

Semileptonic B -decays from lattice QCD

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Based on “*Lattice QCD study of inclusive semileptonic decays of heavy mesons*” [[hep-lat](#)] 2203.11762, *J. High Energ. Phys.* 2022, 83 (2022)

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- 1 Introduction
- 2 Inclusive decays formalism
- 3 Inclusive decays on the lattice
 - HLT algorithm
 - Lattice computation
- 4 OPE comparison
- 5 Conclusion



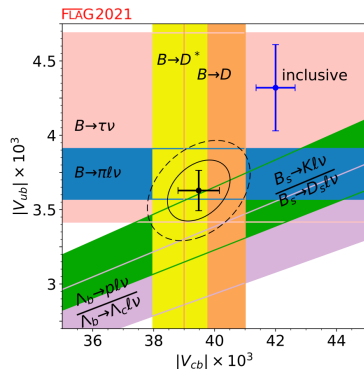
Anomalies & tensions

Some interesting anomalies in Flavour Physics:

Lepton Flavour Universality Violation ($\simeq 3\sigma$), $g - 2$ (4σ ?)

A persistent tension

The tension between the **exclusive** and **inclusive** determination of the CKM parameter V_{cb} is $\sim 3\sigma$

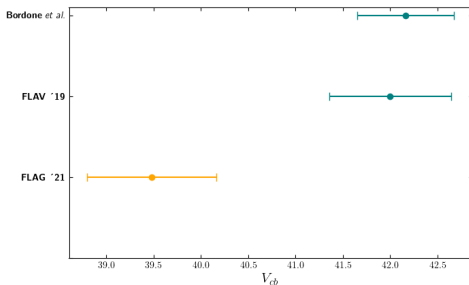


V_{cb} problem

$$|V_{cb}|_{inc.} = 42.00(64)10^{-3} \text{ [HFLAV '19]}$$

$$|V_{cb}|_{inc.} = 42.16(51)10^{-3} \text{ [Bordone, Capdevila, Gambino '21]}$$

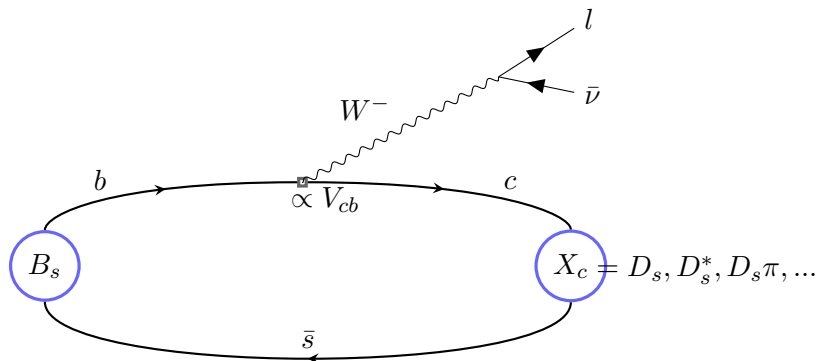
$$|V_{cb}|_{exc.} = 39.48(68)10^{-3} \text{ [FLAG '21]}$$



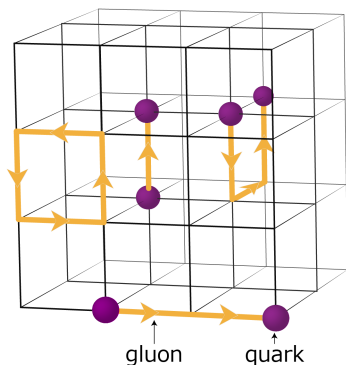
Inclusive $B_s \rightarrow X_c l \nu$

non-perturbative calculation

Weak decays of hadrons involve two mass scales: the intermediate W boson and the QCD low-energy scale dominating the hadron structure.



Lattice QCD



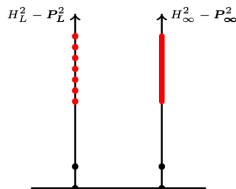
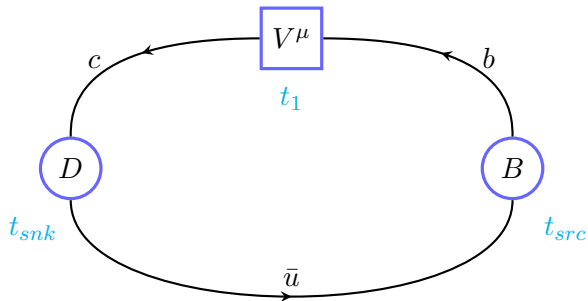
- systematically improvable tool for non-perturbative QCD calculations
- work in a discretised $4D$ Euclidean spacetime in a finite volume $L^3 \times T$
- The action is interpreted as the Boltzmann weight factor which allows for MC simulations
- Extract physics from Correlation functions of operators

$$\langle O \rangle = \mathcal{Z}^{-1} \int [DU] e^{-S_{QCD}[U]} O[U] \approx \frac{1}{N} \sum_{i=1}^N O[U_i]$$



Gauge ensembles

On the lattice we can generate lots of different gauge configs collected in what is called an *ensemble* $\{U_i\}$. On this ensemble one can compute Euclidean correlators



Decay rate

we start by considering the differential decay rate:

$$\frac{d\Gamma}{dq^2 dq^0 dE_l} = \frac{G_F^2 |V_{cb}|^2}{8\pi^2} L_{\mu\nu} W^{\mu\nu}$$

with:

$$W_{\mu\nu}(w, \mathbf{q}) = \frac{(2\pi)^3}{2m_{B_s}} \langle \bar{B}_s(\mathbf{0}) | J_\mu^\dagger(0) \delta(\hat{H} - w) \delta^3(\hat{P} + \mathbf{q}) J_\nu | \bar{B}_s(\mathbf{0}) \rangle$$

Integrating over E_l :

$$\Gamma = \frac{G_F^2 |V_{cb}|^2}{24\pi^3} \int_0^{q_{max}^2} dq^2 \sqrt{q^2} \sum_{l=0}^2 (\sqrt{q^2})^{2-l} Z^{(l)}(q^2)$$



Introducing the kernel

$$\Theta^{(l)}(w_{max} - w) = (w_{max} - w)^l \theta(w_{max} - w)$$

We can write

$$Z^{(l)}(\mathbf{q}^2) = \int_0^\infty dw \Theta^{(l)}(w_{max} - w) T^{(l)}(w, \mathbf{q}^2)$$

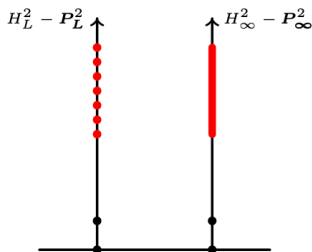
with:

$$T^{(0)} = W^{00} + \sum_{i,j=1}^3 \hat{n}_i \hat{n}_j W^{ij} - \sum_{i=1}^3 \hat{n}_i (W^{0i} + W^{i0})$$



Spectral density

$$\begin{aligned} \frac{d\Gamma}{dq^2} \propto Z^{(l)}(q^2) &= \lim_{\sigma \rightarrow 0} \left(\lim_{V \rightarrow \infty} \right) \int_0^\infty dw T^{(l)}(w, q^2) \Theta_\sigma^{(l)}(w_{max} - w) \\ &= \lim_{\sigma \rightarrow 0} \left(\lim_{V \rightarrow \infty} \right) m_{B_s}^l \sum_\tau^\infty g_\tau^{(l)}(w_{max}, \sigma) G^{(l)}(a\tau, q) \end{aligned}$$



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Smooth function

One can connect Euclidean correlators to the spectral density

$$G^{(l)}(a\tau; \mathbf{q}) = \int_0^\infty dw T^{(l)}(w, \mathbf{q}) e^{-aw\tau}$$

If a function is infinitely differentiable (smooth) then we can approximate it numerically:

$$f(w) = \sum_{\tau} g_{\tau} e^{-aw\tau}$$

So that:

$$\sum_{\tau} g_{\tau} G^{(l)}(a\tau; \mathbf{q}) = \int_0^\infty dw T^{(l)}(w, \mathbf{q}) f(w)$$



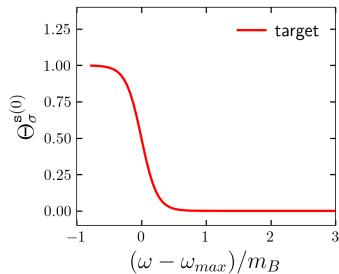
Smeared kernel

Problem

The θ -function is definitely NOT smooth

Solution

We need to introduce a *smeared* kernel θ_σ



$$\begin{aligned}\Theta_\sigma^{(l)}(w_{max} - w) &= (w_{max} - w)^l \theta_\sigma(w_{max} - w) \\ &= m_{B_s}^l \sum_{\tau=1}^{\infty} g_\tau^{(l)}(w_{max}, \sigma) e^{-aw\tau}\end{aligned}$$



Smeared spectral density

$$\sum_{\tau}^{\infty} g_{\tau}(w_{max}, \sigma) G^{(l)}(a\tau; \mathbf{q}) = \int_0^{\infty} dw T^{(l)}(w, \mathbf{q}) \Theta_{\sigma}^{(l)}(w_{max} - w)$$

$$Z^{(l)}(\mathbf{q}^2) = \int_0^{\infty} dw \Theta^{(l)}(w_{max} - w) T^{(l)}(w, \mathbf{q}^2)$$

$$\begin{aligned} Z_{\sigma}^{(l)}(\mathbf{q}^2) &= \int_0^{\infty} dw \Theta_{\sigma}^{(l)}(w_{max} - w) T^{(l)}(w, \mathbf{q}^2) \\ &= \sum_{\tau}^{\infty} g_{\tau}(w_{max}, \sigma) G^{(l)}(a\tau; \mathbf{q}) \end{aligned}$$



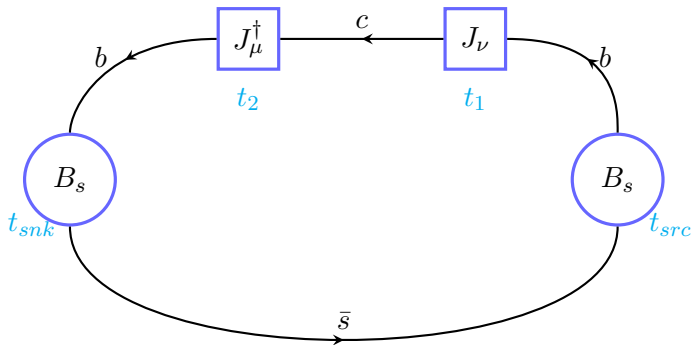
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Euclidean 4-point Correlator

[Gambino, Hashimoto'20, Phys. Rev. Lett. 125, 032001, hep-lat/2005.13730]

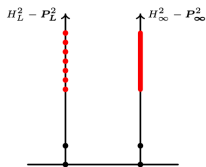
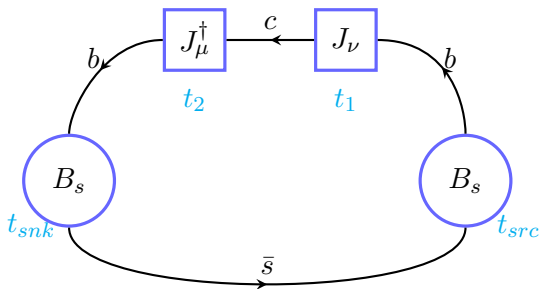
$$C_{\mu\nu}(t_{snk}, t_2, t_1, t_{src}; \mathbf{q}) = \int d^3x e^{i\mathbf{q}\cdot\mathbf{x}} T \langle 0 | \tilde{\phi}_{B_s}(\mathbf{0}, t_{snk}) J_\mu^\dagger(\mathbf{x}, t_2) J_\nu(\mathbf{0}, t_1) \tilde{\phi}_{B_s}^\dagger(\mathbf{0}, t_{src}) | 0 \rangle$$



Euclidean 4-point Correlator

[Gambino, Hashimoto'20, Phys. Rev. Lett. 125, 032001, hep-lat/2005.13730]

$$C_{\mu\nu}(t_{snk}, t_2, t_1, t_{src}; \mathbf{q}) = \int d^3x e^{i\mathbf{q}\cdot\mathbf{x}} T \langle 0 | \tilde{\phi}_{B_s}(\mathbf{0}, t_{snk}) J_\mu^\dagger(\mathbf{x}, t_2) J_\nu(\mathbf{0}, t_1) \tilde{\phi}_{B_s}^\dagger(\mathbf{0}, t_{src}) | 0 \rangle$$



HLT algorithm

The details can be found in:

[Hansen, Lupo, Tantalò '19, Phys. Rev. D 99, 094508]

[hep-lat/1903.06476]

we want to reconstruct:

$$\Theta_{\sigma}^{(l)}(w_{max} - w) = \sum_{\tau=1}^{\tau_{max}} g_{\tau} e^{-aw\tau}$$

$$W_{\lambda}[g] = (1 - \lambda) \frac{A[g]}{A[0]} + \lambda B[g]$$

$$\left. \frac{\partial W_{\lambda}[g]}{\partial g_{\tau}} \right|_{g_{\tau}=g_{\tau}^{\lambda}} = 0$$



$A[g^\lambda]$ is the reconstruction bias

$$A[g] = a \int_{E_0}^{\infty} dw \left\{ \Theta_{\sigma}^{(l)} - \sum_{\tau=1}^{\tau_{max}} g_{\tau} e^{-aw\tau} \right\}^2$$

$B[g^\lambda]$ is the statistical variance

$$B[g] = \sum_{\tau, \tau'=1}^{\tau_{max}} g_{\tau} g_{\tau'} \frac{Cov[G^{(l)}(a\tau), G^{(l)}(a\tau')]}{[G^{(l)}(0)]^2}$$

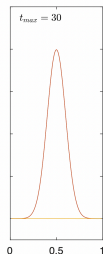
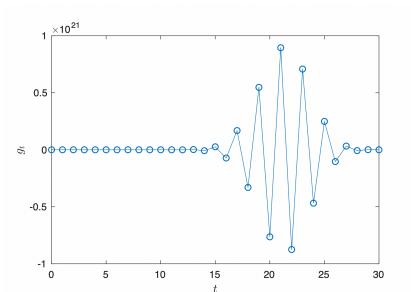


g_τ^λ defines the approximation of the kernel

$$\Theta_\sigma^\lambda = \sum_{\tau=1}^{\tau_{max}} g_\tau^\lambda e^{-aw\tau}$$

the best choice for λ can be found via:

$$\left. \frac{\partial W(\lambda)}{\partial \lambda} \right|_{\lambda=\lambda_*} = 0$$



Smearing kernel reconstruction

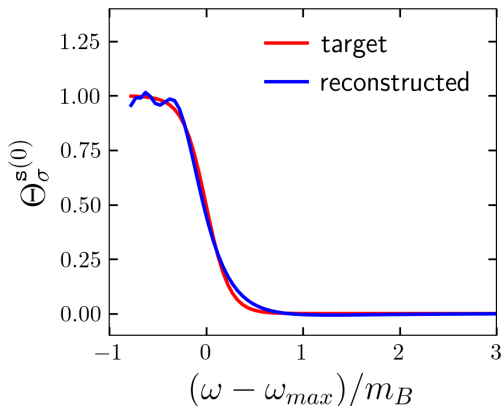


Figure: $\lambda = \lambda_*$



Smeared Kernel λ

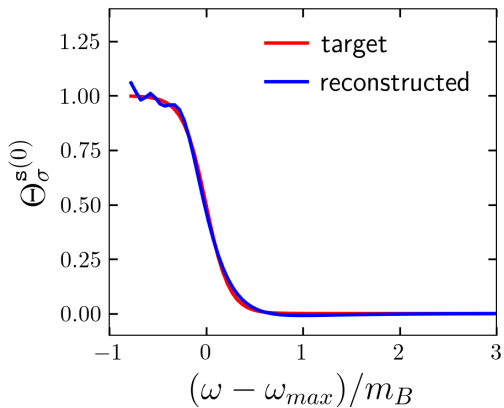


Figure: $\lambda < \lambda_*$

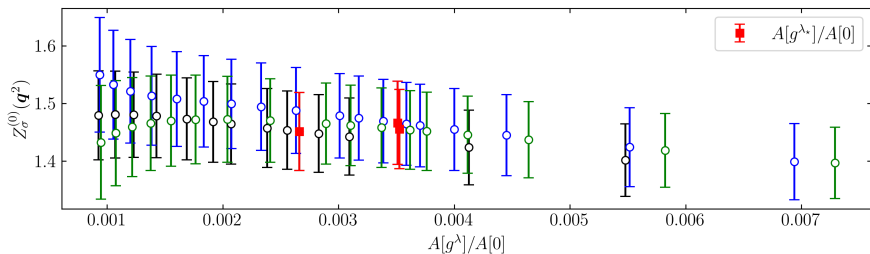


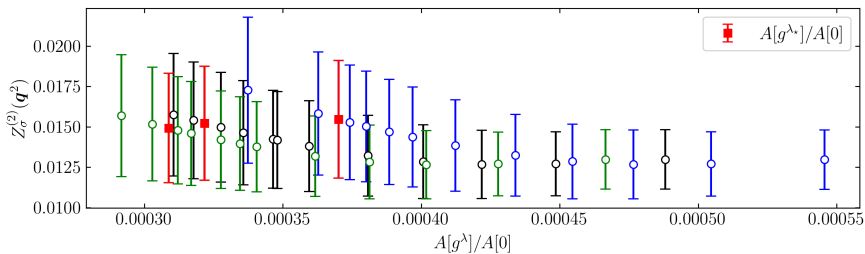
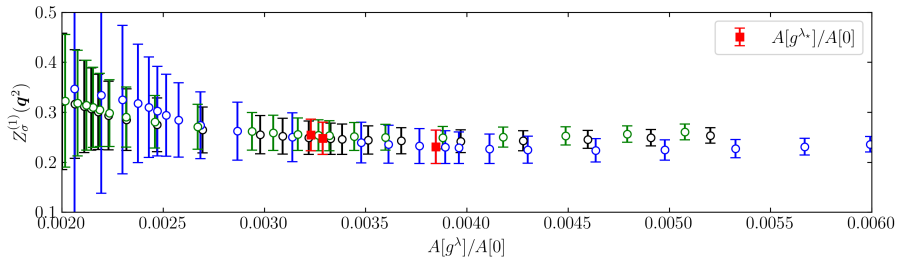
$$\begin{aligned} Z_{\sigma}^{(l)\lambda_*}(\mathbf{q}^2) &= \int_0^{\infty} dw \Theta_{\sigma}^{(l)\lambda_*}(w_{max} - w) T^{(l)}(w, \mathbf{q}^2) \\ &= \sum_{\tau}^{\infty} g_{\tau}^{\lambda_*}(w_{max}, \sigma) G^{(l)}(a\tau; \mathbf{q}) \end{aligned}$$



Smeared spectral density

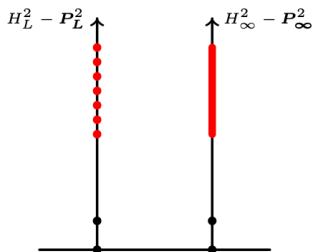
Now we can apply the smeared kernel to $Z_\sigma^{(l)}(\mathbf{q})$ which is obtained by applying the coefficients g_τ^λ to the correlator $G^{(l)}(t, \mathbf{q}^2)$ and perform the integral over the allowed phase space



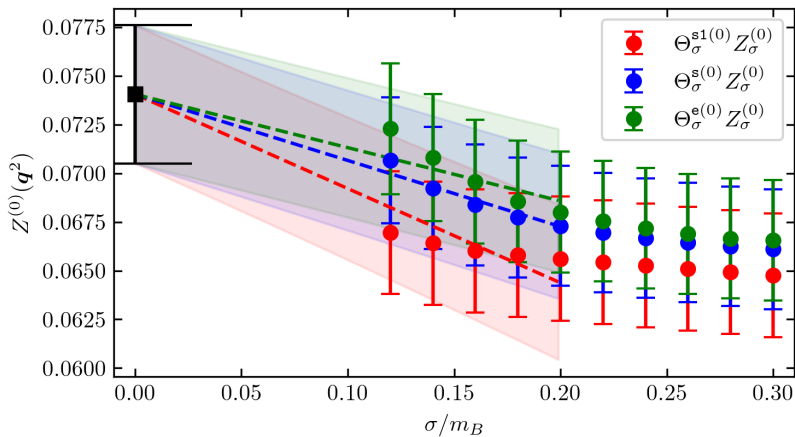


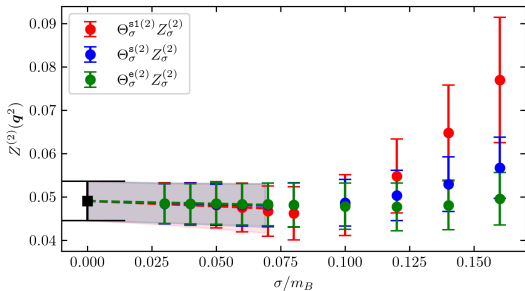
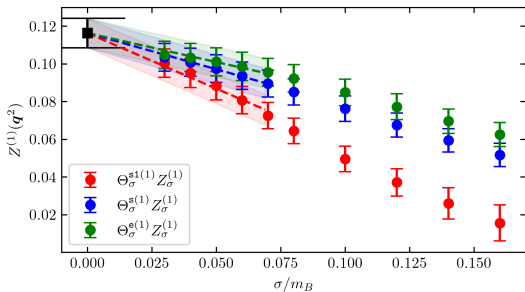
$\sigma \rightarrow 0$ limit

$$\begin{aligned}
 Z^{(l)}(\mathbf{q}^2) &= \lim_{\sigma \rightarrow 0} \left(\lim_{V \rightarrow \infty} \right) \int_0^\infty dw T^{(l)}(w, \mathbf{q}^2) \Theta_\sigma^{(l)}(w_{max} - w) \\
 &= \lim_{\sigma \rightarrow 0} \left(\lim_{V \rightarrow \infty} \right) m_{B_s}^l \sum_\tau^\infty g_\tau^{(l)}(w_{max}, \sigma) G^{(l)}(a\tau, \mathbf{q})
 \end{aligned}$$



Extrapolation to $\sigma = 0$





Differential decay rate

$$\Gamma = \frac{G_F^2 |V_{cb}|^2}{24\pi^3} \int_0^{q_{max}^2} dq^2 \sqrt{q^2} \sum_{l=0}^2 (\sqrt{q^2})^{2-l} Z^{(l)}(q^2)$$

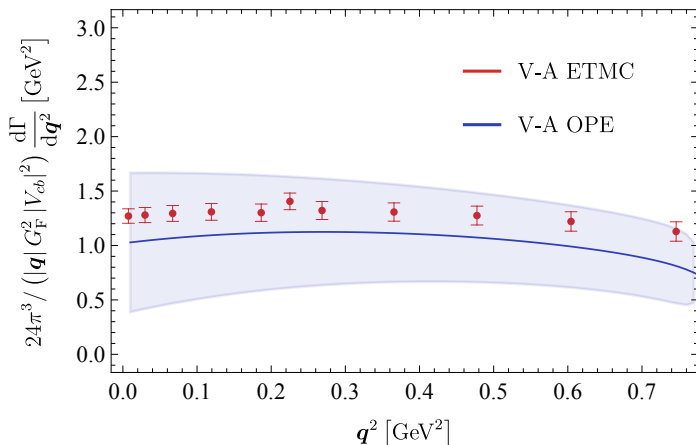


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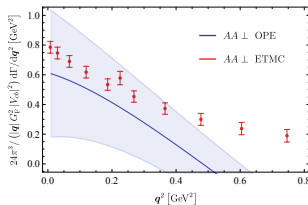
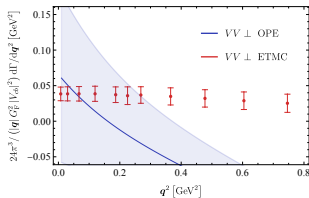
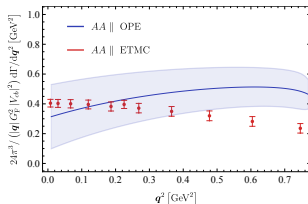
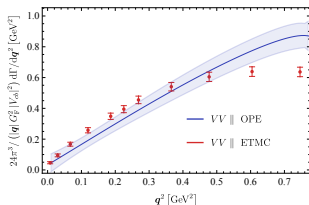
Differential Decay Rate comparison

After we take the $\sigma \rightarrow 0$ limit, we can confront our results with the analytic results from the OPE



Channel decomposition

One can decompose the contribution to the differential decay rate into V_{\parallel} , V_{\perp} , A_{\parallel} , A_{\perp}



Final Results

The last thing to do is to perform the integration over q^2

	ETMC	OPE
$\Gamma/ V_{cb}^2 \times 10^{13}$ (GeV)	0.987(60)	1.20(46)
$\langle E_\ell \rangle$ (GeV)	0.491(15)	0.441(43)
$\langle E_\ell^2 \rangle$ (GeV ²)	0.263(16)	0.207(49)
$\langle E_\ell^2 \rangle - \langle E_\ell \rangle^2$ (GeV ²)	0.022(16)	0.020(8)
$\langle M_X^2 \rangle$ (GeV ²)	3.77(9)	4.32(56)

Table: Total width and moments in the ETMC case.



Future prospects

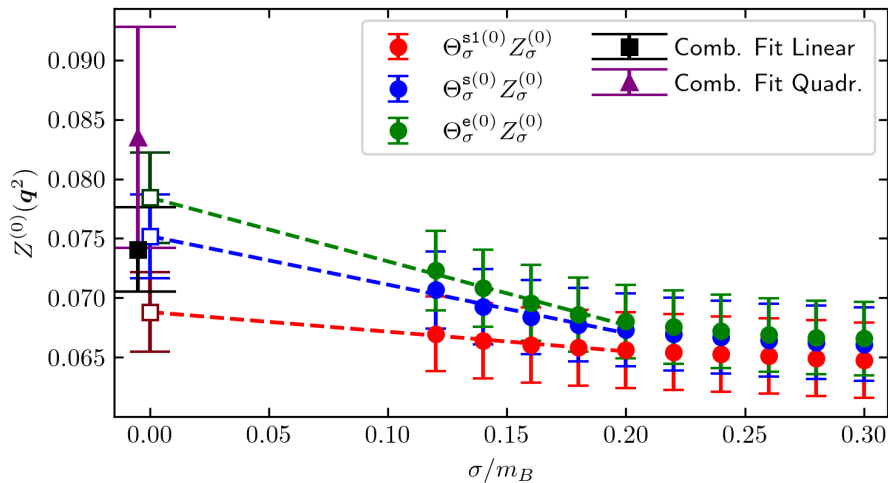
- Perform the analysis with several values of lattice spacing, volumes and b -quark masses in order to perform the extrapolations: $a \rightarrow 0$, $V \rightarrow \infty$, $m_b \rightarrow m_b^{phys}$
- Apply this method to other Semileptonic decays for which we have good experimental results for Branching ratios, such as those involving the $D_{(s)}$ -meson
- Extend this method to semileptonic decays needed to estimate V_{ub} such as $B \rightarrow X_u l \nu$
- Calculate quantities needed for an accurate estimate of the inclusive value of V_{cb}



BACKUP SLIDES



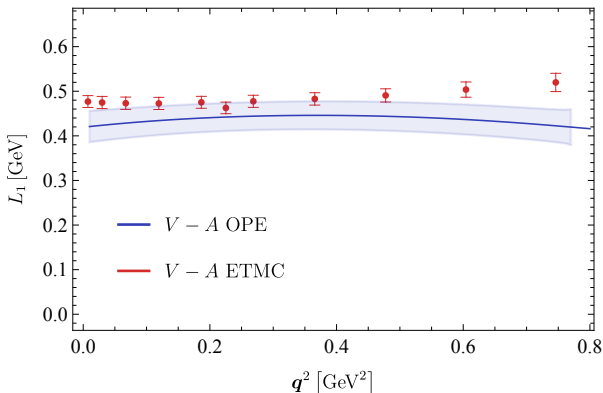
Extrapolation



Lepton Energy Moment

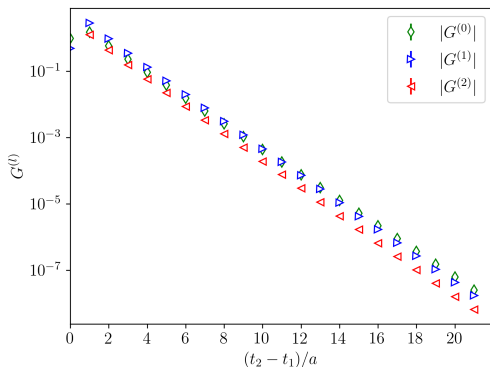
We also looked at the Lepton Energy Moment defined as

$$L_1(\mathbf{q}^2) = \frac{\int dq_0 dE_l E_l \left[\frac{d\Gamma}{dq^2 dq_0 dE_l} \right]}{\int dq_0 dE_l \left[\frac{d\Gamma}{dq^2 dq_0 dE_l} \right]}$$



G correlators

$$G^{(l)}(t_2 - t_1; \mathbf{q}) = M_{\mu\nu}(t_2 - t_1; \mathbf{q}) = Z_{B_s} \lim_{\substack{t_{\text{snk}} \rightarrow +\infty \\ t_{\text{src}} \rightarrow -\infty}} \frac{C_{\mu\nu}(t_{\text{snk}}, t_2, t_1, t_{\text{src}}; \mathbf{q})}{C(t_{\text{snk}} - t_2)C(t_1 - t_{\text{src}})}$$



ETMC ensemble

B55.32 ensemble

Uses the Twisted Mass QCD action with $N_f = 2 + 1 + 1$ sea quarks.

$L^3 \times T$	N_{cnfg}	a		
$32^3 \times 64$	150	0.0815(30)		
m_{ud}^{phys}	m_s^{phys}	m_c^{phys}	m_π	(MeV)
3.70(17)	99.6(4.3)	1176(39)	375(13)	
$a\mu_s$	$a\mu_c$	$a\mu_b$		
0.021	0.25	0.5		

Unphysically light B_s mass

This corresponds to $m_b(\overline{MS} = 2 \text{ GeV}) = 2.4 \text{ GeV}$ which gives a meson mass $M_{B_s} = 3.08(11) \text{ GeV}$