Parton physics of mesons in the limit of large number of colors

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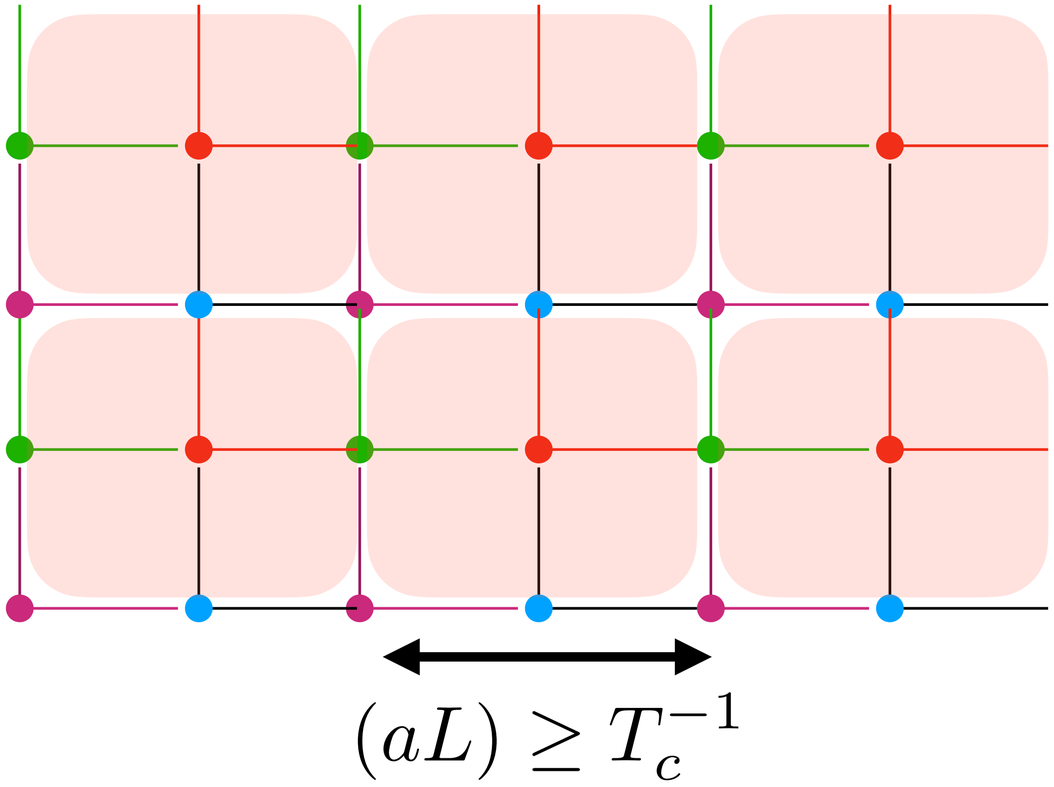
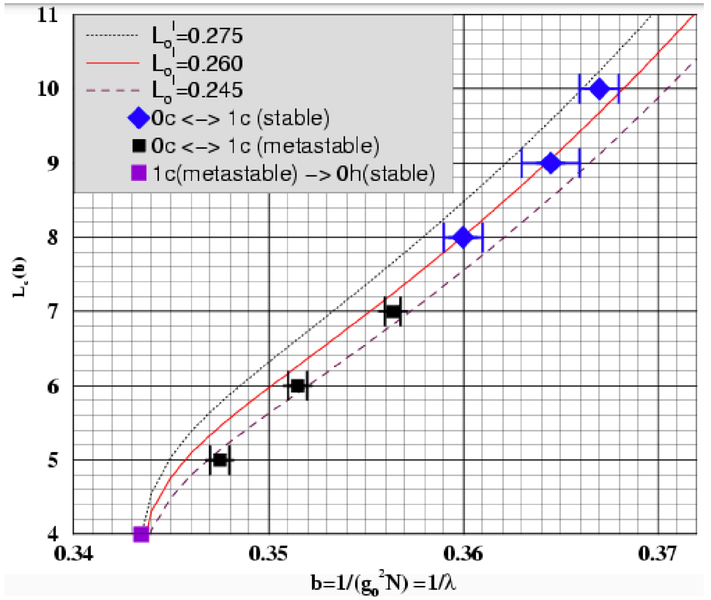
Why is QCD interesting in the limit of large number of colors?

- Strong interactions taken in isolation has no free parameters — non-abelian gauge fields interacting with massless fermions
- A classically scale invariant theory breaks scale invariance upon quantization and generates a rich spectrum of particles
- ’t Hooft suggested using the number of colors as a *free* parameter
- The theory in the limit of large number of colors has all the important features of QCD with three colors
- Numerical simulations of various physical quantities over three decades has shown that three color QCD is close enough to infinite color QCD

Deconfinement phase transition in 4D: partial reduction



$$(aL) \geq T_c^{-1}$$



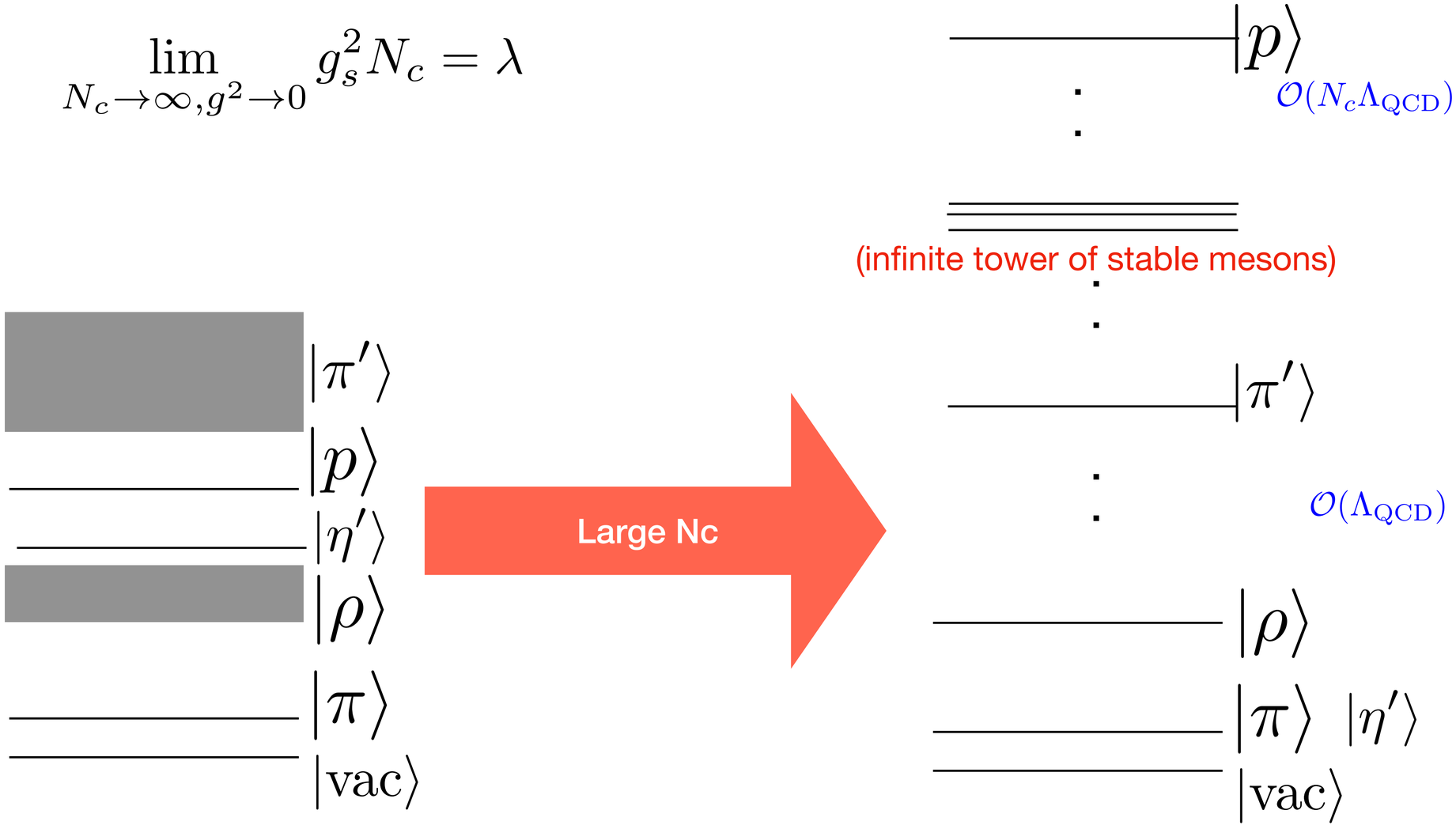
$$a \to 0, L_c \to \infty$$

Practically, one can expect to use L~ 12 or 14 to use very fine lattices and do “OPE without apology” !

Other ideas: e.g., Twisted Eguchi-Kawai to prevent center symmetry-breaking

Idealization of hadronic physics in ’t Hooft limit

$$\lim_{N_c\to\infty, g^2\to 0} g_s^2 N_c = \lambda$$



Long-distance behavior is a tower of noninteracting stable mesons:

Quark-hadron duality exact!

Numerology:
Some identification of scale in QCD and large-Nc required for GeV units.

$\sqrt{\sigma} = 440\ \mathrm{MeV}$

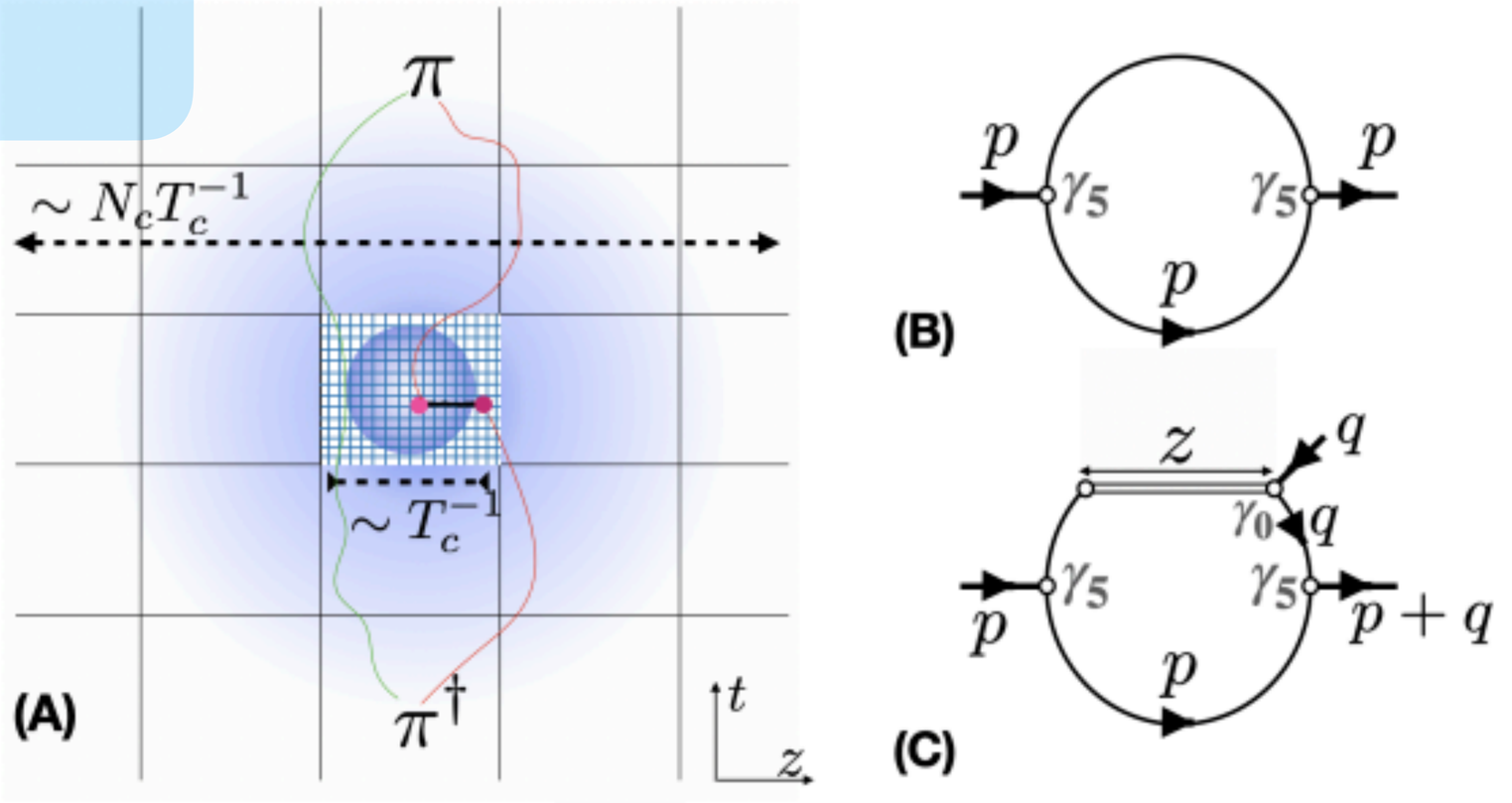


FIG. 1. (A) Schematic of large-N_c continuum reduction for quasi-PDF operator evaluated within a pion. The gauge fields on $\approx N_c\ell$ sized box are obtained as replicas of gauge fields within a ℓ sized box, with $\ell \approx T_c^{-1}$, the deconfinement temperature. The quarks hopping on such crystalline configuration are labeled by their positions in periodic ℓ^4 box and their Bloch momenta. The correlation functions in the larger box can be obtained using lattice implementation of momentum space Feynman diagrams that use quark propagators in ℓ^4 box. (B and C) The momentum space Feynman diagrams implemented directly on the lattice. The lines are quark propagators. The arrows show the off-shell 4-momentum injected at the vertices. The 2-point function of pion is shown in B. The 3-point function of quasi-PDF operator (double line) with pion creation and annihilation operators is shown in C.

Lattice setup for the first calculation

$$L = 8, N_c = 17$$

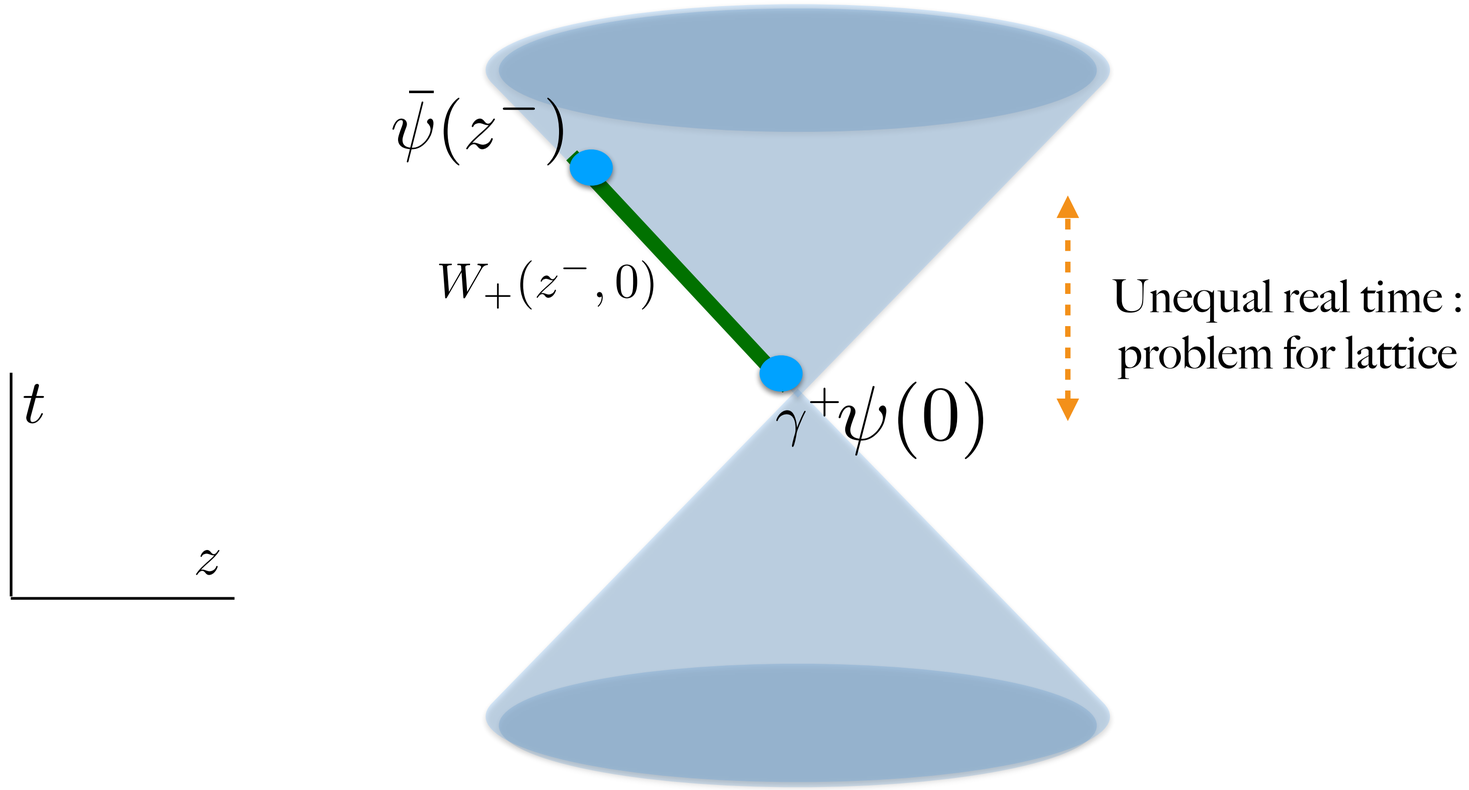
$$\sqrt{\sigma} a = 0.254$$

$$P_z = \frac{2\pi n_z}{(L \times N_c)}$$

$$n_z \in [0, 16]$$

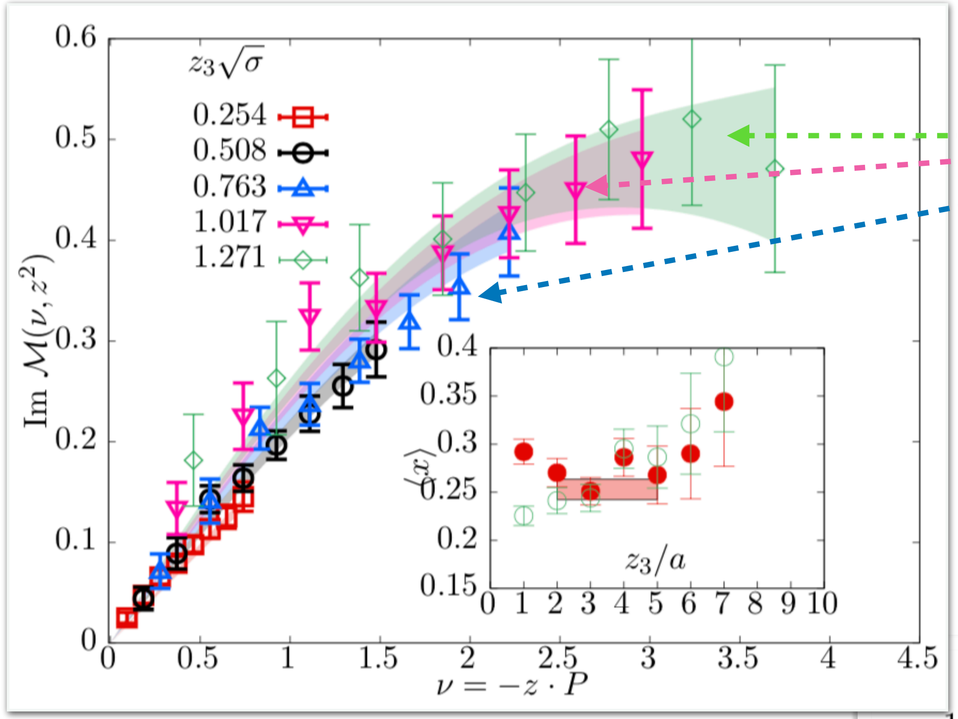
PDF as light-like separated q-$\bar{q}$ correlation

$$\mathcal{M}_{\rm LF}(z^- P^+, \mu) = \frac{1}{2P^+}\langle H(P)|\ \bar{\psi}(z^-) W_+(z^-, 0)\gamma^+\psi(0)\ |H(P)\rangle$$



$$f(x, \mu) = \int \frac{d\nu}{2\pi} e^{-ix\nu}\mathcal{M}_{\rm LF}(\nu, \mu)$$

Lattice data from M.C. and fits



Imaginary part of quasi-PDF matrix element

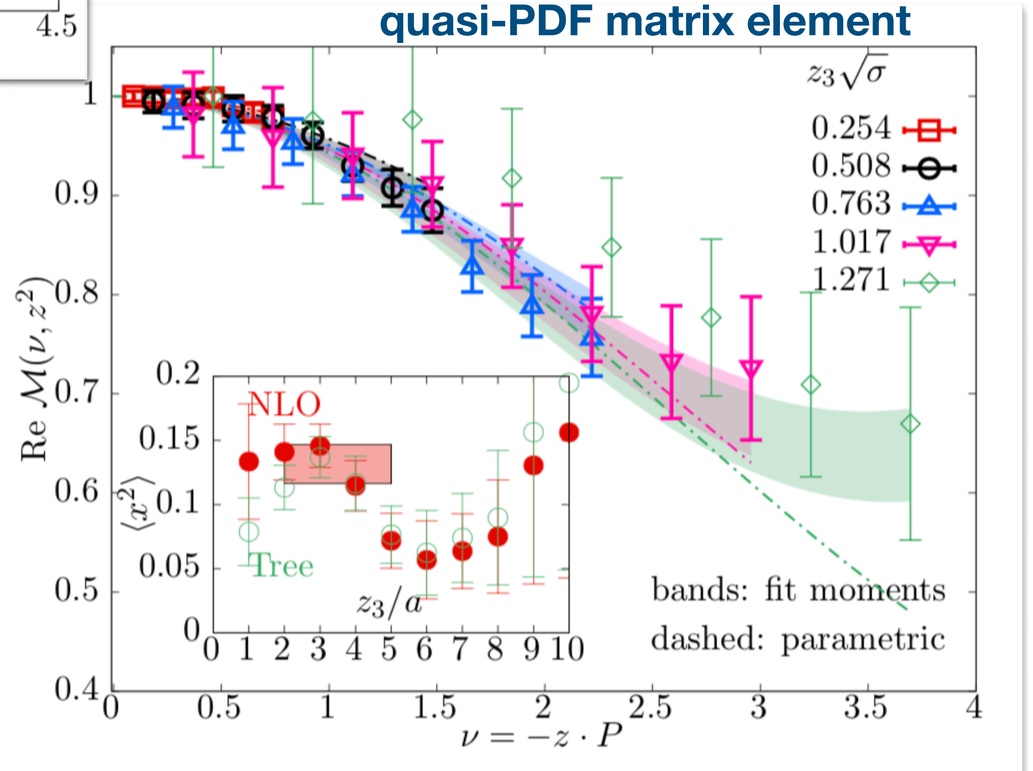
Fits to leading-twist large-N_c NLO OPE

$$\lambda_s = \left(\frac{11}{12\pi} \ln\left(\mu^2/\Lambda^2\right) \right)^{-1}$$

set values similar to QCD

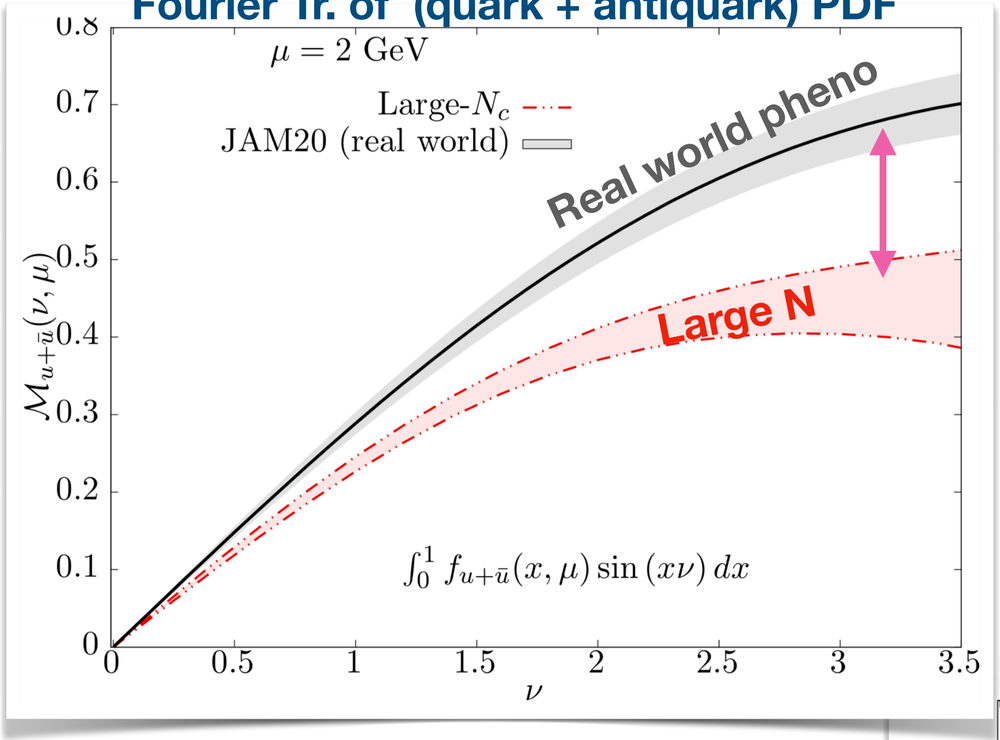
$$\Lambda \sim 0.3 \text{ GeV} \quad \mu = 2 \text{ GeV}$$

Real part of quasi-PDF matrix element



disconnected diagram
1/N_c suppressed.
Not easy in real-world LQCD.

Fourier Tr. of (quark + antiquark) PDF



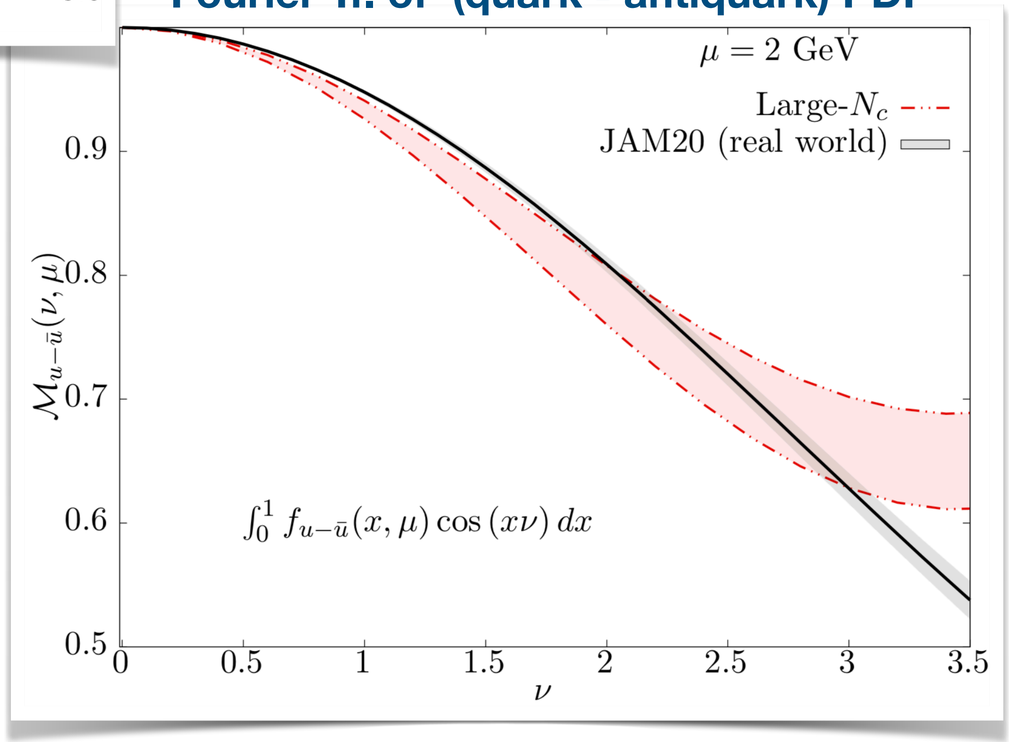
$$2\langle x\rangle_{u+\bar{u}} = 0.60(2)$$
$$\langle x\rangle_{g} = 0.40(2)$$
$$2\langle x\rangle_{u-\bar{u}} = 0.47(2)$$

$$2\langle x\rangle_{u+\bar{u}} = 0.50(2)$$
(infer) $$\langle x\rangle_{g} = 0.50(2)$$

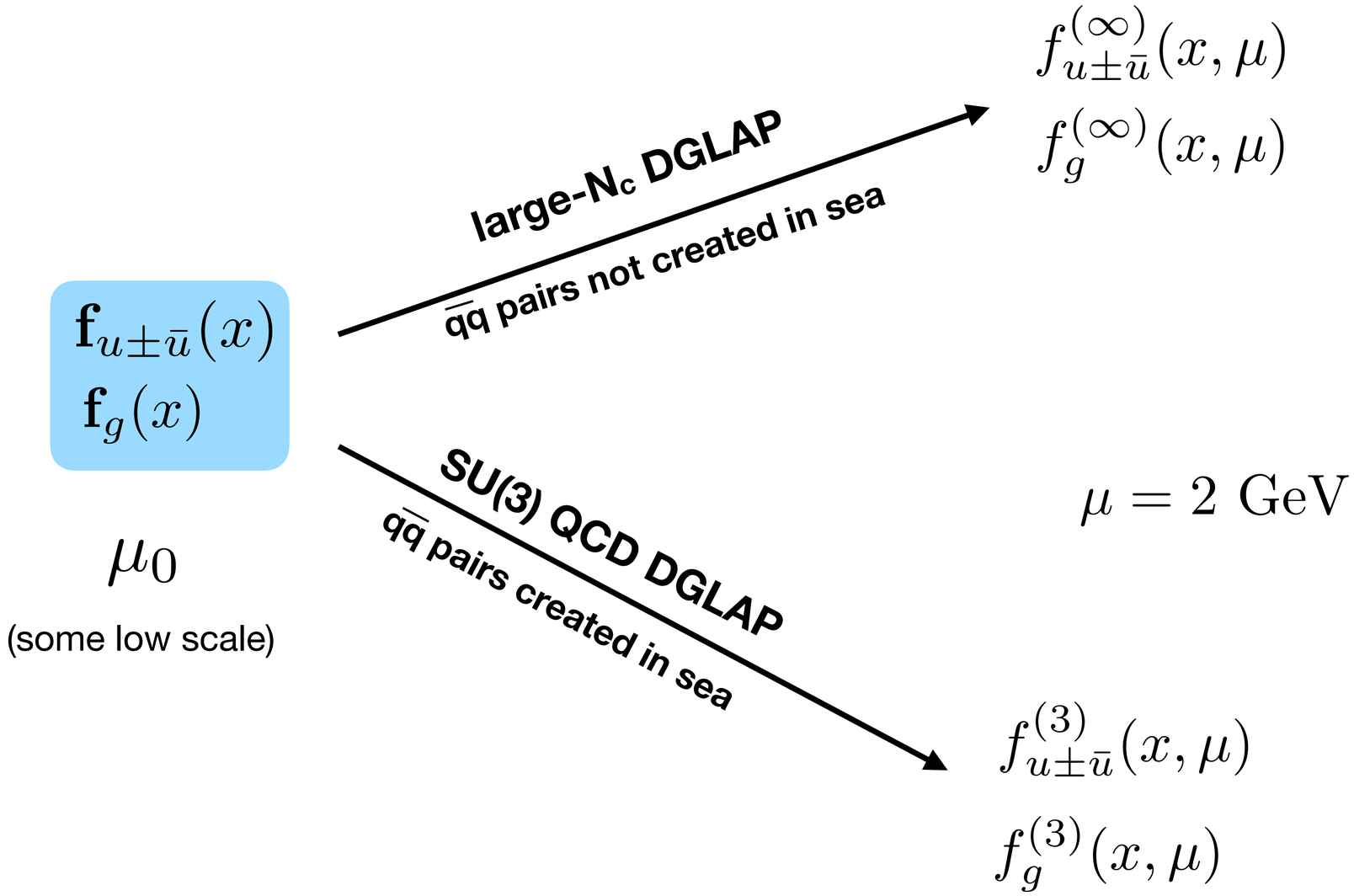
Large-N_c parton phenomenology

- ~50% pion momentum carried by gluons might be quite universal to Yang-Mills theories.
- Quantitative difference because of missing sea quarks in large-N pion?

Fourier Tr. of (quark - antiquark) PDF



Evolution as a mechanism for difference in large-Nc and QCD ?



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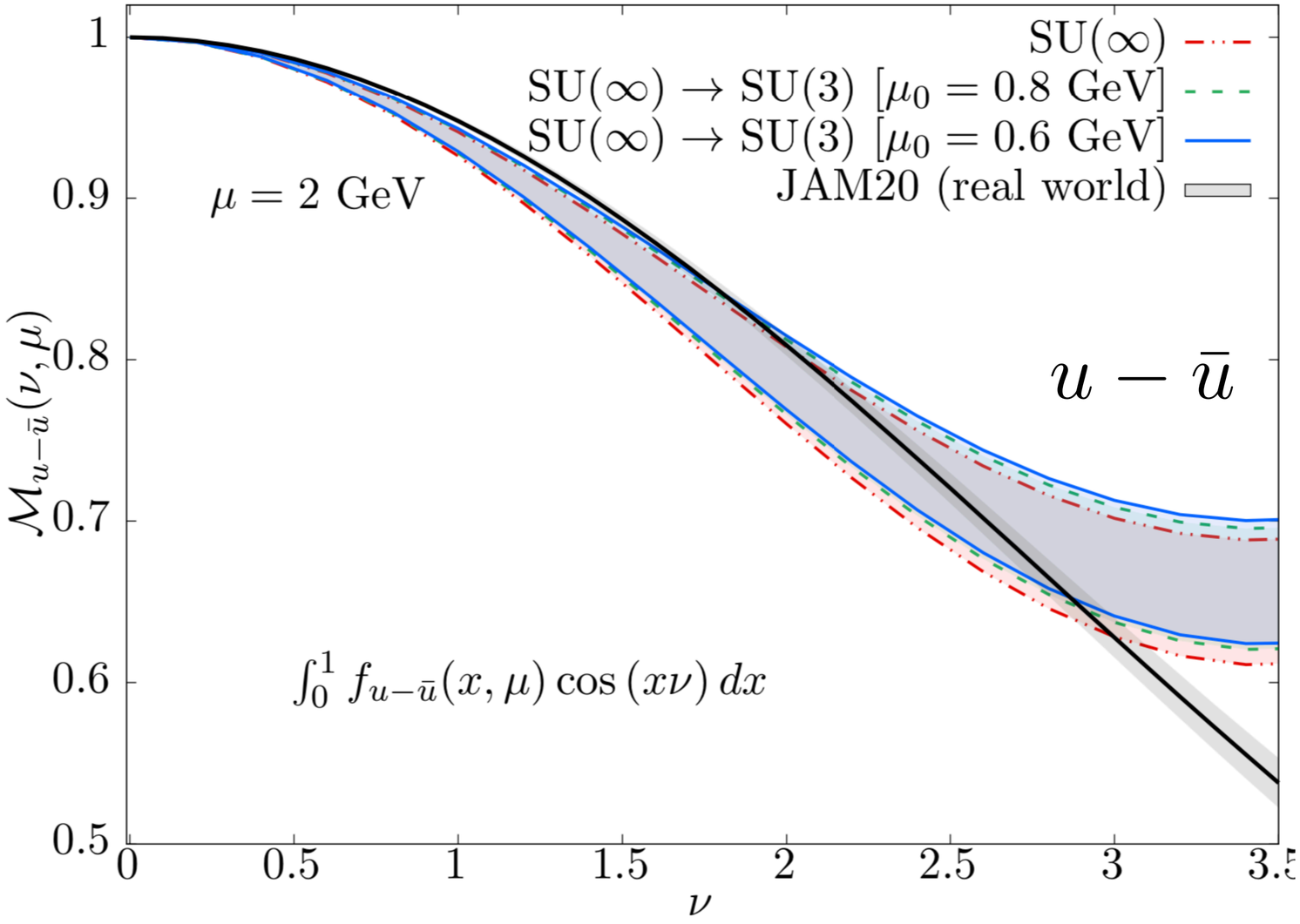
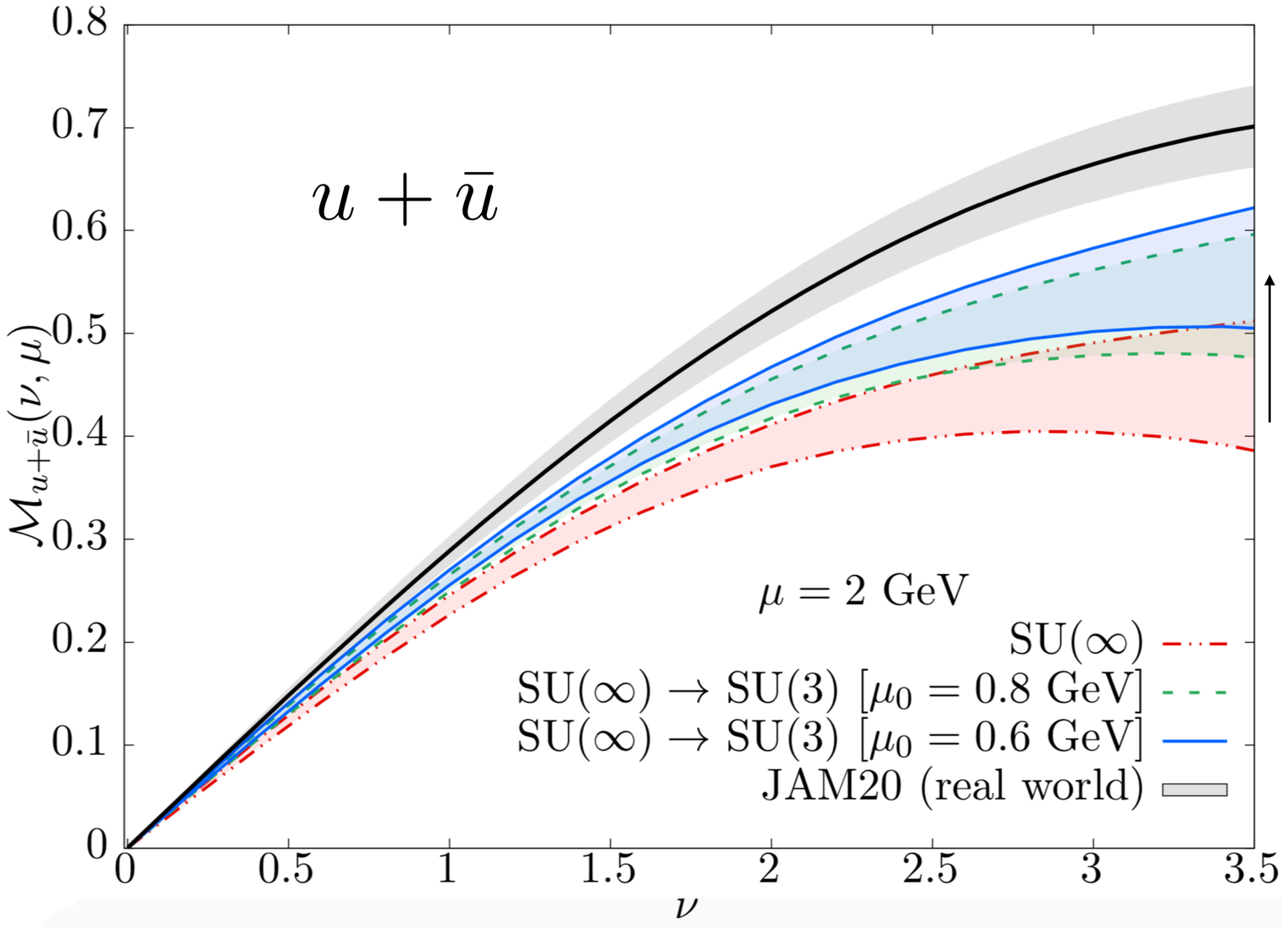


Disagreement small and effect of difference in DGLAP is also small

Evolution as a mechanism for difference in large-Nc and QCD ?



Add sea quarks radiatively

Disagreement substantial — effect of difference in DGLAP is in the right direction