ISMD 2022 Conference

Beyond NNLO in Global PDF fits

MSHTaN^{3}LO Parton Distribution Functions

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Full information in recent article: JM et. al., 2207.04739

What is a theoretical uncertainty?

And also… why do we care?

- Leading source from **Missing Higher Orders** in perturbation theory - many different areas these occur in F_2.

$$F_2(x,Q^2) = \sum_{\alpha\in\{H,q,g\}} \sum_{i\in\{q,g\}} \left(C^{GMVF,n_f+1}_{q,\alpha} \otimes A_{\alpha i}(Q^2/m_h^2) \otimes f_i^{n_f}(Q^2) + C^{GMVF,n_f+1}_{H,\alpha} \otimes A_{\alpha i}(Q^2/m_h^2) \otimes f_i^{n_f}(Q^2) \right)$$

$$\frac{df}{d\ln\mu_f^2} = P \otimes f$$

LO NLO NNLO | N3LO

$$P(x,\alpha_s) = \alpha_s P^{(0)}(x) + \alpha_s^2 P^{(1)}(x) + \alpha_s^3 P^{(2)}(x) \;\Big|\; + \alpha_s^4 P^{(3)}(x) + \dots$$

- Current knowledge is up to NNLO, with **higher orders unknown.**

- Potentially **large corrections** hiding in higher orders beyond **theory truncation**.

- Already **progress** in calculating features at N3LO[1-11].

Theoretical Uncertainties in a Global PDF Fit

$$P(T|D) \propto \exp\left(-\frac{1}{2}(T-D)^T H_0 (T-D)\right) \longrightarrow \begin{cases} P(T|D) \propto \exp\left(-\frac{1}{2}M^{-1}(\theta' - \bar{\theta}')^2 - \frac{1}{2}(T'-D)^T H (T'-D)\right) \\ P(\theta') = \frac{1}{\sqrt{2\pi}\sigma_{\theta'}} \exp(-\theta'^{\,2}/2\sigma_{\theta'}^2) \end{cases}$$

- Do we need to **wait for a full description** of the next order to be able to use the knowledge we have?
- Can attempt to **parameterise the higher order** effects with a nuisance parameter defined by a prior probability distribution[12].
- Allow the fit to move these N^{3}LO parameters (with a **penalty attached** to ensure we stay close to the behaviour already known).
- With these alterations, we follow the **same practice** as set out in the MSHT20 NNLO PDF fit - the exact same global fit is done to approximate N^{3}LO (aN^{3}LO).

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What do we know?

…and what don't we know?

$$f(x \to 0) = \frac{C_A^3}{3\pi^4}\left(\frac{82}{81} + 2\zeta_3\right)\frac{1}{2}\frac{\ln^2 1/x}{x}$$

$$\mathcal{M}[f(x)](N) = \int_0^1 dx x^{N-1} f(x)$$

- **Zero-mass** N[3]LO coefficient functions are known[1].
- Some knowledge of **leading terms** in the $x \to 0$ and large regime[2-11].
- Some **numerical constraints** (Low-integer Mellin moments)[2-11].
- **Intuition** from lower orders/expectations from perturbation theory.
- Other parts, we know a very **limited amount** about ($A^{(3)}_{gg,H}$ and most K-factors)[8-10].

Splitting Functions up to N³LO

…approximately

- Consider we know N_m Mellin moments[1-5].

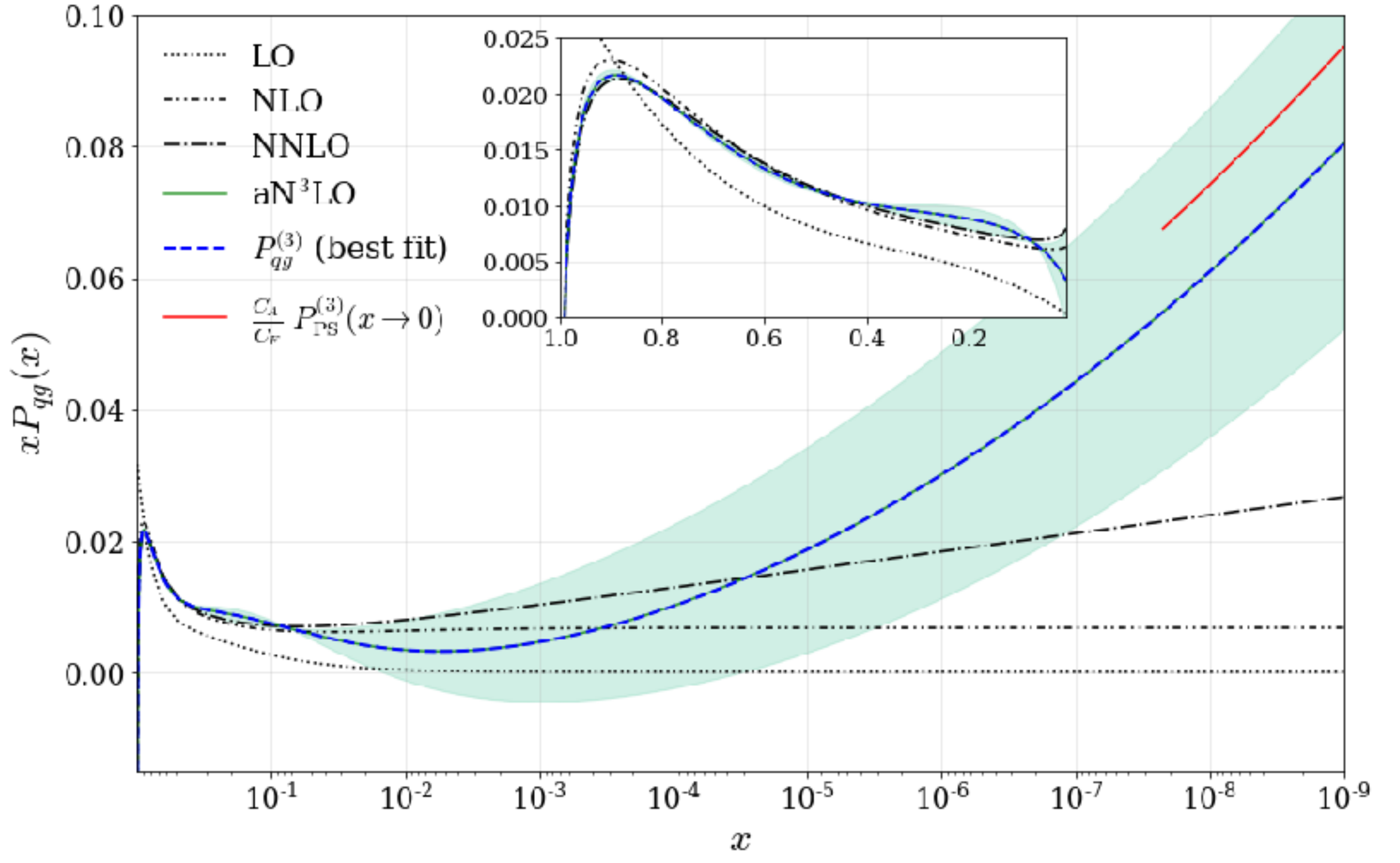
- With N_m constraints, we employ:

$$P(x) = \sum_{i=1}^{N_m} A_i f_i(x) + f_e(x)$$

contains any known information.

- Choose a set of **relevant functions** f_i and solve for A_i.

- To allow control of this function, introduce a **degree of freedom** a. $f_e(x) \rightarrow f_e(x, a)$

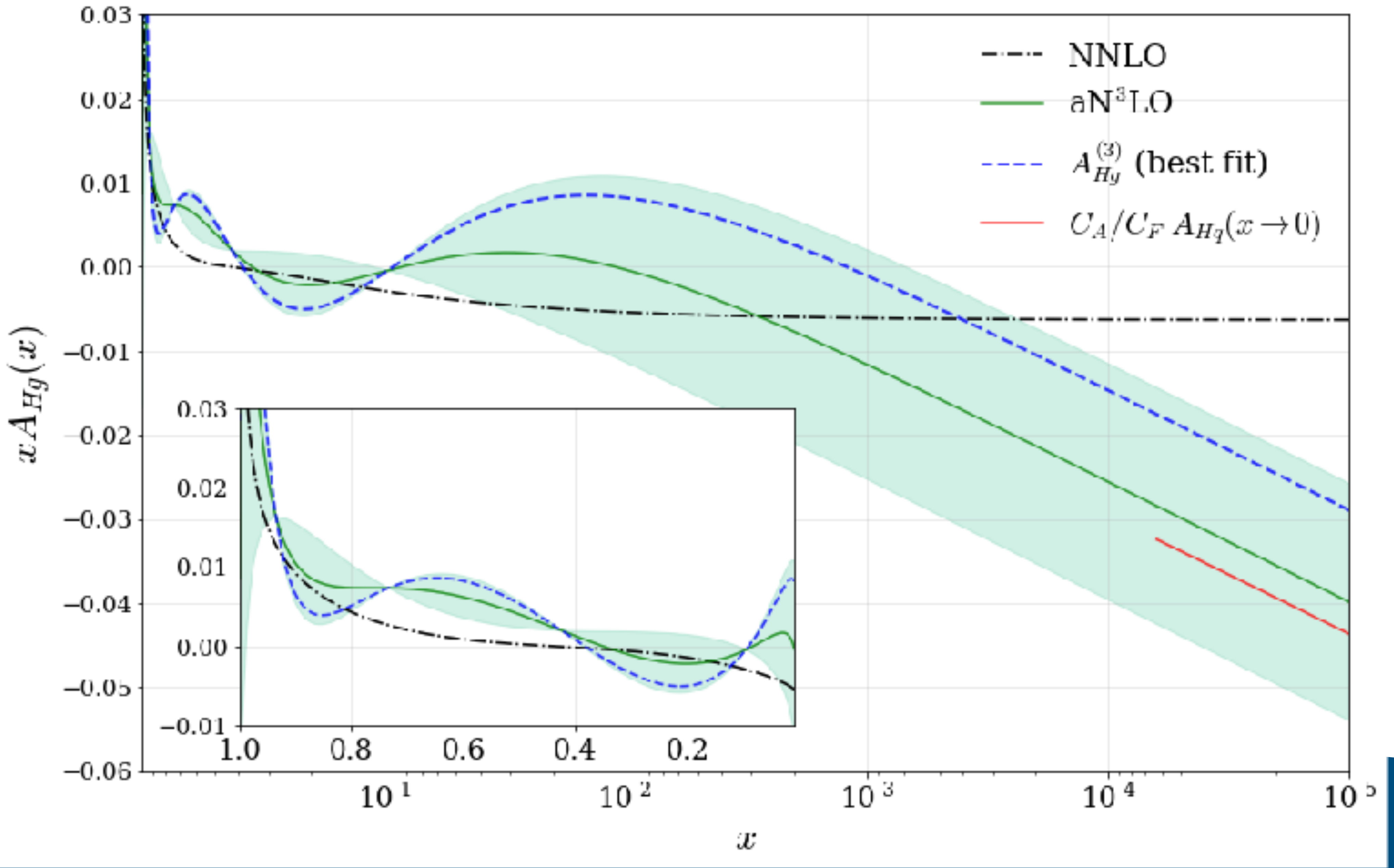


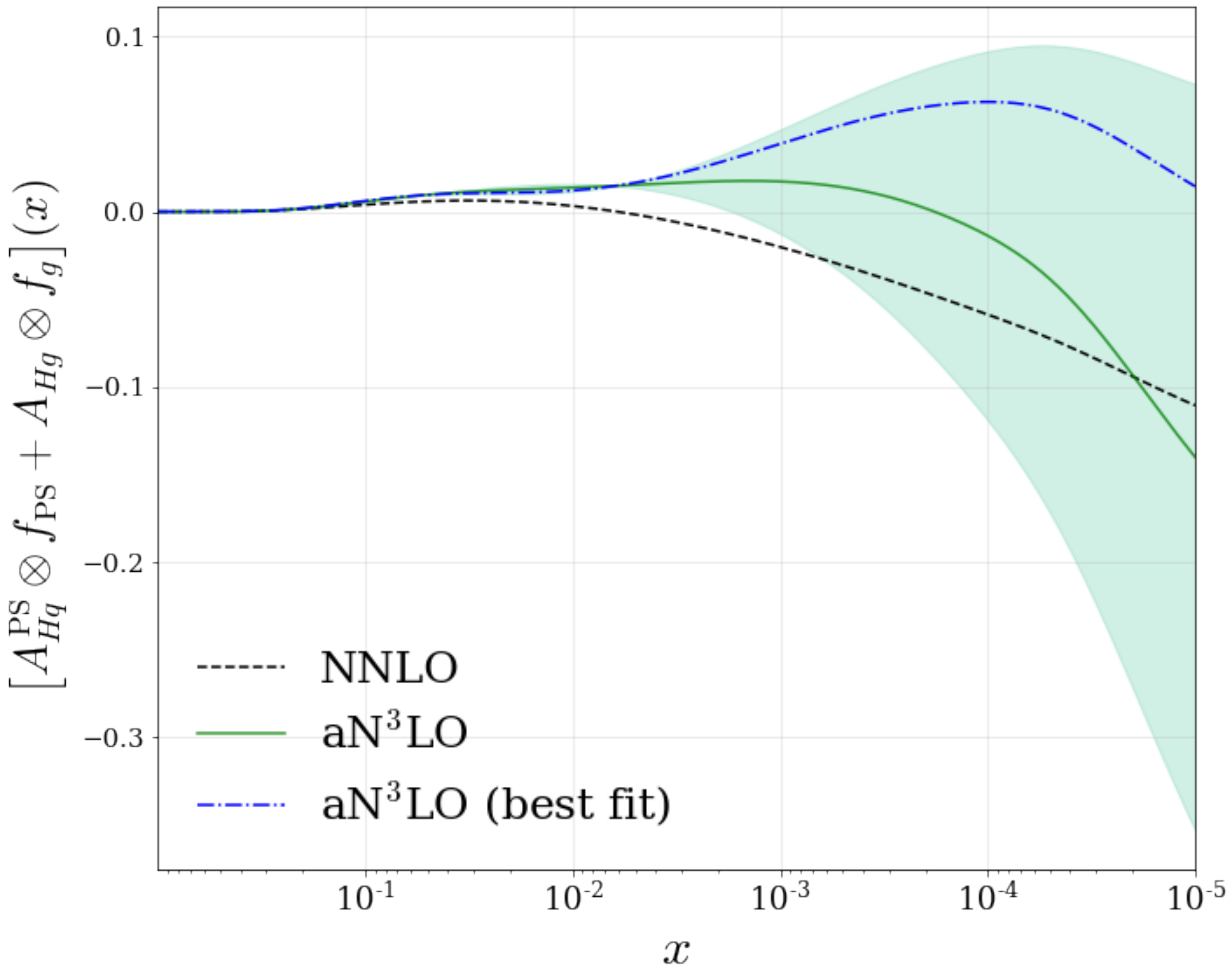
- a interpreted as a **nuisance parameter** allowed to vary in a PDF fit.
- In our treatment a is the **coefficient of the most divergent** unknown small-x term.

Transition Matrix Elements up to N³LO

…approximately

- Following the **same procedure** as for the splitting functions.
- A_{Hg} is the **dominant contribution** to the overall form of $(H + \overline{H})$ shown across.





- A_{Hg} variation is **comparable** to previous results[14].

N^{3}LO K-factors

$$K(y) = 1 + \alpha_s D(y) + \alpha_s^2 E(y) + \alpha_s^3 F(y) + \mathcal{O}(\alpha_s^4)$$

$$K^{\mathrm{N^3LO/LO}} = K^{\mathrm{NNLO/LO}} \left(1 + \alpha_s^3 \hat{a}_1 D + \alpha_s^3 \hat{a}_2 E\right)$$

- Parameterise the N^{3}LO K-factor as a **superposition** of both NNLO and NLO K-factors.
- Allows the fit to **decide on a shape** (based on the shapes of preceding orders) and an overall magnitude.
- Center variational parameters $\hat{a}_1$, $\hat{a}_2$ about 0, so K_{NNLO} is the **central value**.

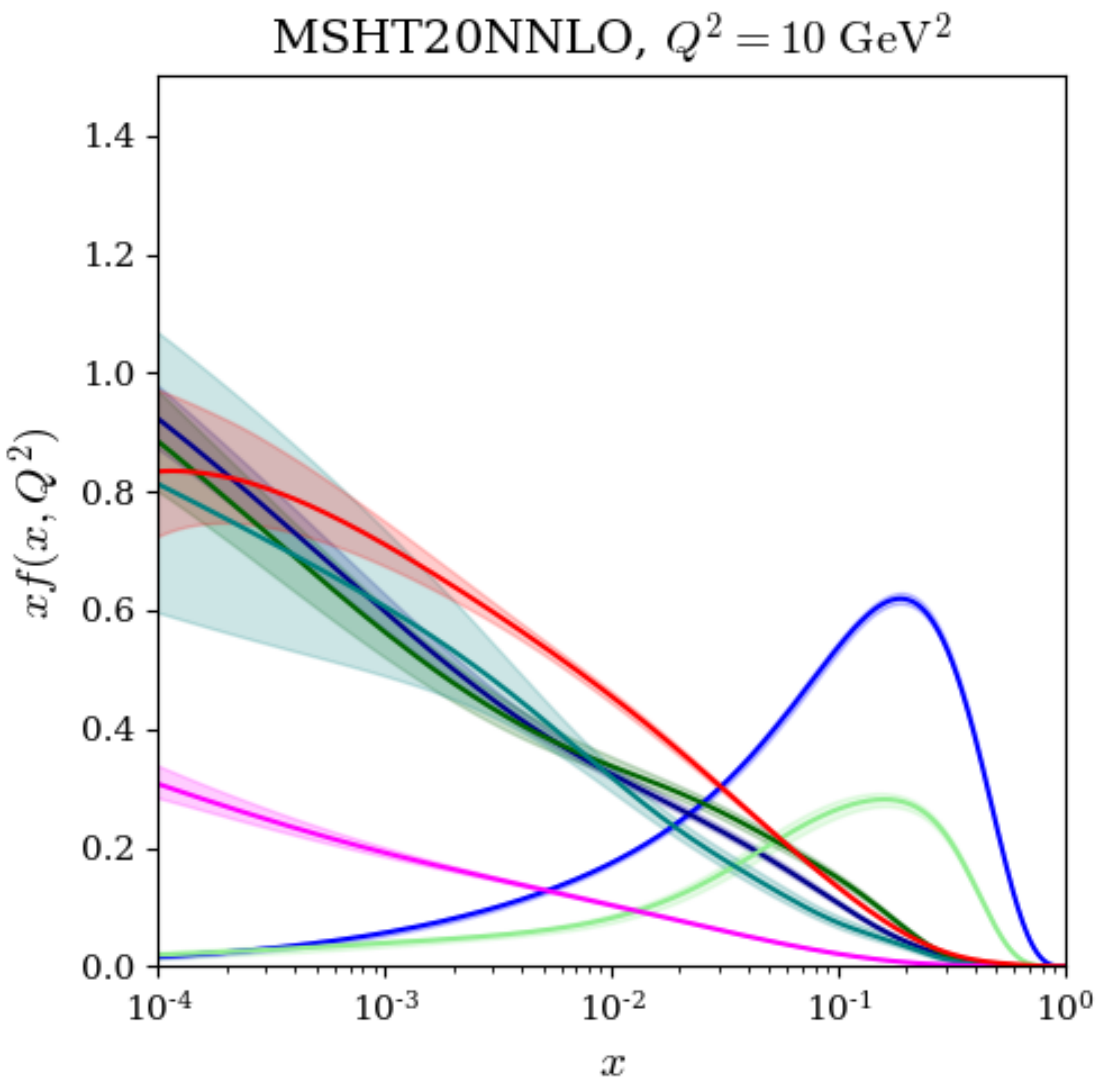
- **Correlated K-factors** for each of the 5 processes: DY, Top, Jets, $Z\ p_T$ & VB Jets and Dimuon *(also dijets)*.
- $\hat{a}_1$, $\hat{a}_2$ could be included as **correlated with PDF parameters** (incl. other N^{3}LO theory parameters) or as **completely decorrelated** from the inclusive DIS process.
  - Ignores some small correlations through DGLAP.

MSHT aN³LO PDFs

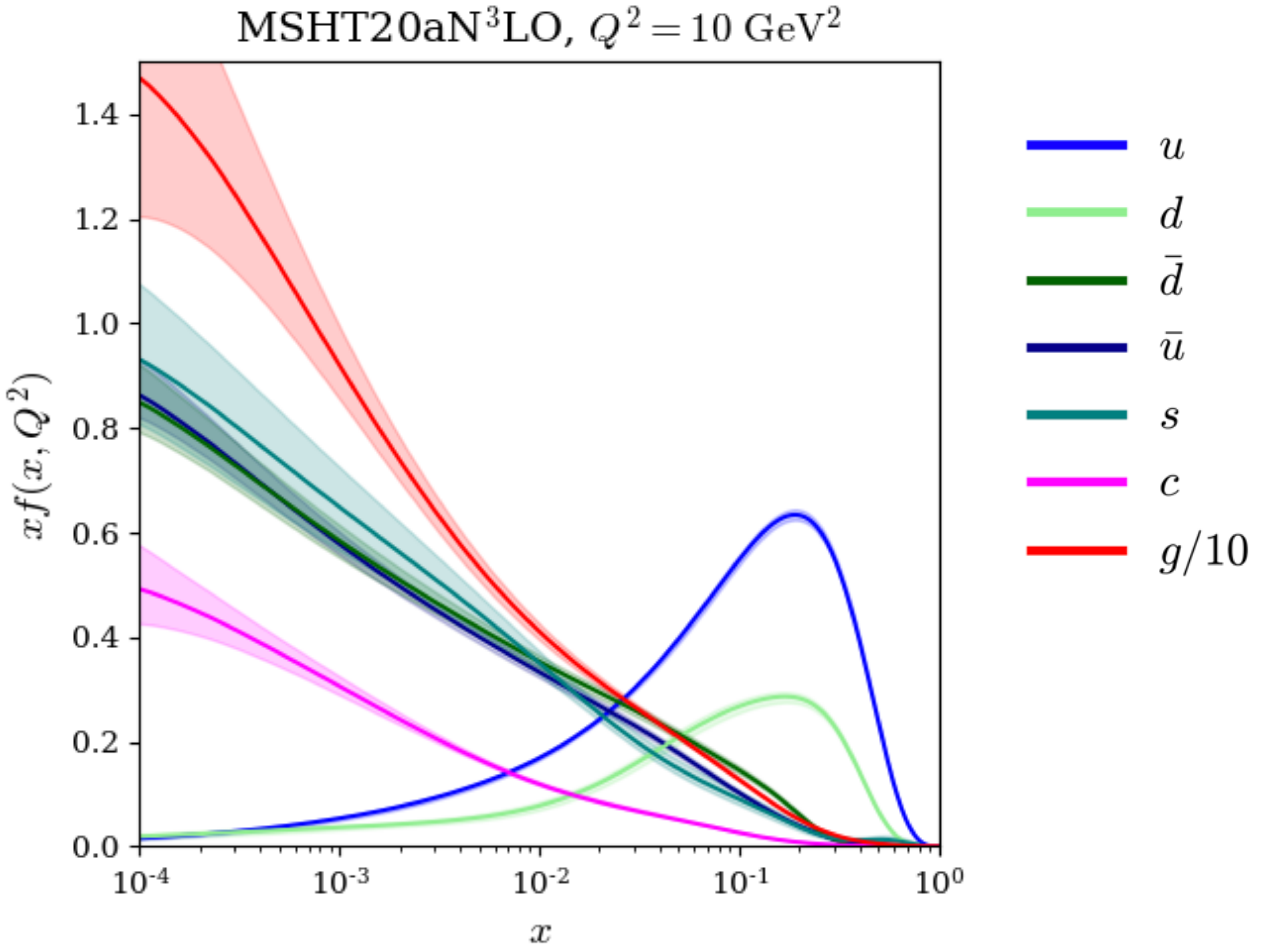
NNLO: $\chi^2 \simeq 5121\ /\ 4363$

aN³LO: $\chi^2 \simeq 4949\ /\ 4363$

NNLO

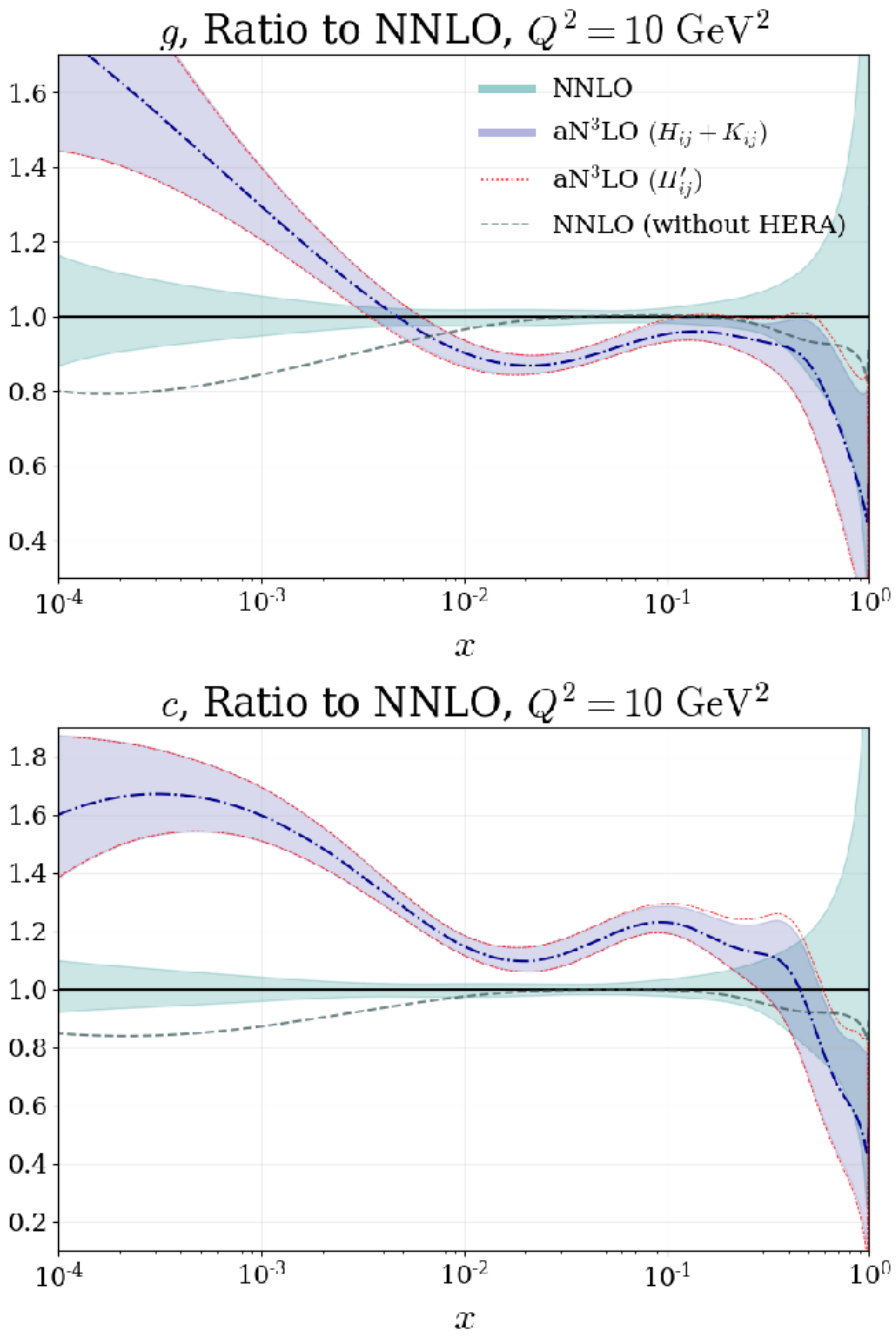


aN³LO



MSHT aN^{3}LO PDFs

- Gluon is enhanced at small-x due to the large logarithms present at **higher orders**.
- Charm receives a sizeable contribution from $A_{Hg}^{(3)}$.
    - A_{Hg} at high-x and the gluon at small-x involved in **convolution**.
- At high-Q^2 there are less drastic effects, however charm remains more similar to *CT18 NNLO PDF* than *MSHT20 NNLO PDF*.



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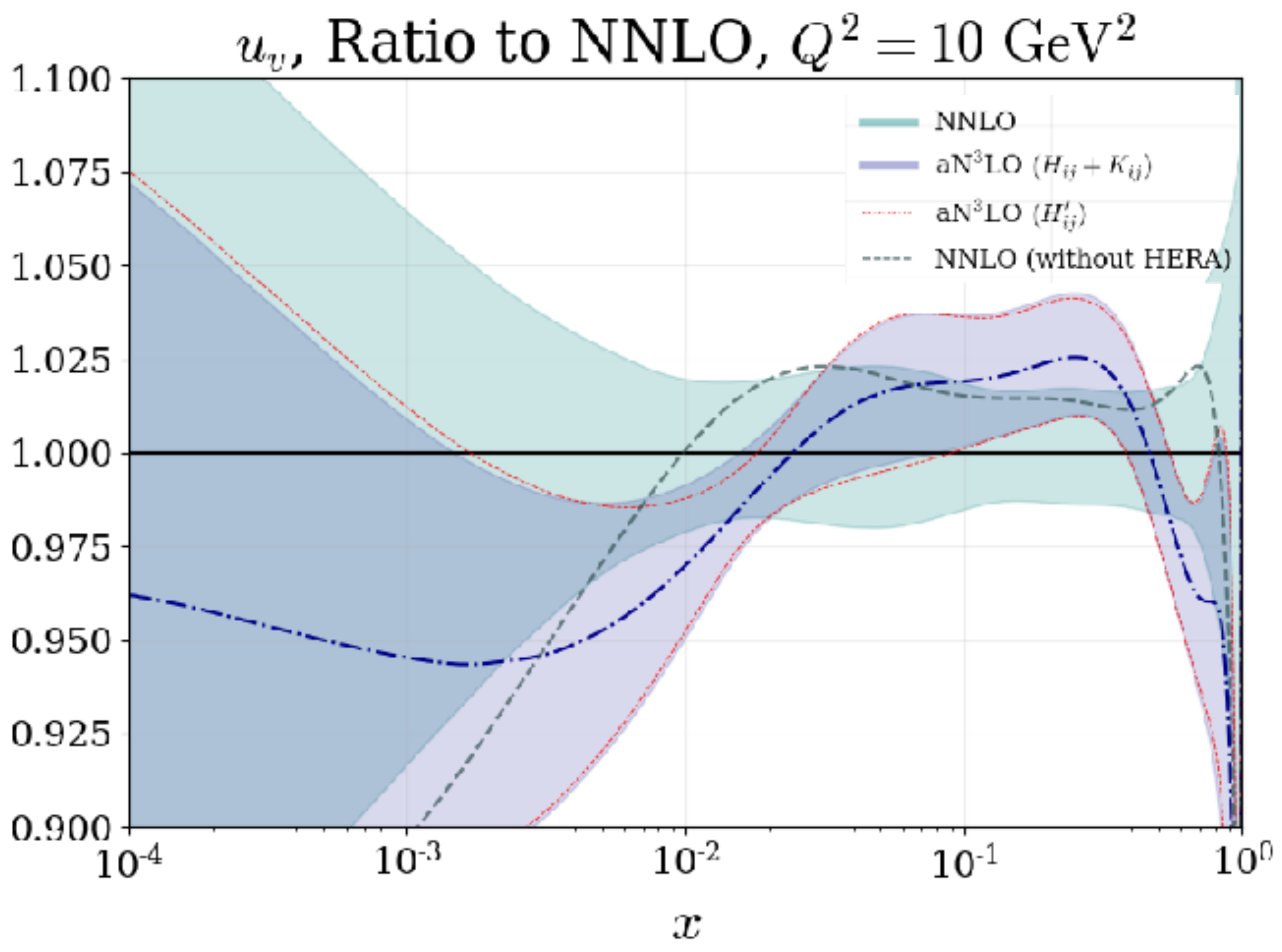
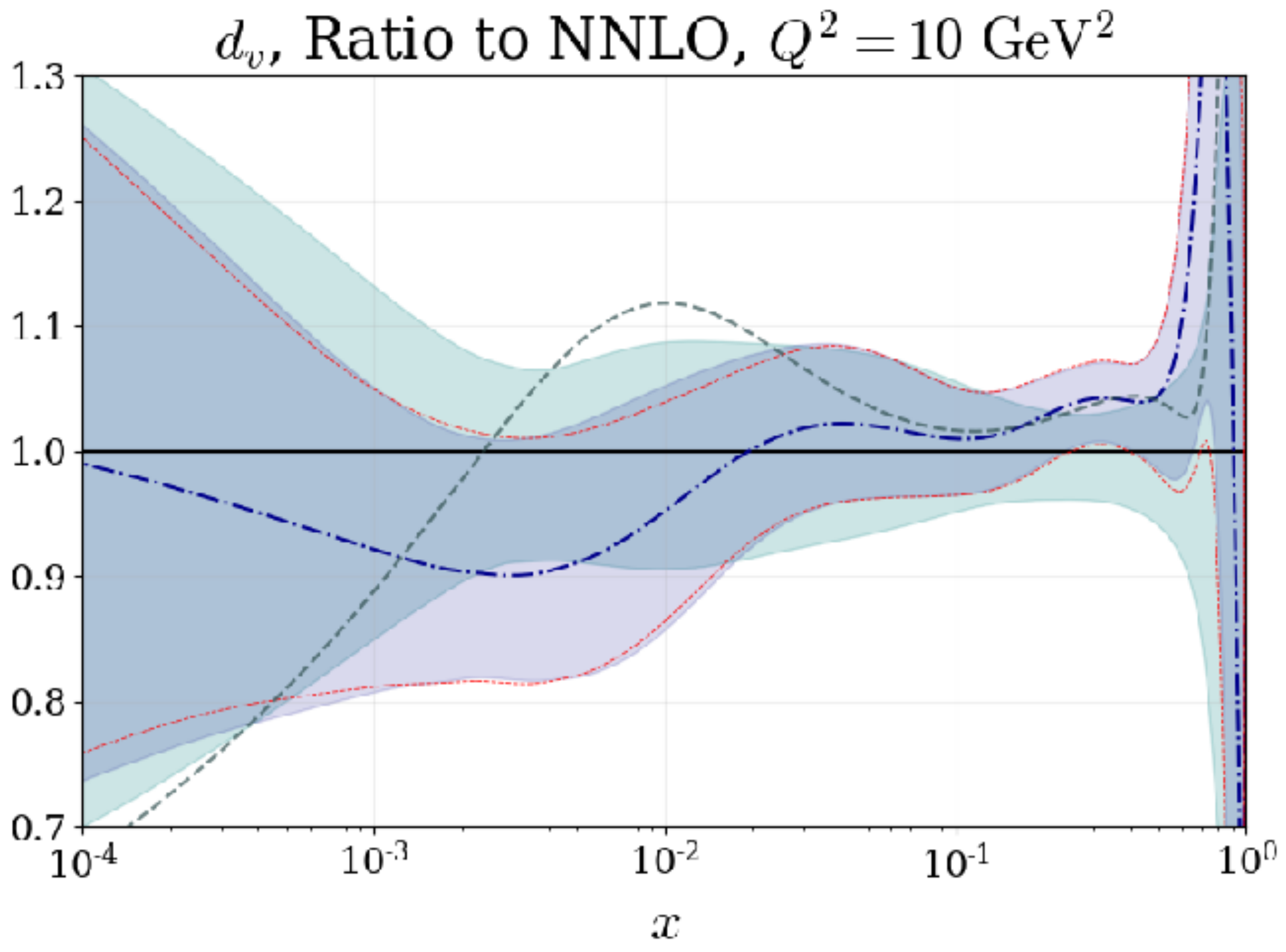
MSHT aN^{3}LO PDFs

- Correlated and uncorrelated K-factors show **consistent uncertainty** predictions across all (x, Q^2).
- Quarks are reduced at large and small-x to accommodate the gluon.
- aN^{3}LO follows more closely the NNLO fit to non-HERA datasets at high-x, demonstrating a **reduction in tension** between the small-x HERA data and other datasets.





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χ^2 Results

- We see a **reduction** in χ^2 from NNLO across all datasets ($\Delta\chi^2 = -172.5$ for 20 extra parameters).
- ATLAS 8 TeV Z p_T [15] sees a huge reduction in χ^2_{NNLO}/npts ~ 1.82 to $\chi^2_{aN^3LO}$/npts ~ 1.02.
- This is a **similar reduction** found at NNLO when HERA datasets were not included[17].
- In the aN^{3}LO fit, we also see a **reduction** in the HERA data χ^2.

Dataset	N_{pts}	χ^2	$\Delta\chi^2$ from NNLO
HERA ep F_2^{charm}	79	134.7	+2.4
NMC/BCDMS/SLAC/HERA F_L	57	45.5	−23.0
HERA e^+p CC	39	51.8	−0.1
HERA e^-p CC	42	66.3	−3.8
HERA e^+p NC 820 GeV	75	83.8	−6.0
HERA e^-p NC 460 GeV	209	247.4	−0.9
HERA e^+p NC 920 GeV	402	476.7	−36.0
HERA e^-p NC 575 GeV	259	248.0	−15.0
HERA e^-p NC 920 GeV	159	243.3	−1.0
ATLAS W^+, W^-, Z	30	30.0	+0.1
CMS double diff. Drell-Yan	132	137.1	−7.4
LHCb 2015 W, Z	67	97.2	−2.2
ATLAS 7 TeV jets	140	214.5	−7.1
ATLAS 7 TeV high prec. W, Z	61	110.5	−6.2
CMS 7 TeV jets	158	189.8	+14.1
ATLAS 8 TeV Z p_T	104	105.8	−82.7
CMS 8 TeV jets	174	272.6	+11.3
ATLAS 8 TeV High-mass DY	48	63.4	+6.3
ATLAS 8 TeV W + jets	30	19.1	+0.9
ATLAS 8 TeV W	22	55.1	−2.3
CMS 2.76 TeV jet	81	113.9	+11.1
DY data Total	864	1044.8	−43.1
Top data Total	71	73.4	−5.9
Jets data Total	739	972.9	+30.8
p_T Jets data Total	144	137.1	−78.1
Dimuon data Total	170	124.6	−1.6
DIS data Total	2375	2585.2	−86.4
Total	4363	4948.6	−172.5

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χ^2 Results

	LO	NLO	NNLO	aN^{3}LO
χ^2/N_{pts}	2.57	1.33	1.17	1.13

- The overall χ^2 follows the **general trend** one may expect from perturbation theory.
- Evidence that including aN^{3}LO has reduced tensions between small and large-x.
- χ^2 reduction is **mostly due** to new theory, not just from K-factors included in fit.
- Average **penalty** for included 20 aN^{3}LO parameters is ~ 0.53.

Low-Q^2 Coefficient	χ^2	Low-Q^2 Coefficient	χ^2
c_q^{NLL}	0.000	c_g^{NLL}	0.862
Transition Matrix Elements	χ^2	**Transition Matrix Elements**	χ^2
a_{Hg}	0.526	$a_{qq,H}^{\text{NS}}$	0.022
$a_{gg,H}$	1.091		
Splitting Functions	χ^2	**Splitting Functions**	χ^2
ρ_{qq}^{NS}	0.007	ρ_{gq}	0.935
ρ_{qq}^{PS}	0.255	ρ_{gg}	4.280
ρ_{qg}	0.000		
K-factors	χ^2	**K-factors**	χ^2
DY_{NLO}	0.061	DY_{NNLO}	0.003
Top_{NLO}	0.105	Top_{NNLO}	0.659
Jet_{NLO}	0.063	Jet_{NNLO}	0.517
$p_T\text{Jets}_{\text{NLO}}$	0.438	$p_T\text{Jets}_{\text{NNLO}}$	0.011
$\text{Dimuon}_{\text{NLO}}$	0.481	$\text{Dimuon}_{\text{NNLO}}$	0.363
N^{3}LO Penalty Total	10.7 / 20	Average Penalty	0.534
		Total	4948.6 / 4363
		$\Delta\chi^2$ from NNLO	−172.5

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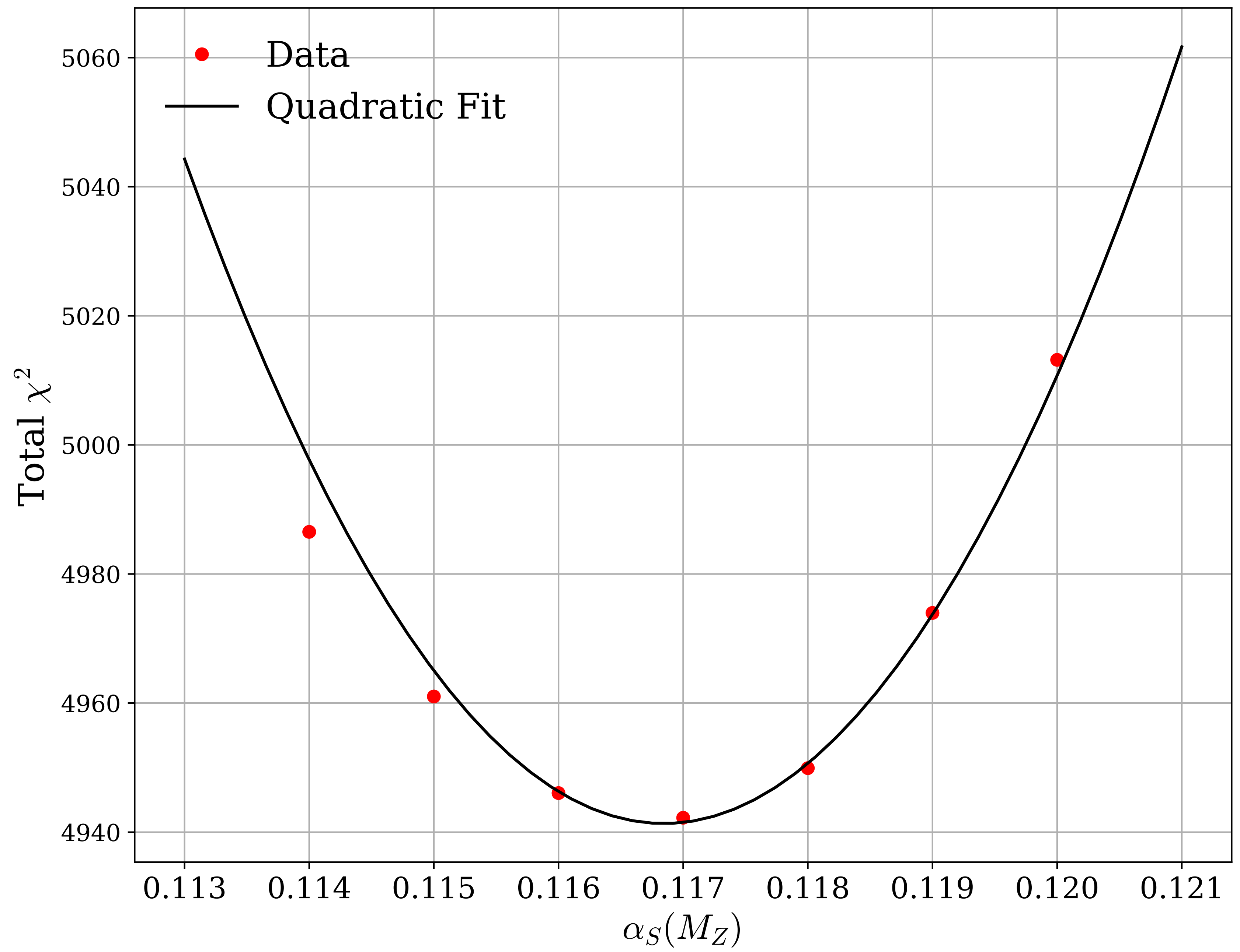
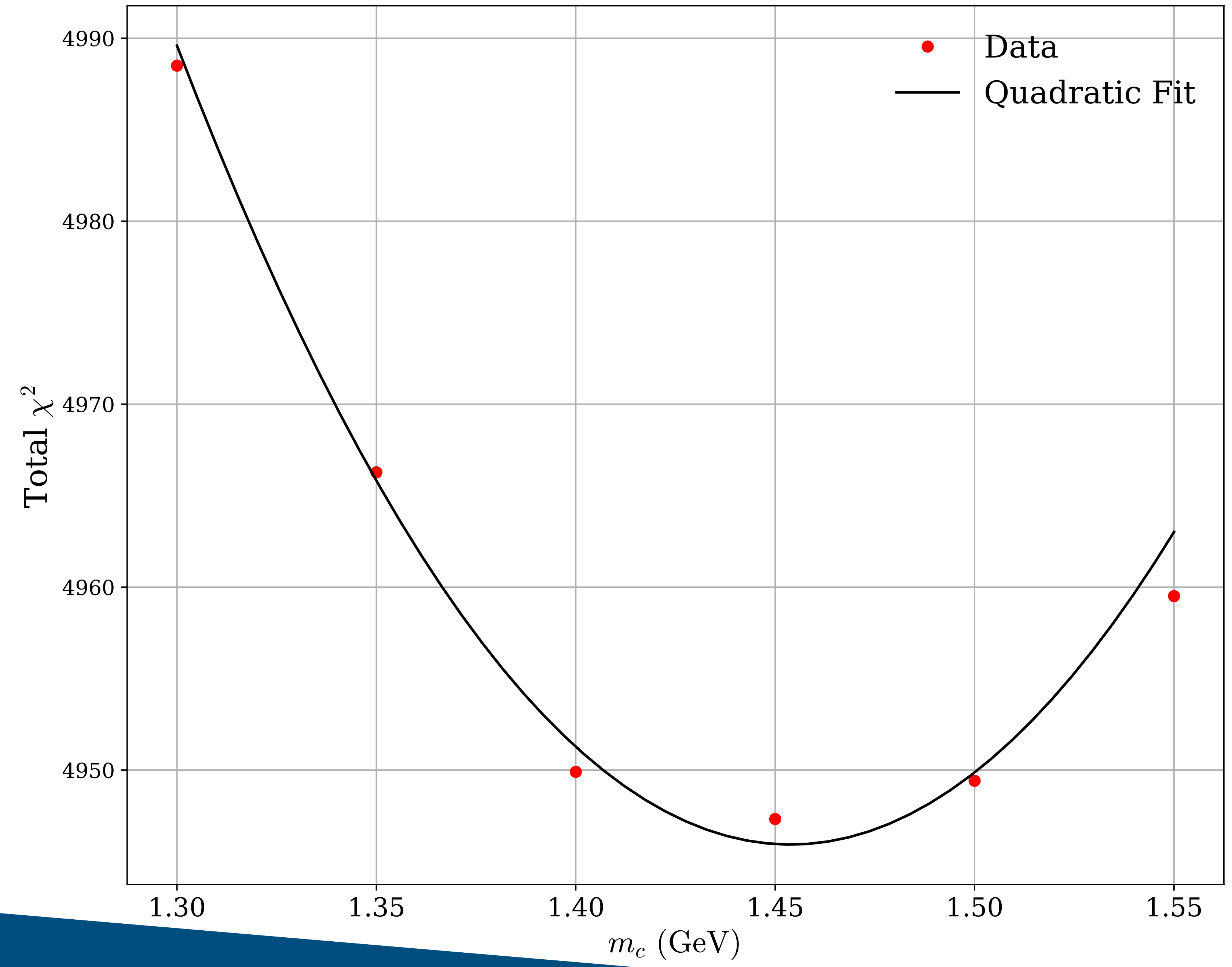
$\alpha_S(M_Z^2)$ and m_c

- Both $\alpha_S(M_Z^2)$ and m_c show a **quadratic behaviour** around their respective minima.
- Best fit of $\alpha_S(M_Z^2)$ is settling $\simeq 0.1167$.
  - MSHT20 NNLO: $\alpha_S(M_Z^2) = 0.1174 \pm 0.0013$
  - MSHT20 NLO: $\alpha_S(M_Z^2) = 0.1203 \pm 0.0015$
- Both these results suggest that the fit is preferring a **slight suppression** of the PDFs, particularly the enhanced gluon and charm.
- With a **future full analysis** we expect the aN3LO result to overlap with the NNLO world average within uncertainties.





N^3LO Drell-Yan Processes

(K-factors up to N^3LO)

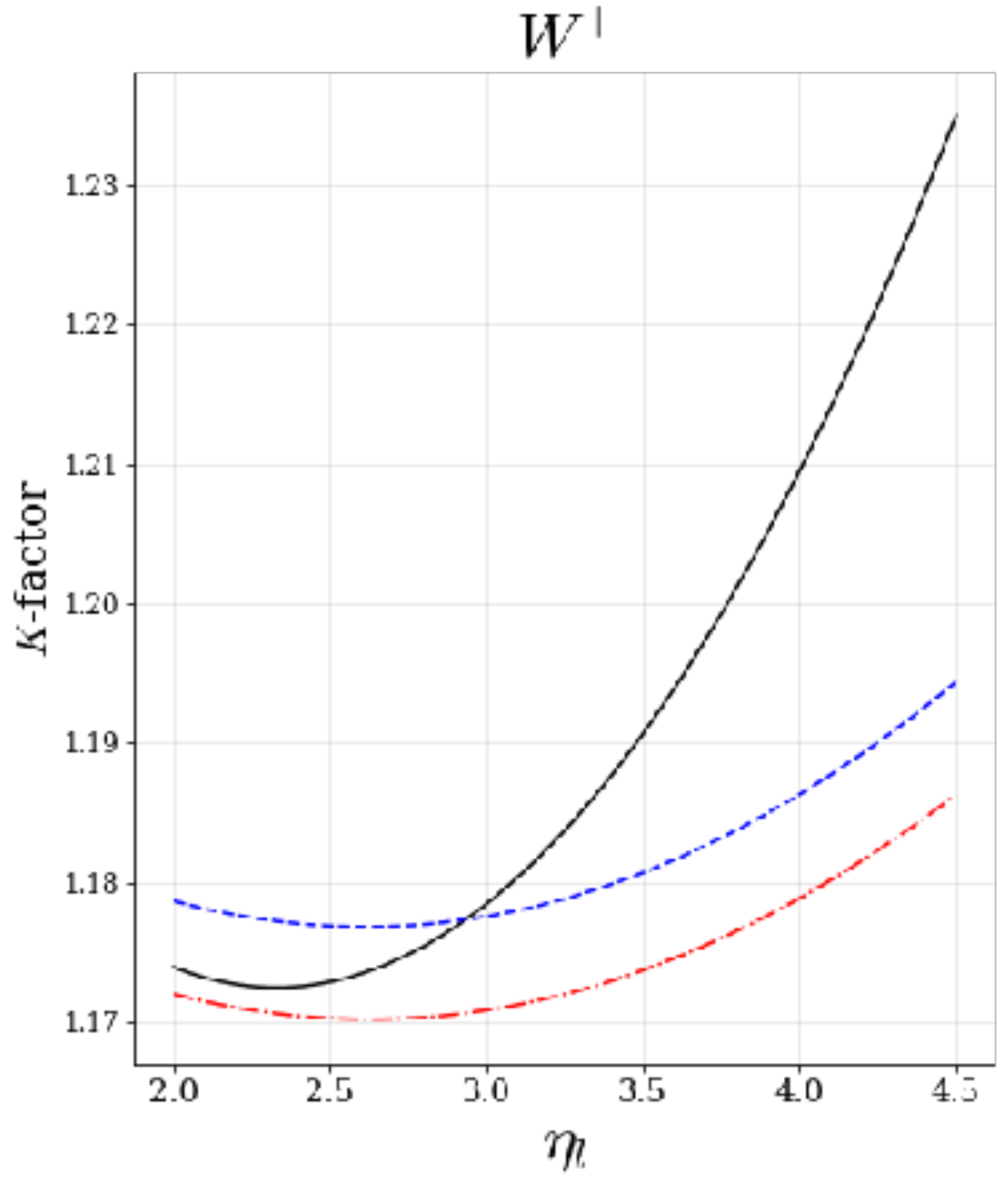
- K-factors transform the hard cross section between orders.

$$K(y) = 1 + \alpha_s D(y) + \alpha_s^2 E(y) + \alpha_s^3 F(y) + \mathcal{O}(\alpha_s^4)$$

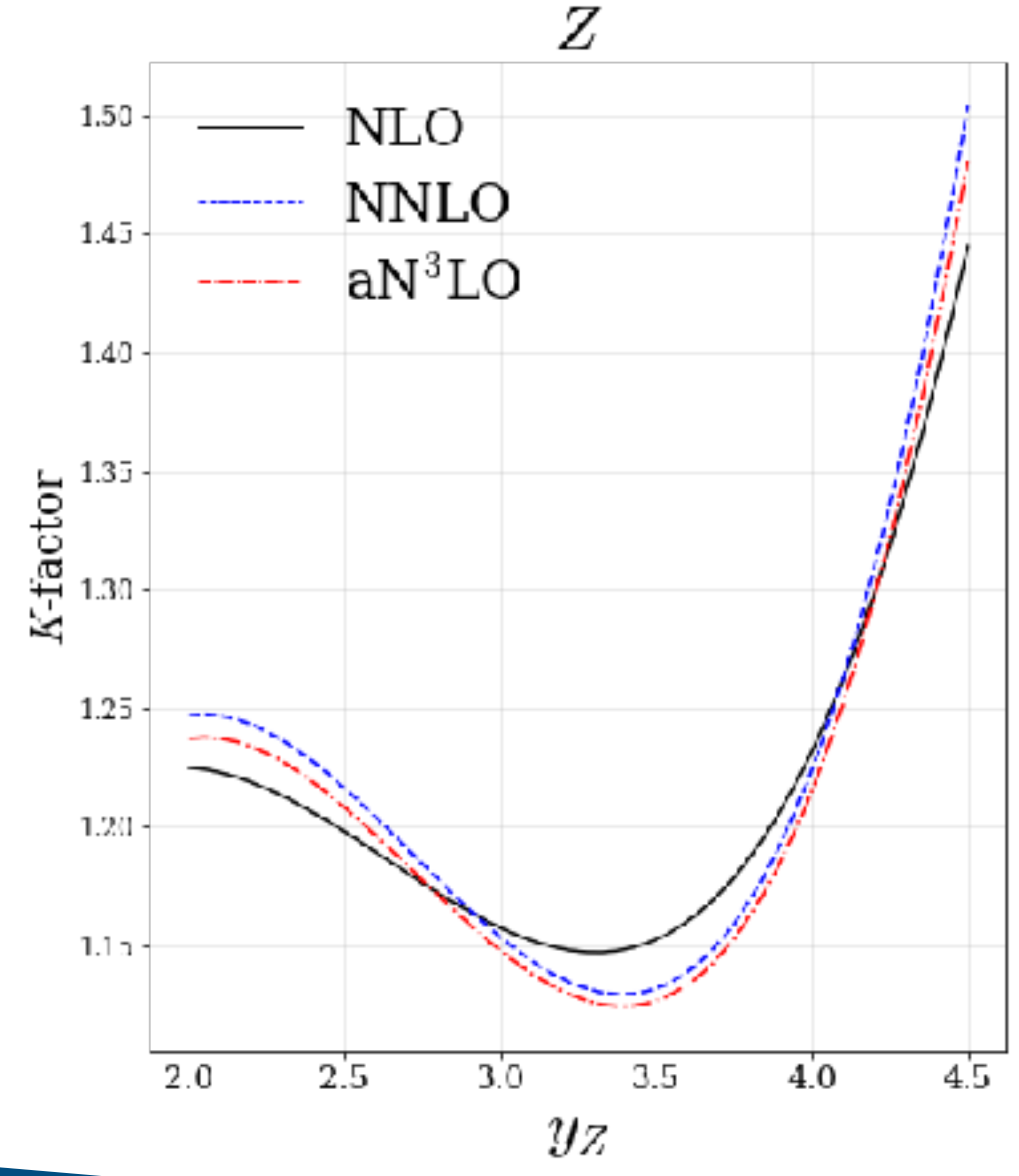
$$K^{\mathrm{aN^3LO/LO}} = K^{\mathrm{NNLO/LO}}\left(1 + \alpha_s^3 \hat{a}_1 D + \alpha_s^3 \hat{a}_2 E\right)$$

- Allowed to vary about the NNLO central value.
- Predict a **~1% decrease** in the DY K-factors from NNLO.
- In agreement with **recent results** found using NNLO PDFs with aN^3LO cross section[15].

LHCb 2015 W, Z dataset results



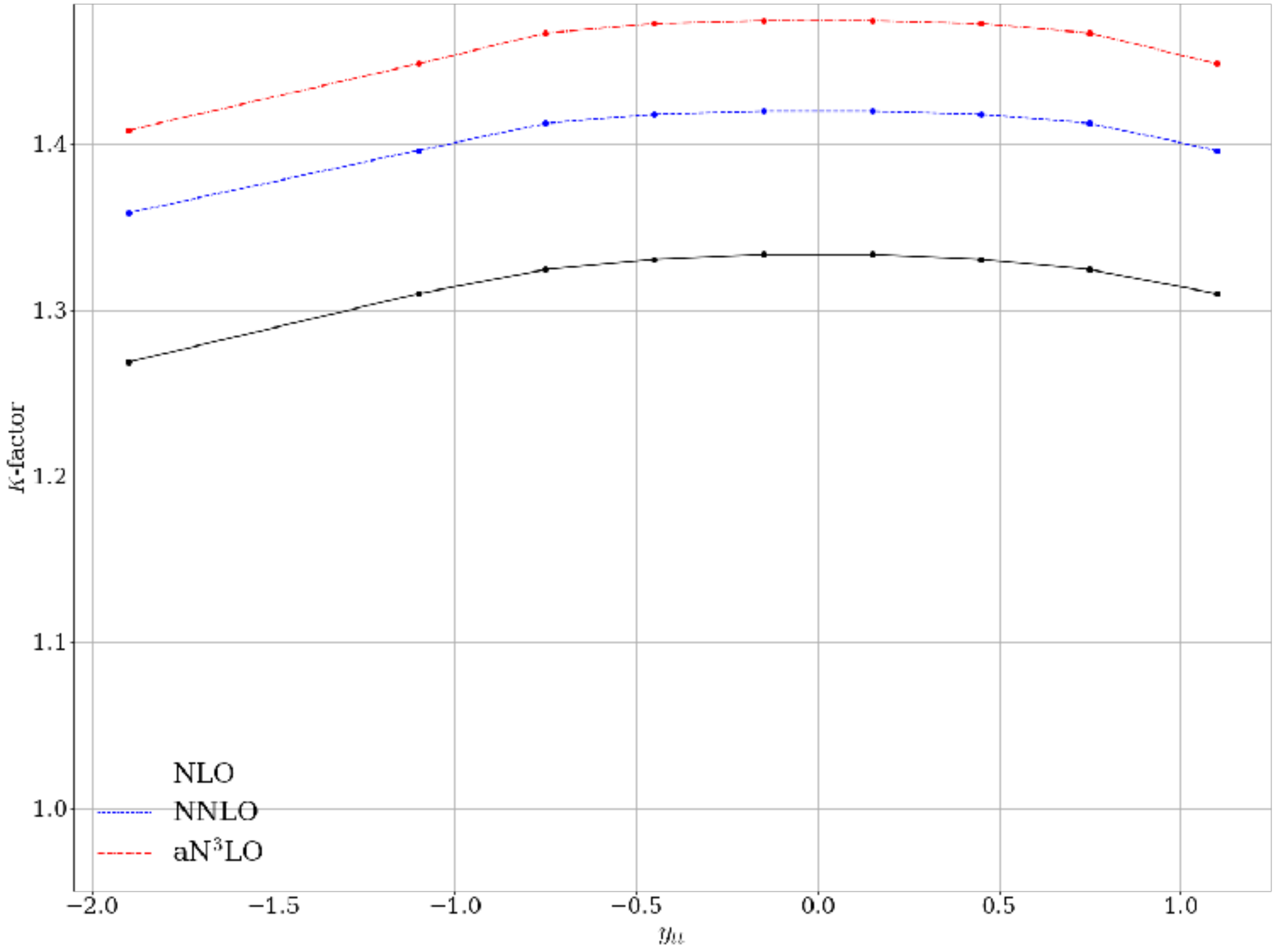
	NNLO	aN^3LO
$\chi^2_{\mathrm{DY}}/N_{\mathrm{pts}}$	1.26	1.21



N^3LO Top Processes

(K-factors up to N^3LO)

- Top K-factors see an **overall increase** in magnitude, consistent with recent results[16].
- χ^2 results show a **marginally better** fit overall.
- K-factors have successfully accounted for the **theory changes** in the F_2 structure function theory.
- K-factor for CMS 8 TeV single diff. $t\bar{t}$ shown here.

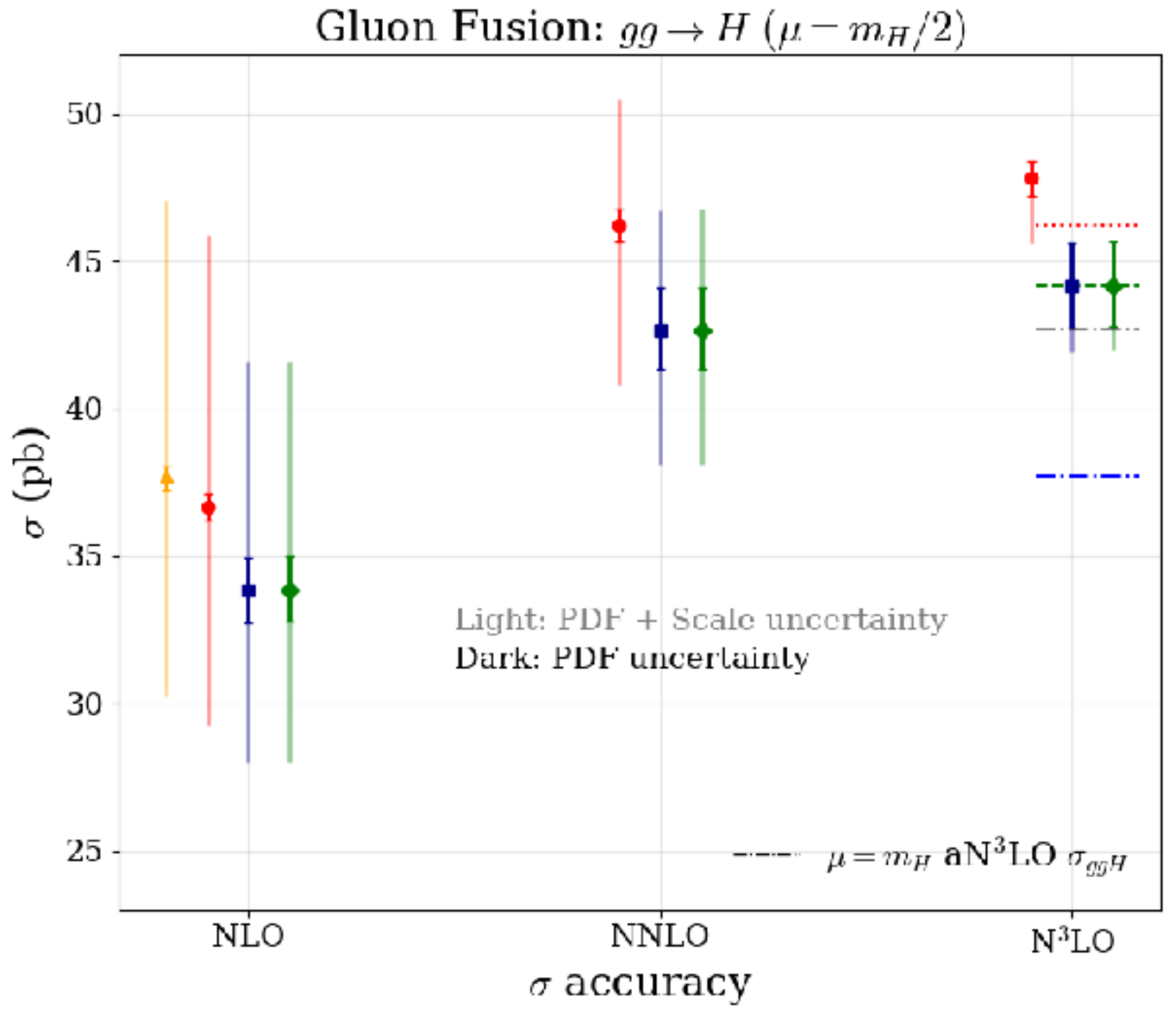


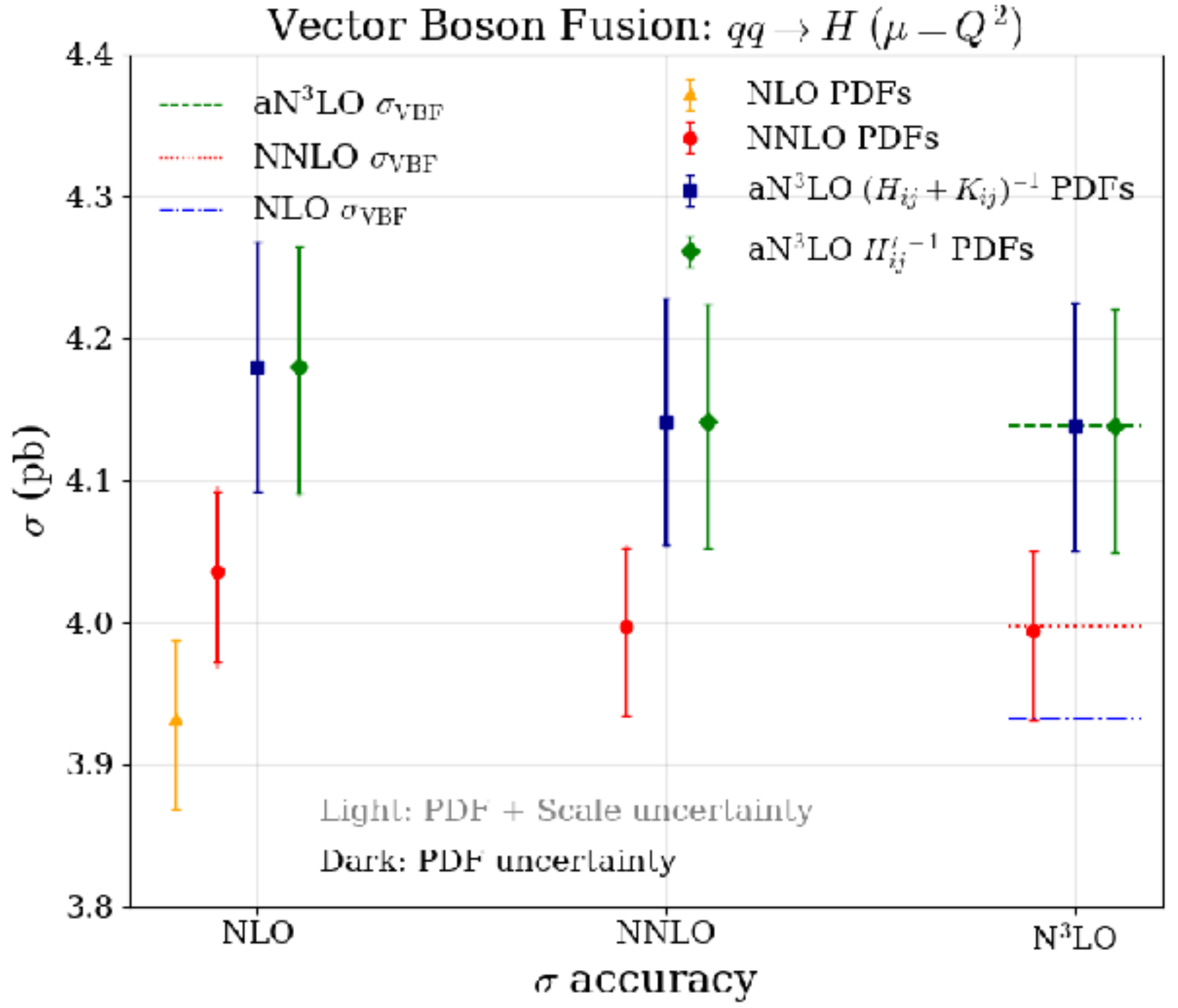
	NNLO	aN^3LO
$\chi^2_{\mathrm{top}}/N_{\mathrm{pts}}$	1.12	1.03

Higgs Predictions

For gluon fusion and Vector Boson Fusion (VBF)

- Good agreement between NNLO and aN^{3}LO for **gluon fusion** (top).
- Cancellation between N^{3}LO cross section and PDFs **not guaranteed**.
- **Less cancellation** for VBF (bottom).
- However **variation between orders** is smaller for VBF σ.





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Dijet data at the LHC

Preliminary

- Fit quality to dijet data at NNLO shows a **good improvement** from jet data.
    - Particularly better fit to $Z\ p_T$.
    - Slightly worse fit to top data.
- Fit quality is also better when fitting to dijet data at aN^{3}LO.
    - Fit quality to all **other data** (incl. $Z\ p_T$ and top datasets) becomes marginally better $\Delta\chi^2_{\text{other}} \sim -20$.
- Full analysis to follow in the **near future**.

- CMS 8 TeV dijet data is fit with leading colour K-factors - full colour results will follow soon.

Jets data

	N_{pts}	χ^2/N_{pts}	
		NNLO	aN^{3}LO
ATLAS 7 TeV jets	140	1.58	1.53
CMS 7 TeV jets	158	1.11	1.20
CMS 8 TeV jets	174	1.50	1.57
Total	472	1.39	1.43

Dijets data

	N_{pts}	χ^2/N_{pts}	
		NNLO	aN^{3}LO
ATLAS 7 TeV dijets	90	1.05	1.14
CMS 7 TeV dijets	54	1.43	1.40
CMS 8 TeV dijets	122	1.04	0.84
Total	266	1.12	1.06

Summary

- **Approximate N³LO PDFs** are available and we encourage their use.
  - Available as *LHAPDF* grids at *www.hep.ucl.ac.uk/msht/* (see publication for usage instructions).
  - Full information is available in the article *JM et. al.,* 2207.04739
- Provide an intuitive and controllable way to **include theoretical uncertainties** into PDFs.
- Results show **good agreement** with current N³LO predictions.
- Stay tuned for further developments regarding **dijets** (and **SeaQuest**) in an aN³LO global fit.

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For an exhaustive list please refer to
J. McGowan et. al., (2022)
2207.04739

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Full χ^2 Breakdown

Dataset	N_{pts}	χ^2	$\Delta\chi^2$ from NNLO
BCDMS μp F_2 [114]	163	180.7	+0.5
BCDMS μd F_2 [114]	151	144.0	−2.0
NMC μp F_2 [115]	123	119.2	−4.9
NMC μd F_2 [115]	123	106.5	−6.2
SLAC ep F_2 [116, 117]	37	32.0	−0.0
SLAC ed F_2 [116, 117]	38	21.6	−1.4
E665 μd F_2 [118]	53	64.3	+4.7
E665 μp F_2 [118]	53	67.1	+2.4
NuTeV νN F_2 [119]	53	38.7	+0.4
NuTeV νN xF_3 [119]	42	34.3	+3.6
NMC $\mu n/\mu p$ [120]	148	128.4	−2.4
E866 / NuSea pp DY [60]	184	208.8	−16.2
E866 / NuSea pd/pp DY [61]	15	7.7	−2.6
HERA ep $F_2^{\rm charm}$ [121]	79	135.8	+3.6
NMC/BCDMS/SLAC/HERA F_L [114, 115, 117, 122–124]	57	45.5	−23.0
CCFR $\nu N \to \mu\mu X$ [113]	86	69.0	+1.3
NuTeV $\nu N \to \mu\mu X$ [113]	84	55.3	−3.1
CHORUS νN F_2 [125]	42	32.9	+2.7
CHORUS νN xF_3 [125]	28	19.5	+1.0
HERA e^+p CC [126]	39	51.6	−0.4
HERA e^-p CC [126]	42	66.3	−3.8
HERA e^+p NC 820 GeV [126]	75	84.0	−5.8
HERA e^-p NC 460 GeV [126]	209	247.1	−1.2
HERA e^+p NC 920 GeV [126]	402	476.2	−36.5
HERA e^-p NC 575 GeV [126]	259	247.9	−15.1
HERA e^-p NC 920 GeV [126]	159	243.4	−1.0
CDF II $p\bar{p}$ incl. jets [82]	76	68.7	+8.3
DØ II Z rap. [62]	28	16.8	+0.5
CDF II Z rap. [63]	28	39.6	+2.5
DØ II $W \to \nu\mu$ asym. [64]	10	16.7	−0.6
CDF II W asym. [65]	13	20.1	+1.1

Dataset	N_{pts}	χ^2	$\Delta\chi^2$ from NNLO
DØ II $W \to \nu e$ asym. [66]	12	29.0	−5.0
DØ II $p\bar{p}$ incl. jets [83]	110	113.6	−6.7
ATLAS W^+, W^-, Z [67]	30	29.9	−0.0
CMS W asym. $p_T > 35$ GeV [68]	11	7.0	−0.8
CMS W asym. $p_T > 25, 30$ GeV [69]	24	7.5	+0.1
LHCb $Z \to e^+e^-$ [70]	9	20.6	−2.1
LHCb W asym. $p_T > 20$ GeV [71]	10	12.9	+0.4
CMS $Z \to e^+e^-$ [72]	35	17.3	−0.6
ATLAS High-mass Drell-Yan [73]	13	18.6	−0.3
Tevatron, ATLAS, CMS $\sigma_{t\bar{t}}$ [97–109]	17	14.1	−0.5
CMS double diff. Drell-Yan [74]	132	136.8	−7.7
LHCb 2015 W, Z [57, 58]	67	97.1	−2.3
LHCb 8TeV $Z \to ee$ [75]	17	26.9	+0.7
CMS 8 TeV W [76]	22	12.1	−0.6
ATLAS 7 TeV jets [84]	140	214.0	−7.6
CMS 7 TeV $W + c$ [88]	10	12.2	+3.6
ATLAS 7 TeV high prec. W, Z [59]	61	110.4	−6.2
CMS 7 TeV jets [81]	158	189.9	+14.1
DØ W asym. [77]	14	8.8	−3.3
ATLAS 8 TeV Z p_T [87]	104	106.3	−82.2
CMS 8 TeV jets [85]	174	271.9	+10.6
ATLAS 8 TeV sing. diff. $t\bar{t}$ [110]	25	25.0	−0.7
ATLAS 8 TeV sing. diff. $t\bar{t}$ dilep. [111]	5	2.2	−1.2
ATLAS 8 TeV High-mass DY [78]	48	63.8	+6.6
ATLAS 8 TeV W + jets [89]	30	19.2	+1.1
CMS 8 TeV double diff. $t\bar{t}$ [112]	15	23.8	+1.3
ATLAS 8 TeV W [79]	22	54.8	−2.6
CMS 2.76 TeV jet [86]	81	113.7	+10.8
CMS 8 TeV sing. diff. $t\bar{t}$ [91]	9	8.3	−4.9
ATLAS 8 TeV double diff. Z [80]	59	81.5	−4.1

Comparison with/without K-factors

Dimuon Dataset	χ^2	$\Delta\chi^2$ from NNLO	$\Delta\chi^2$ from NNLO (NNLO K-factors)
CCFR $\nu N \to \mu\mu X$ [113]	69.0 / 86	+1.3	+2.6
NuTeV $\nu N \to \mu\mu X$ [113]	55.3 / 84	−3.1	−3.1
Total	124.3 / 170	−1.8	−0.5

DY Dataset	χ^2	$\Delta\chi^2$ from NNLO	$\Delta\chi^2$ from NNLO (NNLO K-factors)
E866 / NuSea pp DY [60]	208.8 / 184	−16.2	−11.6
E866 / NuSea pd/pp DY [61]	7.7 / 15	−2.6	−2.9
DØ II Z rap. [62]	16.8 / 28	+0.5	+0.3
CDF II Z rap. [63]	39.6 / 28	+2.5	+1.3
DØ II $W \to \nu\mu$ asym. [64]	16.7 / 10	−0.6	−0.5
CDF II W asym. [65]	20.1 / 13	+1.1	+0.8
DØ II $W \to \nu e$ asym. [66]	29.0 / 12	−5.0	−5.3
ATLAS W^+, W^-, Z [67]	29.9 / 30	−0.0	+0.3
CMS W asym. $p_T > 35$ GeV [68]	7.0 / 11	−0.8	−0.6
CMS W asym. $p_T > 25, 30$ GeV [69]	7.5 / 24	+0.1	−0.1
LHCb $Z \to e^+e^-$ [70]	20.6 / 9	−2.1	−1.6
LHCb W asym. $p_T > 20$ GeV [71]	12.9 / 10	+0.4	+1.0
CMS $Z \to e^+e^-$ [72]	17.3 / 35	−0.6	−0.6
ATLAS High-mass Drell-Yan [73]	18.6 / 13	−0.3	−1.1
CMS double diff. Drell-Yan [74]	136.8 / 132	−7.7	+11.9
LHCb 2015 W, Z [57, 58]	97.1 / 67	−2.3	−2.8
LHCb 8TeV $Z \to ee$ [75]	26.9 / 17	+0.7	−0.2
CMS 8 TeV W [76]	12.1 / 22	−0.6	+0.2
ATLAS 7 TeV high prec. W, Z [59]	110.4 / 61	−6.2	−18.7
DØ W asym. [77]	8.8 / 14	−3.3	−1.8
ATLAS 8 TeV High-mass DY [78]	63.8 / 48	+6.6	+2.8
ATLAS 8 TeV W [79]	54.8 / 22	−2.6	−1.1
ATLAS 8 TeV double diff. Z [80]	81.5 / 59	−4.1	−1.9
Total	1044.6 / 864	−43.2	−32.1

DIS Dataset	χ^2	$\Delta\chi^2$ from NNLO	$\Delta\chi^2$ from NNLO (NNLO K-factors)
BCDMS μp F_2 [114]	180.7 / 163	+0.5	+0.1
BCDMS μd F_2 [114]	144.0 / 151	−2.0	−1.1
NMC μp F_2 [115]	119.2 / 123	−4.9	−7.0
NMC μd F_2 [115]	106.5 / 123	−6.2	−10.2
SLAC ep F_2 [116, 117]	32.0 / 37	−0.0	+0.5
SLAC ed F_2 [116, 117]	21.6 / 38	−1.4	−1.4
E665 μp F_2 [118]	64.3 / 53	+4.7	+5.7
E665 μd F_2 [118]	67.1 / 53	+2.4	+2.8
NuTeV νN F_2 [119]	38.7 / 53	+0.4	+1.7
NuTeV νN xF_3 [119]	34.3 / 42	+3.6	+1.9
NMC $\mu n/\mu p$ [120]	128.4 / 148	−2.4	−2.6
HERA ep F_2^{charm} [121]	135.8 / 79	+3.6	+9.1
NMC/BCDMS/SLAC/HERA F_L [114, 115, 117, 122–124]	45.5 / 57	−23.0	−23.3
CHORUS νN F_2 [125]	32.9 / 42	+2.7	+3.0
CHORUS νN xF_3 [125]	19.5 / 28	+1.0	+1.1
HERA e^+p CC [126]	51.6 / 39	−0.4	+0.3
HERA e^-p CC [126]	66.3 / 42	−3.8	−3.0
HERA e^+p NC 820 GeV [126]	84.0 / 75	−5.8	−5.5
HERA e^-p NC 460 GeV [126]	247.1 / 209	−1.2	−0.4
HERA e^+p NC 920 GeV [126]	476.2 / 402	−36.5	−33.3
HERA e^-p NC 575 GeV [126]	247.9 / 259	−15.1	−14.4
HERA e^-p NC 920 GeV [126]	243.4 / 159	−1.0	−1.0
Total	2587.0 / 2375	−84.7	−76.8

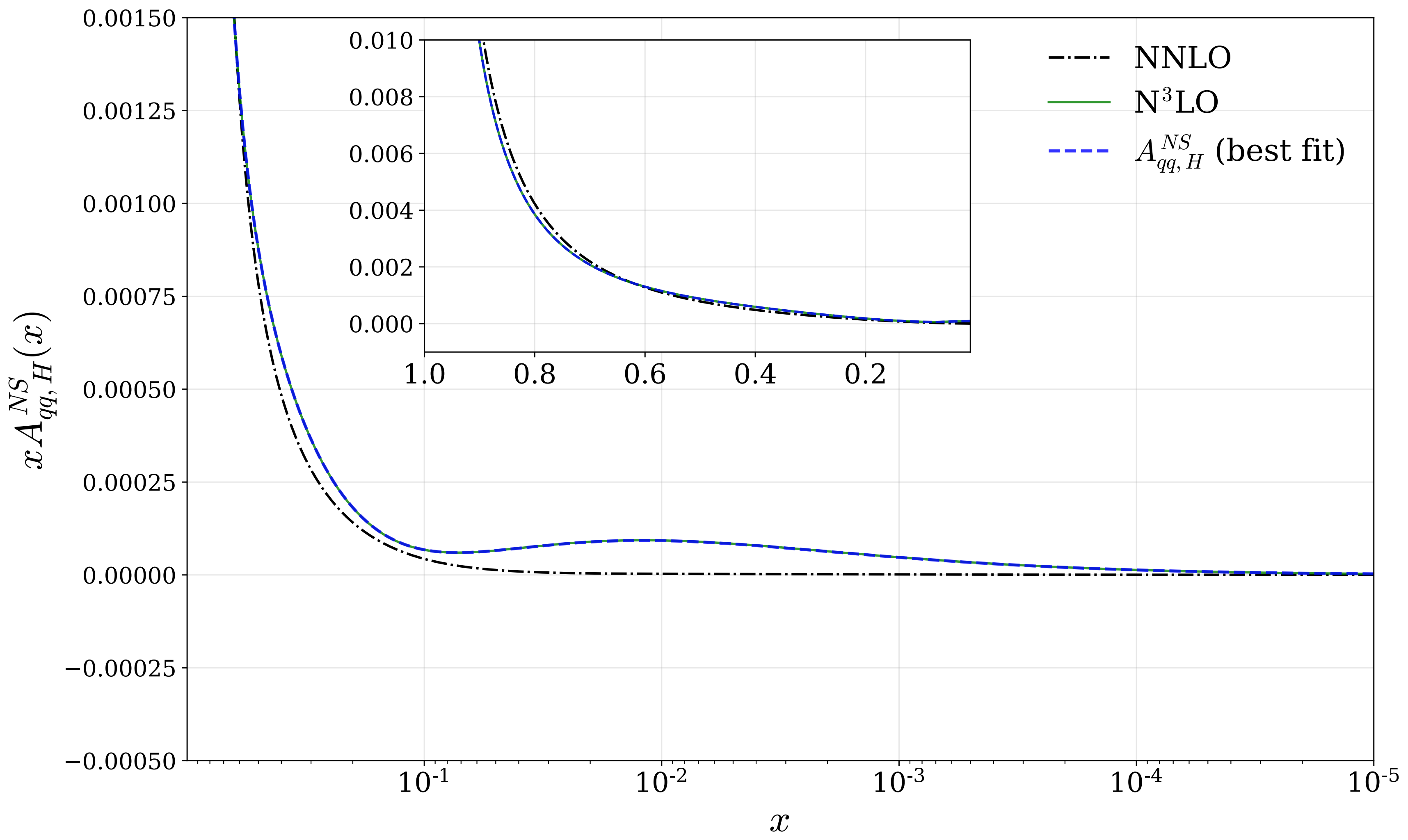
Top Dataset	χ^2	$\Delta\chi^2$ from NNLO	$\Delta\chi^2$ from NNLO (NNLO K-factors)
Tevatron, ATLAS, CMS $\sigma_{t\bar{t}}$ [97–109]	14.1 / 17	−0.5	−0.7
ATLAS 8 TeV single diff. $t\bar{t}$ [110]	25.0 / 25	−0.7	−0.0
ATLAS 8 TeV single diff. $t\bar{t}$ dilep. [111]	2.2 / 5	−1.2	−0.7
CMS 8 TeV double diff. $t\bar{t}$ [112]	23.8 / 15	+1.3	+4.9
CMS 8 TeV single diff. $t\bar{t}$ [91]	8.3 / 9	−4.9	−5.4
Total	73.3 / 71	−6.0	−2.0

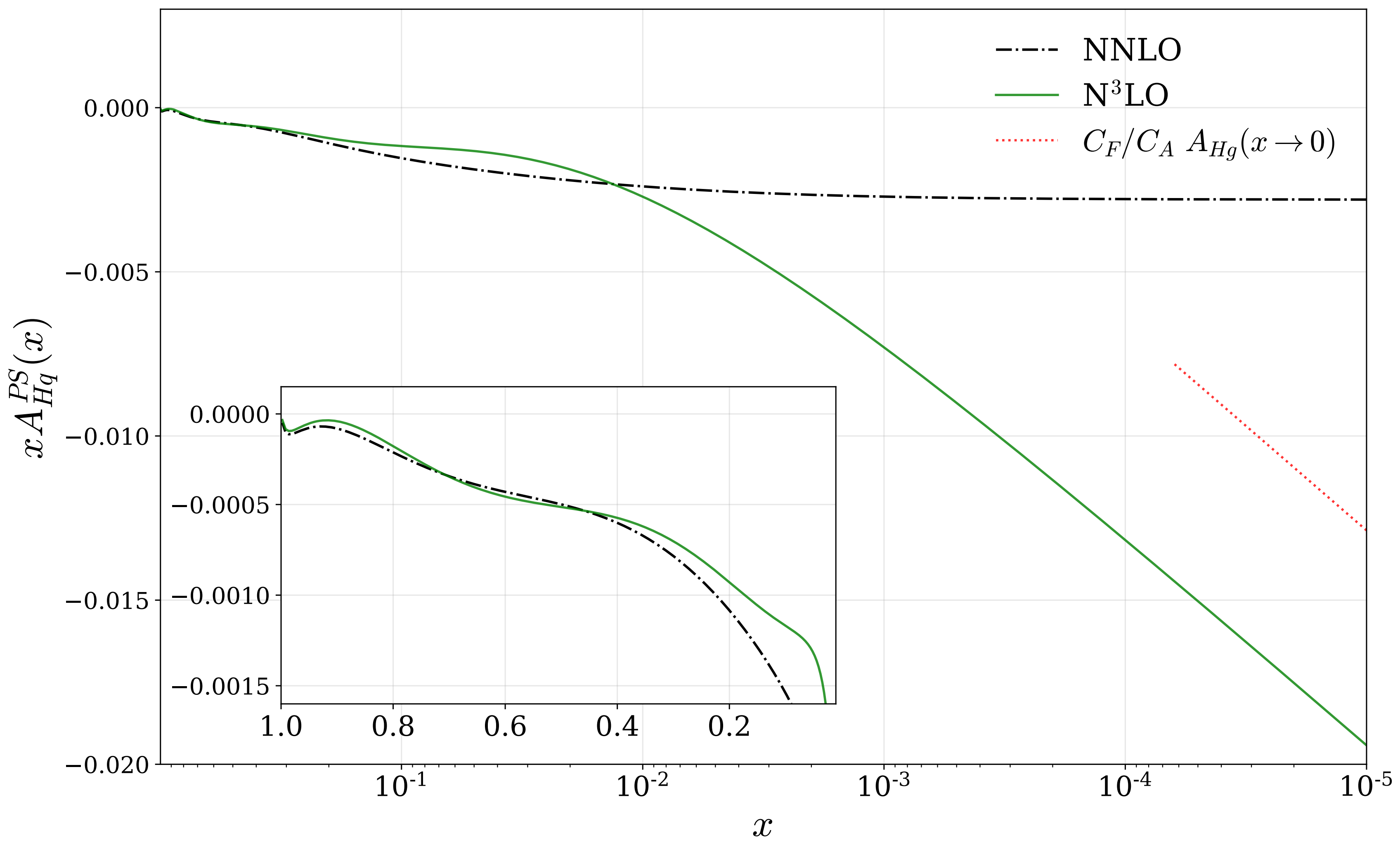
Jets Dataset	χ^2	$\Delta\chi^2$ from NNLO	$\Delta\chi^2$ from NNLO (NNLO K-factors)
CDF II $p\bar{p}$ incl. jets [82]	68.7 / 76	+8.3	+0.6
DØ II $p\bar{p}$ incl. jets [83]	113.6 / 110	−6.7	−3.5
ATLAS 7 TeV jets [84]	214.0 / 140	−7.6	+2.4
CMS 7 TeV jets [81]	189.9 / 158	+14.1	+14.5
CMS 8 TeV jets [85]	271.9 / 174	+10.6	+22.9
CMS 2.76 TeV jet [86]	113.7 / 81	+10.8	+13.5
Total	971.7 / 739	+29.6	+50.3

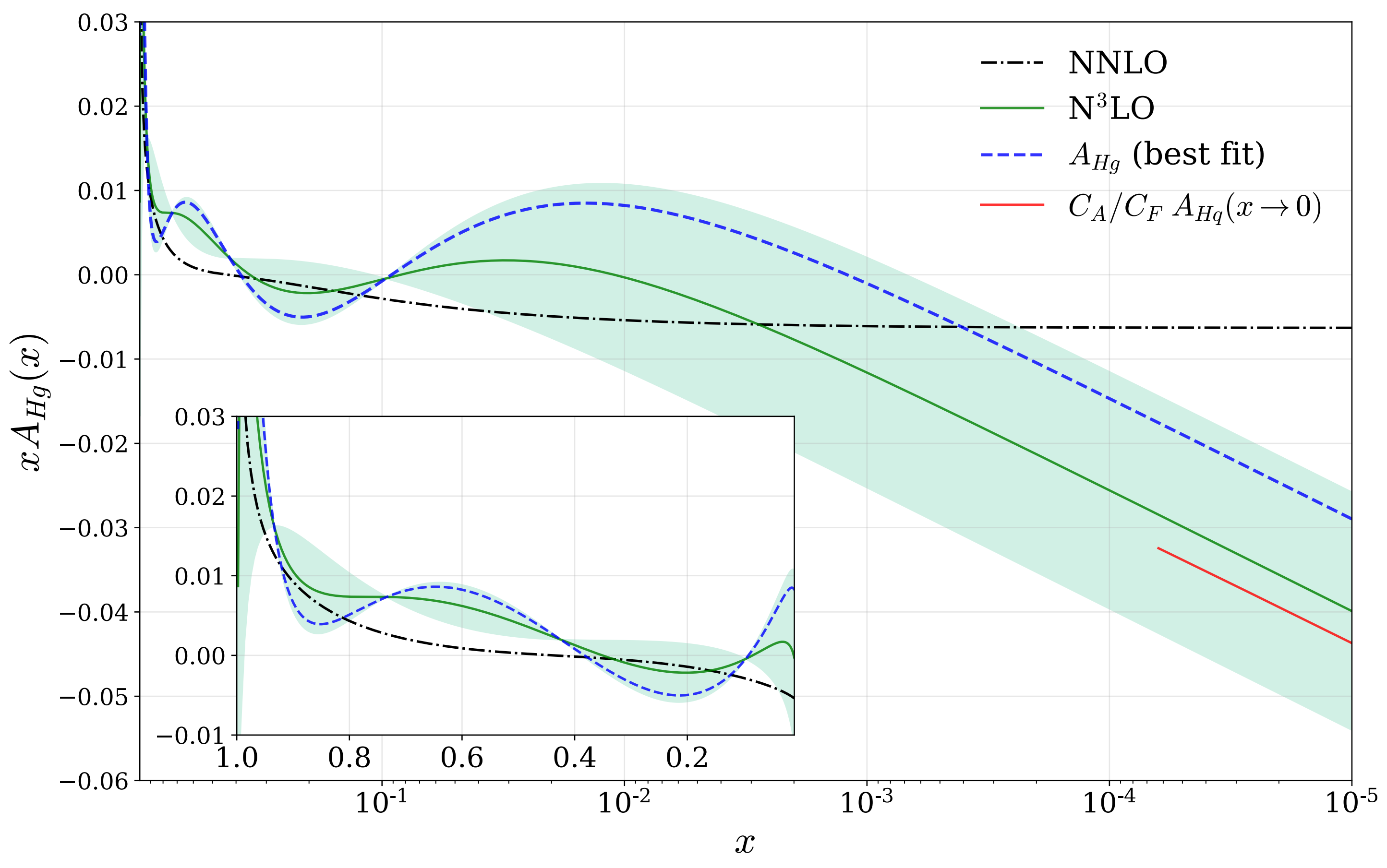
p_T Jets Dataset	χ^2	$\Delta\chi^2$ from NNLO	$\Delta\chi^2$ from NNLO (NNLO K-factors)
CMS 7 TeV $W + c$ [88]	12.2 / 10	+3.6	+1.3
ATLAS 8 TeV Z p_T [87]	106.3 / 104	−82.2	−52.5
ATLAS 8 TeV W + jets [89]	19.2 / 30	+1.1	+0.4
Total	137.7 / 144	−77.5	−50.9

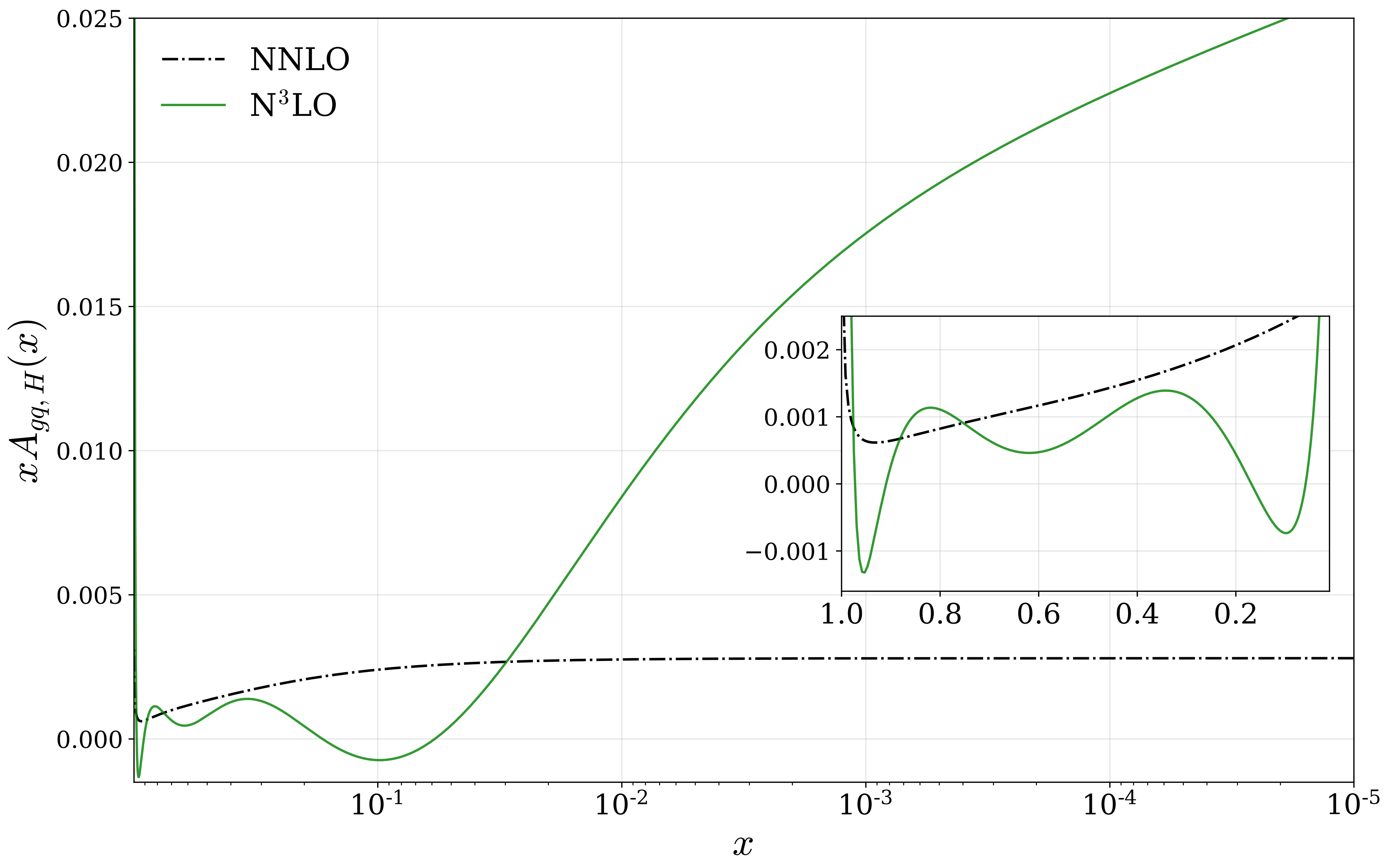
Approximate N³LO Splitting Functions

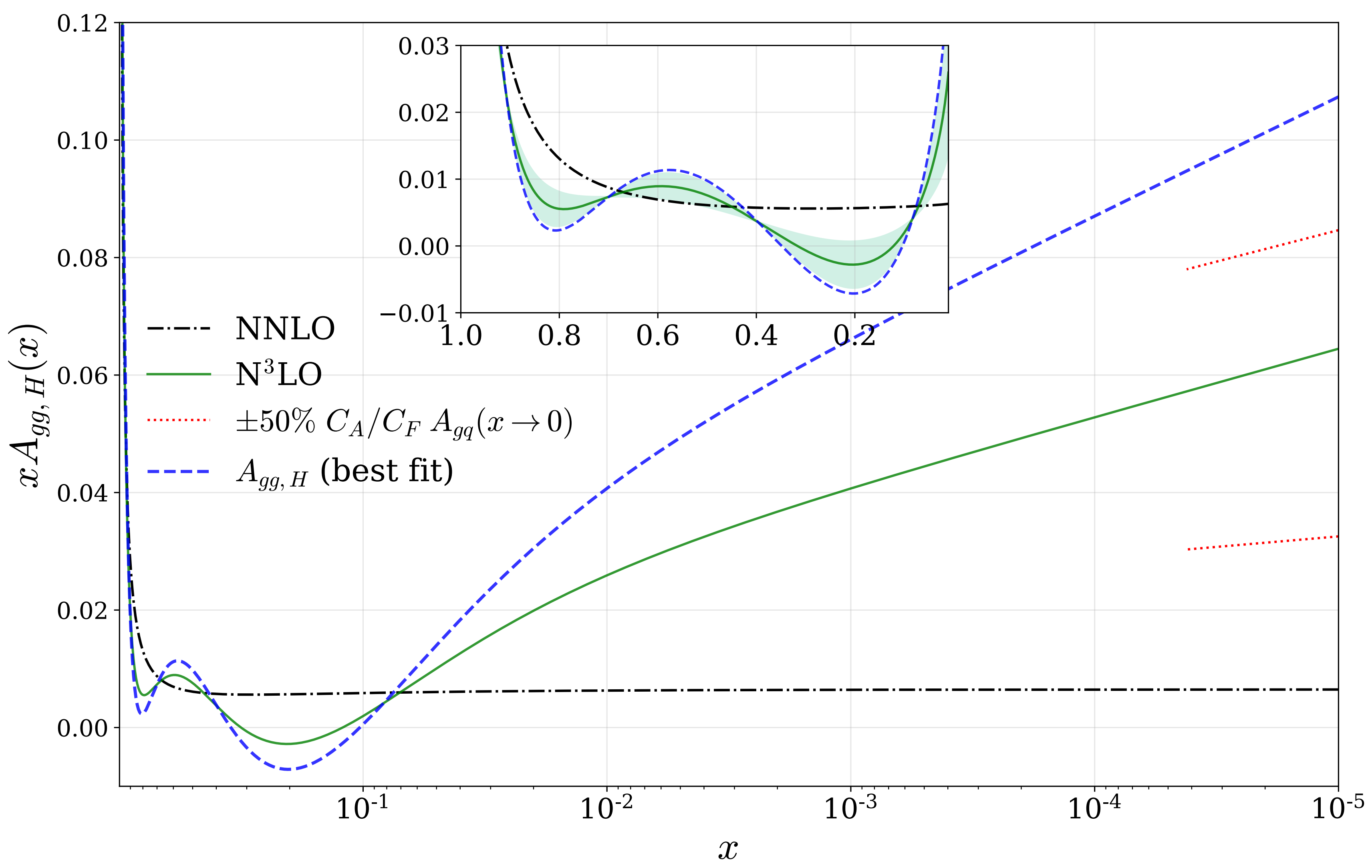
Approximate N^{3}LO Transition Matrix Elements











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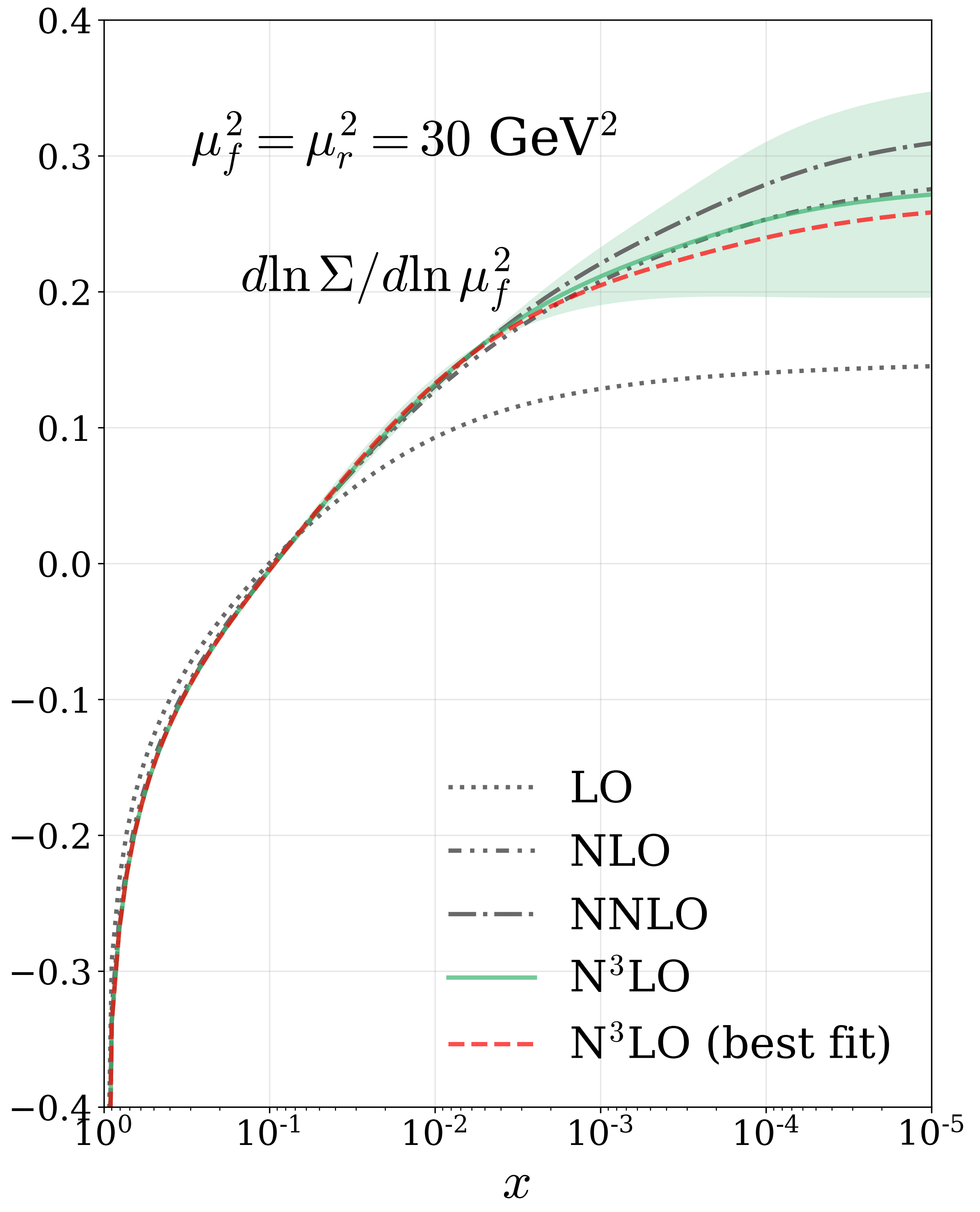
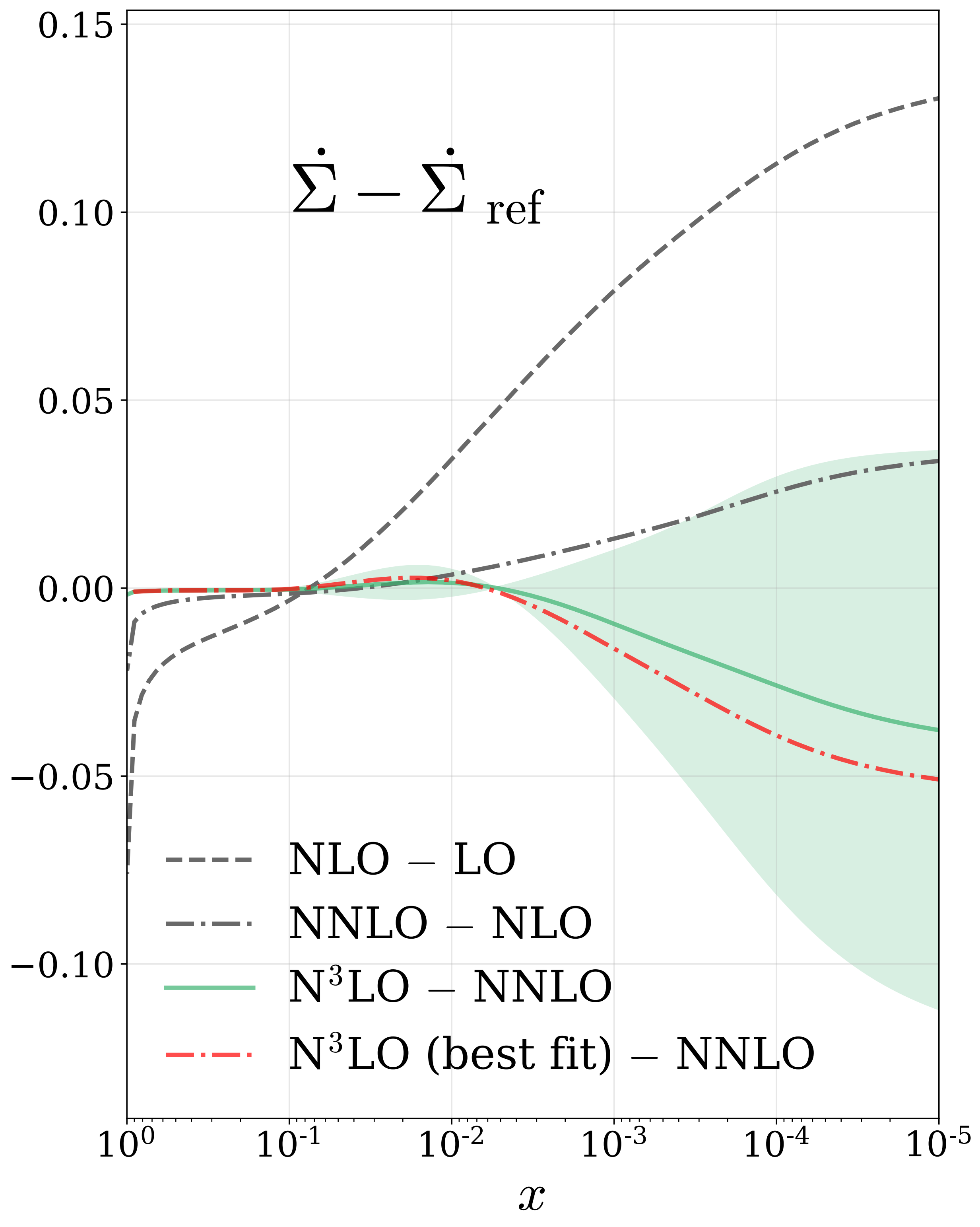
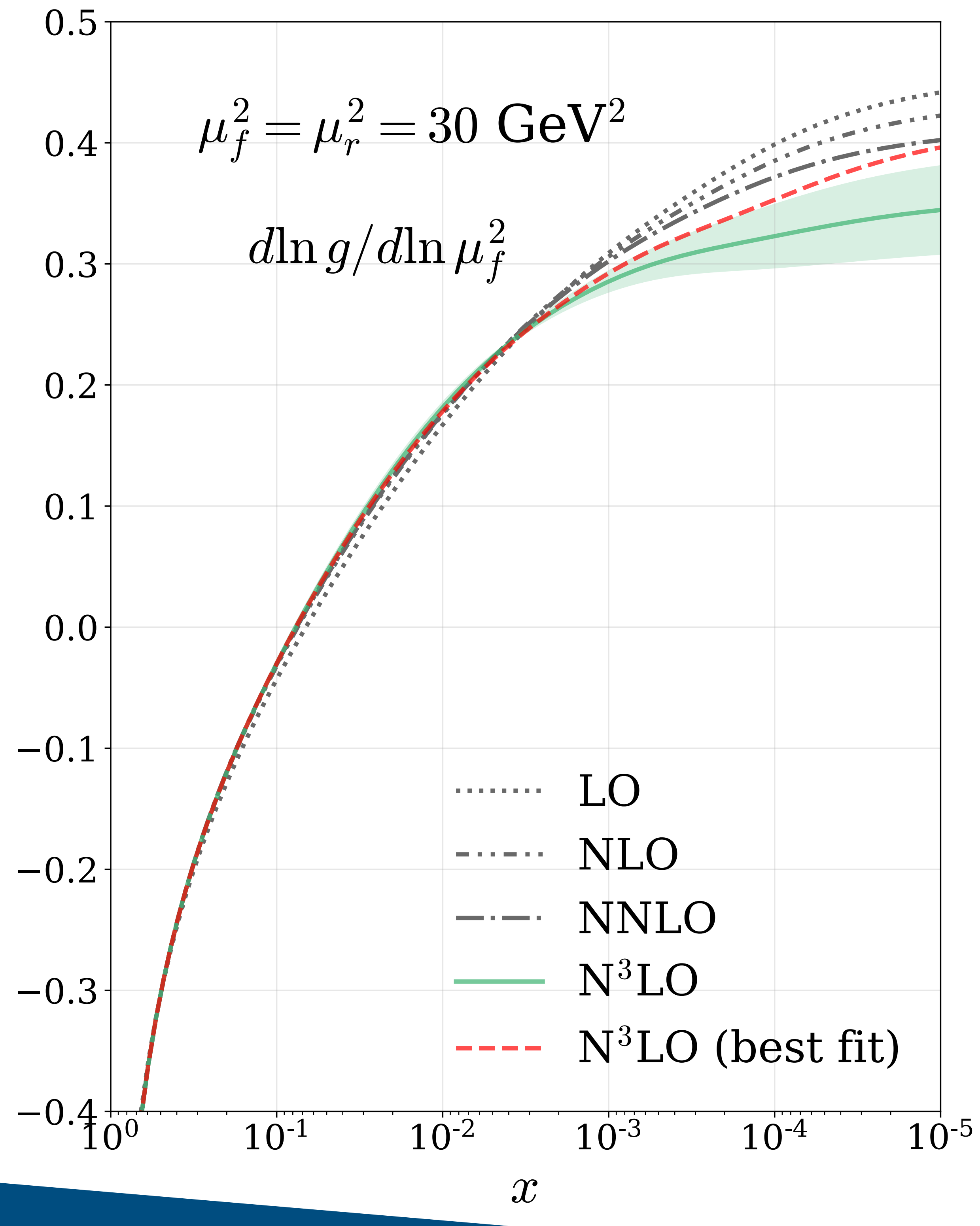
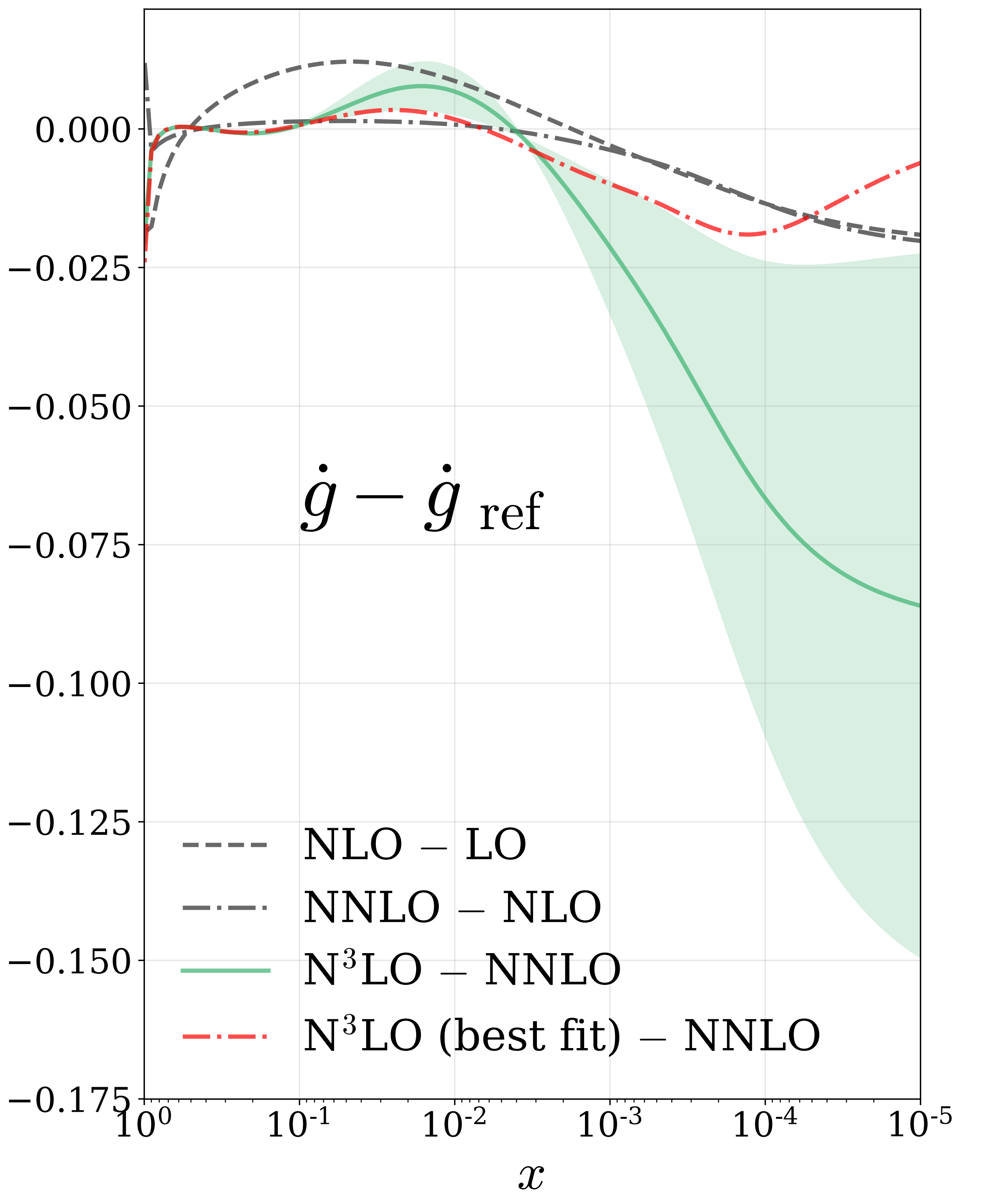
DGLAP Evolution









Usage of aN^{3}LO PDFs

- For DIS processes, using the standard PDF set is advised.
- For any of the other 5 processes included in the fit (which we fit K-factors for), we provide the full details of these fitted aN^{3}LO K-factors.
- For processes not included in the fit, this will be a little more involved.
    - Full details and instructions are provided in *J. McGowan et. al., (2022)* 2207.04739
        - Feel free to contact us with questions about usage.

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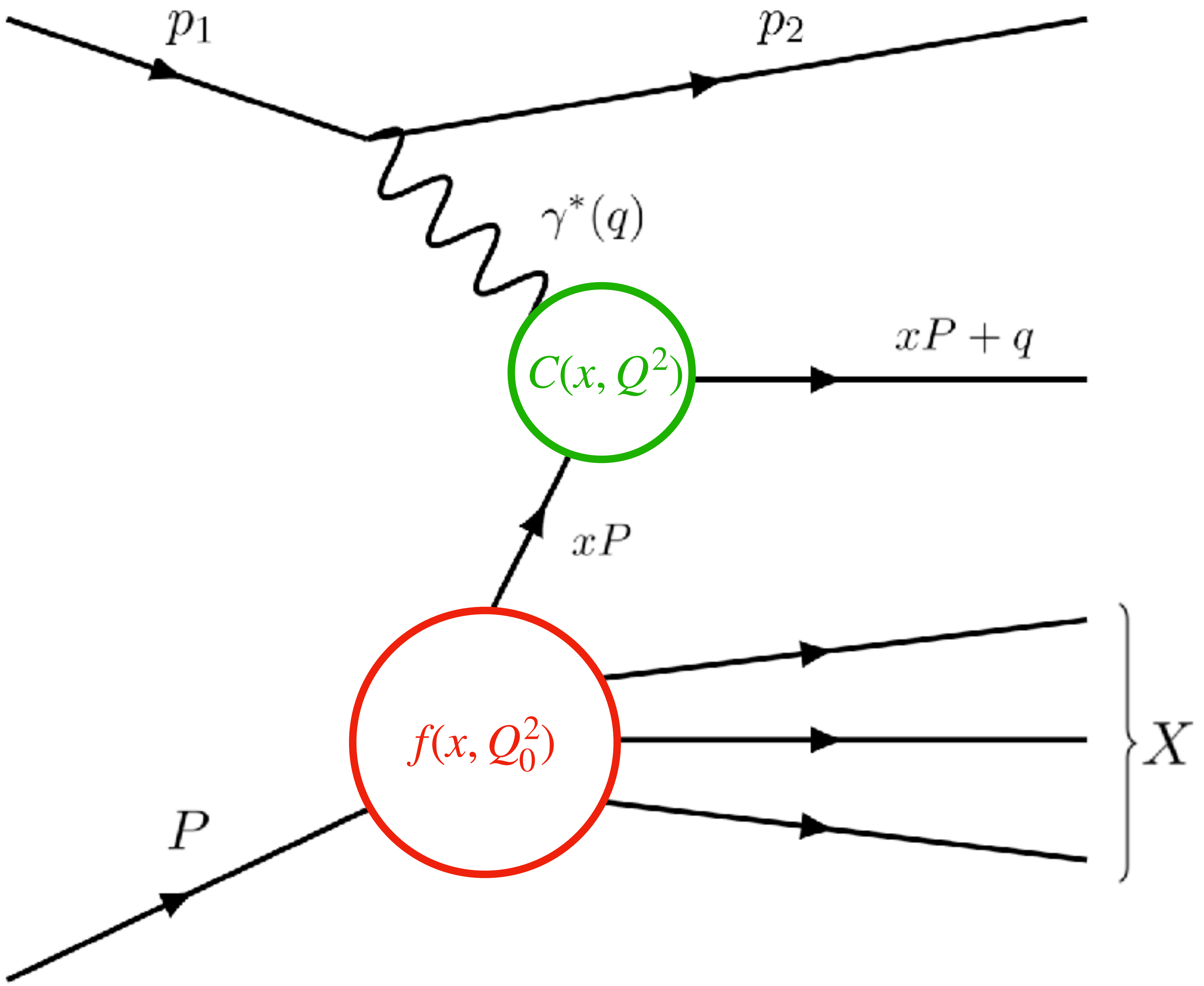
Deep Inelastic Scattering

- PDFs - **probability** of a parton fluctuating out of proton.
- Coefficient function - **perturbatively** calculated.

$$C(x,Q^2) = C^{(0)}(x,Q^2) + \alpha_s C^{(1)}(x,Q^2) + \alpha_s^2 C^{(2)}(x,Q^2) + \alpha_s^3 C^{(3)}(x,Q^2) + \dots$$

- PDFs are **determined from experiment** using complex parameterisations.



- ‘Global’ fit using many different data sets and **processes**.

Deep Inelastic Scattering

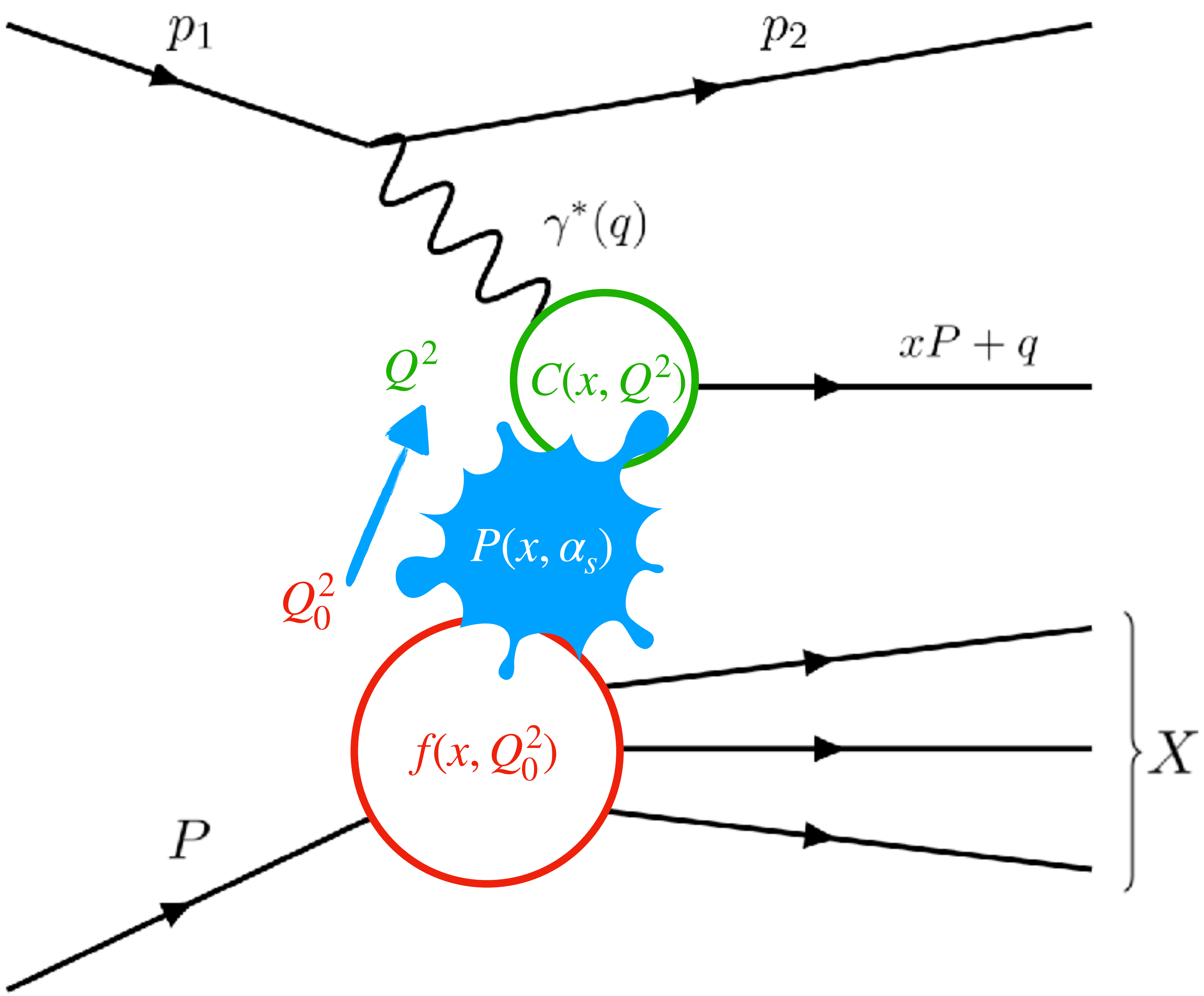
- Scale dependence of PDFs is **also calculable** in QCD perturbation theory!

$$\mu^2 \frac{d}{d\mu^2} f(x,\mu^2) = P(x,\alpha_s(\mu^2)) \otimes f(x,\mu^2)$$

$$P(x,\alpha_s) = \alpha_s P^{(0)}(x) + \alpha_s^2 P^{(1)}(x) + \alpha_s^3 P^{(2)}(x) + \alpha_s^4 P^{(3)}(x) + \dots$$

where $P(x,\alpha_s)$ are the **splitting functions.**

- PDFs parameterised at a starting scale Q_0^2 and **evolved** to a desired scale Q^2.



Takeaway: Perturbatively calculable quantities are essential **ingredients** for PDF **determination** (and making predictions using PDFs).

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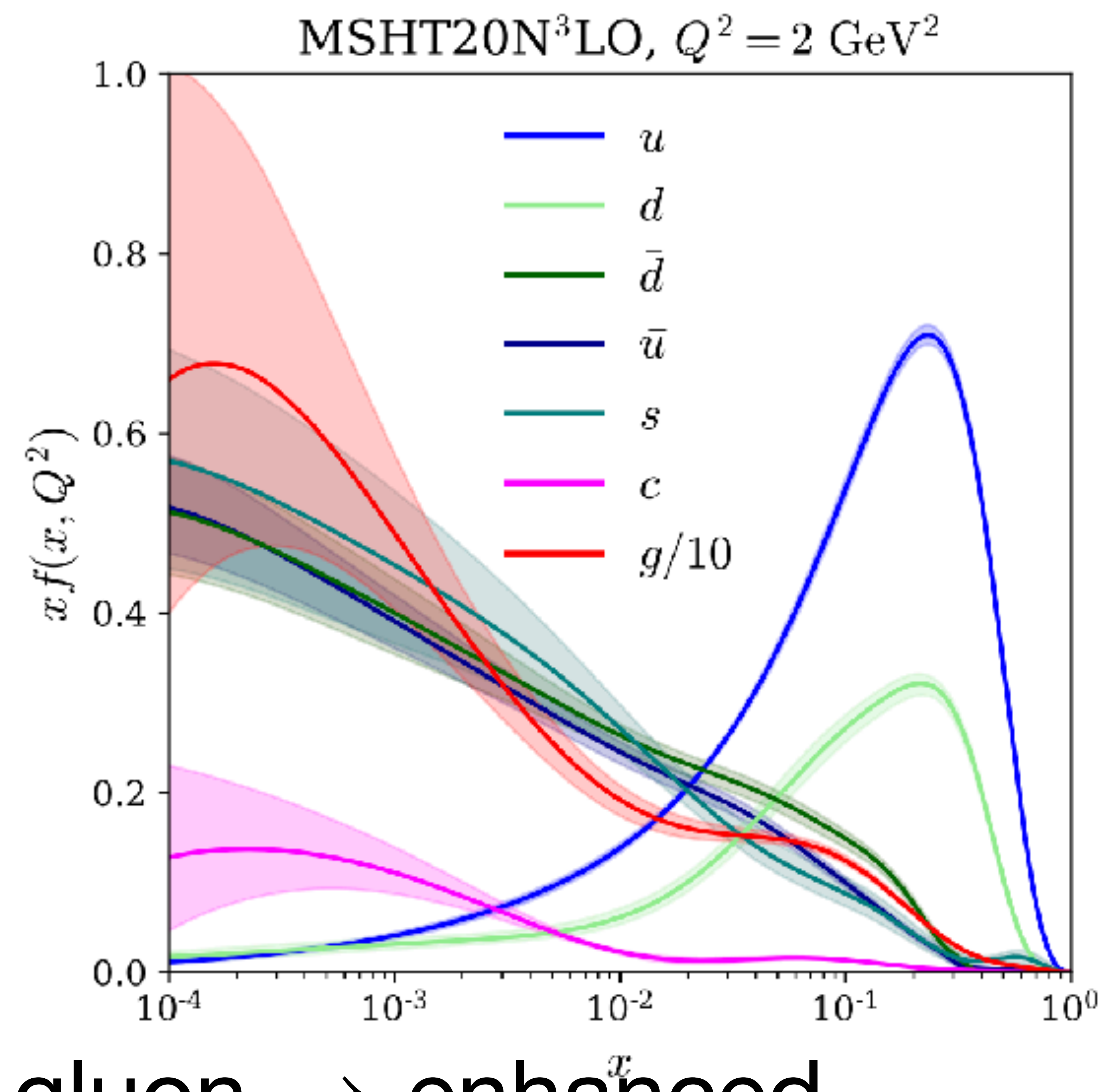
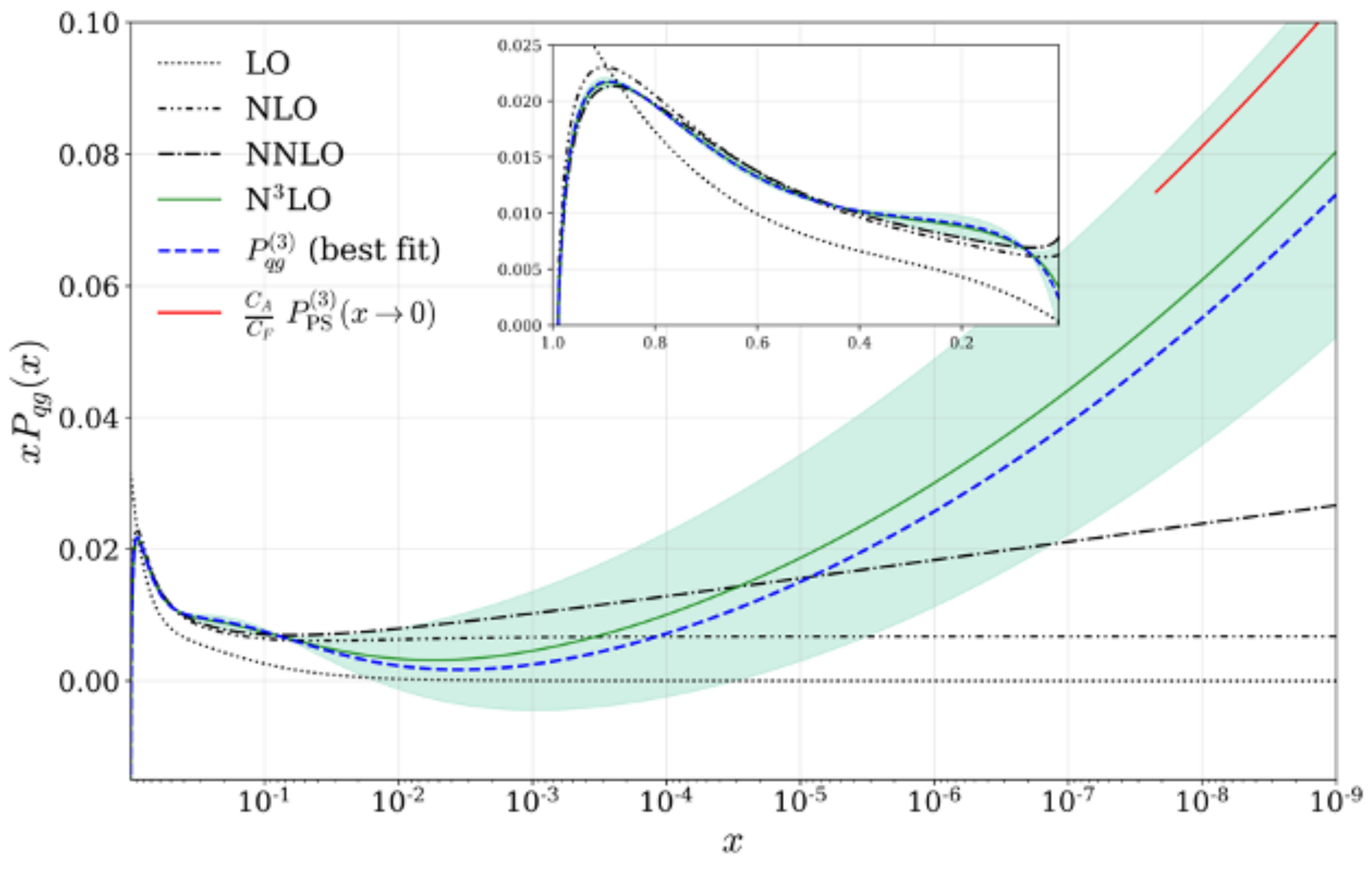
MSHT Approximate N^{3}LO PDFs

- MHOUs are leading source of theoretical uncertainty.
- Parameterisation of N^{3}LO F_2 structure function (incl. N^{3}LO splitting functions) and N^{3}LO K-factors for a consistent aN^{3}LO fit.
- Overall better fit to data - reduced tensions between small and large-x.





- Results show a harder gluon → enhanced charm.
- In agreement with recent N^{3}LO results - DY and Top process K-factors. arXiv: 2107.09085, 2203.03698
- Paper and PDF sets available (very) soon.

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