All-Order Merging of High Energy and Soft-Collinear Resummation

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HEJ+PYTHIA - Exclusive-Exclusive Merging

Fixed order (FO) perturbative calculations are **inclusive** e.g. $pp \to jj$ @LO corresponds to a $pp \to jj + X$ @LO.

Well-proven procedures (e.g. CKKW-L hep-ph/0109231, hep-ph/0112284) exist for merging **inclusive** FO input to parton showers.

We develop a procedure to merge **exclusive** high energy (HE) resummation of *High Energy Jets* (HEJ 2.1 arXiv:2110.15692) with the PYTHIA 8.2 (arXiv:1410.3012) parton shower - allows us to construct **histories** for input HEJ events.

Seek to retain HEJ description of **wide angle** radiation and add the shower description of **transverse momentum** $(p_\perp)$ **hierarchies** and **collinear** splittings.

HEJ+PYTHIA - software implementation of our method to produce showered events from HEJ input.

Calculations in Perturbation Theory

Perturbative expansion in QCD - expansion of the cross section in powers of α_s:

$$\hat{\sigma} = \alpha_{\mathbf{s}}^{\mathbf{2}}\mathbf{c}_0 \ + \ \alpha_{\mathbf{s}}^{\mathbf{3}}\mathbf{c}_1 \ + \ \alpha_{\mathbf{s}}^{\mathbf{4}}\mathbf{c}_2 + \dots$$

Each subsequent term is a **subleading** order correction to the leading order $\alpha_{\mathbf{s}}\mathbf{c_0}$.

Fixed Order (FO) Expansion: Evaluates the series exactly up to a **fixed** power of α_s.

Leading Order (LO)	Next to LO (NLO)	Next to NLO (NNLO)
$\alpha_{\mathbf{s}}^{\mathbf{2}}\mathbf{c}_0$	$\alpha_{\mathbf{s}}^{\mathbf{2}}\mathbf{c}_0$	$\alpha_{\mathbf{s}}^{\mathbf{2}}\mathbf{c}_0$
	$+ \ \alpha_{\mathbf{s}}^{\mathbf{3}}\mathbf{c}_1$	$+ \ \alpha_{\mathbf{s}}^{\mathbf{3}}\mathbf{c}_1$
		$+ \ \alpha_{\mathbf{s}}^{\mathbf{4}}\mathbf{c}_2$

Calculations in Perturbation Theory

Divergences arise in certain kinematic limits when logarithmic terms $\alpha_s^n L^n$ are enhanced:

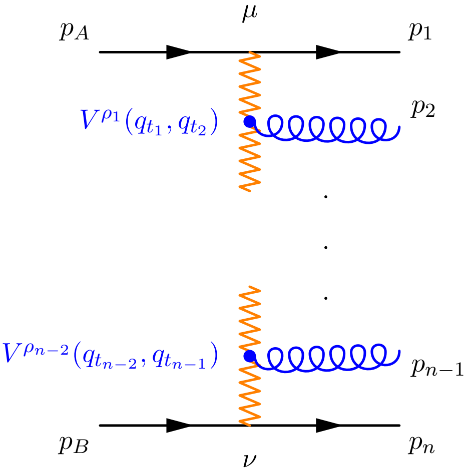
$$\hat{\sigma} = \alpha_\mathbf{s}^\mathbf{2}\mathbf{c_{0,0}} \ + \ \alpha_\mathbf{s}^\mathbf{3}\left(\mathbf{c_{1,1}L} + \mathbf{c_{1,0}}\right) \ + \ \alpha_\mathbf{s}^\mathbf{4}\left(\mathbf{c_{2,2}L^2} + \mathbf{c_{2,1}L} + \mathbf{c_{2,0}}\right) + \dots$$

L is a logarithm of disparate scales, enhanced in the **approximation** of this limit.

Resummation: Expand to **all orders** in α_s the logarithmic terms that dominate.

Leading Order (LO)	Leading Logarithmic (LL)	Next to LL (NLL)
$\alpha_\mathbf{s}^\mathbf{2}\mathbf{c_{0,0}}$	$\alpha_\mathbf{s}^\mathbf{2}\mathbf{c_{0,0}}$	$\alpha_\mathbf{s}^\mathbf{2}\mathbf{c_{0,0}}$
	$+\ \alpha_\mathbf{s}^\mathbf{3}\mathbf{c_{1,1}L} + \alpha_\mathbf{s}^\mathbf{4}\mathbf{c_{2,2}L^2} + \cdots$	$+\ \alpha_\mathbf{s}^\mathbf{3}\mathbf{c_{1,1}L} + \alpha_\mathbf{s}^\mathbf{4}\mathbf{c_{2,2}L^2} + \cdots$
		$+\ \alpha_\mathbf{s}^\mathbf{3}\mathbf{c_{1,0}} \ + \alpha_\mathbf{s}^\mathbf{4}\mathbf{c_{2,1}L} \ + \cdots$

High Energy Jets - High Energy Resummation



Reggeised gluon exchanges
Lipatov effective emission vertices

Define rapidity y_i and generalised Mandelstam variable s_{ij} for particles i, j by:

$$y_i = \frac{1}{2} \log \frac{E_i + p_{i,z}}{E_i - p_{i,z}}, \quad s_{ij} = (p_i + p_j)^2 \ .$$

High Energy Jets resums BFKL logarithms $\log\left(\frac{s_{(AB)}}{p_\perp^2}\right) \to \Delta y$ in the HE limit:

$$y_i \ll y_{i+1} \text{ and } |p_{i\perp}| \sim p_\perp \ \forall i \in \{1, ..., n\}.$$

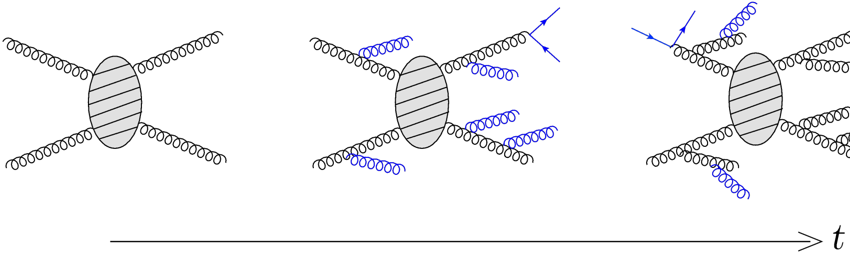
HEJ describes well effects from **hard**, **wide-angle** radiation.

Result is **LO-matched**, resummation ensures $>$ LL accuracy.

Parton Showers - Pythia

Divergences in partonic cross sections arise when partons are **soft**, **collinear** or both - logarithms of $p_\perp$ scales are enhanced which HEJ does not resume:

$$\log\left(\frac{p_{\perp,1}^2}{p_{\perp,2}^2}\right) \quad \sim \quad \log(z_{\text{split}}) \cdot \log(\theta_{\text{split}}).$$



Divergent logs are resummed by **parton showers** - these evolve a hard process down in an **evolution scale** t and add the soft and collinear parton splittings.

Logarithmic Enhancement - Comparison to Data

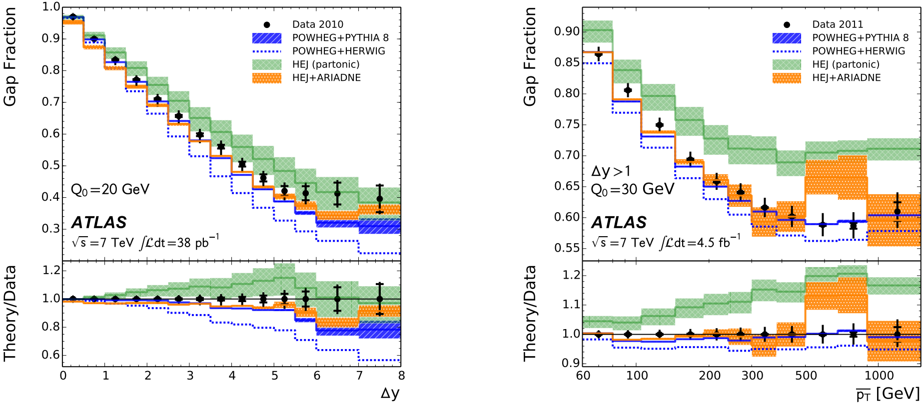


Figure: Gap fraction against Δy (left) and $\overline{p_\perp}$ (right) for HEJ, a previous merging (arXiv:1104.1316) of HEJ with the ARIADNE shower, and two other shower predictions compared to ATLAS data (arXiv:1407.5756). HEJ provides much of the necessary detail to describe the Δy spectrum while the showered predictions resum the necessary higher order effects to describe $\overline{p_\perp}$.

HEJ+PYTHIA- Merging Procedure

Method: Express HEJ resummation in shower language, feed HEJ events into HEJ+PYTHIA and veto shower emissions according to the probability HEJ has already performed them:

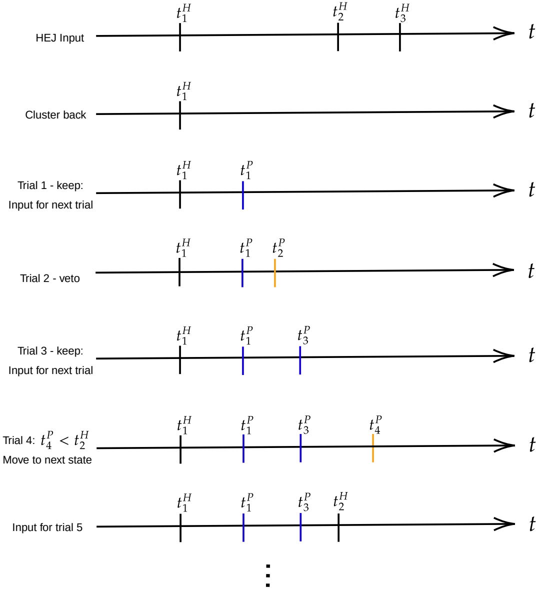
$$\mathcal{P}^{\text{veto}} = \frac{\mathrm{P}^{\texttt{HEJ}}}{\mathrm{P}^{\textsc{Pythia}}}.$$

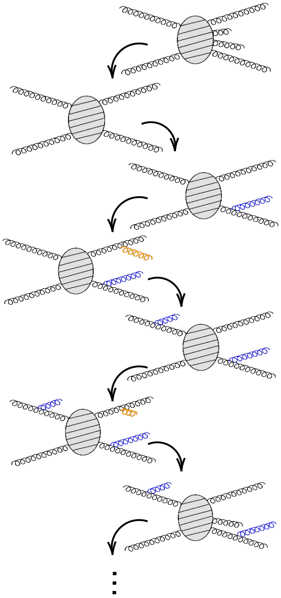
Altarelli-Parisi splitting functions (left) in QCD parametrise splittings in a parton shower - define an analogue for HEJ (right):

$$dk_\perp^2 dz \int d\phi \frac{1}{16\pi^2} \frac{\left|\mathcal{M}^{n+1}\right|^2}{\left|\mathcal{M}^{n}\right|^2} \sim \frac{dk_\perp^2}{\mathbf{k_\perp^2}} dz \frac{\alpha_\mathbf{s}}{\mathbf{2\pi}} \mathbf{P(z)}, \qquad \mathbf{P}^{\texttt{HEJ}} = \frac{1}{2} \frac{1}{16\pi^2} \frac{\overline{\left|\mathcal{M}_{\texttt{HEJ}}^{n+1}\right|^2}}{\overline{\left|\mathcal{M}_{\texttt{HEJ}}^{n}\right|^2}}$$

Additionally, we merge non-HEJ resummable states at LO via CKKWL to retain LO+>LL accuracy in both HE and soft-collinear logarithms throughout.

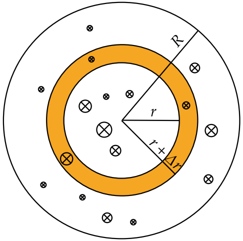
Illustration of Merging Procedure

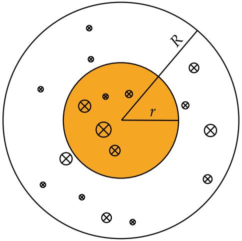




Comparison to Data - Jet Profiles I

Jet profiles - normalised distribution of $p_\perp$ in the (y, ϕ) distribution of a jet.





Differential jet profile $\rho(r)$: normalised sum of $p_\perp$ in annuli of width Δr (left).

Integrated jet profile $\Psi(r)$: integral of ρ from the centre to a radius $r < R$ (right).

We expect PYTHIA to describe these observables well, HEJ mostly predicts narrow jets containing only one parton - validation test of merging procedure.

Comparison to Data - Jet Profiles II

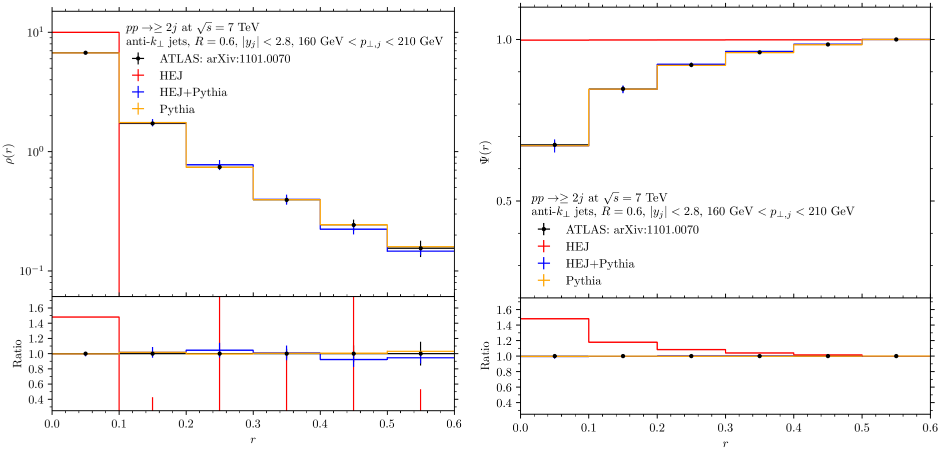


Figure: Differential (left) and integrated (right) jet profiles predicted in HEJ and HEJ+Pythia compared to data from ATLAS (arXiv:1101.0070). HEJ+Pythia successfully adds to the HEJ prediction the missing detail in jet substructure.

Current Standing

HEJ+PYTHIA is still in development, preliminary results show promise.

The interplay between high energy and soft-collinear logarithms is far from trivial, naïvely considering the two regimes as opposites does not yield potent results.

Aim to produce predictions for pertinent observables: m_{jj}, H_T, $p_{\perp,j}$, Δy_{jj} ...

Aim to extend method to include H, W, Z, ... production with jets.

For these we hope to retain the shower description of distributions enhanced by events with $\mathbf{p}_\perp$ **hierarchies** and the HEJ description of dijet **rapidity differences** and **invariant masses** in **cross sections** and **ratios** e.g. $R32$ etc.

Suggestions for Future Measurements

To validate the necessity of such a tool, we make gentle recommendations for experimental measurements:

1. Wide rapidity selection e.g. $|y| < 4.5$ rather than restricting to the central region $|y| < 2.8$ — tests `HEJ` corrections are applied.
2. Hard jets with $p_\perp$-hierarchy in jet selection e.g. $p_{\perp,j} > 60$ GeV, with $p_{\perp,j_1} > 80, 100, 120$ GeV — tests shower corrections are applied.
3. Jets of varying radii ($R = 0.4,\ 0.6$) — tests effect of broadening jet cone on respective resummation schemes.

Thank You

Backup

HEJ+Pythia - Merging Procedure

For non HEJ resummable input events, we merge these via CKKWL in Pythia.

For resummable HEJ events input we construct shower **histories** - clustering back to the $2 \to 2$ hard process - and perform a (subtracted) trial shower in Pythia.

For each (trial) emission from Pythia, we veto the emission with probability

$$\mathcal{P}^{\text{veto}} = \frac{\text{P}^{\texttt{HEJ}}}{\text{P}^{\text{Pythia}}}. \tag{1}$$

If we keep the emission from Pythia we append it to the HEJ event record and to **each subsequent state in the history**.

Continue with the trial shower, vetoing and appending emissions as in Eq. (1).

The result is a HEJ event whose history has been dressed with shower emissions; this can be further (trial) showered+hadronised in Pythia.

Recoil Strategy

When we add back an emission from a trial event to an arbitrary later stage in the history, we recoil the additional momentum according to the following global strategy:

1. Reshuffle the excess transverse momentum across the final state partons, conserving the mass and rapidity of each.
2. Rescale all transverse momenta by a constant factor λ such that the invariant mass of the initial state $\sqrt{\hat{s}}$ is conserved and again reassign the E and p_z components of each final state particle such that the rapidities and masses of each are conserved.
3. Sum over positive and negative lightcone components of the final state momenta to find physical analogues for the momenta of the initial state partons.
4. Boost along the z-axis such that that the initial state momenta are the same as they were in the original state, using the momenta in step 3. to derive the rapidity ψ. This ensures that beam energies are not exceeded if many emissions are added.

Identifying Mother Candidates in Subsequent Nodes

To add an emission from a trial event to all subsequent node in the history there is an ambiguity in which parton corresponds to the mother of the splitting.

We address this by by minimising the distance measure between all candidates c, and the mother in the trial m, in an arbitrary subsequent node:

$$\Delta R_{\mathsf{extended}}(p_m, p_c) = \sqrt{(y_c - y_m)^2 + (\phi_c - \phi_m)^2 + \left(\frac{p_{\perp,c} - p_{\perp,m}}{p_{\perp,m}}\right)^2}.$$

HEJ Splitting Probabilities

HEJ **splitting kernel** contains extra factor of $1/2$:

$$dk_\perp^2 dz \int d\phi \frac{1}{16\pi^2} \frac{\left|\mathcal{M}^{n+1}\right|^2}{\left|\mathcal{M}^{n}\right|^2} \sim \frac{dk_\perp^2}{\mathbf{k_\perp^2}} dz \frac{\alpha_\mathbf{s}}{\mathbf{2}\pi} \mathbf{P}(\mathbf{z}), \qquad \mathbf{P}^{\texttt{HEJ}} = \frac{1}{2} \frac{1}{16\pi^2} \frac{\overline{\left|\mathcal{M}_{\texttt{HEJ}}^{n+1}\right|^2}}{\overline{\left|\mathcal{M}_{\texttt{HEJ}}^{n}\right|^2}}.$$

This accounts for the only two possible colour configurations that dominate in the HE limit - the case that the colour connections can be unwound into two 'ladders' without compromising the rapidity ordering.

Comparison to Data - Jet Profiles for Softer Jets

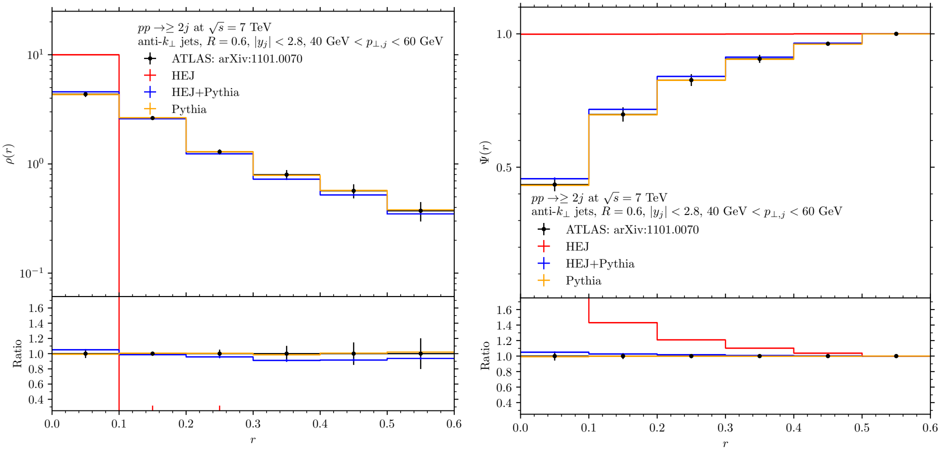


Figure: Differential (left) and integrated (right) jet profiles predicted in HEJ and HEJ+Pythia compared to data from ATLAS (arXiv:1101.0070). HEJ+Pythia describes well the jet profiles of soft jets just as we were able to for harder jets earlier.