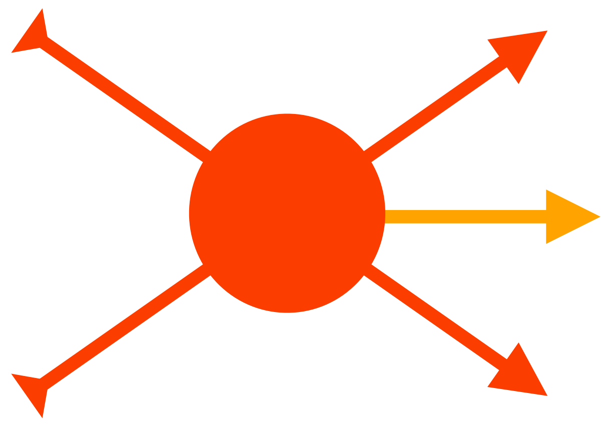
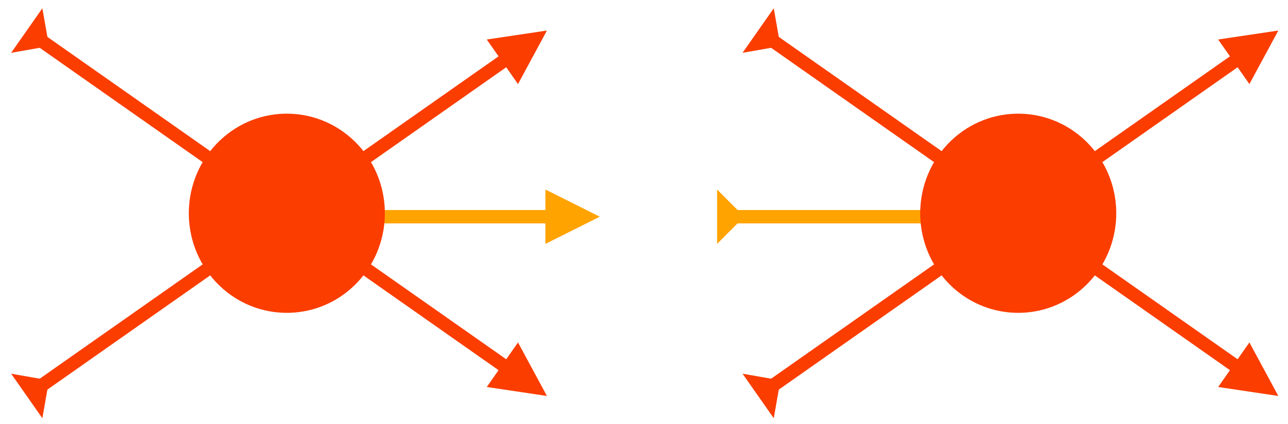


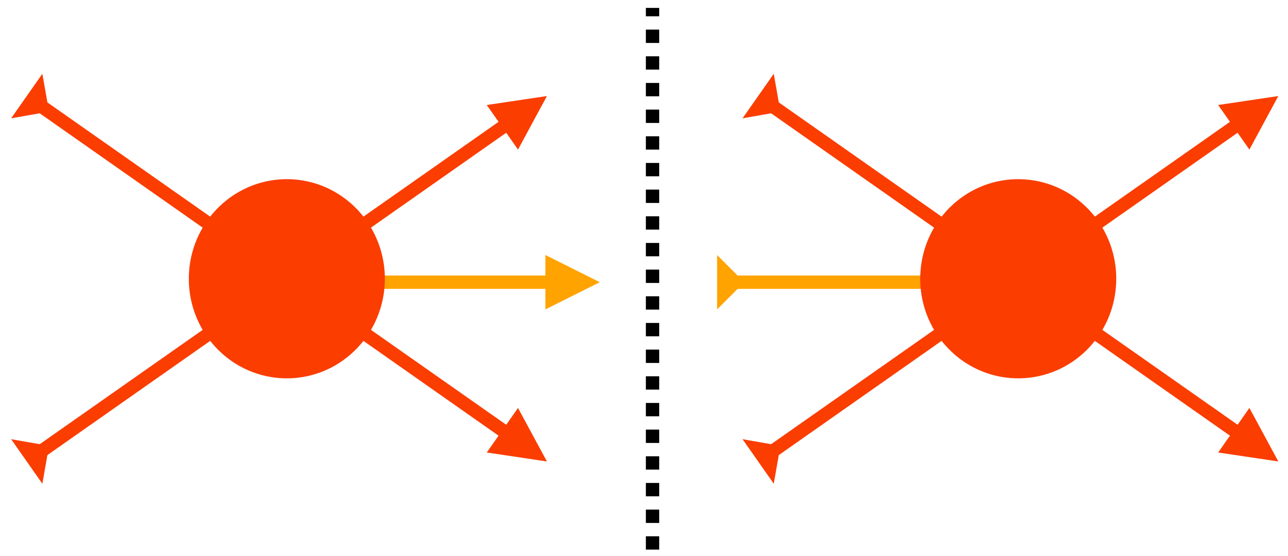
$$\langle f|\hat{S}|i\rangle$$

Amplitude evolution basics



$$\langle f|\hat{S}|i\rangle \qquad \langle i|\hat{S}^\dagger|f\rangle$$

Amplitude evolution basics



$$\langle f|\hat{S}|i\rangle \qquad \langle i|\hat{S}^\dagger|f\rangle$$

$$p_i\delta(o - o_f)\mathrm{d}\phi_f$$

Amplitude evolution basics

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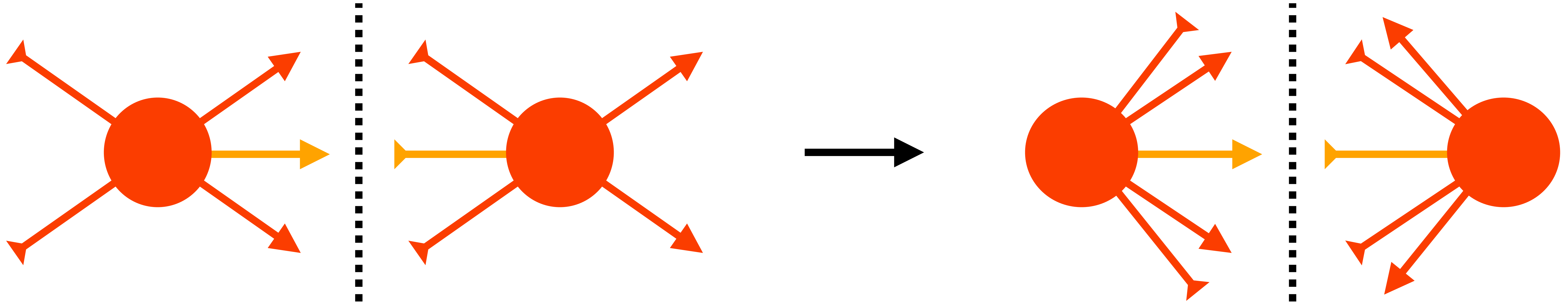


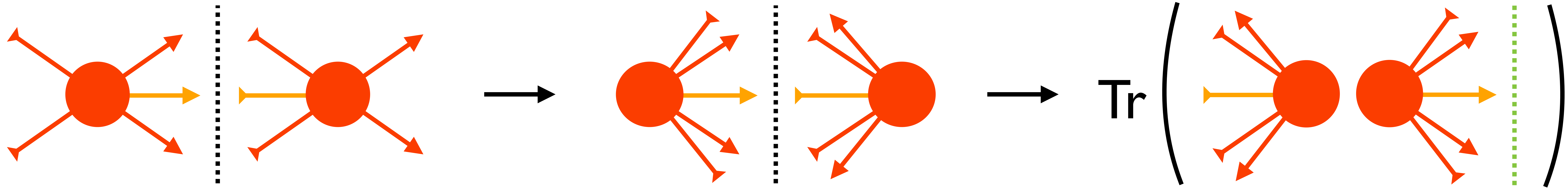
$$\langle f|\hat{S}|i\rangle \qquad \langle i|\hat{S}^\dagger|f\rangle \qquad \{\mathcal{M}|i,f\} \quad \{i,f|\mathcal{M}\}$$

$$p_i\delta(o-o_f)\mathrm{d}\phi_f$$

Correlation function of field operators.

External wave functions.

Amplitude evolution basics



$\langle f|\hat{S}|i\rangle \quad \langle i|\hat{S}^\dagger|f\rangle$

$p_i\delta(o-o_f)\mathrm{d}\phi_f$

$\{\mathcal{M}|i,f\} \quad \{i,f|\mathcal{M}\}$

$|\mathcal{M}\}\{\mathcal{M}| \quad |i,f\}\{i,f|$

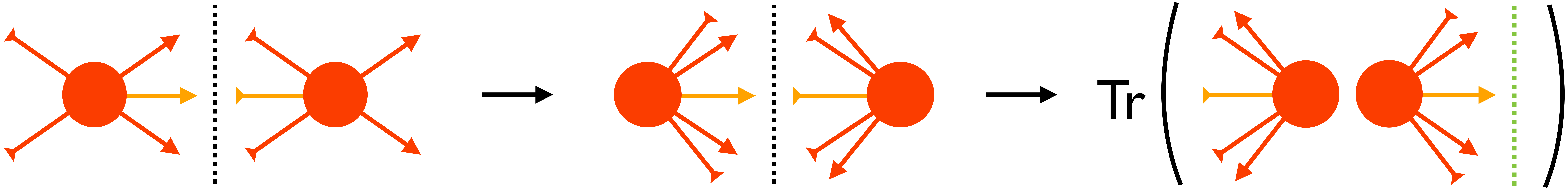
Correlation function of field operators.

External wave functions.

Cross section density operator.

Measurement projector.

Amplitude evolution basics



$\langle f|\hat{S}|i\rangle \quad \langle i|\hat{S}^\dagger|f\rangle$ $\quad \{\mathcal{M}|i,f\} \quad \{i,f|\mathcal{M}\}$ $\quad |\mathcal{M}\}\{\mathcal{M}| \quad |i,f\}\{i,f|$

$$p_i\delta(o-o_f)\mathrm{d}\phi_f$$

Correlation function of field operators.

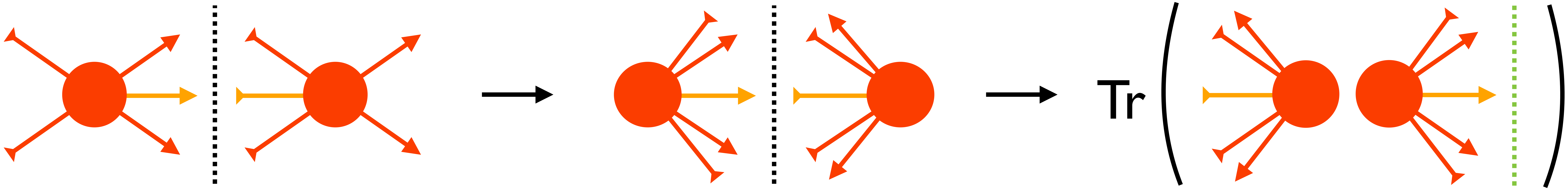
External wave functions.

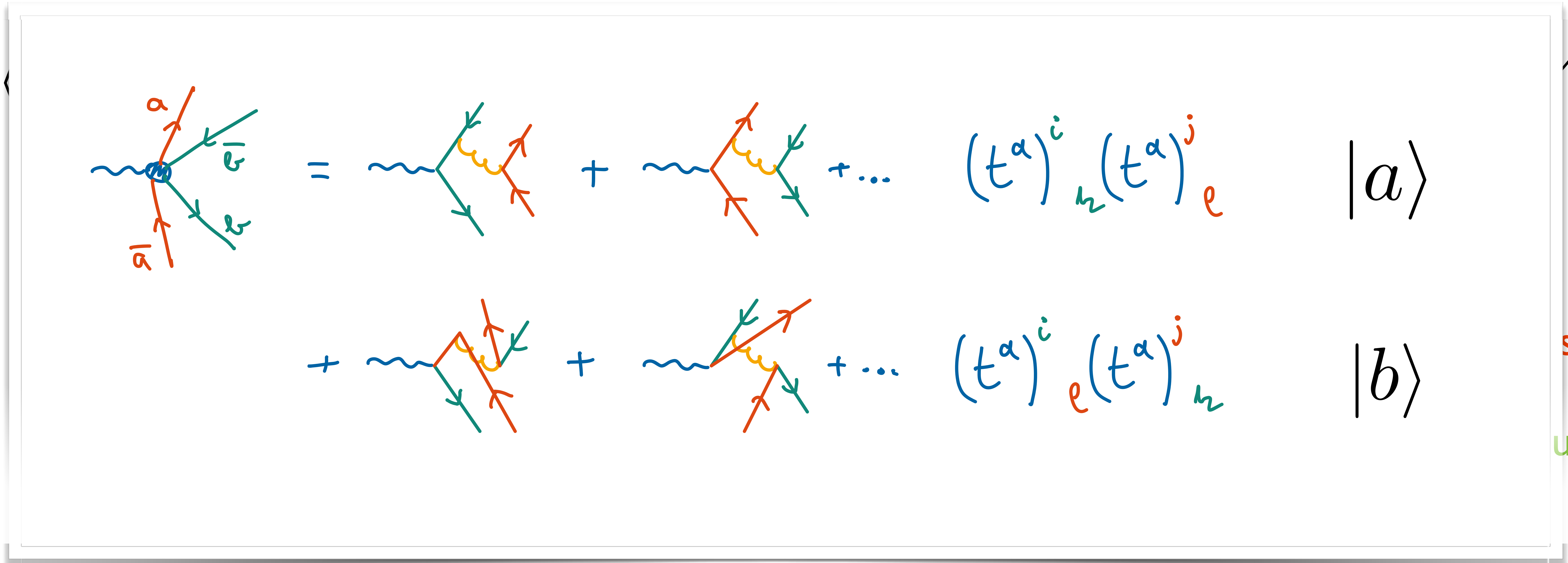
Cross section density operator.

Measurement projector.

Unless stated otherwise: $|\mathcal{M}\} \to |\mathcal{M}\rangle$

Amplitude evolution basics





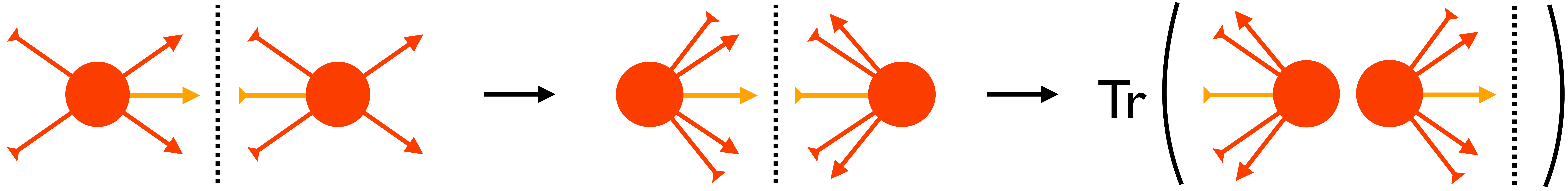
$1| \quad |i,f\}\{i,f|$

sity operator.

urement projector.

Unless stated otherwise: $|\mathcal{M}\} \to |\mathcal{M}\rangle$

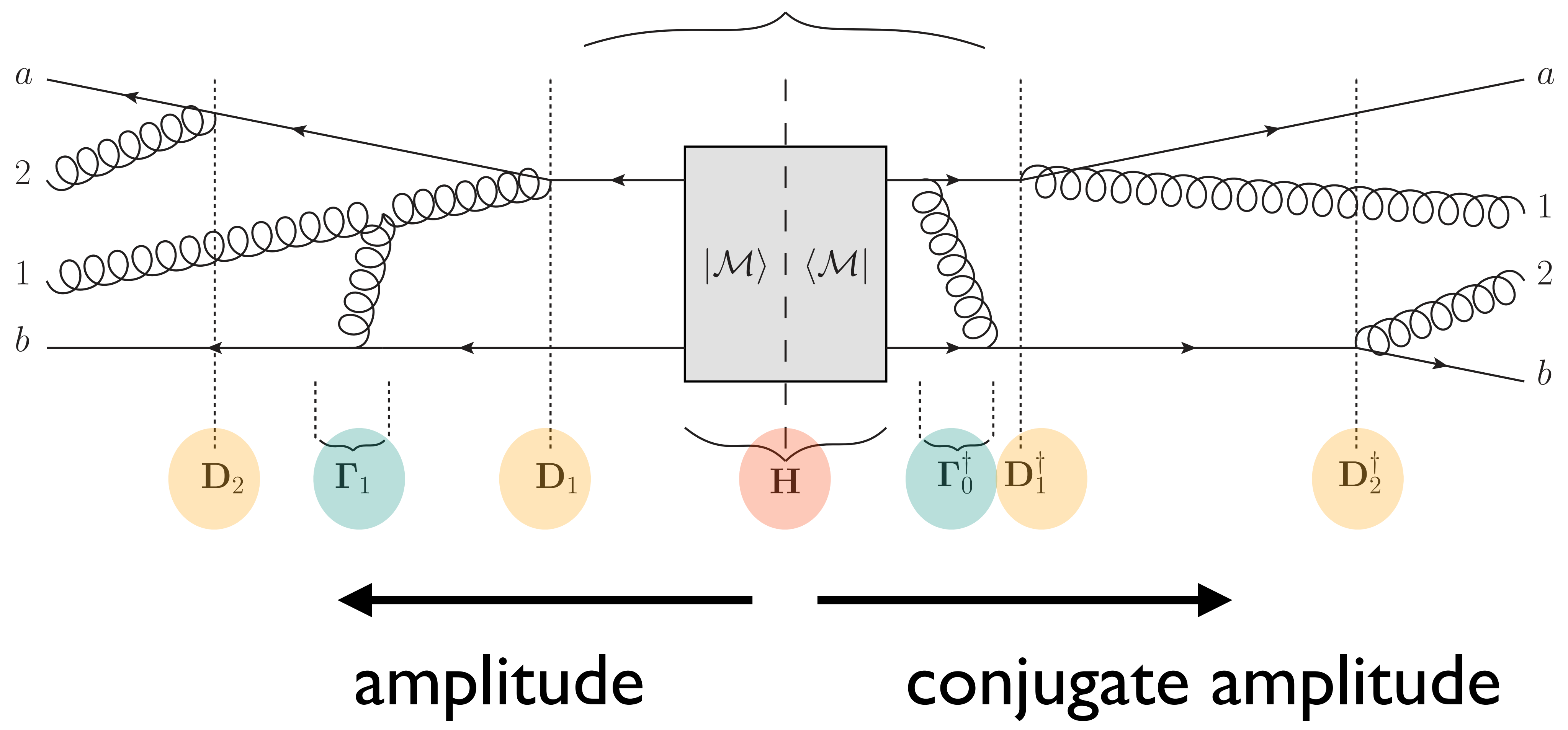
Amplitude evolution basics



$$\mathbf{A}_n(q) = \int_q^Q \frac{\mathrm{d}k}{k}\; \mathrm{P}e^{-\int_q^k \frac{\mathrm{d}k'}{k'}\boldsymbol{\Gamma}(k')}\; \mathbf{D}_n(k)\; \mathbf{A}_{n-1}(k)\; \mathbf{D}_n^\dagger(k)\; \overline{\mathrm{P}}e^{-\int_q^k \frac{\mathrm{d}k'}{k'}\boldsymbol{\Gamma}^\dagger(k')}$$

Markovian algorithm at the amplitude level:
Iterate gluon exchanges and emission.

Different histories in amplitude and conjugate amplitude needed to include interference.

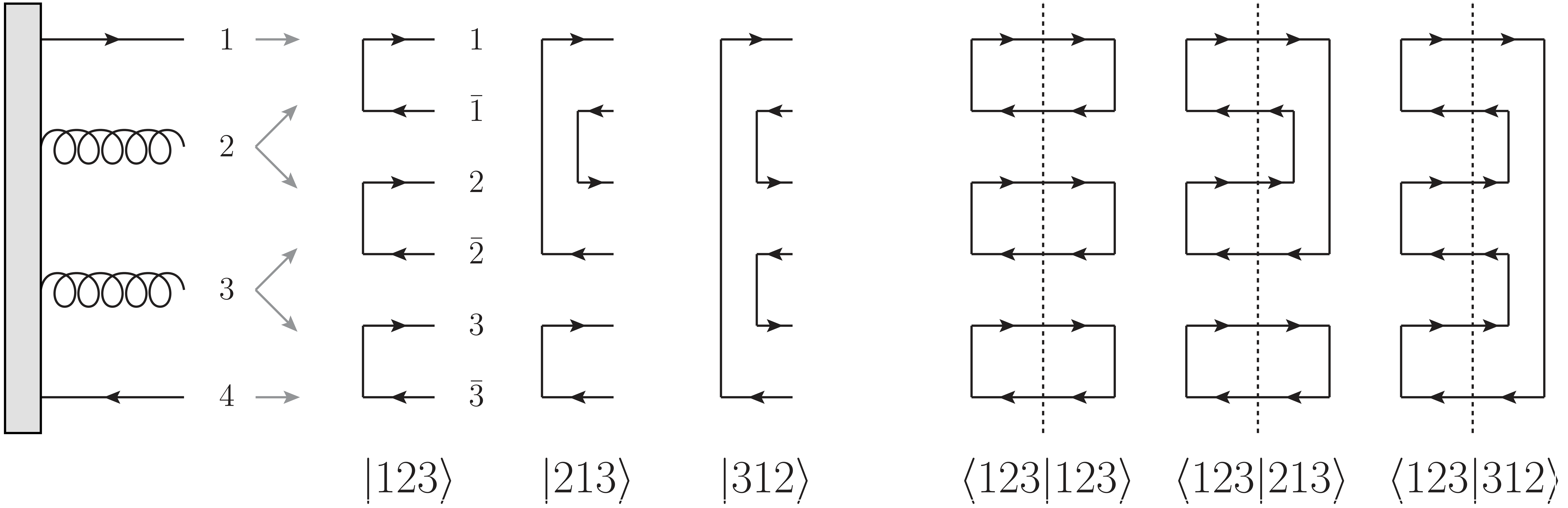


[Angeles, De Angelis, Forshaw, Plätzer, Seymour – JHEP 05 (2018) 044]
[Forshaw, Holguin, Plätzer – JHEP 1908 (2019) 145]

Tracking colour charge

Decompose amplitudes in flow of colour charge.

$$\mathrm{Tr}\left[\mathbf{A}_n\right] = \sum_{\sigma,\tau} A_{\tau\sigma} \langle \sigma | \tau \rangle$$



$$(t^a)^i{}_k (t^a)^j{}_l = T_R \left(\delta^i_l \delta^j_k - \frac{1}{N} \delta^i_k \delta^j_l \right)$$

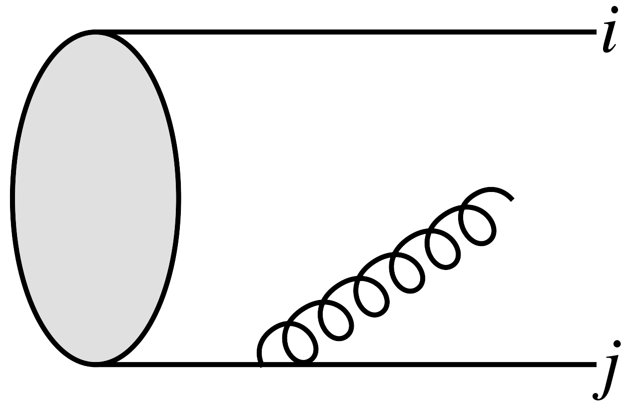
N^3 N^2 N

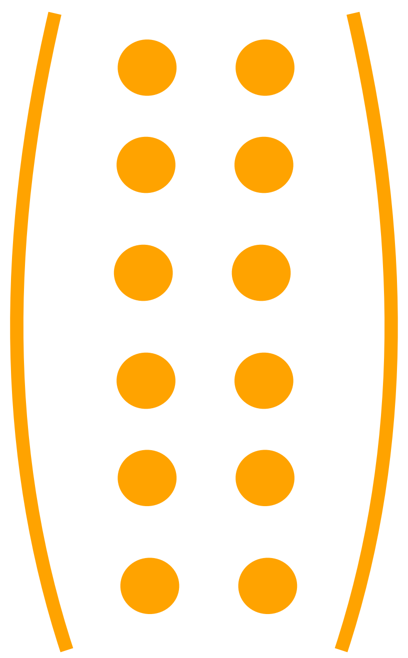
[Plätzer – EPJ C 74 (2014) 2907]
[Angeles, De Angelis, Forshaw, Plätzer, Seymour – JHEP 05 (2018) 044]

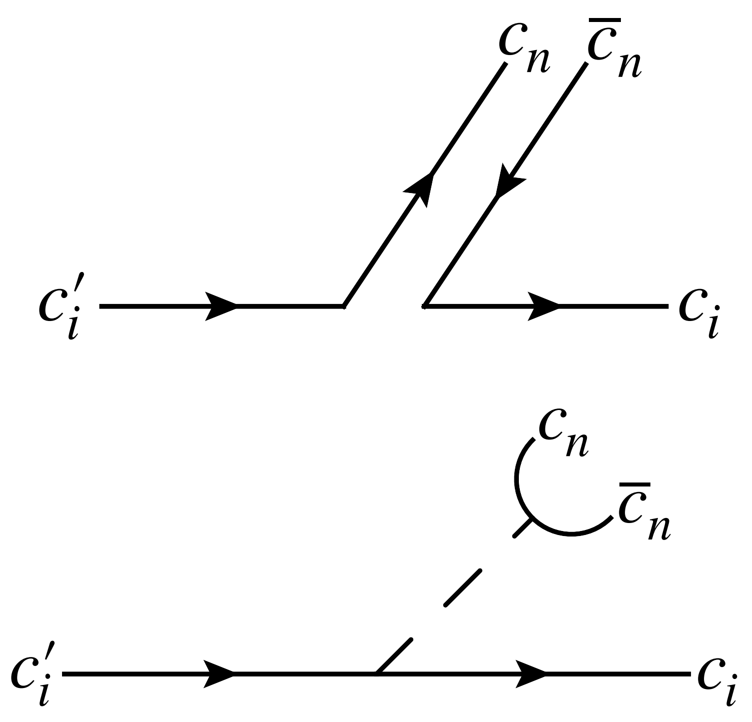
Tracking colour charge

Gluon emission

$$\mathbf{D}_n(k)$$



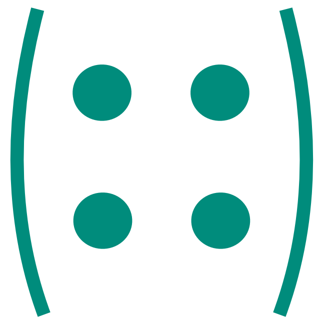


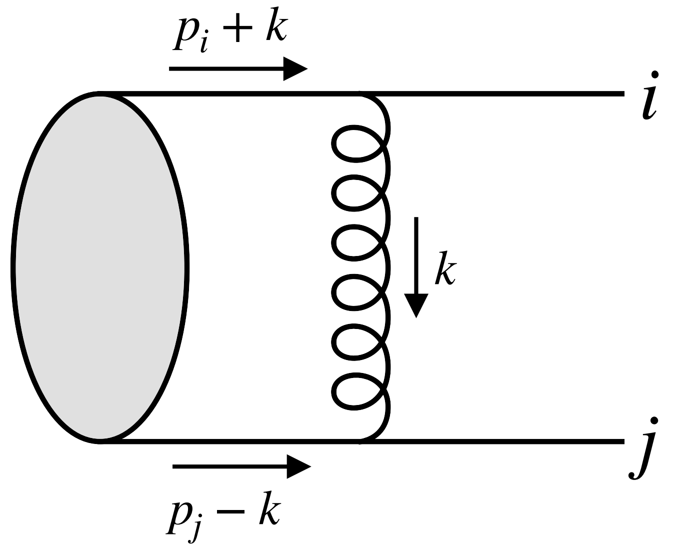


Explicit suppression in 1/N

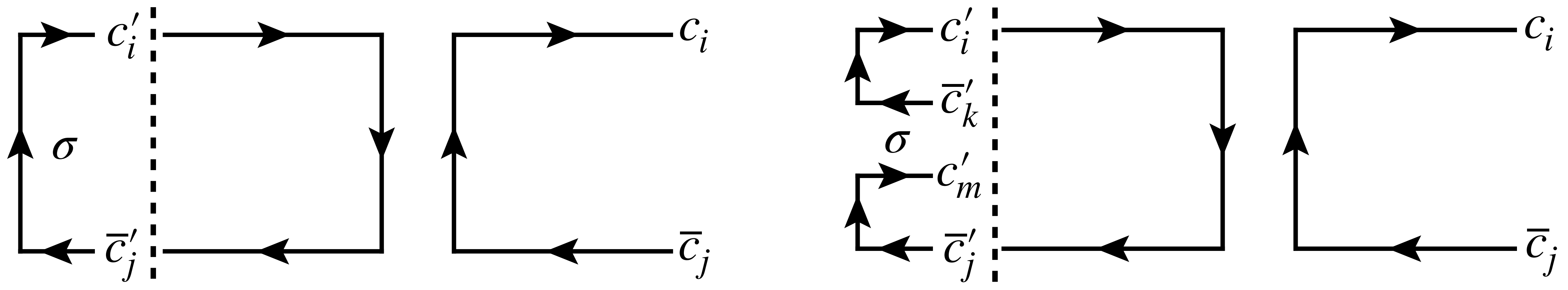
Gluon exchange

$$\mathrm{P}e^{-\int_q^k \frac{\mathrm{d}k'}{k'}\mathbf{\Gamma}(k')}$$





$$[\tau|\mathbf{\Gamma}|\sigma\rangle = (\alpha_s N)[\tau|\mathbf{\Gamma}^{(1)}|\sigma\rangle + (\alpha_s N)^2[\tau|\mathbf{\Gamma}^{(2)}|\sigma\rangle + \ldots$$



$$[\tau|\mathbf{\Gamma}^{(1)}|\sigma\rangle = \left(\Gamma_\sigma^{(1)} + \frac{1}{N^2}\rho^{(1)}\right)\delta_{\sigma\tau} + \frac{1}{N}\Sigma_{\sigma\tau}^{(1)}$$

dipole flips — implicit suppression in 1/N

Systematically expand around large-N limit summing towers of terms enhanced by $\alpha_S N$

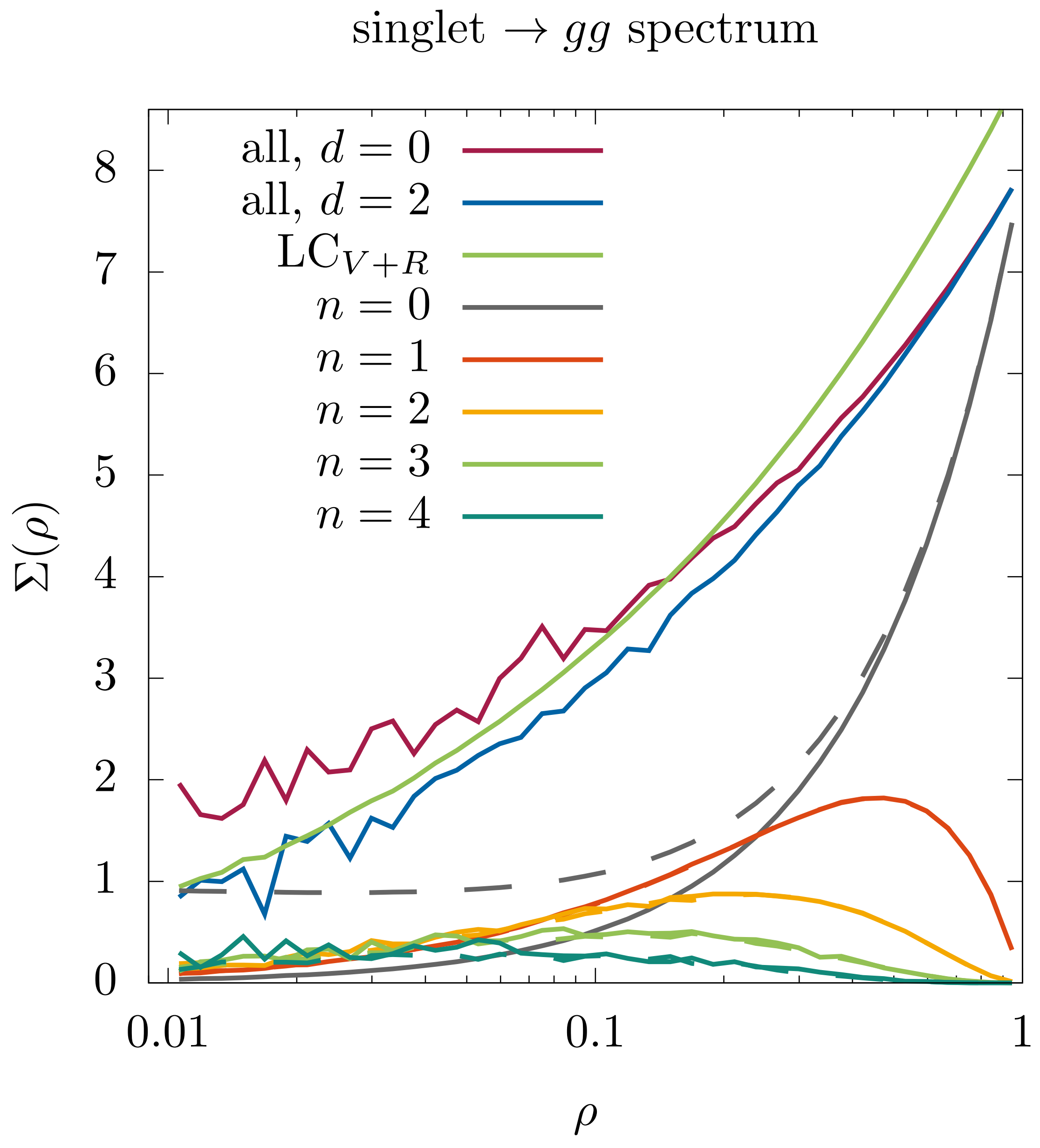
[Plätzer – EPJ C 74 (2014) 2907] — diagrams from [Ruffa, MSc thesis 2020]

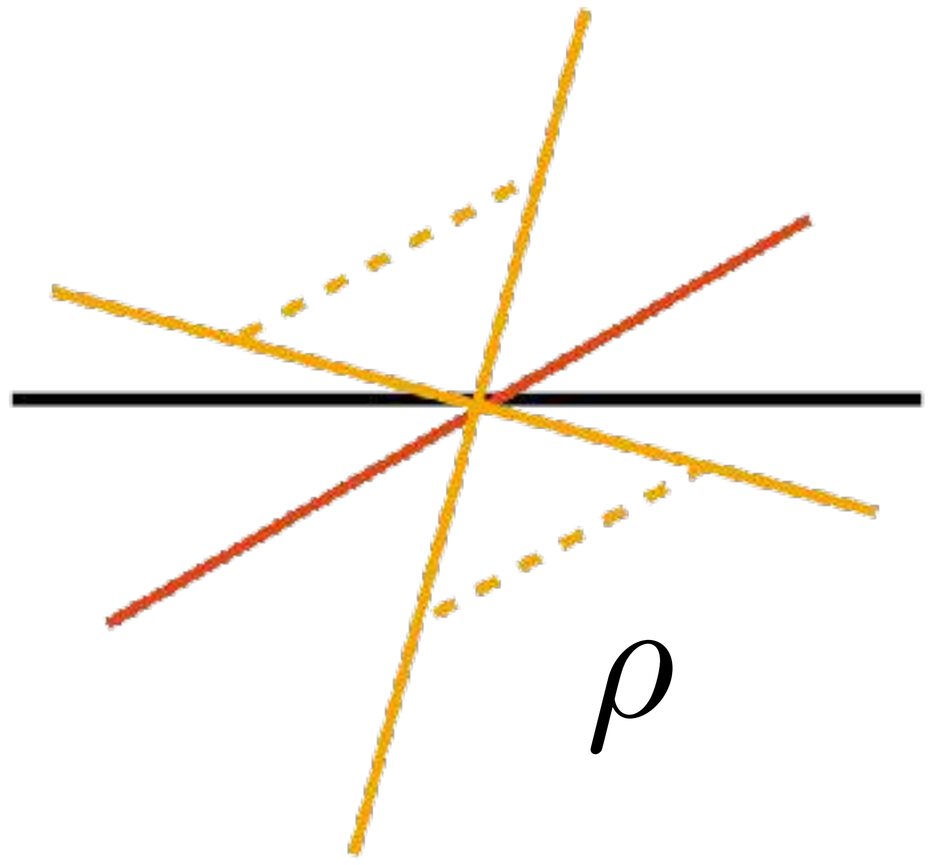
Simulations

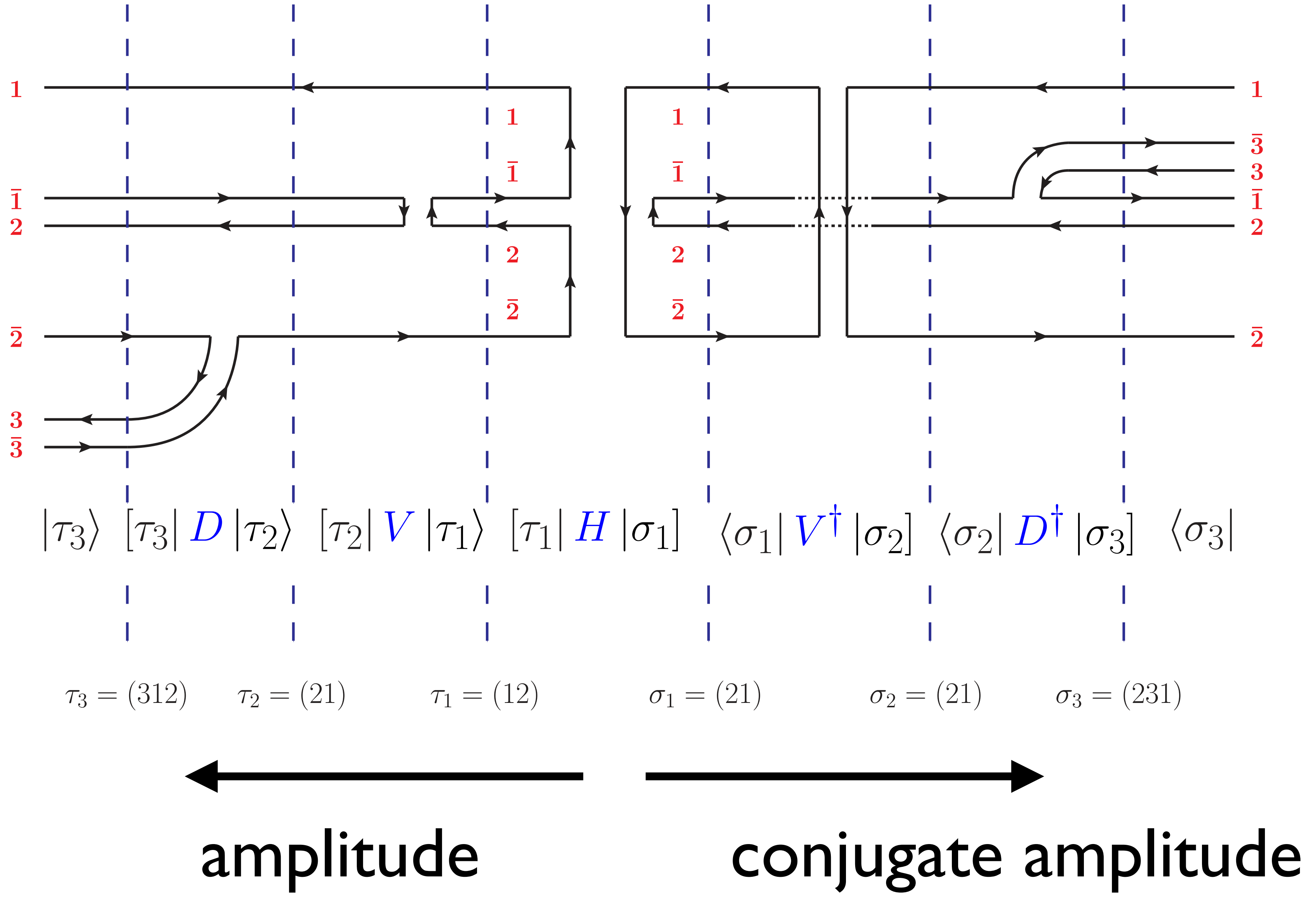
CVolver solves evolution equations in colour flow space

[De Angelis, Forshaw, Plätzer — PRL 126 (2021) 11]
[Plätzer – EPJ C 74 (2014) 2907]

$$E\frac{\partial}{\partial E}\mathbf{A}_n(E) = \mathbf{\Gamma}_n(E)\mathbf{A}_n(E) + \mathbf{A}_n(E)\mathbf{\Gamma}_n^\dagger(E) - \sum_k \mathbf{R}_n^{(k)}(E)\mathbf{A}_{n-k}(E)\mathbf{R}_n^{(k),\dagger}(E)$$

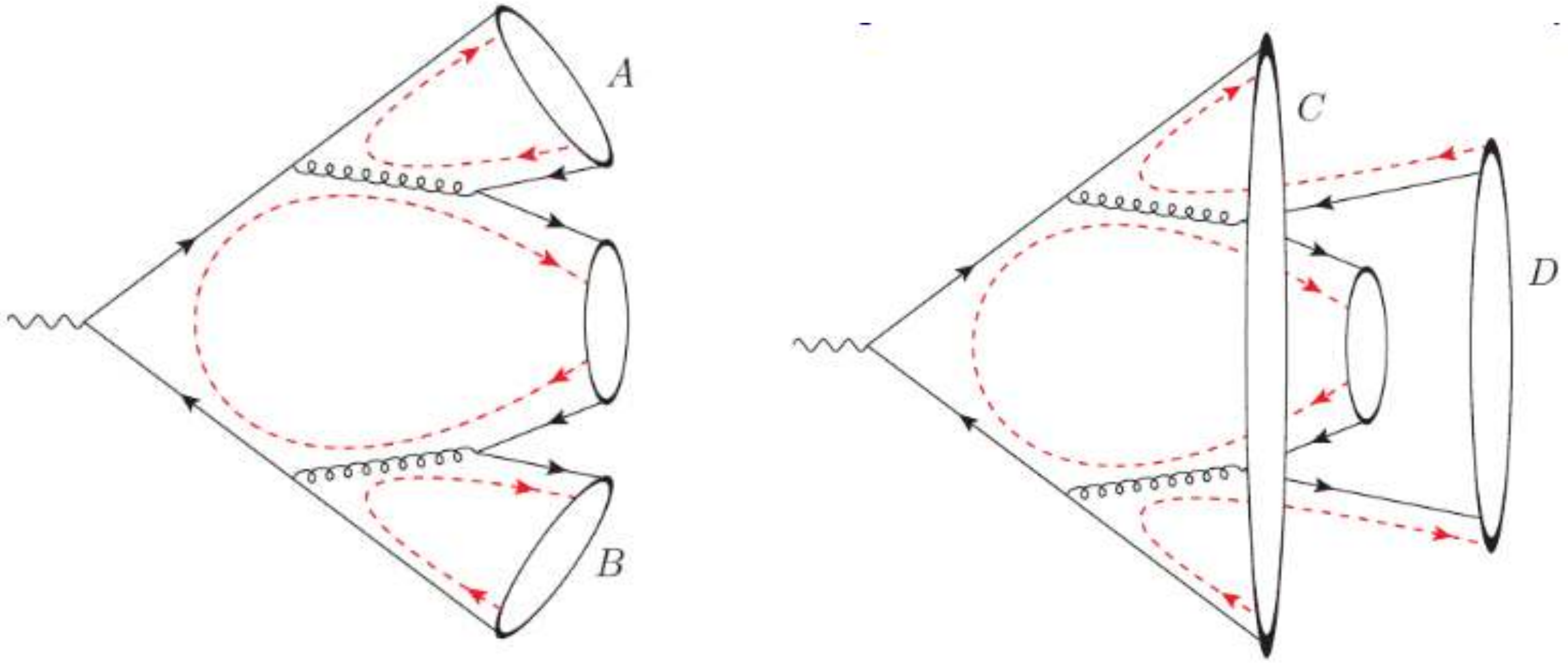


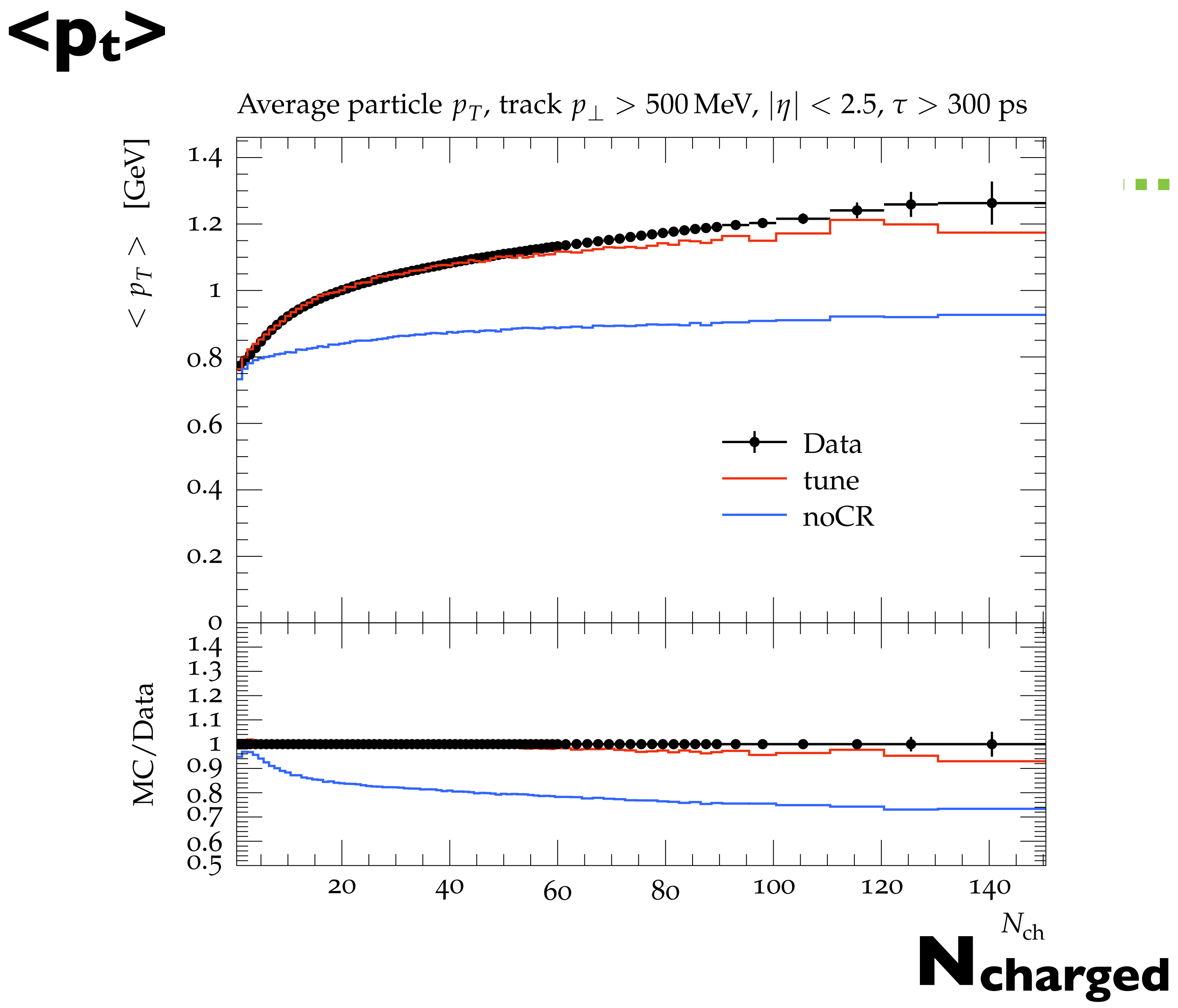


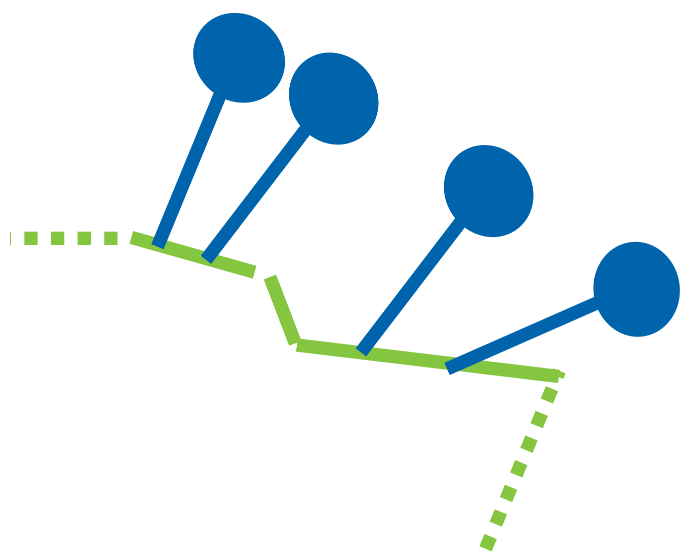


Agrees with [Hatta et al. — Nucl.Phys.B 962 (2021) 115273] using equivalent Langevin formulation.
Much more to come … thanks to Patrick Kirchgaesser, first and foremost.

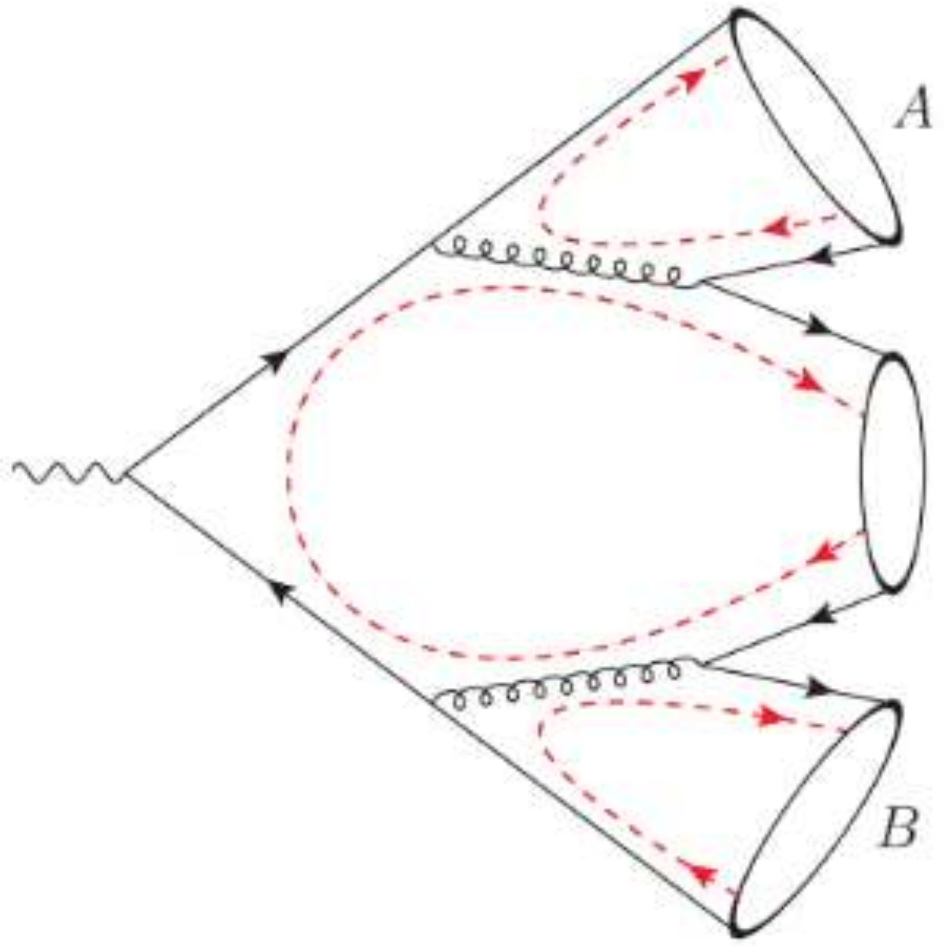
Would subleading-N matter?

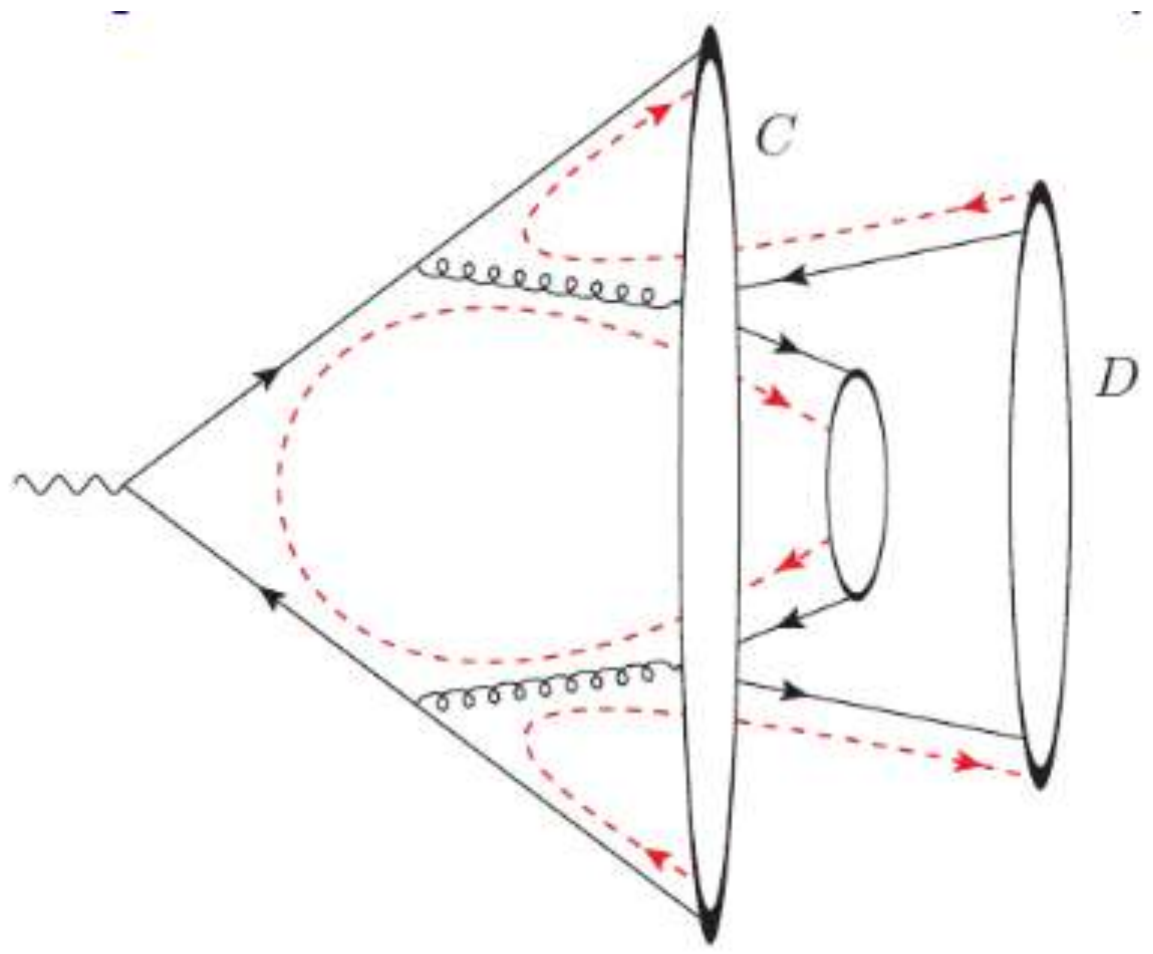






Would subleading-N matter?





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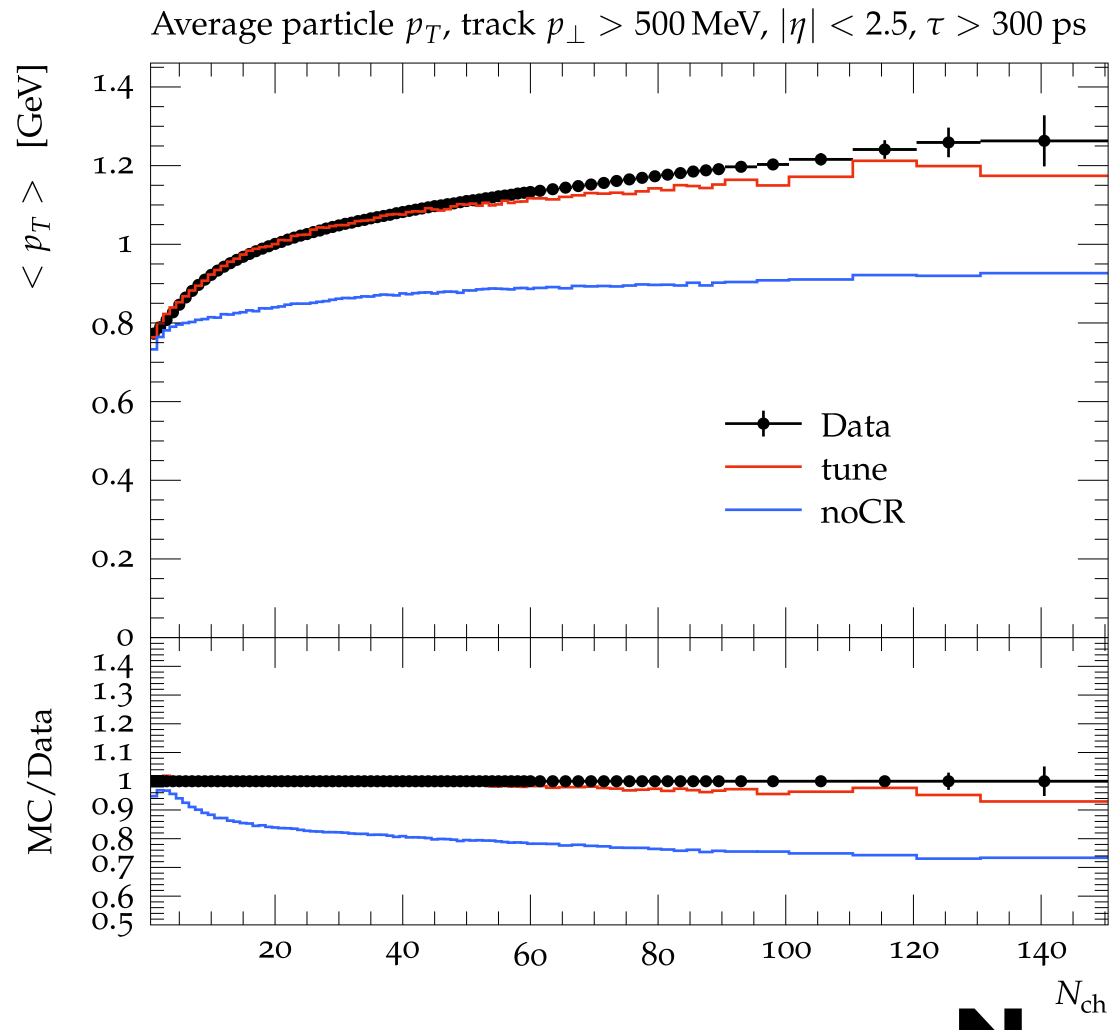
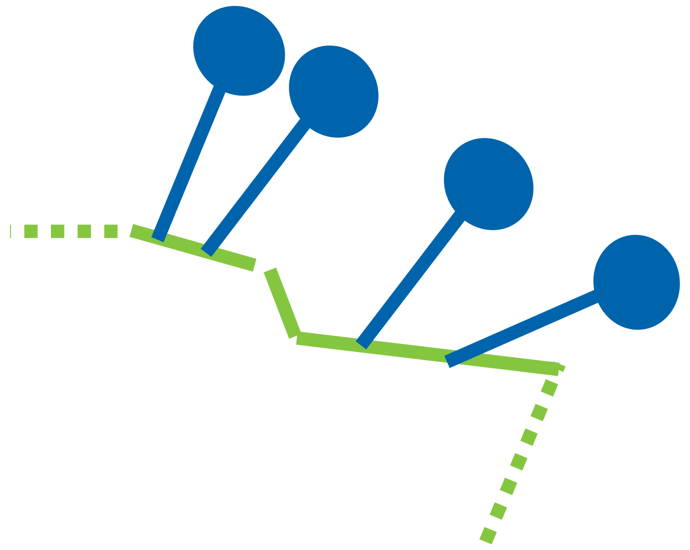


N_{charged}

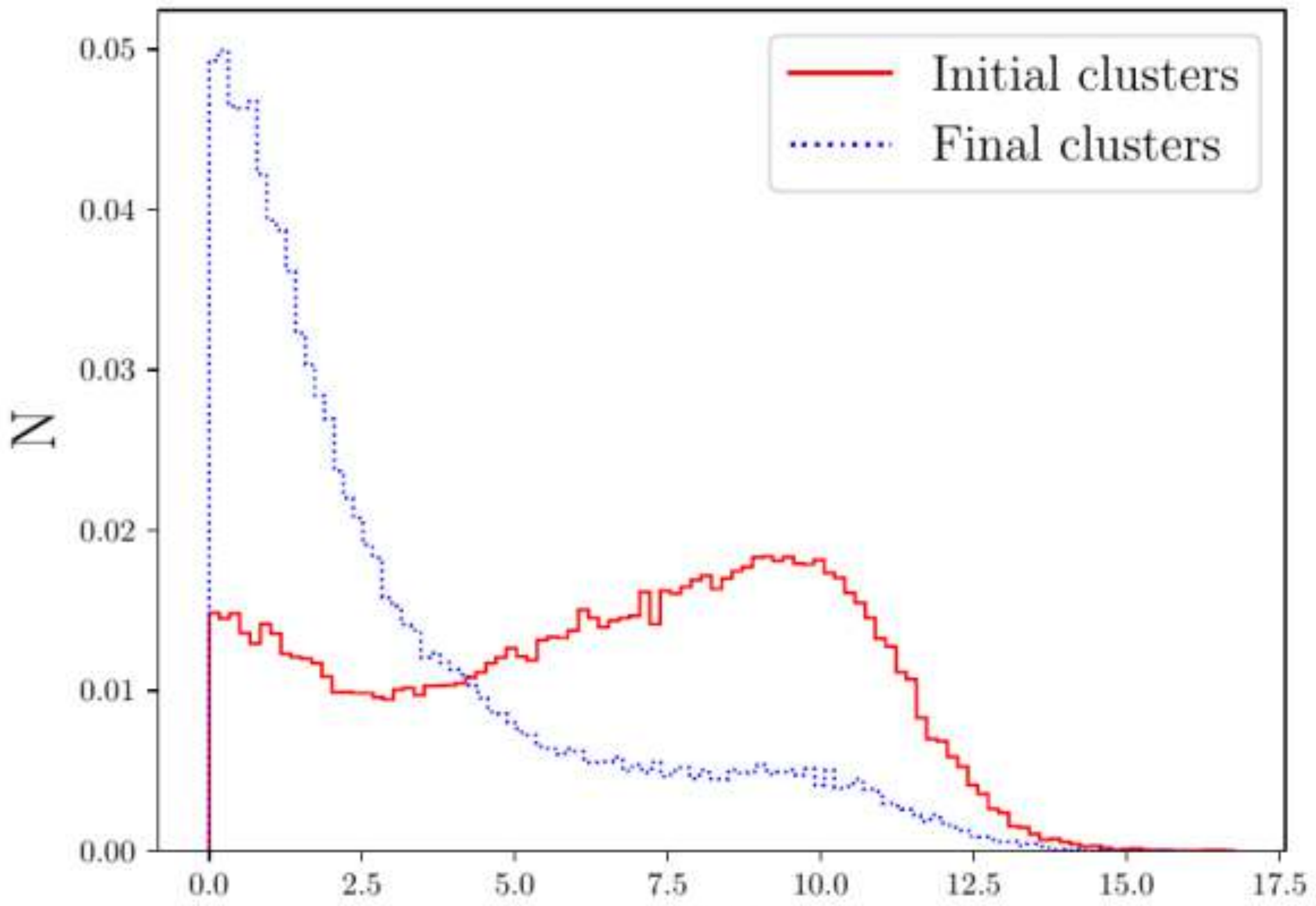


Approach colour reconnection from colour evolution: perturbative component?

$$P e^{-\int_q^k \frac{\mathrm{d}k'}{k'}\mathbf{\Gamma}(k')}$$

Reconnection amplitude

$$\mathcal{A}_{\tau\to\sigma} = \langle\sigma|\mathbf{U}\left(\{p\},\mu^2,\{M_{ij}^2\}\right)|\tau\rangle$$



~ cluster size

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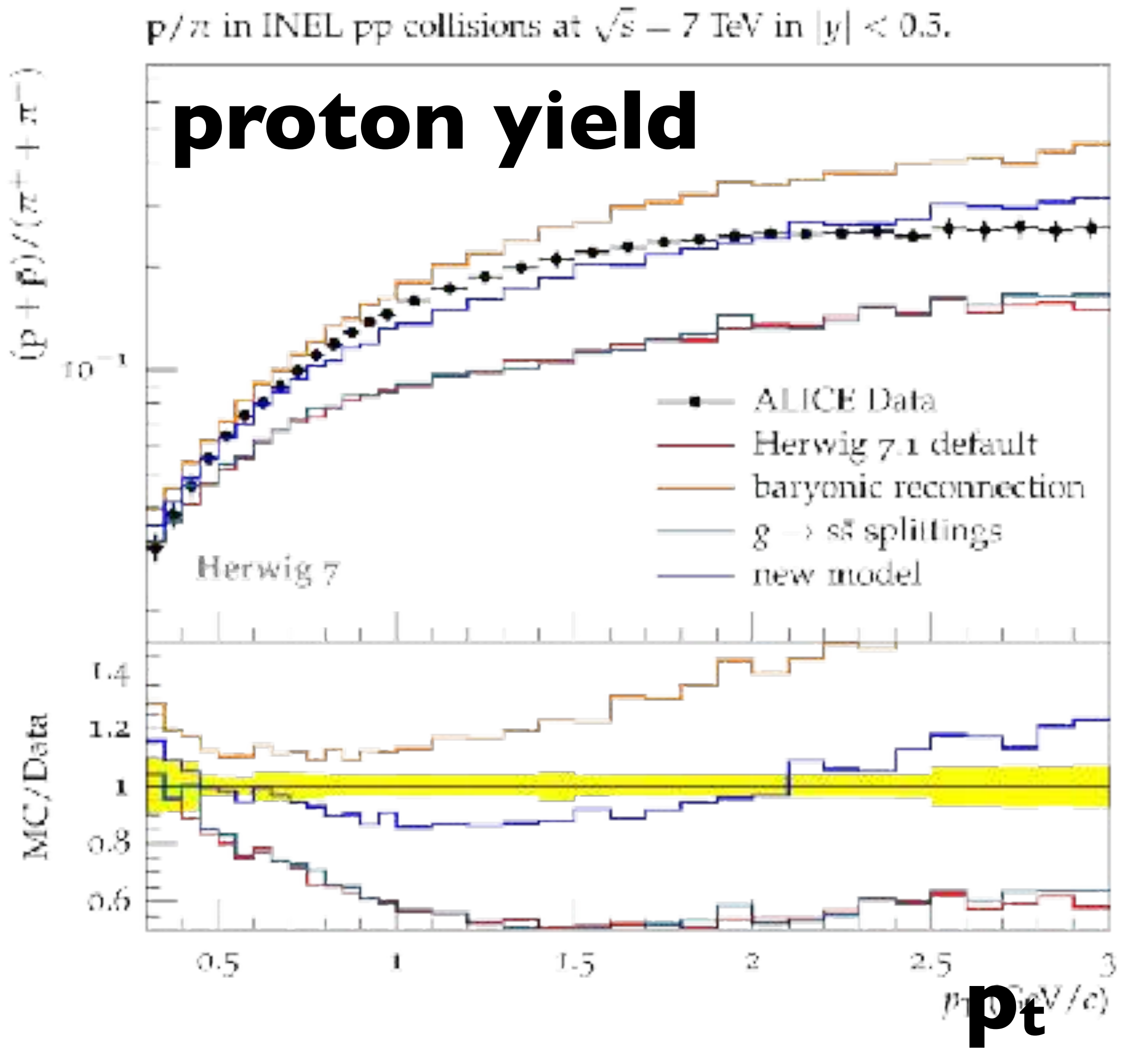
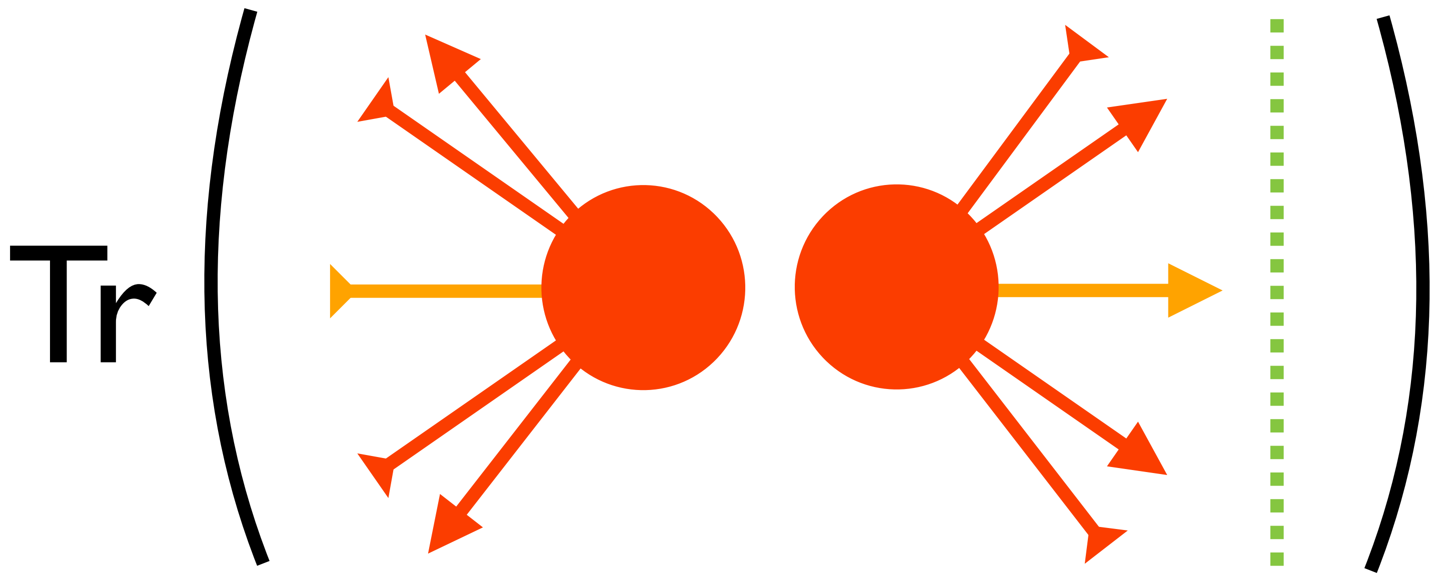


[Gieseke, Kirchgaesser, Plätzer – EPJ C 78 (2018) 99]
[Gieseke, Kirchgaesser, Plätzer, Siodmok – JHEP 11 (2018) 149]

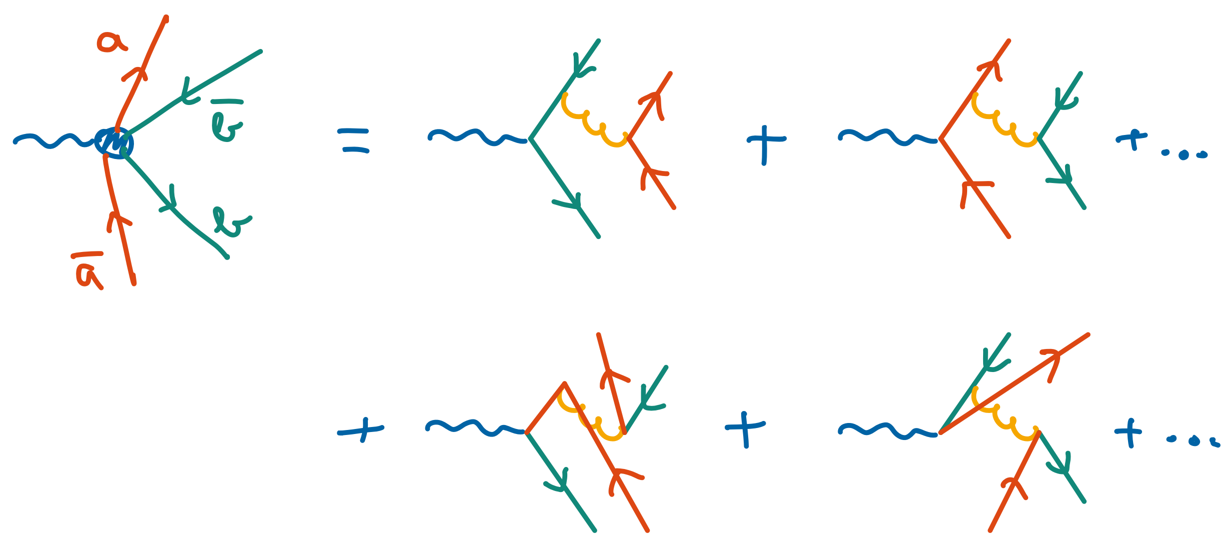
Constructing amplitude evolution

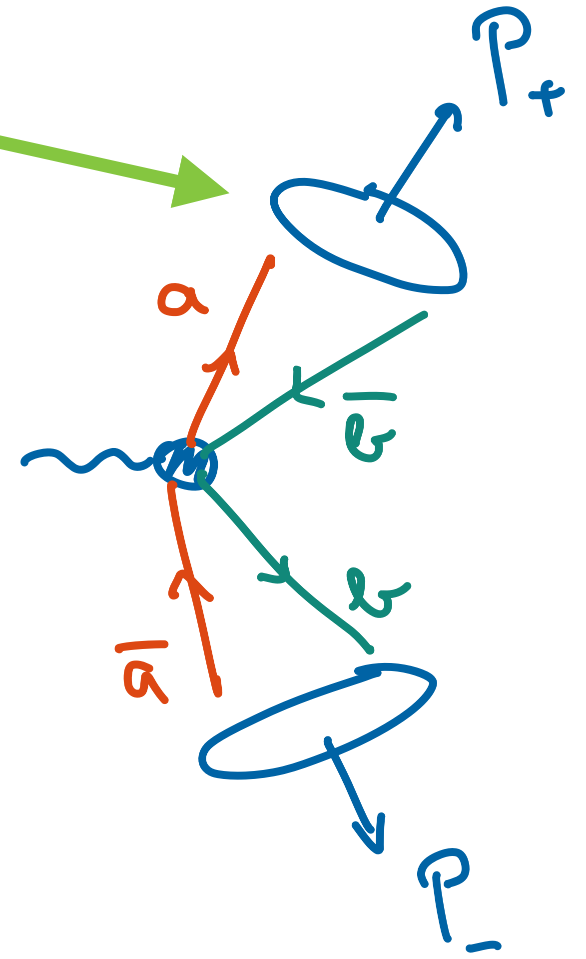
Consider factorisation to a partonic cross section, supplied by a very general measurement operator including possible projections onto colour singlet systems. Somehow needs to happen for hadronziation, but other observables fit, as well.



$$\sigma[\mathbf{U}] = \sum_n \int \alpha_0^n \, \mathrm{Tr}\left[\mathbf{M}_n(Q;p_1,...,p_n)\mathbf{U}_n(Q;p_1,...,p_n)\right] \mathrm{d}\phi(Q) \prod_{i=1}^{n} (4\pi\mu^2)^\epsilon [\mathrm{d}p_i]\tilde{\delta}(p_i)$$

Rely on factorisation properties of amplitudes to isolate divergent contributions.





Physical cross section finite: resort to RGE methodology.

[Plätzer — arXiv:2204.06956]

Constructing amplitude evolution

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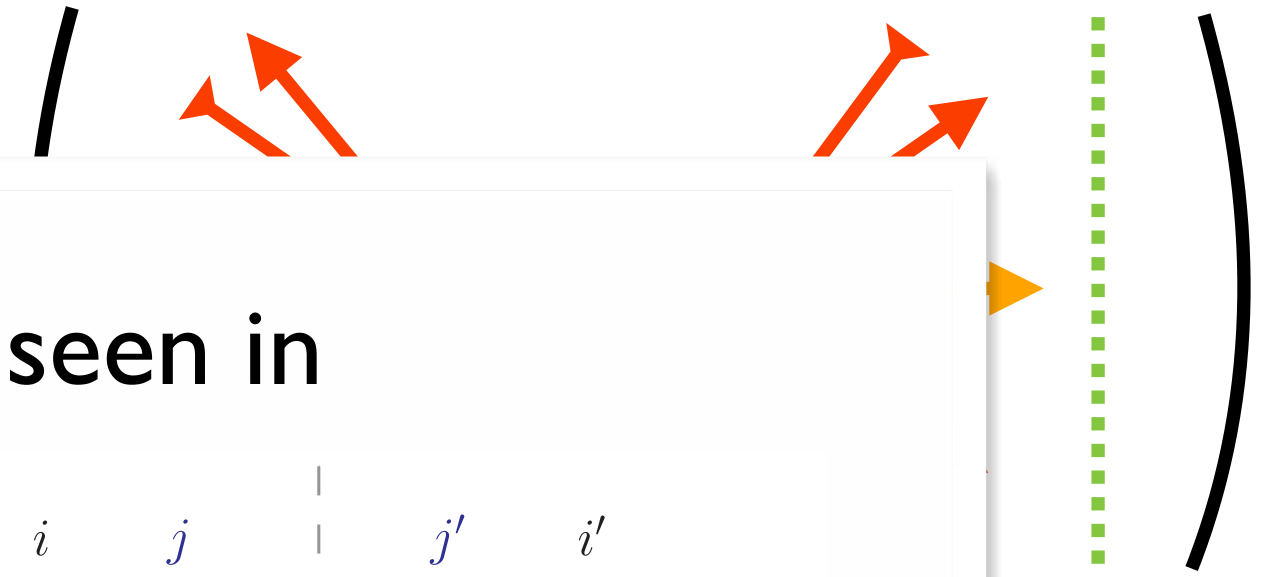


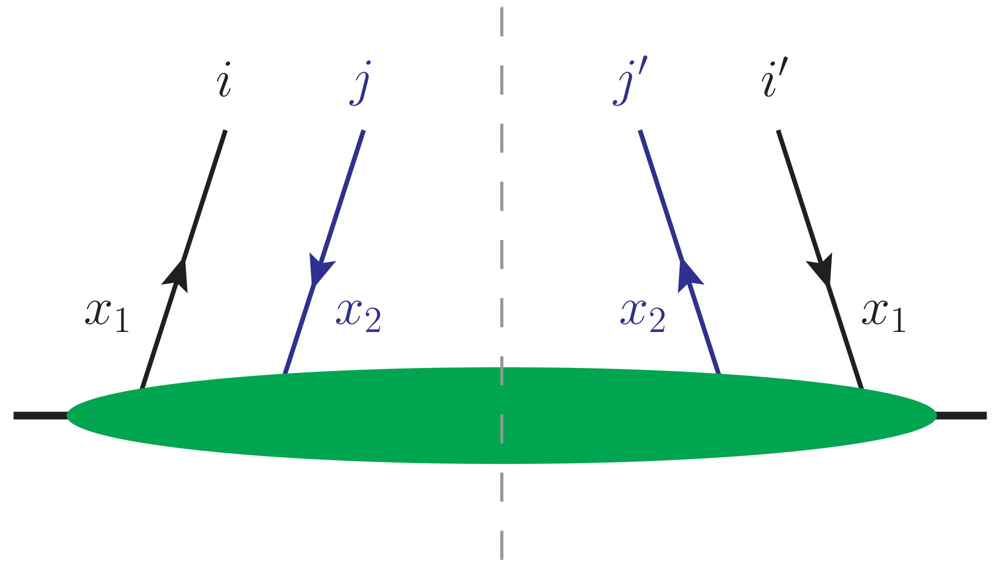
Consider factorisation to a partonic cross section, supplied by a very general measurement operator including po… colour singlet systems. Somehow needs to l… but other observables fit, as well.

$$\sigma[\mathbf{U}] = \sum_n \int \alpha_0^n \ \mathrm{Tr}\left[\mathbf{M}_n(Q;p_1, \ldots \quad p_i)\right.$$

Rely on factorisation properties of amplitudes to isolate divergent contributions.

Physical cross section finite: resort to RGE methodology.

U as colour space operator also seen in double parton distributions



$$\begin{pmatrix} F_{qq}^{\bar{3}3} \\ F_{qq}^{66} \end{pmatrix}(x_1, x_2, y, \mu_1, \mu_2, \zeta_p) = \mathbf{U}_{qq}(\alpha) \begin{pmatrix} F_{qq}^{\bar{3}3} \\ F_{qq}^{66} \end{pmatrix}(x_1, x_2, y, \mu_1, \mu_2, \zeta_0)$$

$$\mathbf{U}_{qq}(\alpha) = \exp\left(-\alpha\, \hat{\mathbf{J}}_{qq}\right) = \frac{1}{3}\begin{pmatrix} 1 + 2e^{-\alpha} & 2(1 - e^{-\alpha}) \\ 1 - e^{-\alpha} & 2 + e^{-\alpha} \end{pmatrix}$$

[Diehl, Gaunt, Pichini, Plössl — Eur.Phys.J.C 81 (2021) 11]

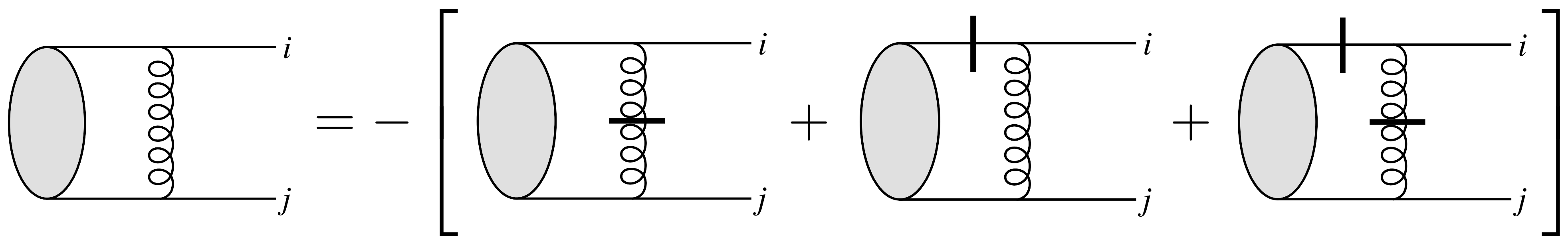
[Plätzer — arXiv:2204.06956]

Factorisation of amplitudes

Factorisation of virtual contributions

$$\sigma[\mathbf{U}] = \sum_n \int \alpha_0^n \ \mathrm{Tr}\,[\mathbf{M}_n(Q;p_1,...,p_n)\mathbf{U}_n(Q;p_1,...,p_n)]\,\mathrm{d}\phi(Q)\prod_{i=1}^{n}(4\pi\mu^2)^{\epsilon}[\mathrm{d}p_i]\tilde{\delta}(p_i)$$

$$\mathbf{M}_n^{(l)} = \mathbf{V}^{(1)}\mathbf{M}_n^{(l-1)} + \mathbf{M}_n^{(l-1)}\mathbf{V}^{(1)\dagger} + \mathbf{V}^{(1)}\mathbf{M}_n^{(l-2)}\mathbf{V}^{(1)\dagger} + \mathbf{V}^{(2)}\mathbf{M}_n^{(l-2)} + \mathbf{M}_n^{(l-2)}\mathbf{V}^{(2)\dagger} + \ldots$$

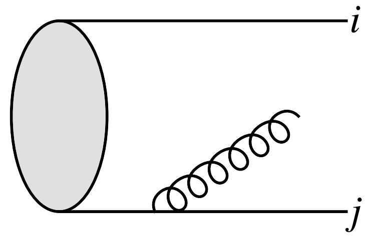


Coefficient	Diagram	Colour-factor
$\Omega_{ij}^{(2)}$		$(\mathbf{T}_i\cdot\mathbf{T}_j)(\mathbf{T}_i\cdot\mathbf{T}_j)$
$\tilde{\Omega}_{ij}^{(2)}$		$\mathbf{T}_i^a\mathbf{T}_i^b\mathbf{T}_j^b\mathbf{T}_j^a$
$\Omega_{ijl}^{(2)}$		$(\mathbf{T}_i\cdot\mathbf{T}_l)(\mathbf{T}_i\cdot\mathbf{T}_j)$
$\hat{\Omega}_{ijl}^{(2)}$		$if^{abc}\mathbf{T}_i^a\mathbf{T}_j^b\mathbf{T}_l^c$
$\Omega_{ij,\mathrm{self-en.}}^{(2)}$		$T_R(\mathbf{T}_i\cdot\mathbf{T}_j)$
$\Omega_{ij,\mathrm{vertex-corr.}}^{(2)}$		$T_R(\mathbf{T}_i\cdot\mathbf{T}_j)$
$\hat{\Omega}_{ij}^{(2)}$		$\mathbf{T}_i^b\mathbf{T}_i^a\mathbf{T}_i^b\mathbf{T}_j^a$

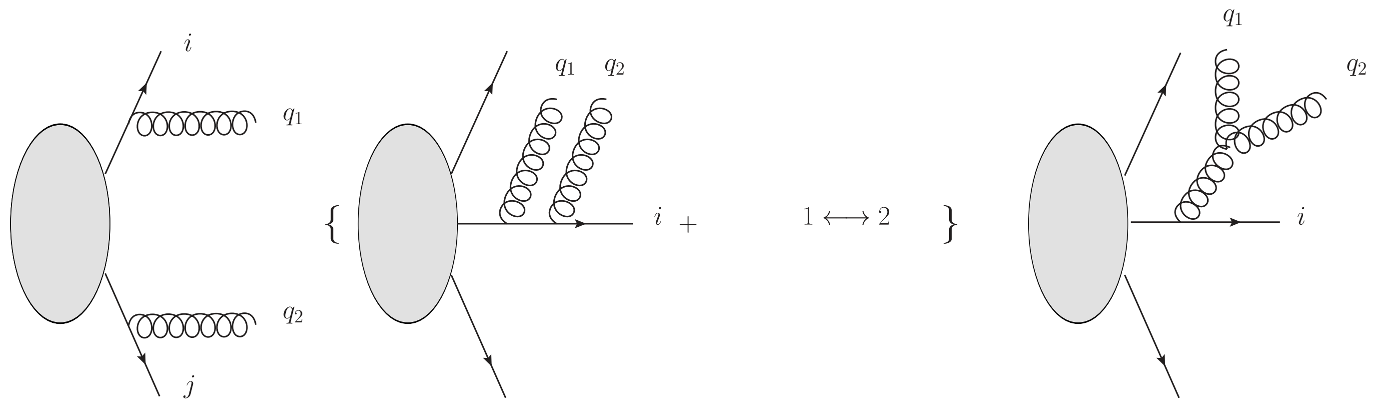
Systematically convert loops into phase-space type integrals.

[Plätzer, Ruffa — JHEP 06 (2021) 007]

Factorisation of real contributions



$$\mathbf{M}_n^{(l)} = \mathbf{D}_n^{(1,0)}\mathbf{M}_{n-1}^{(l)}\mathbf{D}_n^{(1,0)\dagger} + \mathbf{D}_n^{(1,1)}\mathbf{M}_{n-1}^{(l-1)}\mathbf{D}_n^{(1,0)\dagger} + \mathbf{D}_n^{(1,0)}\mathbf{M}_{n-1}^{(l-1)}\mathbf{D}_n^{(1,1)\dagger} + \mathbf{D}_n^{(2,0)}\mathbf{M}_{n-2}^{(l)}\mathbf{D}_n^{(2,0)\dagger} + \ldots$$



$$\sum_{(a,b),(c,d)}\ \sum_{i,j,k,l=1}^{n} \omega_{ijkl}^{abcd}\ \mathrm{T}_i^{(a)}\mathrm{T}_j^{(b)} \circ \mathrm{T}_k^{(c),\dagger}\mathrm{T}_l^{(d),\dagger}$$

[Majcen — M.Sc. thesis 2022]

Redefinitions of "bare" operators

Path to a finite cross section: renormalisation & subtractions.

$$\sigma[\mathbf{U}] = \sum_n \int \alpha_0^n \, \mathrm{Tr}\left[\mathbf{M}_n(Q; p_1, ..., p_n)\mathbf{U}_n(Q; p_1, ..., p_n)\right] \mathrm{d}\phi(Q) \prod_{i=1}^{n} (4\pi\mu^2)^\epsilon [\mathrm{d}p_i]\tilde{\delta}(p_i)$$

$$\mathbf{M}_n = \sum_{l=0}^{\infty} \alpha_0^l \; \mathbf{M}_n^{(l)}$$

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$$\mathbf{U}_n = \mathbf{X}_n^\dagger \mathbf{S}_n \mathbf{X}_n - \sum_{s=1}^{\infty} \alpha_S^s \int \mathbf{F}_{n+s}^{(s)\dagger} \mathbf{S}_{n+s} \mathbf{F}_{n+s}^{(s)} \prod_{i=n+1}^{n+s} \mu_R^{2\epsilon} [\mathrm{d}p_i]\tilde{\delta}(p_i)$$

$$\mathbf{X}_n = 1 - \sum_{k\geq 1} \alpha_S^k \mathbf{X}_n^{(k)} \qquad \mathbf{F}_n^{(s)} = \sum_{k\geq 0} \alpha_S^k \mathbf{F}_n^{(s,k)}$$

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$$\sigma[\mathbf{U}]|_{n_0,n_c} = \sum_{n\geq n_0} \int \alpha_S^n \, \mathrm{Tr}\left[\hat{\mathbf{M}}_n \mathbf{S}_n\right] \mathrm{d}\phi(Q) \prod_{i=1}^{n} \mu_R^{2\epsilon} [\mathrm{d}p_i]\tilde{\delta}(p_i) + \mathcal{O}(\alpha_S^{n_0+n_c+1})$$

$$\hat{\mathbf{M}}_n^{(0)} = \mathbf{M}_n^{(0)} - \mathcal{S}_{\text{tree}}\left[\mathbf{M}_n^{(0)}\right]$$

$$\hat{\mathbf{M}}_n^{(1)} = \mathbf{M}_{n,R}^{(1)} - \mathcal{S}_{\text{tree}}\left[\mathbf{M}_{n,R}^{(1)}\right] - \mathcal{S}_{\text{1-loop}}\left[\mathbf{M}_n^{(0)}\right]$$

$$\hat{\mathbf{M}}_n^{(2)} = \mathbf{M}_{n,R}^{(2)} - \mathcal{S}_{\text{tree}}\left[\mathbf{M}_{n,R}^{(2)}\right] - \mathcal{S}_{\text{1-loop}}\left[\mathbf{M}_{n,R}^{(1)}\right] - \mathcal{S}_{\text{2-loop}}\left[\mathbf{M}_n^{(0)}\right]$$

UV & IR finite — upon fixed order expansion and redefinition of the observable.

Systematic build-up of IR subtractions separately for virtual and real corrections.

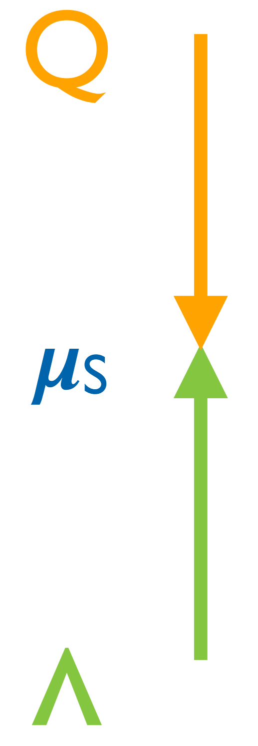
Hard & soft evolution

[Plätzer — arXiv:2204.06956]

We can now obtain the evolution equations we asked for:

$$\partial_S \mathbf{A}_n = \mathbf{\Gamma}_{n,S}\mathbf{A}_n + \mathbf{A}_n\mathbf{\Gamma}^\dagger_{n,S} - \sum_{s\geq 1} \alpha_S^s \mathbf{R}^{(s)}_{S,n}\mathbf{A}_{n-s}\mathbf{R}^{(s)\dagger}_{S,n}$$

$$\partial_S \equiv \partial/\partial \log \mu_S$$



$$\partial_S \mathbf{S}_n = -\tilde{\mathbf{\Gamma}}^\dagger_{S,n}\mathbf{S}_n - \mathbf{S}_n\tilde{\mathbf{\Gamma}}_{S,n} + \sum_{s\geq 1}\alpha_S^s \int \tilde{\mathbf{R}}^{(s)\dagger}_{S,n+s}\mathbf{S}_{n+s}\tilde{\mathbf{R}}^{(s)}_{S,n+s}\prod_{i=n+1}^{n+s}[\mathrm{d}p_i]\tilde{\delta}(p_i)$$

Coupled system of evolution equations: For each resolution we have chosen, we get one. Directions of evolution are different in scale and multiplicity.

α_s corrections to tower of logarithms present in A

soft evolution ~ hadronization model

$$\sigma[\mathbf{U}_n] = \sum_n \int \alpha_S^n \ \mathrm{Tr}\left[(\mathbf{A}_n + \mathbf{\Delta}_n)\mathbf{S}_n\right] \mathrm{d}\phi(Q) \prod_{i=1}^{n} \mu_R^{2\epsilon}[\mathrm{d}p_i]\tilde{\delta}(p_i)$$

dressing of hard process ~ parton shower

Evolution equations

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[Plätzer — arXiv:2204.06956]

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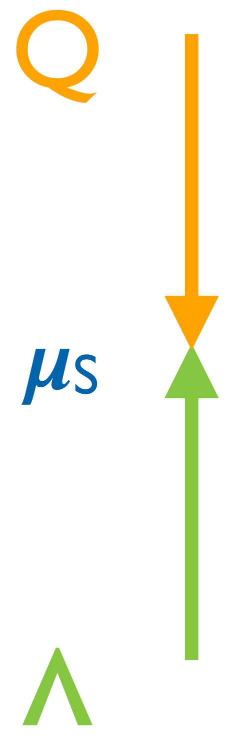

$$\partial_S \mathbf{S}_n = -\tilde{\mathbf{\Gamma}}^\dagger_{S,n} \mathbf{S}_n - \mathbf{S}_n \tilde{\mathbf{\Gamma}}_{S,n} + \sum_{s\geq 1} \alpha_S^s \int \tilde{\mathbf{R}}^{(s)\dagger}_{S,n+s} \mathbf{S}_{n+s} \tilde{\mathbf{R}}^{(s)}_{S,n+s} \prod_{i=n+1}^{n+s} [\mathrm{d}p_i] \tilde{\delta}(p_i)$$

Choosing energy and angular cutoffs essentially gives us the "gaps between jets" algorithms (with similar choices for real emission).

$$\hat{\Xi}^{(ij)}_{n,1,\mathrm{rad}} = 1 - \theta(Q-E)\theta(E-\mu_S)\hat{\Theta}^{(ij)}_\lambda(1-x, 1+x)$$

$$\mathbf{\Gamma}^{(1)}_{S,n,\mathrm{rad}} = -\tilde{\mathbf{\Gamma}}^{(1)}_{S,n,\mathrm{rad}} =$$

$$-\sum_{i<j} \mathbf{T}_i \cdot \mathbf{T}_j \left(\frac{\mu_R}{\mu_S}\right)^{2\epsilon} \theta(Q-\mu_S) \int_{-1}^{1} \mathrm{d}x \frac{(1-x^2)^{-\epsilon}}{1-x^2} \hat{\Theta}^{(ij)}_\lambda(1-x, 1+x) \int \mathrm{d}\Omega^{(d-3)}$$

$$\mathbf{\Gamma}^{(1)}_{R,n,\mathrm{rad}} = \frac{n}{2}\beta_{R,0} \quad \text{and} \quad \tilde{\mathbf{\Gamma}}^{(1)}_{R,n,\mathrm{rad}} = 0$$

[Angeles, De Angelis, Forshaw, Plätzer, Seymour – JHEP 05 (2018) 044]
[De Angelis, Forshaw, Plätzer — PRL 126 (2021) 11] — also see Becher et al. in the large-N limit.

Evolution equations

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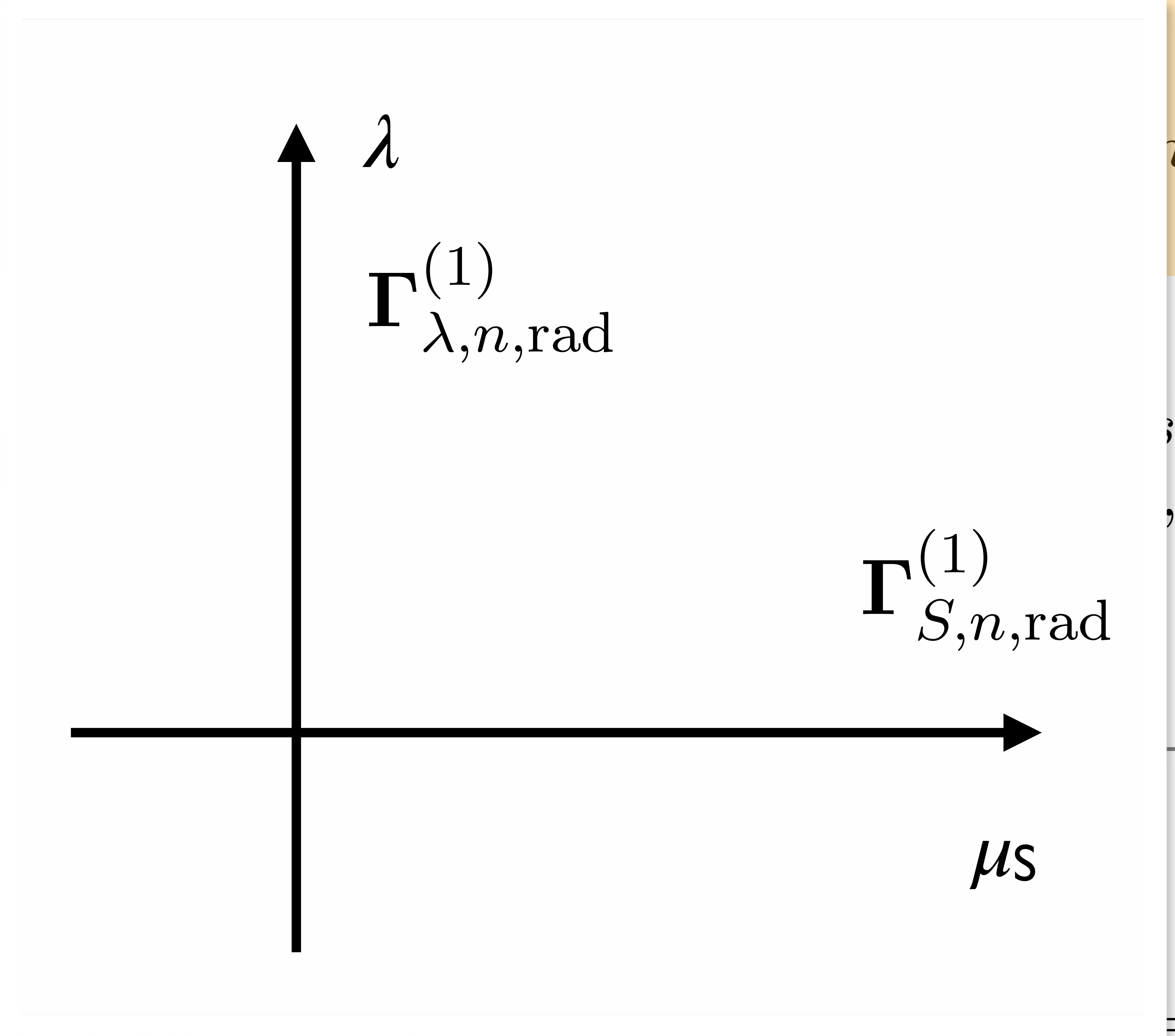
[Plätzer — arXiv:2204.06956]

We can now obtain the evolution equations we asked for:

$$\partial_S \mathbf{A}_n = \mathbf{\Gamma} \ldots {}_{n-s}\mathbf{R}^{(s)\dagger}_{S,n}$$



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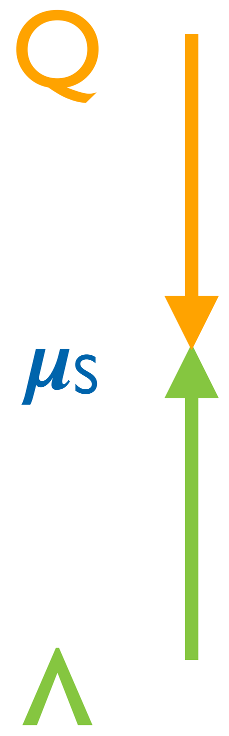

$$\partial_S \mathbf{S}_n = -\ldots {}^{)\dagger}_{,n+s}\mathbf{S}_{n+s}\tilde{\mathbf{R}}^{(s)}_{S,n+s}\prod_{i=n+1}^{n+s}[\mathrm{d}p_i]\tilde{\delta}(p_i)$$

Choosing ene… essentially give… jets" algorithm… for real emission).

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Evolving a hadronization model

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[Plätzer — arXiv:2204.06956]

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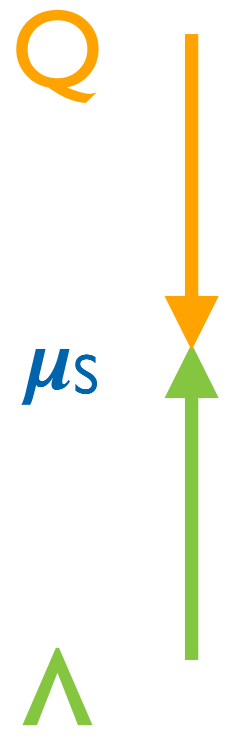
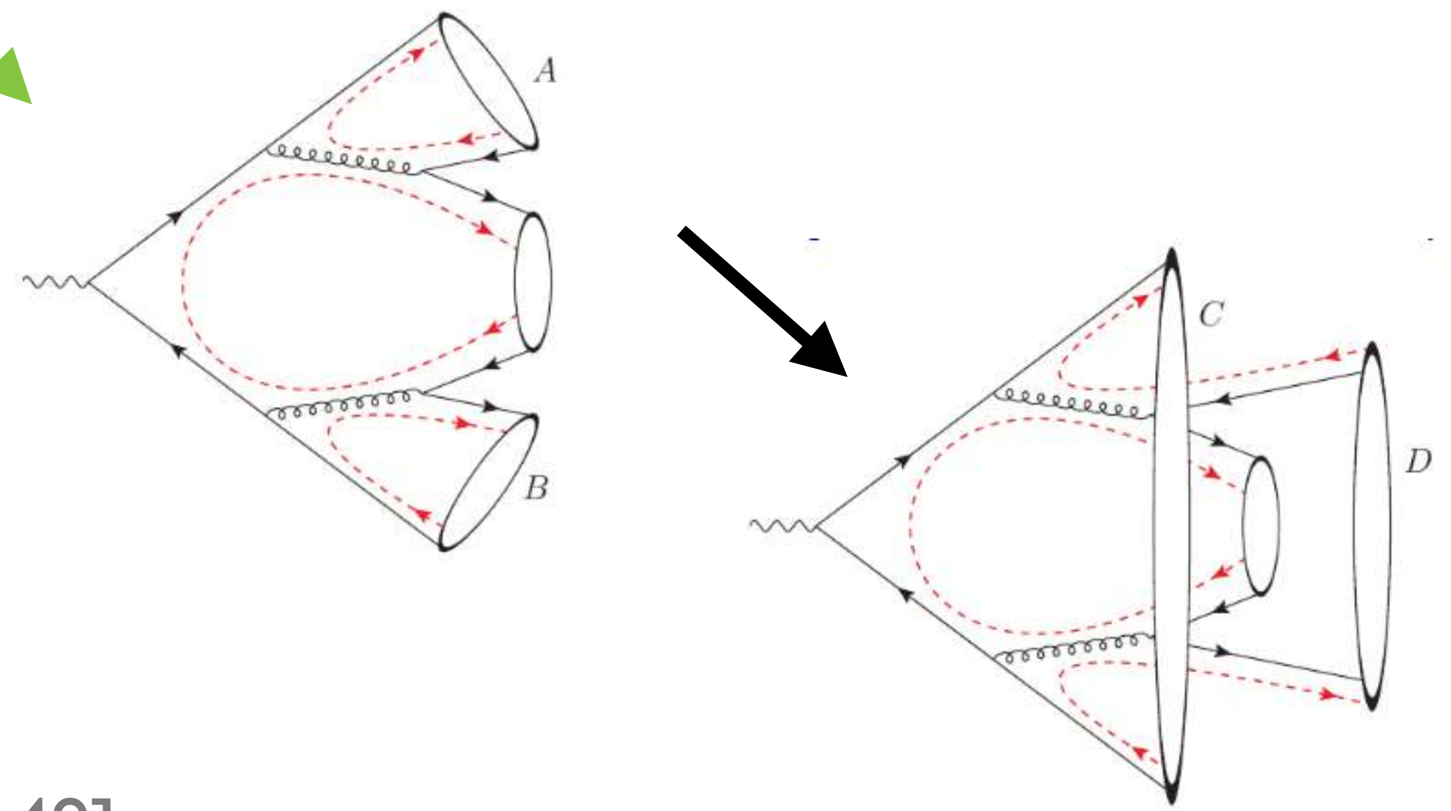
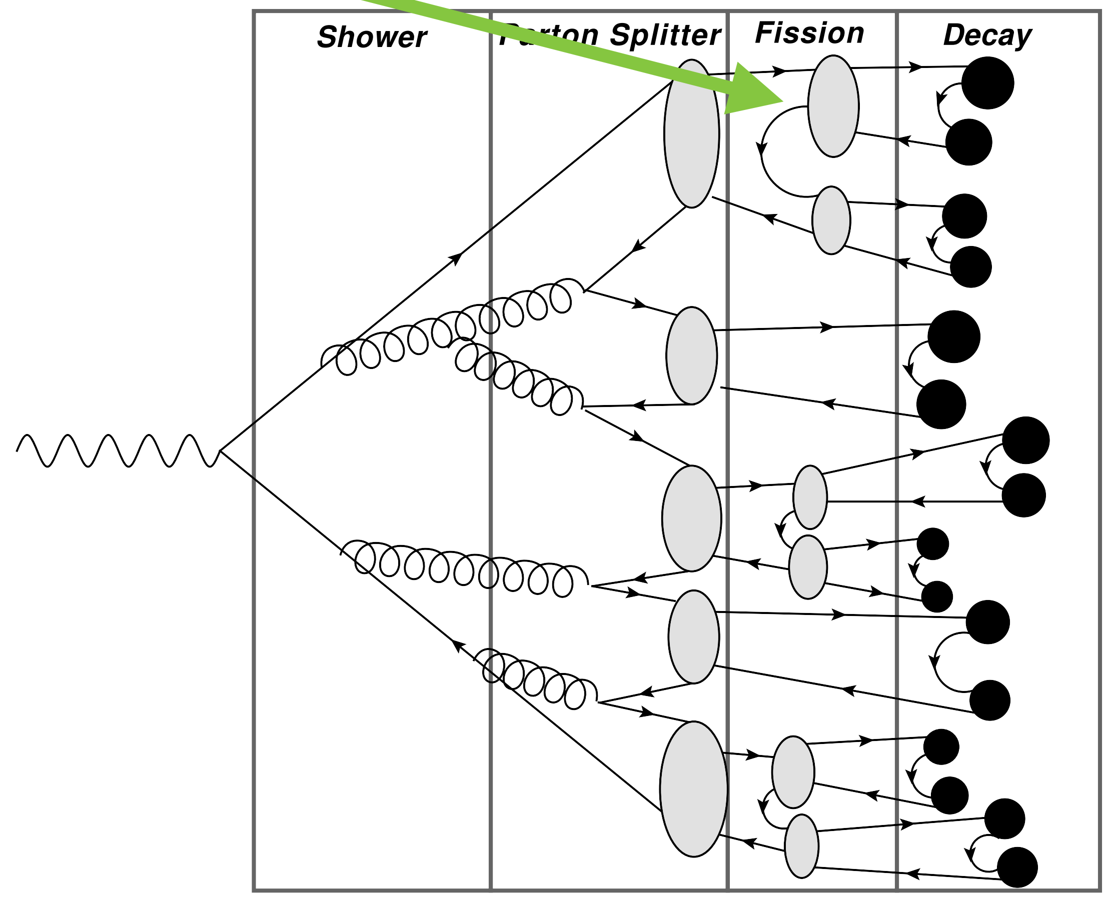


$$\partial_S \mathbf{S}_n = -\tilde{\mathbf{\Gamma}}^\dagger_{S,n}\mathbf{S}_n - \mathbf{S}_n\tilde{\mathbf{\Gamma}}_{S,n} + \sum_{s\geq 1}\alpha_S^s \int \tilde{\mathbf{R}}^{(s)\dagger}_{S,n+s}\mathbf{S}_{n+s}\tilde{\mathbf{R}}^{(s)}_{S,n+s}\prod_{i=n+1}^{n+s}[\mathrm{d}p_i]\tilde{\delta}(p_i)$$

Evolution equation for a hadronization model!

Features which relate to the high-energy dynamics of the Herwig cluster model.





[Gieseke, Kirchgaesser, Plätzer, Siodmok – JHEP 11 (2018) 149]

Summary

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Amplitude evolution:

- A theoretical framework to build parton shower and resummation algorithms.
- A numerical method in its own right to address QM interference beyond the large-N limit.

Open questions beyond state-of-the-art answered:

- How would hadronization and other infrared unsafe observables fit into this framework?
- How would we rigorously construct the evolution beyond the leading order?

Vital input for the construction of better parton shower algorithms and to put constraints on showers and hadronization models where we have no data available, specifically SIDM models.

Much more in progress: hard-collinear contributions, first steps to electroweak evolution …

[Forshaw, Holguin, Plätzer – JHEP 1908 (2019) 145]
[Löschner, Plätzer, Simpson — arXiv:2112.14454]
[Plätzer, Sjödahl — arXiv:2204.03258]