Exotic Hadrons in Unquenched Quark Model

Muhammad Naeem Anwar[1]

[1] Department of Physics
Swansea University, Wales, UK

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Atholl Palace Hotel, Pitlochry, Scottish Highlands

Swansea University
Prifysgol Abertawe

www.swansea.ac.uk
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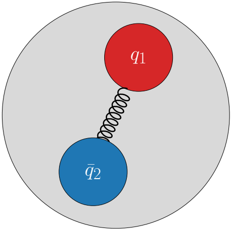
Outline

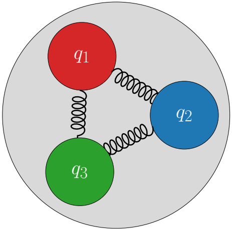
Definitions

Conventional States: in line with the expected QCD spectrum, e.g. quark potential models, Lattice QCD David Wilson, Monday

$\hookrightarrow$: Mesons (valence $q\bar{q}$) and Baryons (valance qqq)

π, ρ, K, ... p, N, Λ, ...





$$P_{q\bar{q}} = (-1)^{L+1}, \; C_{q\bar{q}} = (-1)^{L+S}$$

Definitions

Exotic States: not understood "well-enough", & properties do not match the expected QCD spectra $\hookrightarrow X(3872)$, T_{cc}, $Y(4260)$, $Z_{c,b}$, P_c

Wolfgang Gradl, Monday

1. Quantum numbers; 0^{--}, and $CP = -1$ $(0^{+-}, 1^{-+}, 2^{+-}, \cdots)$
2. Isospin violation, e.g. $\mathcal{B}\big(X(3872) \to \rho^0 J/\psi\big) \sim \mathcal{B}\big(X(3872) \to \omega J/\psi\big)$
3. Anomalously narrow, e.g. $\Gamma(P_{cs}) = 7.0 \pm 1.2$ MeV LHC Seminar July 5, 2022

$\hookrightarrow$Popular Naming Scheme:

1. $Z_{c,b}$: electrically charged $(I = 1)$
2. Y vectors: $J^{PC} = 1^{--}$, produced in e^+e^- via virtual photon
3. X: anything else

$\hookrightarrow$ New naming scheme is recently proposed, LHCb paper: arXiv:2206.15233

Manifestly Exotic

Wolfgang Gradl, Monday

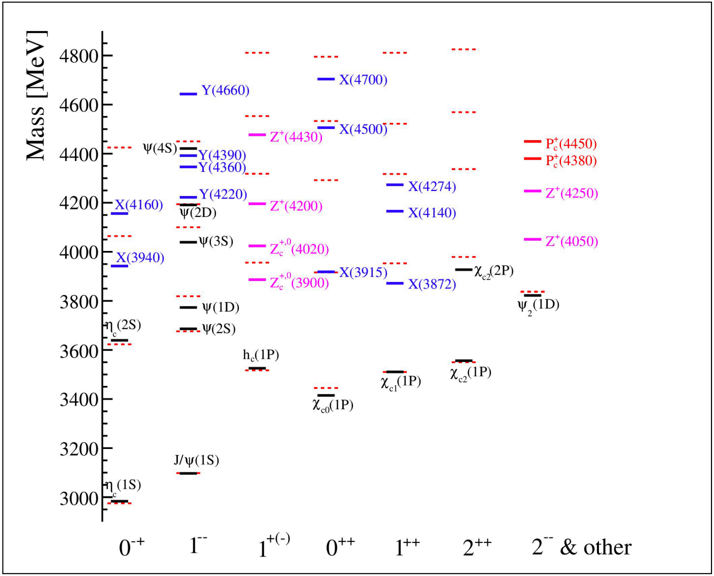
☛ Charged $Z_{c,b}$ are manifestly 4-quark states

$$\mathrm{Z}_c^+(3900) \to J/\psi\pi^+$$

- $c\bar{c}$ production from QCD vacuum is suppressed
- π^+ doesn't have the same quantum number as the vacuum

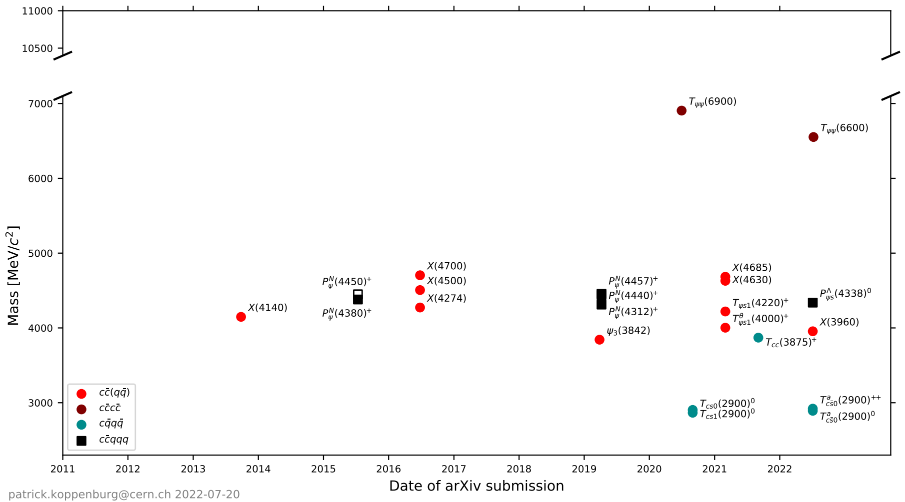
Exotics: Charmonium(-like)

Olsen et al. Rev. Mod. Phys. 90, 15003 (2018)



New Exotic at LHC: An Example

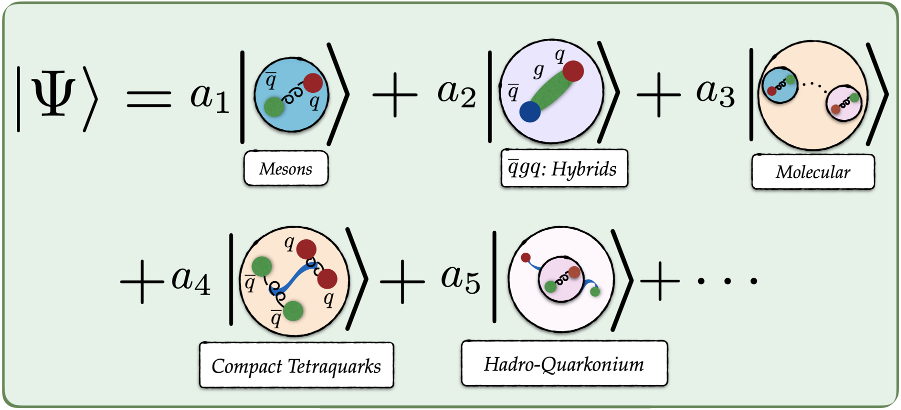
https://www.nikhef.nl/%7Epkoppenb/particles.html'



Exotics States: Possibilities?

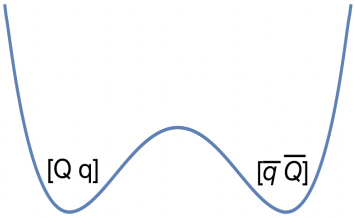
- QCD allows for more complex structures than $q\bar{q}$ or qqq
- Realized by Gell-Mann and Zweig (quark models, 1964)

$$|\Psi\rangle = a_1 |q\bar{q}\rangle + a_2 |qg\bar{q}\rangle + a_3 |[q\bar{q}][q\bar{q}]\rangle + a_4 |(qq)(\bar{q}\bar{q})\rangle + a_5 |(q\bar{q})(q\bar{q})\rangle + \cdots$$



Overview

Compact Tetraquark: Diquark



- Generated by short distance QCD forces
- Exist in the QCD spectrum at $r \sim 1$ fm

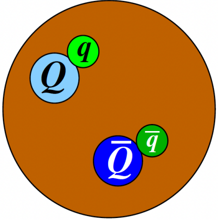
↪ Field operator description

$$T(x) \sim \overbrace{\left(\epsilon_{ijk} Q^j(x) q^k(x)\right)}^{\text{diquark} \in \bar{3}_c} \underbrace{\left(\epsilon^{imn} \bar{Q}_m(x) \bar{q}_n(x)\right)}_{\text{antidiquark} \in 3_c}$$

↪ Features   Maiani and Pilloni, arXiv:2207.05141

- dominant spin-spin interactions are those inside the diquark
- neglect the interaction between the quarks of different diquarks
- diquarks are at relatively large distance compare to their size

Diquark Model Maiani, Polosa, Riquer, PRD89, 114010 (2014)



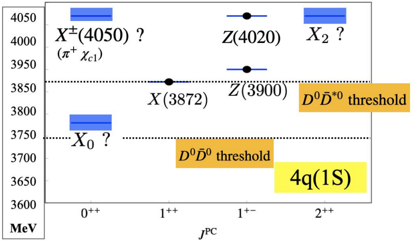
$\hookrightarrow$General prescription for the spectrum

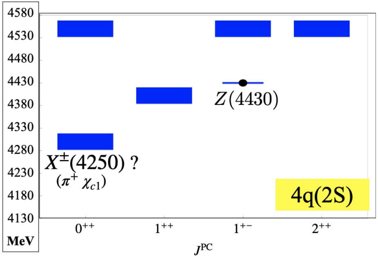
$$H_{\text{eff}} = 2m_{[Qq]} + \underbrace{\sum_{i<j} \kappa_{ij}(s_i \cdot s_j)}_{\text{inside diquark}} + \underbrace{a\ \mathbf{L}\cdot\mathbf{S} + b\ \frac{S_{12}}{4}}_{\text{inter diquarks}} + c\ \mathbf{L}^2$$

$\mathbf{S} = S_{[Qq]} + S_{[\bar{Q}\bar{q}]}$, $\mathbf{L}$ orbital angular momentum, κ_{ij} chromomagnetic coupling

☛ Fit the parameters to some observed exotic state, then predict spectrum

e.g. $1S$-wave spectrum is characterised by two quantities, $m_{[Qq]}$, and κ_{ij}

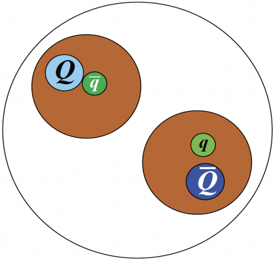




Overview

Hadronic Molecules

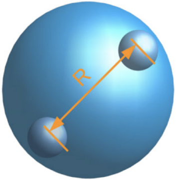
$\hookrightarrow$ Analogues of deuteron and other light nuclei



- Composite state of 2 or more hadrons
- Generated by long distance QCD forces (e.g. pion exchanges)
- Naturally explains near threshold exotics

$\hookrightarrow$Expected to be spatially extended, Bohr radius

$$R \sim \frac{1}{\sqrt{2\mu E_b}} \gg r_{\text{confinement}}$$



for near-threshold states

Only narrow hadrons can be considered as components of hadronic molecules

- $\Gamma_{\text{had}} \ll 1/(\text{range of force})$
- Typical range of force $1/m_{\text{ex}}$, one pion exchange, having range $\mathcal{O}\left(\frac{1}{m_\pi}\right)$

Hadronic Molecules: Prominent Features

1. Realization in terms of color-neutral objects, no color mixing
2. Explore via hadron-hadron interaction/scattering

   Low-energy EFT applicable, model-independent statements (for S-wave)
3. Decay predominantly to nearby hadronic channel(s), get coupling

Compositeness: probability of the physical state being a 2-body bound state

$$|g_{\rm NR}|^2 \approx \underbrace{(1-Z)}_{\text{compositeness}} \frac{2\pi}{\mu^2}\sqrt{2\mu E_b} \leq \frac{2\pi}{\mu^2}\sqrt{2\mu E_b}$$

(composite) $0 \leq Z \leq 1$ (elementary) $1-Z$ Quantifies molecular component in physical state

The structure information is hidden in the effective coupling $|g_{\rm NR}|^2$, extracted from experiment, independent of the phenomenology

Weinberg Compositeness Criterion Weinberg PR130, 776 (1963); 131, 440 (1963)

Scattering amplitude expressed in terms of effective range expansion

$$a = -\frac{2(1-Z)}{(2-Z)\sqrt{2\mu E_b}} + \mathcal{O}(1/R), \qquad r_e = -\frac{Z}{(1-Z)\sqrt{2\mu E_b}} + \mathcal{O}(1/R)$$

	scattering length a	effective range r_e	
Molecule: $Z \to 0 \implies$	large $\sim -\frac{1}{\sqrt{2\mu E_b}}$	small $\approx \mathcal{O}\left(\frac{1}{R}\right)$	$\lvert a\rvert \gg \lvert r_e\rvert$
Compact: $Z \to 1 \implies$	small $\approx \mathcal{O}\left(\frac{1}{R}\right)$	large and negative	$\lvert a\rvert \ll \lvert r_e\rvert$

Example: deuteron (pn); $E_b = -2.2$ MeV, $a_{^3S_1} = -5.41$ fm, $r_e = 1.75$ fm

Theory : $a_{Z=1} = 0$, fm $a_{Z=0} = -4.3 \pm 1.4$ fm $\implies \underbrace{\text{large } a, r_e \sim \mathcal{O}(1/m_\pi)}_{\text{molecule}}$

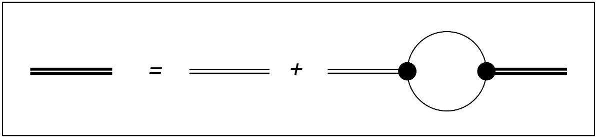
☛ Important application is $X(3872)$: issue; coupled-channels involved, extension is done recently Baru et al. PLB, 833, 137290, (2022)

Part II

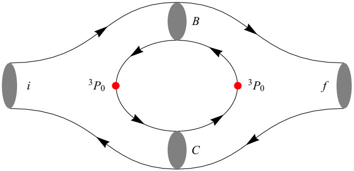
Unquenched Quark Model

Unquenched Effects

$\hookrightarrow$Dyson equation



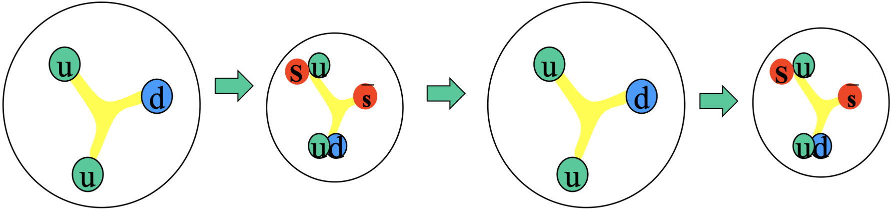
Physical mass = bare mass + mass shift (self energy corrections)



MNA, Lu, Zou, PRD99, 094005 (2019)

- Below threshold: mass shift, mixing of same J^{PC} states
- Above threshold: Open-flavor strong decays, mass shifts, mixing of same J^{PC} states
- Probabilities of multi-quark/molecular components can be worked out

◆ **to understand hadron spectrum, quark model needs to be unquenched, with large hadronic molecule components when close to some thresholds.**



© Bing-Song Zou, T_{cc} Workshop, Lyon

Breathing mode of hadrons: Pentaquark components were essential to reproduce the obervables of nucleons An and Zou, EPJA 39, 195 (2009)

Advantage of UQM: it encapsulates both long- and short-distance information, structure of near-threshold state can be fully explored

Unquenched Effects: Theory

↪The physical state (using non-rel. QM description)

$$|\Psi\rangle = \begin{pmatrix} c_0 \, |\psi_0\rangle \\ c_{BC}(p) \, |BC, p\rangle \end{pmatrix}$$

$|\psi_0\rangle$=bare state, $|BC, p\rangle$ tow-meson continuum

⇒ sum over all possible BC mesons

↪The System is described by the Hamiltonian

$$H = \begin{pmatrix} H_0 & H_I \\ H_I & H_{BC} \end{pmatrix}$$

- H_0 is the bare $Q\bar{Q}$ Hamiltonian
- H_{BC} is the energy of the two-hadron continuum
- H_I dynamical piece, mix the bare state with the continuum

↪Solve the eigen value problem

$$\boxed{\mathrm{H}|\Psi\rangle = E\,|\Psi\rangle}$$

General Prescription

☛ Choose best physical ansatz

$$|\Psi\rangle = c_0\,|A\rangle + \sum_{BC}\int d^3p\, c_{BC}(p)\,|BC;p\rangle$$

Solve $\boxed{H\,|\Psi\rangle = E\,|\Psi\rangle}$ with $\boxed{M = M_0 + \Delta M}$

Physical state normalized to one

$$|c_0|^2 + \int d^3p|c_{BC}|^2 = 1$$

Probability of molecular component

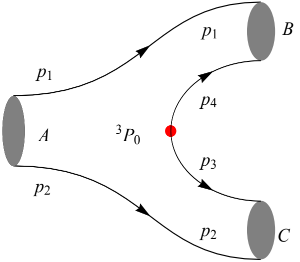
$$P_{BC} := \int d^3p|c_{BC}|^2 = \int d^3p\,\frac{|\langle BC;p|\,H_I\,|A\rangle|^2}{(M - E_{BC})^2}$$

The key quantity is transition form factor $\langle BC;p|\,H_I\,|A\rangle$

P_{BC} is related to Weinberg's Z, $\boxed{P_{BC} = 1 - Z}$, Weinberg PR130, 776 (1963)

Unquenched Quark Model

↪Where you make choice on H_I, H_0, and H_{BC}



↪Interaction Hamiltonian, $^3P_0$ vertex

$q\bar{q}$ from QCD vacuum

$$H_I = 2m_q\gamma \int d^3x \bar{\psi}_q \psi_q$$

m_q quark mass, γ is coupling strength

↪ Bare Hamiltonian

$$H_0 = 2m_c + \frac{p^2}{m_c} + V(r) + V_s(r)$$

$$V(r) = -\frac{4}{3}\frac{\alpha}{r} + \lambda r + c$$

α strong coupling, λ string tension, c renormal., $V_s(r)$ spin dependent

↪ $H_{BC}\left|BC;p\right\rangle = E_{BC}\left|BC;p\right\rangle$, $\qquad E_{BC} = \sqrt{m_B^2 + p^2} + \sqrt{m_C^2 + p^2}$

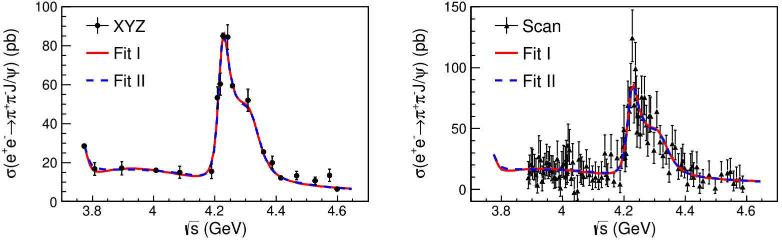
Structure of $Y(4230)$ and spin partners

MNA, Lu, PRD 104, 094006 (2021)
Lu, MNA, Zou, PRD 96, 114022 (2017)

$\psi(4230)$ aka $Y(4230)$ was $Y(4260)$

$\hookrightarrow Y(4260)$ observed at BaBar in ISR process $e^+e^- \to \pi^+\pi^- J/\psi$

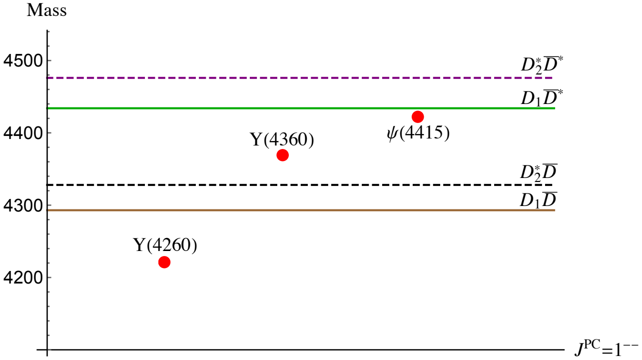
- Quantum numbers, $J^{PC} = 1^{--}$ Wolfgang Gradl, Weimin Song, Monday
- Also seen $\mathcal{B}(\gamma X(3872))$, $\mathcal{B}(\pi^+\pi^- h_c)$, $\mathcal{B}(\pi^+ D^{*-} D^0)$, $\mathcal{B}(K^+K^- J/\psi)$
- Not seen in any open-charm decays $\mathcal{B}(D\bar{D})$
- BESIII help to refine its mass, PDG M =4222.7 MeV, Γ = 49 MeV



- $cs\bar{c}\bar{s}$ Tetraquark Maiani et al. PRD72, 031502 (2005)
- $c\bar{c}g$ hybrid meson S.-L. Zhu, PLB625, 212 (2005)
- $c\bar{c} - q\bar{q}$ Hydro-Charmoniuum M. Voloshin, PLB666, 344 (2008)
- $D_1\bar{D}$ Molecule Q. Wang et al., PRL111, 132003 (2013)

$\psi(4360)$ aka $Y(4360)$

$\hookrightarrow Y(4360)$ observed at BaBar in $e^+e^- \to \psi(2S)\pi^+\pi^-$
$\hookrightarrow$ recently in $e^+e^- \to J/\psi\pi^+\pi^-$ at BESIII PRL118, 092001 (2017)
Several Theoretical Interpretations Chen et al., Phys. Rep., 639 (2016)



- $\psi(4415)$ is mainly referred to $c\bar{c}$ excited state (4^3S_1)

$Y(4230)$: Our Analysis

Physical ansatz are

$$|Y(4230)\rangle = c_0\,|A\rangle + \sum_{BC}\int d^3p\, c_{BC}(p)\,|BC;p\rangle$$

$A \in \{\psi(2S), \psi(3S), \psi(2D), \psi(3D)\}$ bare components

BC mesons, all nearby thresholds, S-wave and P-wave combinations

- Probability (unnormalized), coupling strength

$$P_{BC} := \int d^3p\, \frac{|\langle BC;p|\,H_I\,|\psi_0\rangle\,|^2}{(M - E_{BC})^2}$$

Probabilities are well-defined only for states below thresholds

Kalashnikova, PRD 72, 034010 (2005)

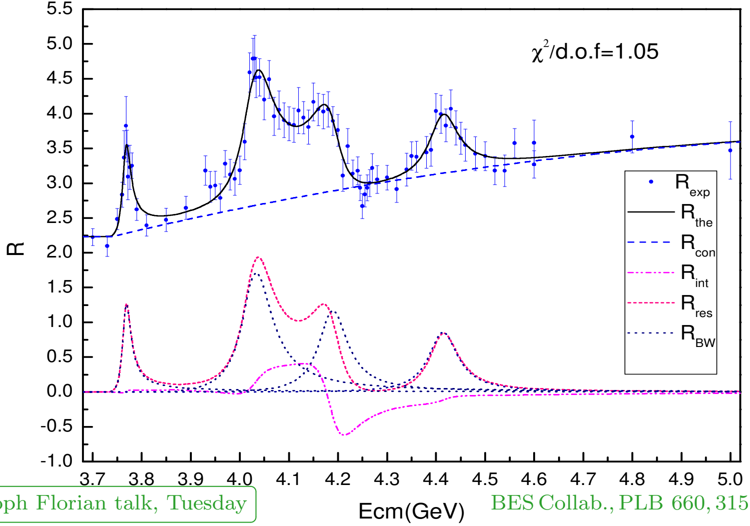
1. Short-distance $c\bar{c}$ core is likely a D-wave
2. $Y(4230)$ couples to $D_1\bar{D}$ in both S-wave and D-wave
3. R ratio dip around 4230 MeV naturally described

TABLE II. Coupling strength of the $D_1\bar{D}$ and $D_1'\bar{D}$ channels for the different charmonium states in the HQSS limit.

	Channels	$\psi(3S)$	$\psi(4S)$	$\psi(2D)$	$\psi(3D)$
Parameters in Table I	$D_1\bar{D}$	2.83	1.48	9.05	3.32
	$D_1'\bar{D}$	0.75	0.62	0.68	0.36
Parameters in Ref. [47]	$D_1\bar{D}$	1.05	0.33	5.17	1.09
	$D_1'\bar{D}$	1.44	0.54	0.69	0.37

R Value: $R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$

$\sigma_{LO}(e^+e^- \to \psi_{c\bar{c}}(nD)) \sim |\Psi(0)|^2$



Christoph Florian talk, Tuesday

BES Collab., PLB 660, 315 (2008)

Dip around 4230 naturally explains the D-wave $c\bar{c}$ core of $Y(4230)$

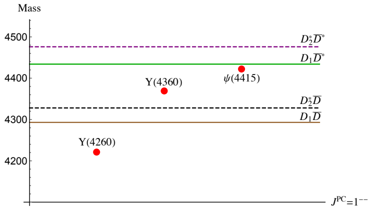
HQSS Partners of $Y(4230)$

- Hadronic interactions are independent of the heavy spin
- HQSS partners of LHCb P_c pentaquarks, and Z_bs
- $Y(4230)$ is dominant by $D_1\bar{D}$, emergence $D_1\bar{D}^*$ and $D_2^*\bar{D}^*$ structures

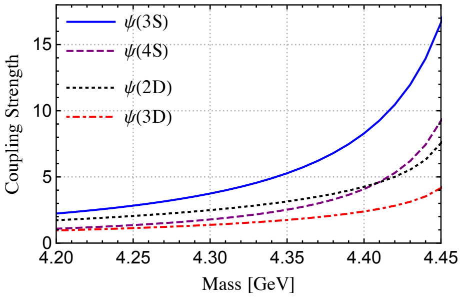
Motivation

$$\underbrace{M_{Y(4360)} - M_{Y(4230)}}_{=150.7\text{ MeV}} \simeq \underbrace{M_{D^*} - M_D}_{=140.6\text{ MeV}}$$

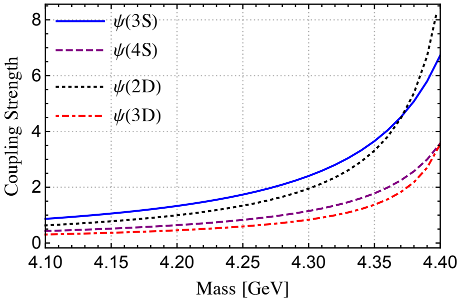
- Equal level spacing, consequence of HQSS
- In a given multiplet, same binding energy of molecules
- In heavy quark limit, $D_1\bar{D}$, $D_1\bar{D}^*$ and $D_2^*\bar{D}^*$ share the same potential



Results for $\psi(4415)$, $Y(4360)$



Coupling strength of $D_2^*\bar{D}^*$ with the $\psi(4415)$



Coupling strength of $D_1\bar{D}^*$ with the $Y(4360)$

- $\psi(4415)$ is likely to have a sizeable $D_2^*\bar{D}^*$ comp.
- But with S-wave $c\bar{c}$ core, prominent in R-value

- $Y(4360)$ showed largest coupling to $D_1\bar{D}^*$
- Its $c\bar{c}$ short-distance core is likely to be in D-wave

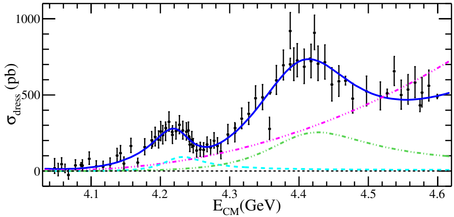
A Possible Test

- The natural decay of a molecular state is into its constituents
- $D_1 \to D^*\pi$ only; However, $D_2^* \to D^*\pi$ and $D_2^* \to D\pi$

State	$D^*\bar{D}\pi$	$D^*\bar{D}^*\pi$
$Y(4230)/D_1\bar{D}$	✓	✗
$Y(4360)/D_1\bar{D}^*$	✗	✓
$\psi(4415)/D_2^*\bar{D}^*$	✓	✓

What is measured so far? $e^+e^- \to D^*\bar{D}\pi$ BESIII Collaboration, PRL 122, 102002 (2019)

State	$D^*\bar{D}\pi$	$D^*\bar{D}^*\pi$
$Y(4230)$	✓ ✓	
$Y(4360)$	✗ ✗	
$\psi(4415)$	✓ ✓	



Key channel to measure $e^+e^- \to D^*\bar{D}^*\pi$

Summary

Hadron Spectroscopy:

- Future is very exciting with lots of new experimental data combined with exciting theory understanding!

Unquenched Quark Model:

- UQM is a powerful tool to explore the heavy quarkonia(-like) near thresholds
- Long- and short-distance structure of near-threshold states can be explored
- Predictions for $Y(4230)$ as $D_1\bar{D}$ are fully consistent with exp. measurements
- HQSS partners, identified as $\underbrace{Y(4360)}_{D_1\bar{D}^*}$ and $\underbrace{\psi(4415)}_{D_2^*\bar{D}^*}$, are awaiting for the measurement of $e^+e^- \to D^*\bar{D}^*\pi$

Thanks

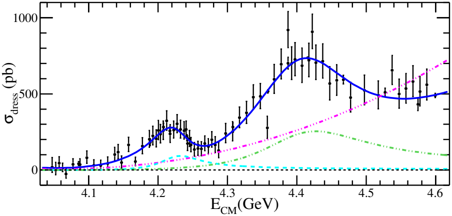
Extras

A Possible Test

- The natural decay of a molecular state is into its constituents
- In $D_1\bar{D}$ configuration, $\boxed{Y(4230) \to D^*\bar{D}\pi}$ is the dominant decay mode; recently observed by BESIII, providing a strong support
- Natural decay of $D_1\bar{D}^*$ molecule will be via $D_1 \to D^*\pi$, expected dominant decay mode of $\boxed{Y(4360) \to D^*\bar{D}^*\pi}$
- $Y(4360) \to D\bar{D}^*\pi$ is highly suppressed, not observed at BESIII and Belle
- However, $D_2^* \to D^*\pi$ and $D_2^* \to D\pi$. Therefore, $\psi(4415)/D_2^*\bar{D}^*$ must leave strong imprints in the $D\bar{D}^*\pi$ and $D^*\bar{D}^*\pi$ final states

What is measured so far? $e^+e^- \to D\bar{D}^*\pi$ BESIII Collaboration, PRL 122, 102002 (2019)

- $\boxed{Y(4230) \to D^*\bar{D}\pi}$ ✓
- $\boxed{Y(4360) \to D^*\bar{D}\pi}$ ✗
- $\boxed{\psi(4415) \to D^*\bar{D}\pi}$ ✓



Key channel to measure $\boxed{e^+e^- \to D^*\bar{D}^*\pi}$