Transverse single spin asymmetry from $g_T(x)$

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SB, Hatta, Li, Yang Phys. Rev. D 100 (2019) 9, 094027
SB, Hatta, Kaushik, Li Phys. Rev. D 104 (2021) 9, 094027
SB, Hatta, Kaushik, in preparation
SB, Kaushik, Vivoda, in preparation

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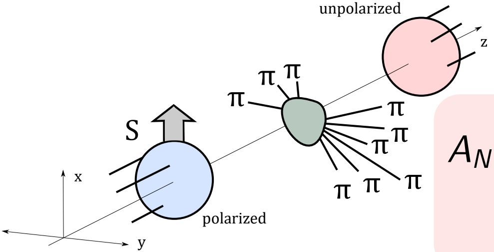
HRZZ
Croatian Science Foundation

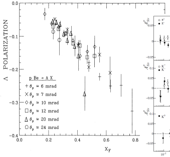
Transverse single spin asymmetry

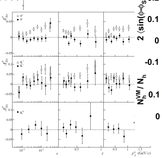
- hadron production is left-right asymmetric

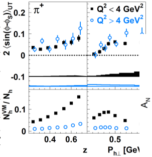


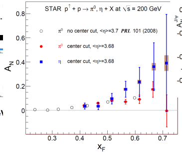
$$A_N \equiv \frac{\sigma(\uparrow) - \sigma(\downarrow)}{\sigma(\uparrow) + \sigma(\downarrow)}$$
$$\sim \sin(\phi_h - \phi_S)$$

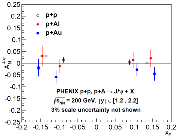
- observed in *ep*, *pp*, *pA* etc.. $A_N \sim$ a few %







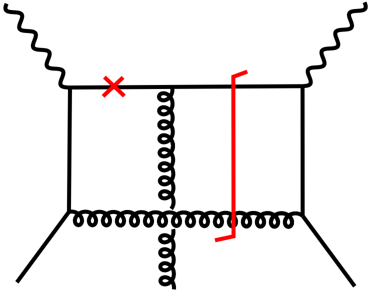




Origin of (T)SSA?

- amplitude is complex - SSA sensitive to the phase

  → interference diagrams



- we need twist-3 quantities such as quark-gluon-quark correlations

Origin of SSA?

- some known sources in collinear framework

ETQS functions: soft gluonic pole, soft fermionic pole, hard pole...

(TMD: closely related to the Sivers function)
Efremov, Teryaev, Sov. J. Nucl. Phys. **36**, 140 (1982)
Qiu, Sterman, Phys. Rev. D **59**, 014004 (1999)

twist-3 fragmentation functions

(TMD: closely related to the Collins function)
Yuan, Zhou Phys. Rev. Lett. **103** (2009) 052001
Kang, Yuan, Zhou Phys. Lett. B **691** (2010) 243
Kanazawa, Koike Phys. Rev. D **88** (2013) 074022

...

$\rightarrow$ require new PDFs and/or FFs..

Revisiting a 40 year old estimate

- the first pQCD estimate of SSA: Kane, Pumplin, Repko (1978) consider the two-loop box diagram

VOLUME 41, NUMBER 25 PHYSICAL REVIEW LETTERS 18 DECEMBER 1978

Transverse Quark Polarization in Large-p_T Reactions, e^+e^- Jets, and Leptoproduction: A Test of Quantum Chromodynamics

G. L. Kane
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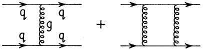
Physics Department, Michigan State University, East Lansing, Michigan 48823

(Received 5 July 1978)

Consider $qq \to qq$. In QCD the leading contribution to each helicity amplitude is given by single-gluon exchange, and the next order is the two-gluon–exchange box diagram, plus crossed box, etc., as shown in Fig. 1. There is a nonzero imaginary amplitude from the box diagram, and thus

$$P \propto g^4 \times g^2/g^4.$$

For $\alpha_s = g^2/4\pi$ of order $\frac{1}{5}$, there could be sizable polarization. However, because QCD is a vector-gluon theory the quark helicities are preserved for zero quark mass (m_q) so that $P = 0$.



(a)

$$A_N \sim \frac{\alpha_s m_q}{P_{hT}}$$

$\rightarrow$ believed to be negligible because $m_q \rightarrow 0$

$\rightarrow$ for $>$ 40 years there has been no attempt to go beyond this simple parametric estimate!

Revisiting a 40 year old estimate

- the first pQCD estimate of SSA: Kane, Pumplin, Repko (1978) consider the two-loop box diagram

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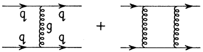
$$P \propto g^4 \times g^2/g^4.$$

For $\alpha_s = g^2/4\pi$ of order $\frac{1}{3}$, there could be sizable polarization. However, because QCD is a vector-gluon theory the quark helicities are preserved for zero quark mass (m_q) so that $P = 0$.



(a)

$$A_N \sim \frac{\alpha_s m_q}{P_{hT}}$$

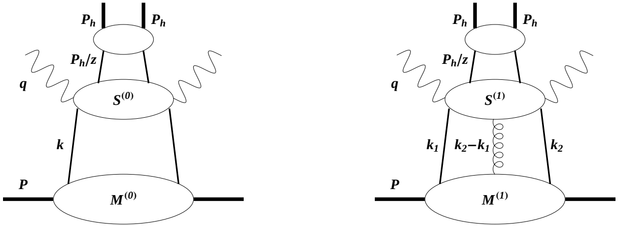
→ believed to be negligible because $m_q \to 0$

→ for $>$ 40 years there has been no attempt to go beyond this simple parametric estimate!

- **this work**: explicit computation

SB, Hatta, Li, Yang Phys. Rev. D 100 (2019) 9, 094027
SB, Hatta, Kaushik, Li, Phys. Rev. D 104 (2021) 9, 094027

SIDIS: Hadronic tensor



- hadronic tensor $W_{\mu\nu} = \int_z \frac{D(z)}{z^2} w_{\mu\nu}$

$$w_{\mu\nu} = \int_k M^{(0)}(k) S^{(0)}_{\mu\nu}(k) + \int_{k_1 k_2} M^{(1)}_\sigma(k_1, k_2) S^{(1)\sigma}_{\mu\nu}(k_1, k_2)$$

- two parton: $M^{(0)} \sim \langle P, S|\psi\bar{\psi}|P, S\rangle$
- three parton: $M^{(1)}_\sigma \sim \langle P, S|\psi A_\sigma \bar{\psi}|P, S\rangle$

Hadronic tensor

- hard part - collinear expansion

$$k^\mu = xP^\mu + k_T^\mu \qquad S_{\mu\nu}^{(0)}(k) = S_{\mu\nu}^{(0)}(xP) + \frac{dS_{\mu\nu}^{(0)}}{dk_T^\alpha}(xP)k_T^\alpha + \dots$$

- soft part

$$\begin{aligned} M^{(0)}(x) \quad &\sim \quad \not{P} f(x) + M_N e(x) + M_N (S\cdot n)\not{P}\gamma_5 \Delta f(x) \\ &+ \quad M_N^2 (S\cdot n)[\not{P}, \not{n}]\gamma_5 h_L(x) + [\not{P}, \not{S}_T]\gamma_5 h_1(x) + \\ &+ \quad M_N \not{S}_T \gamma_5 g_T(x) \end{aligned}$$

- no contribution from $g_T(x)$ at Born level

substitution $M^{(0)}(x) \to \gamma_5 \not{S}_\perp g_T(x)/2$ does not contribute to the cross section: the spinor trace Tr$[\cdots]$ involving γ_5 produces the factor i to the first term in (26) and the result is contracted with the real tensor $L_{\mu\nu}$ of (20) to derive the contribution to (19); but this yields a real quantity for the cross section only when another factor i is provided by the hard part $S^{(0)}(xp, q, P_h/z)$, which is impossible for the Born subprocess. Similarly, it is straightforward to see that the second term

Eguchi, Koike, Tanaka Nucl. Phys. B **763**, 198, (2007)

- higher-orders $\longrightarrow$ non-zero result

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Hadronic tensor

- all-order gauge invariant result

$$w_{\mu\nu} = \frac{M_N}{2}\int_x g_T(x)\mathrm{Tr}\left[\gamma_5 \not{S}_T S^{(0)}_{\mu\nu}(x)\right]$$
$$-\frac{M_N}{4}\int_x \tilde{g}(x)\mathrm{Tr}\left[\gamma_5 \not{P} S_T^\alpha \left.\frac{\partial S^{(0)}_{\mu\nu}(k)}{\partial k_T^\alpha}\right|_{k=xP}\right]$$
$$+\frac{iM_N}{4}\int_{x_1,x_2}\mathrm{Tr}\left[\left(\not{P}\epsilon^{\alpha P n S_T}\frac{G_F(x_1,x_2)}{x_1-x_2} + i\gamma_5\not{P}S_T^\alpha\frac{\tilde{G}_F(x_1,x_2)}{x_1-x_2}\right)S^{(1)}_{\mu\nu\alpha}(x_1,x_2)\right]$$

Ratcliffe Nucl. Phys. B 264, 493 (1986)
SB, Hatta, Li, Yang Phys. Rev. D 100 (2019) 9, 094027

- intrinsic $g_T \sim \langle\bar{\psi}\psi\rangle$
- kinematical $\tilde{g} \sim \langle\bar{\psi}\partial^\mu\psi\rangle$ ($\sim$ first moment of worm-gear TMD)
- dynamical $G_F \sim \langle\bar{\psi}F^{\mu\nu}\psi\rangle$

Wandzura-Wilczek approximation

- g_T and $\tilde{g}$ have a twist-two piece

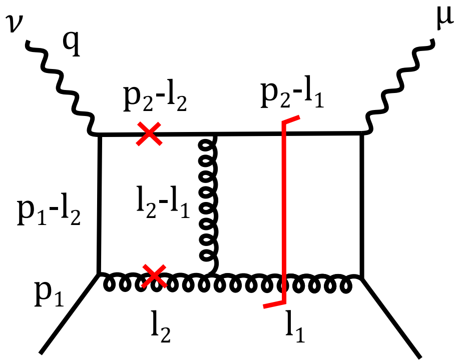
$$g_T(x) = \int_x^1 \frac{dx_1}{x_1} \Delta q(x_1) + \ldots \quad \tilde{g}(x) = -2x \int_x^1 \frac{dx_1}{x_1} \Delta q(x_1) + \ldots .$$

- SSA from twist-two PDFs and twist-two FFs

$$\Delta\sigma \sim \Delta q \otimes H \otimes D_1$$

- Sivers and Collins asymmetry NOT from Sivers and Collins functions
- Δq - twist-two helicity PDF
- D_1 - twist-two unpolarized FF

$S^{(0)}_{\mu\nu}$ at two loops - return of the box



→ can get a phase

- check all cuts, check all diagrams

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Hard coefficients

- all hard coefficients $\Delta\hat{\sigma}$ extracted in completely closed form expressions

$$\Delta\hat\sigma^{qq}_{D8} = \frac{(N_c^2-1)\,\hat x\hat z}{2N_c^2 Q(1-\hat z)^2}\left[(1-\hat z)\left(1-\hat x+\hat z-3\hat x\hat z+N_c^2(1-\hat x-\hat z+3\hat x\hat z)\right)+2(1-2\hat x)\,\hat z\log(\hat z)\right],$$

$$\Delta\hat\sigma^{qq}_{D9} = \frac{(N_c^2-1)(1-\hat x)\,\hat x\hat z}{2N_c^2 q_T(1-\hat z)^2}\left[(1-\hat z)\left(3N_c^2(1-\hat z)+3\hat z-1\right)-2(1-2\hat z)\log(\hat z)\right],$$

$$\Delta\hat\sigma^{qq}_{1} = \frac{N_c^2-1}{2N_c^2 q_T(1-\hat z)\,\hat z}\Big[(1-\hat z)\left(\hat z\hat x\left(\hat x(3+10\hat z)-3(1+\hat z)\right)-1\right) + N_c^2\hat z\left(\hat x^2\left(3+2\hat z(5\hat z-6)\right)-1-3\hat x(1-\hat z)^2\right)+6\hat x(2\hat x-1)\,\hat z^2\log(\hat z)\Big],$$

$$\Delta\hat\sigma^{qq}_{2} = \frac{(N_c^2-1)\,\hat x}{N_c^2 q_T(1-\hat z)}\left[(1-\hat z)\left((1+N_c^2)(-1+\hat x)+(N_c^2-1)(1-3\hat x)\,\hat z\right)+2(2\hat x-1)\,\hat z\log(\hat z)\right],$$

$$\Delta\hat\sigma^{qq}_{3} = \frac{(N_c^2-1)\,\hat x}{4N_c^2 Q(1-\hat x)(1-\hat z)^2}\Big[(1-\hat z)\Big\{(1-\hat x)\left(5\hat x+N_c^2(2-11\hat x)\right)\hat z-(1+N_c^2)(1-\hat x)^2 - (N_c^2-1)\left(1+\hat x(14\hat x-13)\right)\hat z^2\Big\}-2\hat z\left(1-\hat z-\hat x\left(5-4\hat x-8(1-\hat x)\hat z\right)\right)\log(\hat z)\Big],$$

$$\Delta\hat\sigma^{qq}_{4} = \frac{(N_c^2-1)\,\hat x}{2N_c^2 q_T(1-\hat z)^2}\Big[(1-\hat z)\Big\{3(1-\hat z)^2+\hat x\left(-3+(5-4\hat z)\hat z\right) - N_c^2(1-\hat z)\left(2-3\hat z+\hat x(-2+4\hat z)\right)\Big\}-2\left(\hat x-(1-\hat z)^2-2\hat x(1-\hat z)\,\hat z\right)\log(\hat z)\Big],$$

$$\Delta\hat\sigma^{qq}_{8} = \frac{(N_c^2-1)\,\hat x}{4N_c^2 Q(1-\hat x)(1-\hat z)^2}\Big[(1+N_c^2)(1-\hat x)^2+(1-\hat x)\left(5-10\hat x+N_c^2(6\hat x-5)\right)\hat z - \left(9+\hat x(15\hat x-26)-N_c^2\left(7+9(\hat x-2)\hat x\right)\right)\hat z^2 - (N_c^2-1)\left(3+\hat x(4\hat x-9)\right)\hat z^3-2\hat z\left(\hat x(7-4\hat x-2\hat z)+\hat z-3\right)\log(\hat z)\Big],$$

$$\Delta\hat\sigma^{qq}_{9} = \frac{(N_c^2-1)\,\hat x}{2N_c^2 q_T(1-\hat z)^2}\Big[(1-\hat z)\left(2N_c^2\left(1-\hat x(1-\hat z)-2\hat z\right)(1-\hat z)-3+\hat z(9-4\hat z)+\hat x\left(3-(7-2\hat z)\hat z\right)\right) - 2\left(1+(\hat z-3)\hat z+\hat x(2\hat z-1)\right)\log(\hat z)\Big],$$

$$\Delta\hat\sigma^{qg}_{D8} = \frac{(N_c^2-1)\,\hat x}{2N_c^2 Q\hat z}\left[\hat z\left(-2+\hat z-N_c^2\hat z+\hat x\left(4-3\hat z+N_c^2(-2+3\hat z)\right)\right)-2(1-2\hat x)(1-\hat z)\log(1-\hat z)\right],$$

$$\Delta\hat\sigma^{qg}_{D9} = \frac{(N_c^2-1)(1-\hat x)\,\hat x(1-\hat z)}{2{N_c}^2 q_T\hat z^2}\left[\hat z\left(2+3(N_c^2-1)\,\hat z\right)+(2-4\hat z)\log(1-\hat z)\right],$$

$$\Delta\hat\sigma^{qg}_{1} = \frac{N_c^2-1}{2N_c^2 q_T\hat z^2}\Big[\hat z\left(1+\hat x^2\left(-13+23\hat z-10\hat z^2\right)+3\hat x\left(2-3\hat z+\hat z^2\right)\right) - N_c^2(1-\hat z)\left(-1-3\hat x\hat z^2+\hat x^2\left(1-8\hat z+10\hat z^2\right)\right)-6\hat x(-1+2\hat x)(1-\hat z)^2\log(1-\hat z)\Big],$$

$$\Delta\hat\sigma^{qg}_{2} = \frac{(N_c^2-1)\,\hat x(1-\hat z)}{N_c^2 q_T\hat z^2}\left[\hat z\left(2-\hat z+N_c^2\hat z-\hat x\left(4-3\hat z+{N_c}^2(-2+3\hat z)\right)\right)+2(1-2\hat x)(1-\hat z)\log(1-\hat z)\right],$$

$$\Delta\hat\sigma^{qg}_{3} = -\frac{(N_c^2-1)\,\hat x}{4{N_c}^2 Q(1-\hat x)\,\hat z^2}\Big[\hat z\Big\{(-1+\hat z)\left(2+(N_c^2-1)\,\hat z\right)+\hat x\left(10-24\hat z+13\hat z^2+N_c^2\left(-4+16\hat z-13\hat z^2\right)\right) + \hat x^2\left(-8+23\hat z-14\hat z^2+N_c^2\left(4-17\hat z+14\hat z^2\right)\right)\Big\} + 2(1-\hat z)\left(-1+\hat x(5-8\hat z)+\hat z+\hat x^2(-4+8\hat z)\right)\log(1-\hat z)\Big],$$

$$\Delta\hat\sigma^{qg}_{4} = -\frac{(N_c^2-1)\,\hat x(1-\hat z)}{2{N_c}^2 q_T\hat z^3}\Big[\hat z\left(2+(4N_c^2-3)\,\hat z-3(N_c^2-1)\,\hat z^2+\hat x\left(-2+(3-2{N_c}^2)\,\hat z+4(N_c^2-1)\,\hat z^2\right)\right) + 2\left((1-\hat z)^2-\hat x\left(1-2\hat z+2\hat z^2\right)\right)\log(1-\hat z)\Big],$$

$$\Delta\hat\sigma^{qg}_{8} = \frac{(N_c^2-1)\,\hat x}{4N_c^2 Q(1-\hat x)\,\hat z^2}\Big[\hat z\Big\{6-(1+{N_c}^2)\,\hat z+3(N_c^2-1)\,\hat z^2+\hat x\left(-22+8\hat z+9\hat z^2+{N_c}^2\left(4-9\hat z^2\right)\right) + \hat x^2\left(16-9\hat z-4\hat z^2+N_c^2\left(-4+3\hat z+4\hat z^2\right)\right)\Big\}+2(1-\hat z)\left(3+8\hat x^2+\hat z-\hat x(11+2\hat z)\right)\log(1-\hat z)\Big],$$

$$\Delta\hat\sigma^{qg}_{9} = \frac{(N_c^2-1)\,\hat x(1-\hat z)}{2N_c^2 q_T\hat z^3}\Big[\hat z\left((4-N_c^2)\,\hat z-4-4(N_c^2-1)\,\hat z^2+\hat x\left(4+3(N_c^2-2)\,\hat z+2(N_c^2-1)\,\hat z^2\right)\right) + 2\left(-2+2\hat x+3\hat z-4\hat x\hat z+\hat z^2\right)\log(1-\hat z)\Big].$$

$$\Delta\hat\sigma^{gq}_{D8} = \frac{2(1-\hat x)\,\hat x}{N_c Q(1-\hat z)^2\hat z}\left[(1-\hat x)\left(\hat z\log(\hat z)-(1-\hat z)\log(1-\hat z)\right)+\hat x\hat z(1-\hat z)(1-2\hat z)\right],$$

$$\Delta\hat\sigma^{gq}_{D9} = \frac{2(1-\hat x)^2\hat x}{N_c q_T(1-\hat z)^2\hat z^2}\left[(1-\hat z)^2\log(1-\hat z)+\hat z^2\log(\hat z)+\hat z(1-\hat z)\left(\hat z^2+(1-\hat z)^2\right)\right],$$

$$\Delta\hat\sigma^{gq}_{1} = \frac{(1-\hat x)}{N_c q_T(1-\hat z)\,\hat z^2}\Big[(1-2\hat z)\left(1+2\hat x^2(1-2\hat z)^2-(1-\hat z)\,\hat z-2\hat x\left(1-(1-\hat z)\,\hat z\right)\right) + 6(1-\hat x)\,\hat x\left((1-\hat z)\log(1-\hat z)-\hat z\log(\hat z)\right)\Big],$$

$$\Delta\hat\sigma^{gq}_{2} = \frac{4(1-\hat x)\,\hat x}{N_c q_T(1-\hat z)\,\hat z^2}\left[(1-\hat z)\left(\hat x(2\hat z-1)\,\hat z+(1-\hat x)\log(1-\hat z)\right)-(1-\hat x)\,\hat z\log(\hat z)\right],$$

$$\Delta\hat\sigma^{gq}_{3} = \frac{\hat x}{N_c Q(1-\hat z)^2\hat z^2}\Big[(1-\hat z)\,\hat z\left(1-2(1-\hat z)\,\hat z+\hat x\left(-6+(13-12\hat z)\,\hat z\right)+\hat x^2\left(5-12(1-\hat z)\,\hat z\right)\right) + (1-\hat x)\left((1-\hat z)\left(1-\hat z+\hat x(-3+4\hat z)\right)\log(1-\hat z)-\hat z\left(1-\hat z-\hat x(1-4\hat z)\right)\log(\hat z)\right)\Big],$$

$$\Delta\hat\sigma^{gq}_{4} = \frac{2(1-\hat x)\,\hat x}{N_c q_T(1-\hat z)^2\hat z^3}\Big[(1-\hat z)\Big\{\hat z\left(\hat x-2\hat x\hat z\left(2+\hat z(-3+2\hat z)\right)-(1-\hat z)\left(1+\hat z(-3+4\hat z)\right)\right) - \left(1-\hat x(1-\hat z)\right)(1-\hat z)\log(1-\hat z)\Big\}-\hat x\hat z^3\log(\hat z)\Big],$$

$$\Delta\hat\sigma^{gq}_{8} = \frac{\hat x}{N_c Q(1-\hat z)^2\hat z^2}\Big[(-1+\hat z)\,\hat z\left(-3+10\hat x-7\hat x^2+(-2+\hat x)\,\hat z+2\left(1+2(-1+\hat x)\,\hat x\right)\hat z^2\right) + (1-\hat x)\left((1-\hat z)\left(3-3\hat z+\hat x(-5+2\hat z)\right)\log(1-\hat z)-\hat z\left(1-3\hat z+\hat x(-1+2\hat z)\right)\log(\hat z)\right)\Big],$$

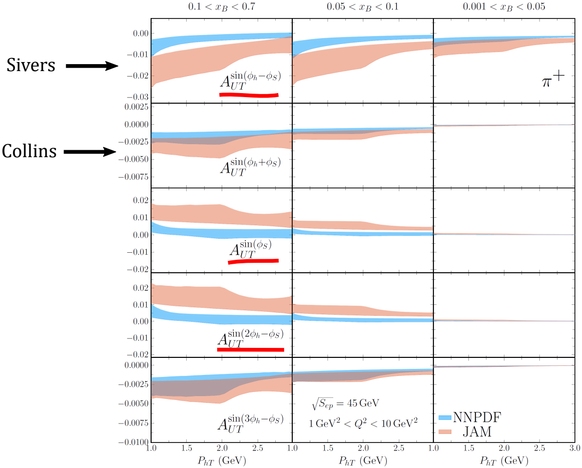
$$\Delta\hat\sigma^{gq}_{9} = \frac{-2(1-\hat x)\,\hat x}{N_c q_T(1-\hat z)^2\hat z^3}\Big[\left(2+\hat x(-2+\hat z)-2\hat z\right)(1-\hat z)^2\log(1-\hat z)+\hat z^2\left(1-\hat x-(2-\hat x)\,\hat z\right)\log(\hat z) - \hat z(1-\hat z)\left(-2+3\hat z+(-1+\hat z)\left(-2\hat x+\hat x\hat z+2(-1+\hat x)\,\hat z^2\right)\right)\Big].$$

- gluon from the loop can be collinear to the proton

  $\rightarrow$ divergence is cancelled between $S^{(0)}_{\mu\nu}$ and $\dfrac{dS^{(0)}_{\mu\nu}}{dk_T^{\alpha}}$

SB, Hatta, Kaushik, Li, Phys. Rev. D 104 (2021) 9, 094027

All A_{UT}s @ EIC



- three A_{UT}s at a few percent level: $\sin(\phi_h - \phi_S)$ (Sivers), $\sin(\phi_S)$ and $\sin(2\phi_h - \phi_S)$

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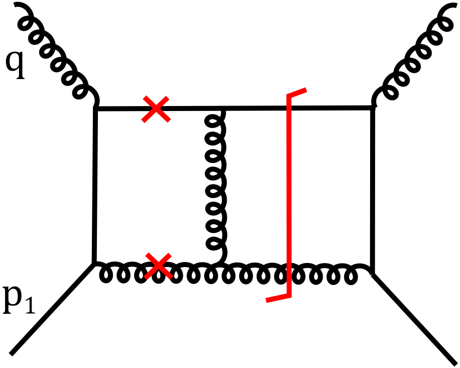
pp: forward limit

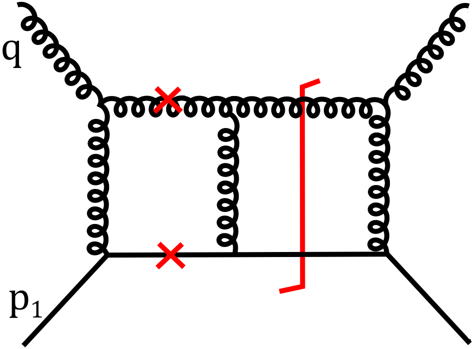
- a diagram similar to SIDIS



SB, Hatta, Kaushik, Li, in preparation

pp: forward limit

- example of an additional diagram



SB, Hatta, Kaushik, Li, in preparation

Hard coefficients

$$E_h \frac{d\Delta\sigma}{d^3P_h} = \frac{\alpha_S^3 M_N}{S} \epsilon^{P n P_h S_T} \int \frac{dz}{z^3} D(z) \int \frac{dx'}{x'} G(x')$$
$$\times \int \frac{dx}{x} \delta\left(\hat{s} + \hat{t} + \hat{u}\right) \left(-\frac{1}{\hat{u}}\right) \left[x^2 \frac{\partial g_T(x)}{\partial x} \Delta\hat{\sigma}_D^{qq} + x g_T(x) \Delta\hat{\sigma}^{qq}\right]$$

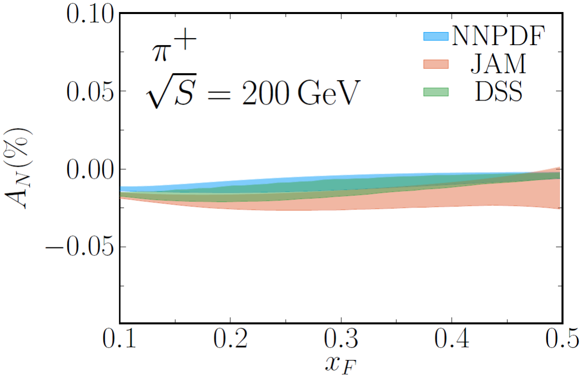
$$\Delta\hat{\sigma}_D^{qq} = 0\,,$$
$$\Delta\hat{\sigma}^{qq} = \frac{N_c}{4}\left(\frac{3\hat{s}^2}{\hat{t}^2} + \frac{8\hat{s}}{\hat{t}} + 9 + \frac{3\hat{t}}{\hat{s}}\right) - \frac{1}{4N_c}\left(\frac{\hat{s}^2}{\hat{t}^2} + 3 + \frac{3\hat{t}}{\hat{s}}\right) + \frac{1}{4N_c^3}\frac{\hat{s}}{\hat{t}}$$

- gluon from the loop can be collinear to either of the two protons

  $\rightarrow$ divergence is cancelled between $S_{\mu\nu}^{(0)}$ and $\frac{dS_{\mu\nu}^{(0)}}{dk_T^\alpha}$

SB, Hatta, Kaushik, Li, in preparation

Results @RHIC



- sub-percent asymmetry??

$\rightarrow$ underlying reason?

SB, Hatta, Kaushik, Li, in preparation

Energy dependence

- SIDIS

$$A_N^{ep} \sim \frac{\alpha_s M_N}{T} \frac{x\Delta f}{f}$$

$\rightarrow$ energy independent!

- forward *pp*

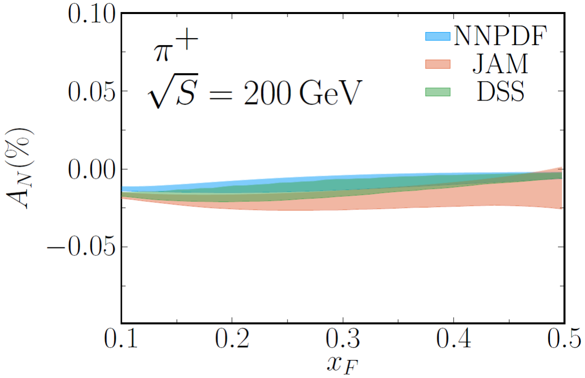
$$A_N^{pp} \sim \frac{\alpha_s M_N}{S} \frac{x\Delta f}{f}$$

$\rightarrow$ suppression with energy!

SB, Hatta, Kaushik, Li, in preparation

Energy dependence

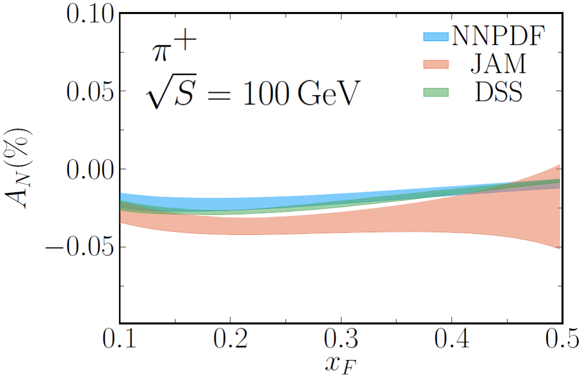
- try lower $\sqrt{S}$



SB, Hatta, Kaushik, Li, in preparation

Energy dependence

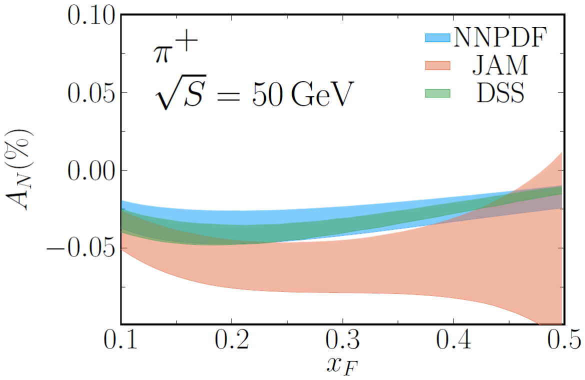
- try lower $\sqrt{S}$



SB, Hatta, Kaushik, Li, in preparation

Energy dependence

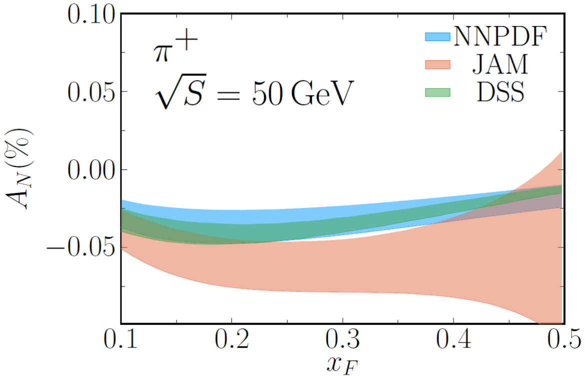
- try lower $\sqrt{S}$



SB, Hatta, Kaushik, Li, in preparation

Energy dependence

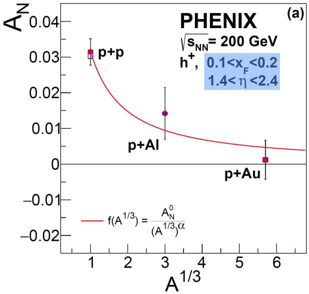
- try lower $\sqrt{S}$

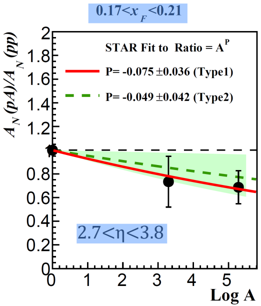


- need other channels
- apply to the old fixed target and BRAHMS data from RHIC?

SB, Hatta, Kaushik, Li, in preparation

SSA in pA: the data



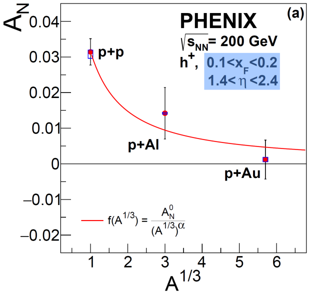


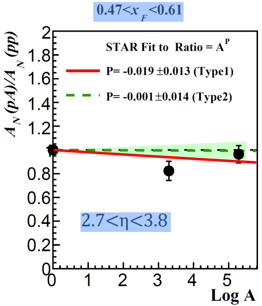
- PHENIX: nuclear suppression of $A_N \sim A^{-1/3}$
- STAR: almost A independent A_N

  covers different kinematics $\to$ not mutually incompatible

PHENIX, Phys. Rev. Lett. 123 (2019) 122001
STAR, Phys. Rev. D 103 (2021) 7, 072005

SSA in pA: the data





- PHENIX: nuclear suppression of $A_N \sim A^{-1/3}$
- STAR: almost A independent A_N

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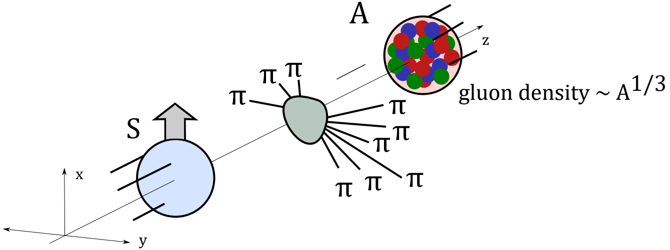
PHENIX, Phys. Rev. Lett. 123 (2019) 122001
STAR, Phys. Rev. D 103 (2021) 7, 072005

SSA in *pA*



- SSA largest in the forward direction

  $\rightarrow$ small-x in the nuclei (Color Glass Condensate)

  $\rightarrow$ interplay between transverse spin physics and small-x physics

Boer, Dumitru, Hayashigaki, Phys. Rev. D 74 (2006) 074018
Kang, Yuan, Phys. Rev. D 84 (2011) 034019
Kovchegov, Sievert, Phys. Rev. D 86 (2012) 034028
Hatta, Xiao, Yoshida, Yuan, Phys. Rev. D 94 (2016) 5, 054013
Hatta, Xiao, Yoshida, Yuan, Phys. Rev. D 95 (2017) 1, 014008
SB, Hatta, Phys. Rev. D 99, no. 9, 094012 (2019)

SSA in pA - models

- ETQS?
  Hatta, Xiao, Yoshida, Yuan, Phys. Rev. D 94 (2016) 5, 054013
  $\rightarrow A_N \sim A^{-1/3}$ but ETQS not favored by global fits
  JAM, Phys. Rev. D 102 (2020) 5, 054002

- twist-3 FFs?
  Hatta, Xiao, Yoshida, Yuan, Phys. Rev. D 95 (2017) 1, 014008
  $\rightarrow A_N \sim A^{-1/3}$ parametrically, favored by global fits but A dependence washed away by high-energy evolution
  SB, Hatta, Phys. Rev. D 99, no. 9, 094012 (2019)

- QCD Odderon?
  Kovchegov, Sievert, Phys. Rev. D 86 (2012) 034028
  $\rightarrow A_N \sim A^{-7/6}$ parametrically!

Odderon mechanism for SSA

PHYSICAL REVIEW D **86,** 034028 (2012)

New mechanism for generating a single transverse spin asymmetry

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(Received 24 February 2012; published 29 August 2012)

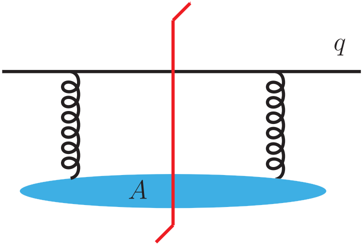
We propose a new mechanism for generating a single transverse spin asymmetry (STSA) in polarized proton-proton and proton-nucleus collisions in the high-energy scattering approximation. In this framework the STSA originates from the $q \to qG$ splitting in the projectile (proton) light-cone wave function followed by a perturbative (C-odd) odderon interaction, together with a C-even interaction, between the projectile and the target. We show that some aspects of the obtained expression for the STSA of the

- dipole gluon distribution of the nuclei

$$\mathcal{S}(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp) = \frac{1}{N_c}\mathrm{tr}\left\langle U(\boldsymbol{x}_\perp)U^\dagger(\boldsymbol{y}_\perp)\right\rangle$$
$$= \mathcal{P}(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp) + i\mathcal{O}(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp)$$

- Kovchegov, Sievert (KS): can the Odderon supply a necessary phase for SSA?

Odderon mechanism at LO



$$\boldsymbol{r}_\perp \equiv \boldsymbol{x}_\perp - \boldsymbol{y}_\perp$$

$$\boldsymbol{b}_\perp \equiv \frac{\boldsymbol{x}_\perp + \boldsymbol{y}_\perp}{2}$$

$$E_q \frac{d\Delta\sigma}{d^3q} \sim \int_{\boldsymbol{r}_\perp \boldsymbol{b}_\perp} e^{i\boldsymbol{q}_\perp \cdot \boldsymbol{r}_\perp} \mathcal{O}(\boldsymbol{r}_\perp, \boldsymbol{b}_\perp) \mathcal{H}^{(0)} = 0$$

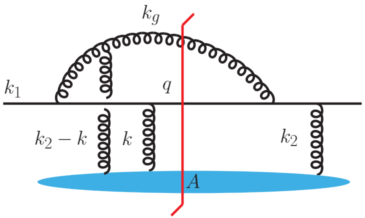
$$\mathcal{O}(\boldsymbol{r}_\perp, \boldsymbol{b}_\perp) \sim (\boldsymbol{r}_\perp \cdot \boldsymbol{b}_\perp) \mathcal{O}_1(\boldsymbol{r}_\perp^2, \boldsymbol{b}_\perp^2)$$

$\rightarrow$ vanishes at leading order

(due to $\boldsymbol{b}_\perp$ integral)

Odderon mechanism at NLO

- KS: non-zero contribution comes from interference diagram



$$E_q \frac{d\Delta\sigma}{d^3 q} \sim \int_{\boldsymbol{k}_\perp \boldsymbol{k}_{2\perp}} \int_{\boldsymbol{r}_\perp \boldsymbol{r}'_\perp \boldsymbol{b}_\perp} e^{i\boldsymbol{k}_\perp \cdot \boldsymbol{r}_\perp} e^{i\boldsymbol{k}_{2\perp} \cdot \boldsymbol{r}'_\perp}$$

$$\times \mathcal{P}(\boldsymbol{r}_\perp, \boldsymbol{b}_\perp)\mathcal{O}(\boldsymbol{r}'_\perp, \boldsymbol{b}'_\perp)\mathcal{H}^{(1)} \qquad \boldsymbol{b}_\perp - \boldsymbol{b}'_\perp = \frac{1}{2}(\boldsymbol{r}_\perp + \boldsymbol{r}'_\perp)$$

$\rightarrow$ $\boldsymbol{b}_\perp$ integral gets convoluted

strong nuclear dependence $A_N \sim A^{-7/6}$

Kovchegov, Sievert, Phys. Rev. D 86 (2012) 034028

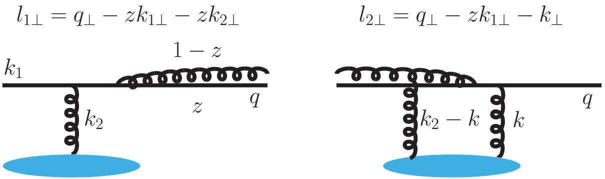
Open questions

1. parton distribution of the projectile?

   $\to$ natural candidate: $\langle \psi\bar{\psi} \rangle \sim \not{S}_T \gamma_5 g_T(x)$

2. hard factor diverges?

$$\mathcal{H}^{(1)}_{g_T} \sim \frac{\boldsymbol{S}_\perp \times \boldsymbol{l}_{1\perp}}{\boldsymbol{l}^2_{1\perp} \boldsymbol{l}^2_{2\perp}}$$



$\to$ surprising initial state ($\boldsymbol{l}_{2\perp} = 0$) collinear divergence!? (regulated by KS with quark mass)

Odderon mechanism in WW approx.

- lesson from SIDIS: a consistent computation should involve at least also the kinematical function $\tilde{g}(x)$
- in WW approx (momentum space)

$$E_q \frac{d\Delta\sigma}{d^3 q} \sim g_T(x) \left(S_\perp^\lambda \frac{\partial}{\partial k_{1\perp}^\lambda} \int_{\boldsymbol{\kappa}_\perp \boldsymbol{\kappa}'_\perp \boldsymbol{\Delta}_\perp} \mathcal{P}(\boldsymbol{\kappa}_\perp, \boldsymbol{\Delta}_\perp) \mathcal{O}(\boldsymbol{\kappa}'_\perp, -\boldsymbol{\Delta}_\perp) \mathcal{H}_{WW}^{(1)} \right)_{\boldsymbol{k}_{1\perp}=0}$$

$$\mathcal{H}_{WW}^{(1)} \sim \frac{\boldsymbol{l}_{1\perp} \times \boldsymbol{l}_{2\perp}}{\boldsymbol{l}_{1\perp}^2 \boldsymbol{l}_{2\perp}^2}$$

$\rightarrow$ manifestly finite!

SB, Kaushik, Vivoda, in preparation

From ∞ to 0

- need a reference vector to get $d\Delta\sigma$
- naively we have $\boldsymbol{k}_{1\perp}$

  ($\boldsymbol{k}_{1\perp}$ is to be replaced by $\boldsymbol{S}_{\perp}$ after the derivative)

- however in $\mathcal{H}_{WW}^{(1)}$ $\boldsymbol{k}_{1\perp}$ enters only through a unique combination

$$\boldsymbol{q}_{1\perp} \equiv \boldsymbol{q}_{\perp} - z\boldsymbol{k}_{1\perp}$$

(recall $\boldsymbol{l}_{1\perp} = \boldsymbol{q}_{1\perp} - z\boldsymbol{k}_{2\perp}$, $\boldsymbol{l}_{2\perp} = \boldsymbol{q}_{1\perp} - \boldsymbol{k}_{\perp}$)

$$\rightarrow \quad d\Delta\sigma_{WW} = 0$$

SB, Kaushik, Vivoda, in preparation

Conclusions

- new contribution to SSA from $g_T(x) \sim \int_x^1 dx_1 \Delta f(x_1)/x_1$ in SIDIS at two loops

$$A_N \sim \frac{\alpha_s M_N}{P_{hT}} \frac{x\Delta f(x)}{f(x)}$$

- involves only (known) twist-2 PDFs and FFs (after WW approx)
- numerical computation in SIDIS reveals up to 2% for the Sivers moment $A_{UT}^{\sin(\phi_h - \phi_S)}$ at the EIC with $P_{hT} > 1$ GeV
- $g_T(x)$ contribution in forward *pp* at sub-percent but possibly larger at lower energies and with remaining channels included
- Odderon mechanism in WW approx vanishes at NLO $\rightarrow$ NNLO and/or beyond WW - dynamical twist-3 functions?