One-loop corrections to dihadron production in DIS at small x

Jamal Jalilian-Marian

Baruch College and the City University of New York Graduate Center

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Outline

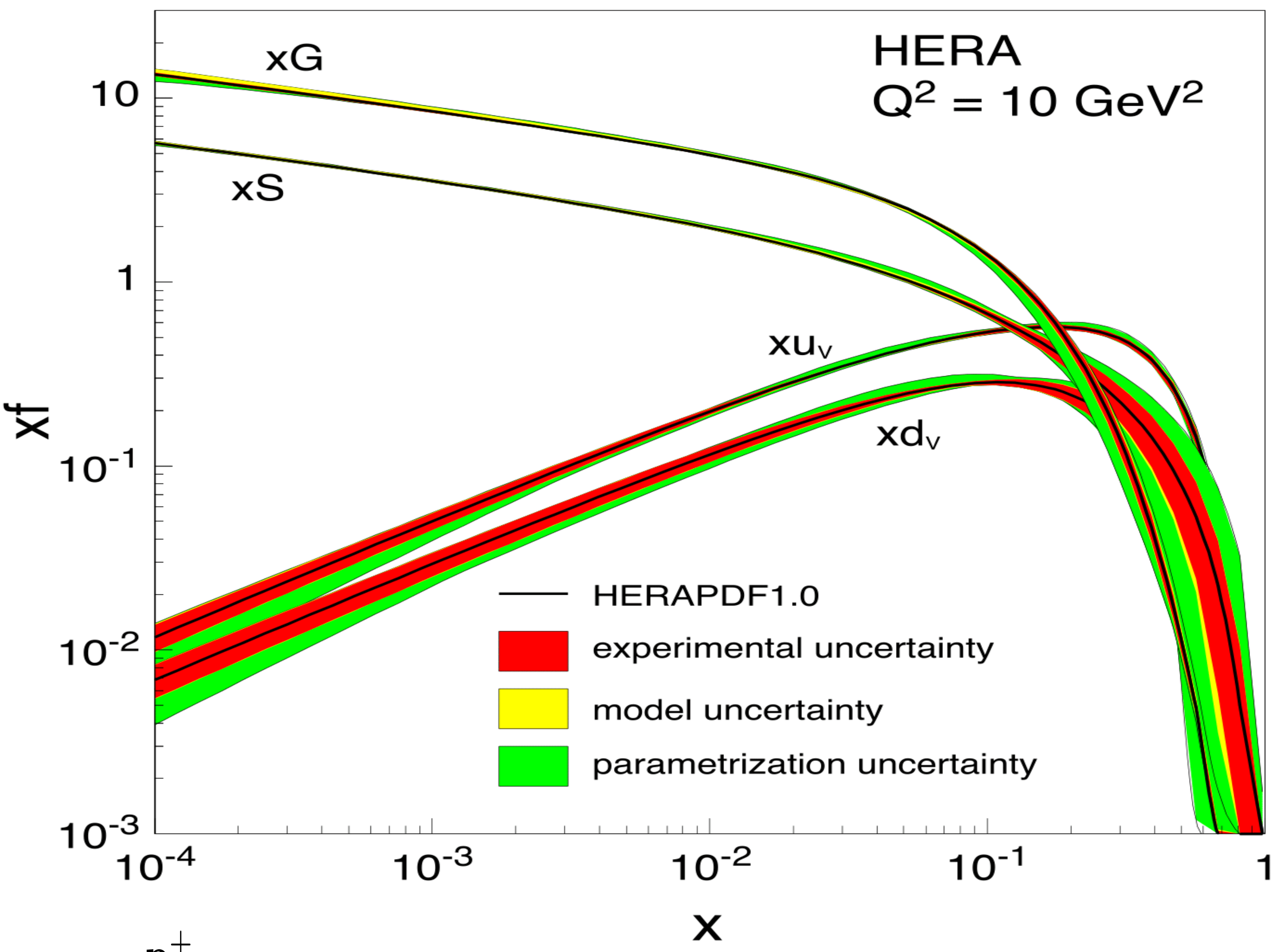
- **Small x: gluon saturation and Color Glass Condensate**

- **Dihadron production at small x:**

  DIS at EIC ⟶ UPC at the LHC

- **Transition from large to small x:**

  Photon-hadron (jet) correlations in pA collisions at RHIC/LHC



$x = \frac{p^+}{P^+}$ ***x** is the fraction of hadron energy carried by a parton*

What drives the growth of parton distributions?

Splitting functions at leading order $O(\alpha_S{}^0)$ $(x \neq 1)$

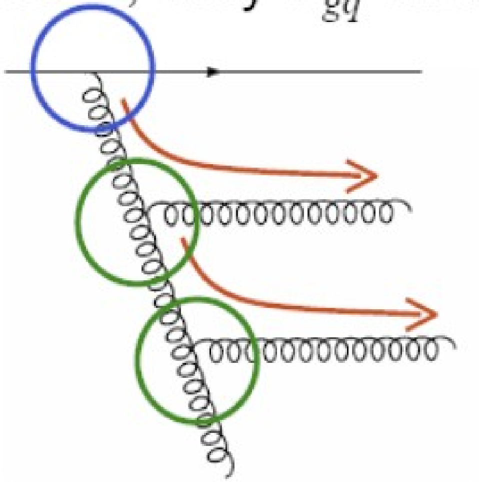
$$P_{qq}^{(0)}(x) = C_F \frac{1+x^2}{1-x}$$

$$P_{qg}^{(0)}(x) = \frac{1}{2}\left[x^2 + (1-x)^2\right]$$

$$P_{gq}^{(0)}(x) = C_F \frac{1+(1-x)^2}{x}$$

$$P_{gg}^{(0)}(x) = 2C_A\left[\frac{x}{1-x} + \frac{1-x}{x} + x(1-x)\right]$$

At small x, only P_{gq} and P_{gg} are relevant.



→ Gluon dominant at small *x*!

The double log approximation (DLA) of DGLAP is easily solved.

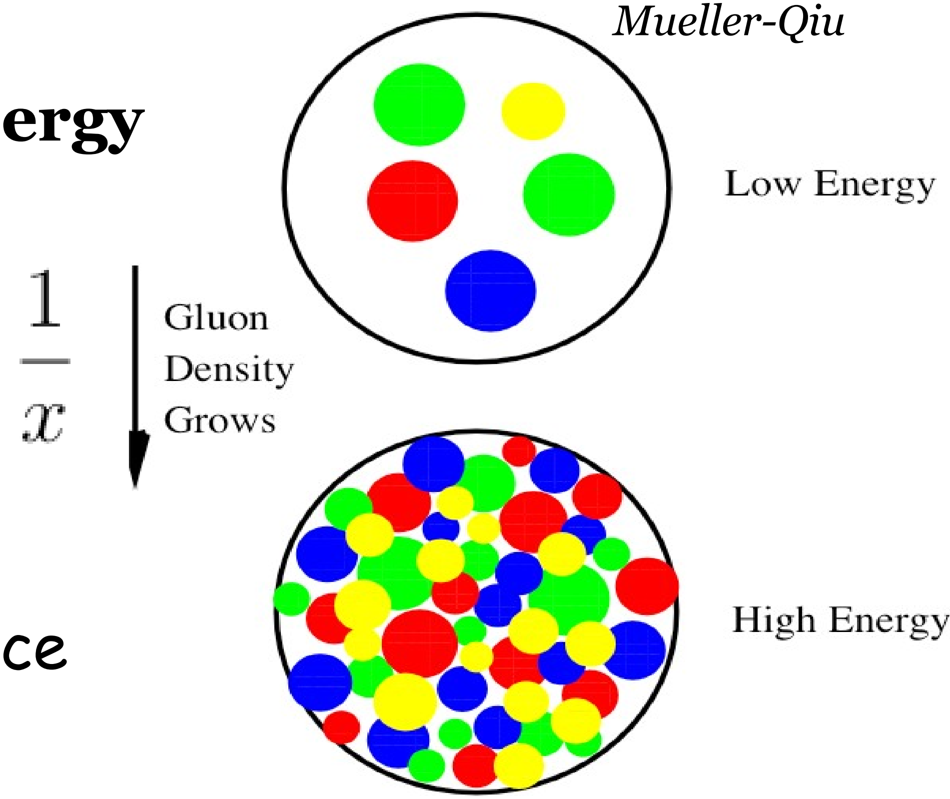
-- increase of gluon distribution at small x

$$\mathbf{xg(x,Q^2) \sim e^{\sqrt{\alpha_s\,(\log 1/x)\,(\log Q^2)}}}$$

new QCD dynamics at small x ?

Gribov-Levin-Ryskin
Mueller-Qiu

A hadron/nucleus at high energy



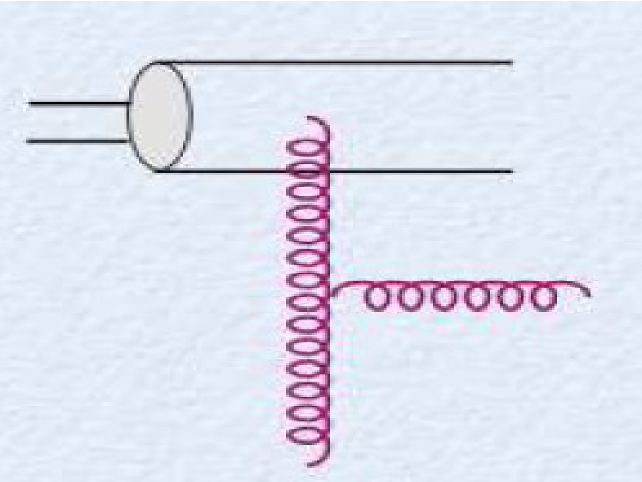
radiated gluons have the same size ($1/Q^2$) - the number of partons increase due to the increased longitudinal phase space

hadron/nucleus becomes a dense system of gluons:
concept of a quasi-free parton is not useful

physics of strong color fields in QCD
novel universal properties of theory in this limit (?)

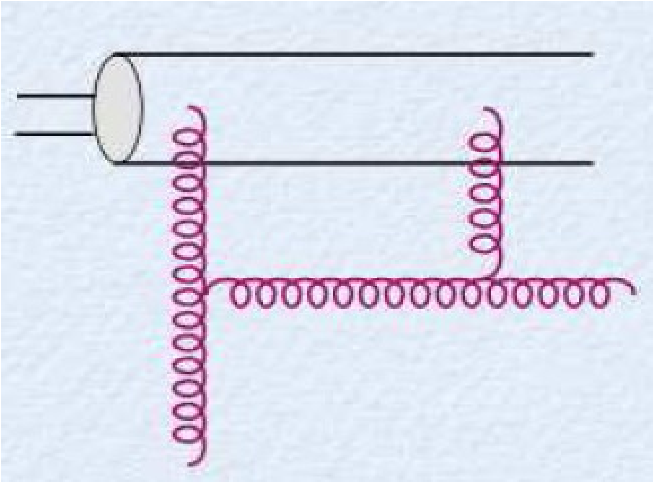
Perturbative QCD breaks down at small x

"attractive" bremsstrahlung *vs.* *"repulsive" recombination*



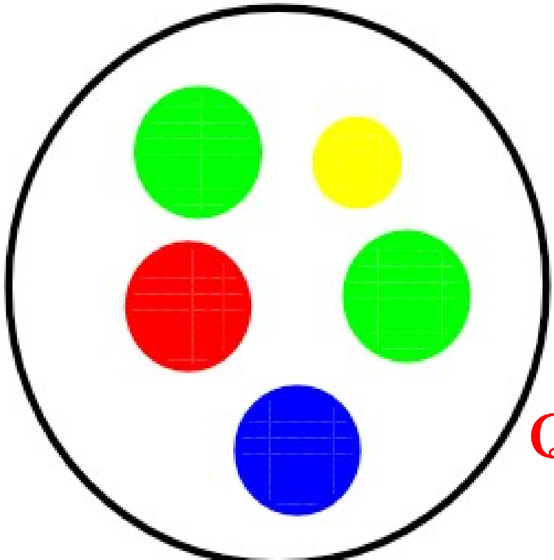
$$S \to \infty, \;\; Q^2 \;\; fixed$$

$$x_{Bj} \equiv \frac{Q^2}{S} \to 0$$



included in pQCD

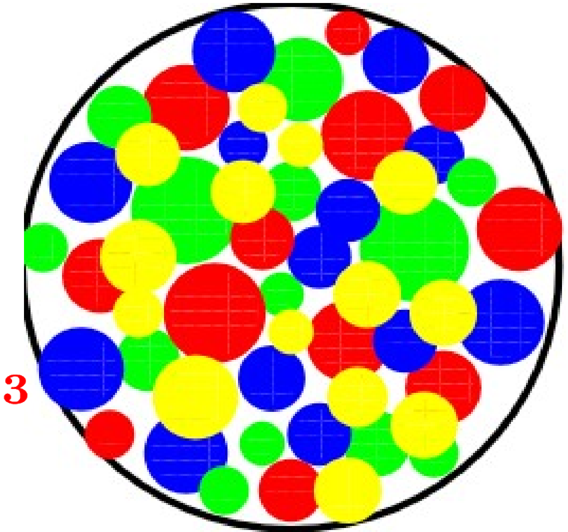
not included in pQCD



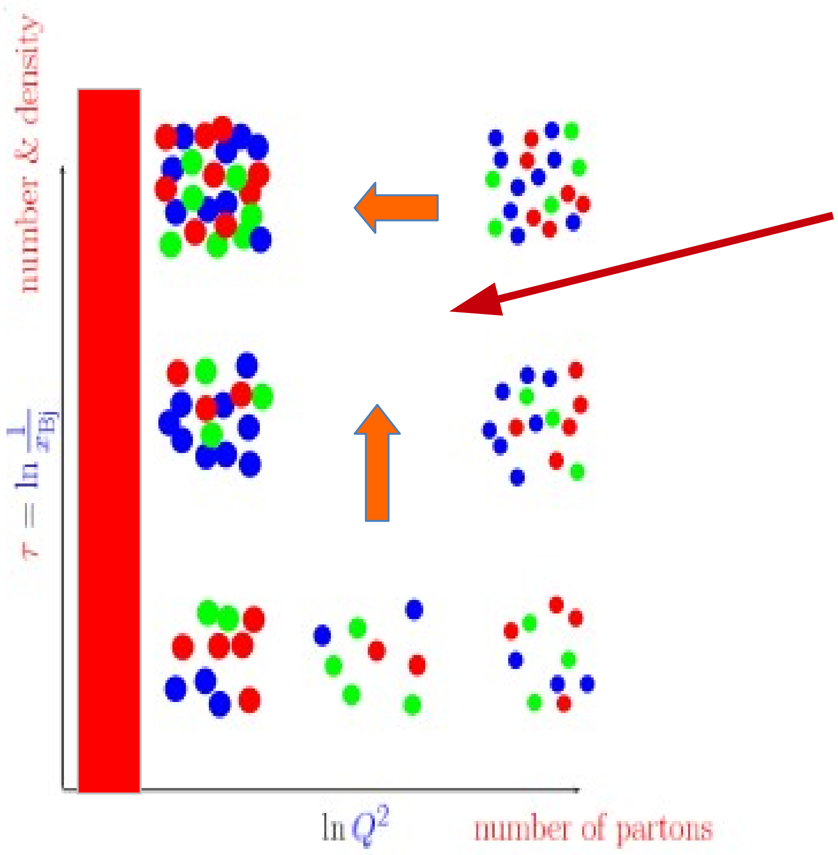
energy ~ 1/x

$$\frac{\alpha_s}{Q^2}\,\frac{xG(x, Q^2)}{\pi r^2} \sim 1$$

$$Q_s^2(x, b_t, A) \sim A^{1/3}\,\left(\frac{1}{x}\right)^{0.3}$$



Many-body dynamics of universal gluonic matter



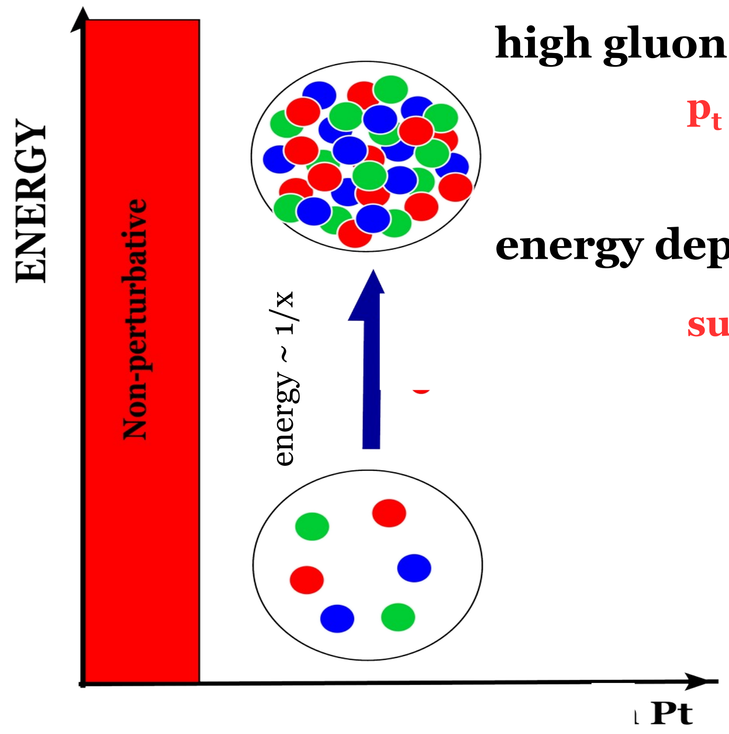
How does this happen ?

How do correlation functions of these evolve ?

Is there a universal fixed point for the RG evolution of d.o.f

Scaling laws ?

QCD at high energy/small x: gluon saturation



high gluon density: **Eikonal multiple scattering**

p_t broadening (generic to multiple scattering)

energy dependence: **x-evolution via JIMWLK/BK**

suppression of spectra/away side peaks

$$\mathbf{Q_s^2(x, b_t, A) \sim A^{1/3}\,(\frac{1}{x})^{0.3}}$$

$$Q_s^2(x = 3 \times 10^{-4}) \sim 1\,GeV^2$$

for a proton target (quarks)

a framework for multi-particle production in QCD at small x/low p_t

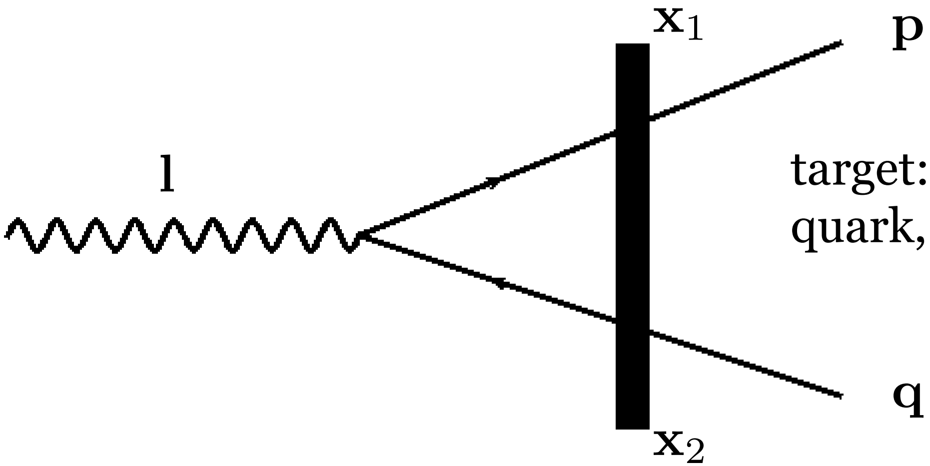
Shadowing/Nuclear modification factor
Azimuthal angular correlations (dihadron,...)
Long range rapidity correlations (ridge,...)
Initial conditions for hydro
Thermalization (?)

$$\mathbf{x \leq 0.01}$$

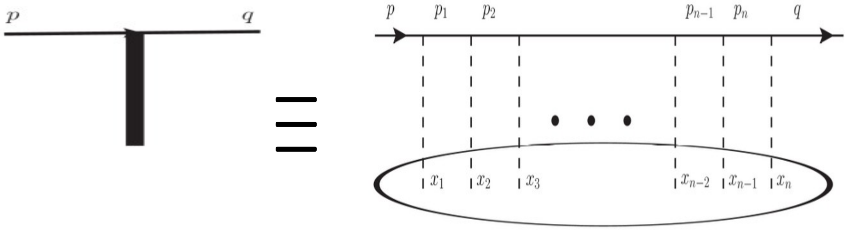
$$\alpha_s \ln{(x_v/x)} \sim 1$$

Dihadron production in DIS at small x

Quark anti-quark production in DIS at small x: LO



target: a classical color field
quark, anti-quark multiply scatter on the target



$$\frac{d\sigma^{\gamma^* A\to q\bar{q}X}}{d^2p\, d^2q\, dy_1\, dy_2} = \frac{e^2Q^2(z_1z_2)^2N_c}{(2\pi)^7}\delta(1-z_1-z_2)$$

$$\int d^8x_\perp e^{ip\cdot(x_1'-x_1)}e^{iq\cdot(x_2'-x_2)}\left[S_{122'1'} - S_{12} - S_{1'2'} + 1\right]$$

$$\Big\{4z_1z_2K_0(|x_{12}|Q_1)K_0(|x_{1'2'}|Q_1)+$$

$$(z_1^2+z_2^2)\,\frac{x_{12}\cdot x_{1'2'}}{|x_{12}||x_{1'2'}|}\,K_1(|x_{12}|Q_1)K_1(|x_{1'2'}|Q_1)\Big\}$$

with

$$\mathbf{S}_{12} \equiv \frac{1}{N_c}Tr\,V(x_1)\,V^\dagger(x_2)$$

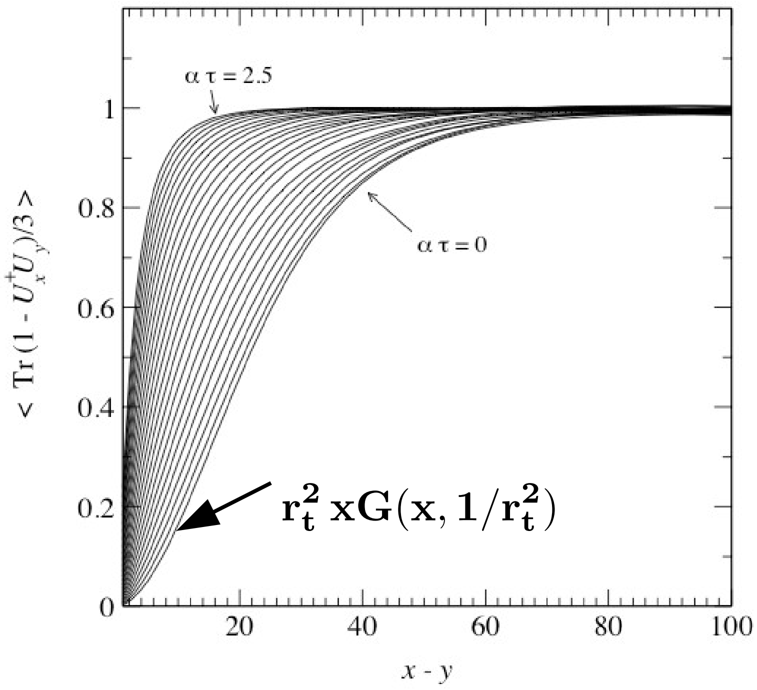
$$\mathbf{x}_{12} \equiv \mathbf{x}_1 - \mathbf{x}_2$$

$$\mathrm{S}_{122'1'} \equiv \frac{1}{N_c}Tr\,V(\mathbf{x}_1)\,V^\dagger(\mathbf{x}_2)\,V(\mathbf{x}_{2'})\,V^\dagger(\mathbf{x}_{1'})$$

Dipoles at large N_c : BK equation

$$\frac{\mathbf{d}}{\mathbf{dy}}\mathbf{T}(\mathbf{x_t}-\mathbf{y_t}) = \frac{\bar{\alpha}_s}{2\pi}\int \mathbf{d^2z_t}\,\frac{(\mathbf{x_t}-\mathbf{y_t})^2}{(\mathbf{x_t}-\mathbf{z_t})^2(\mathbf{y_t}-\mathbf{z_t})^2}\left[\mathbf{T}(\mathbf{x_t}-\mathbf{z_t})+\mathbf{T}(\mathbf{z_t}-\mathbf{y_t})-\mathbf{T}(\mathbf{x_t}-\mathbf{y_t})-\mathbf{T}(\mathbf{x_t}-\mathbf{z_t})\mathbf{T}(\mathbf{z_t}-\mathbf{y_t})\right]$$

$$\mathbf{T}(\mathbf{x_t},\mathbf{y_t}) \equiv \mathbf{1}-\mathbf{S}(\mathbf{x_t},\mathbf{y_t}) = \frac{\mathbf{1}}{\mathbf{N_c}}\mathrm{Tr}\left\langle \mathbf{1}-\mathbf{V}(\mathbf{x_t})\mathbf{V}^\dagger(\mathbf{y_t})\right\rangle$$



$$\tilde{\mathbf{T}}(\mathbf{p_t}) \to \log\left[\frac{\mathbf{Q_s^2}}{\mathbf{p_t^2}}\right]$$ **saturation region**

$$\tilde{\mathbf{T}}(\mathbf{p_t}) \to \frac{\mathbf{1}}{\mathbf{p_t^2}}\left[\frac{\mathbf{Q_s^2}}{\mathbf{p_t^2}}\right]^\gamma$$ **extended scaling region**

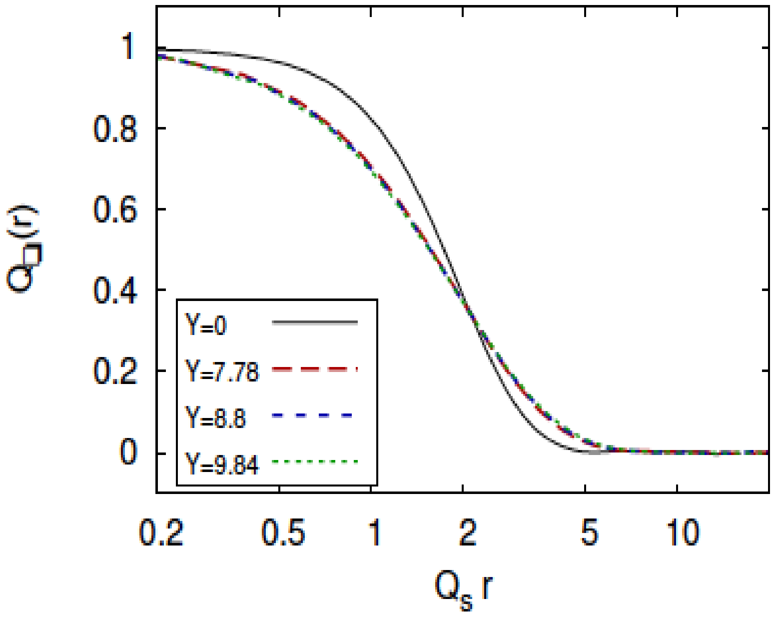
$$\tilde{\mathbf{T}}(\mathbf{p_t}) \to \frac{\mathbf{1}}{\mathbf{p_t^2}}\left[\frac{\mathbf{Q_s^2}}{\mathbf{p_t^2}}\right]$$ **pQCD region**

Rummukainen-Weigert, NPA739 (2004) 183
NLO: Balitsky-Kovchegov-Weigert-Gardi-Chirilli (2007-2008)

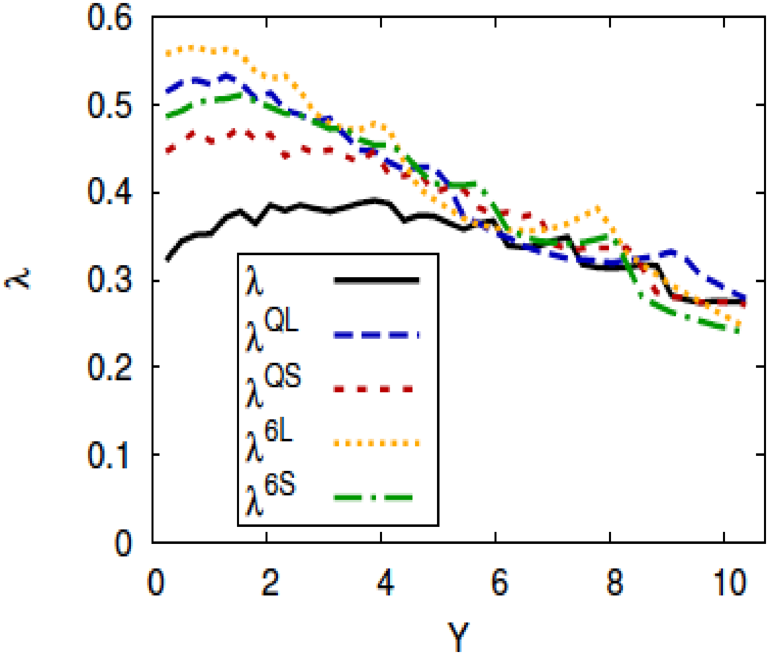
Quadrupole evolution

Dumitru-Jalilian-Marian-Lappi-Schenke-Venugopalan:
PLB706 (2011) 219

$$< Q(r,\bar{r},\bar{s},s) > \equiv \frac{1}{N_c} \; < Tr\, V(r)\, V^\dagger(\bar{r})\, V(\bar{s})\, V^\dagger(s) >$$



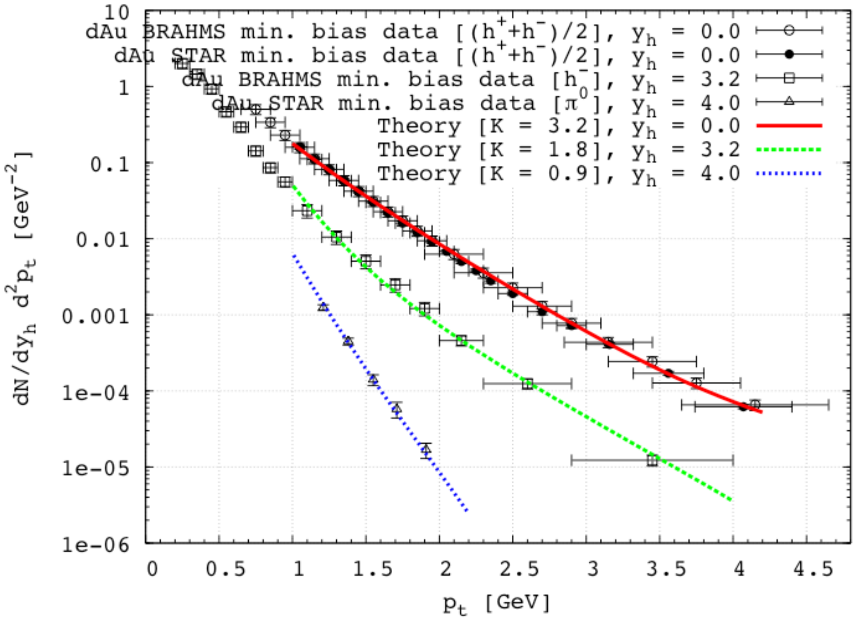
scaling



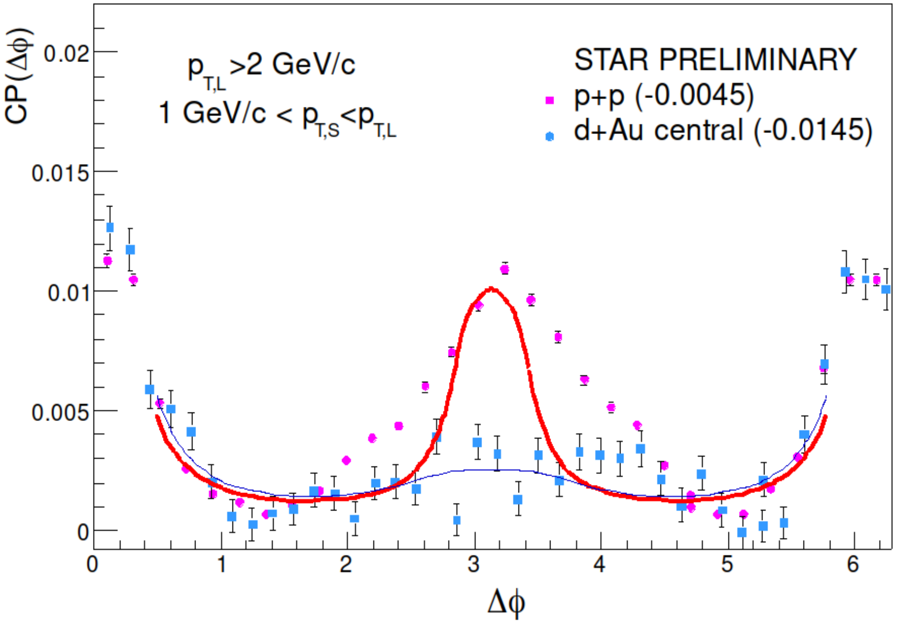
energy dependence

CGC at RHIC

Single and double inclusive hadron production in dA collisions

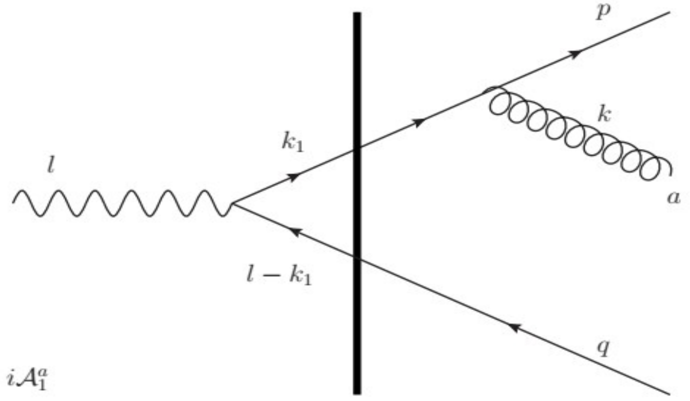


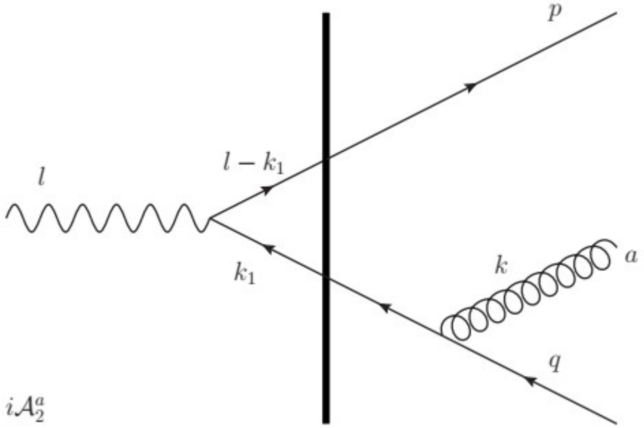
DHJ, NPA770 (2006) 57

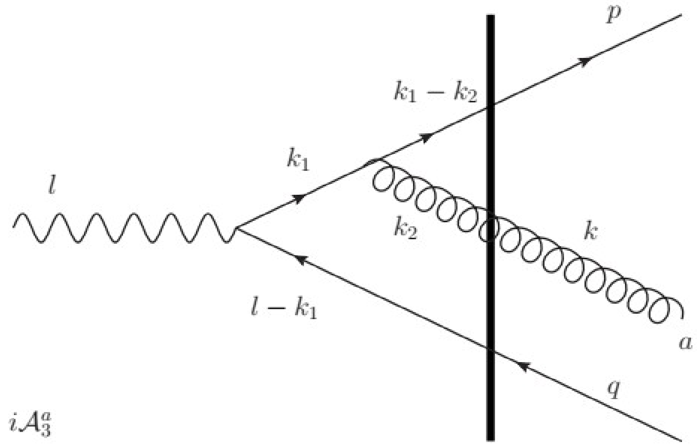


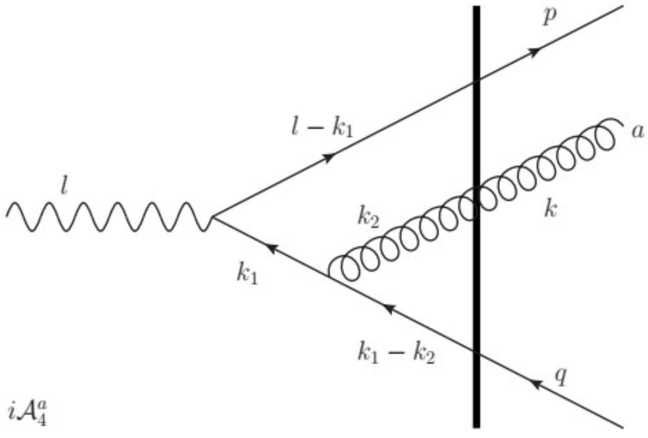
Albacete+Marquet
PRL105 (2010) 162301

Toward precision: one loop corrections - real diagrams



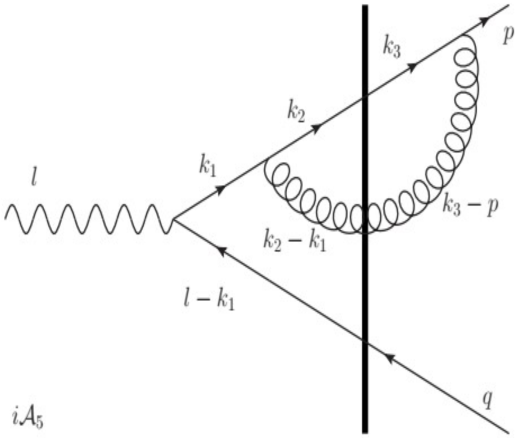


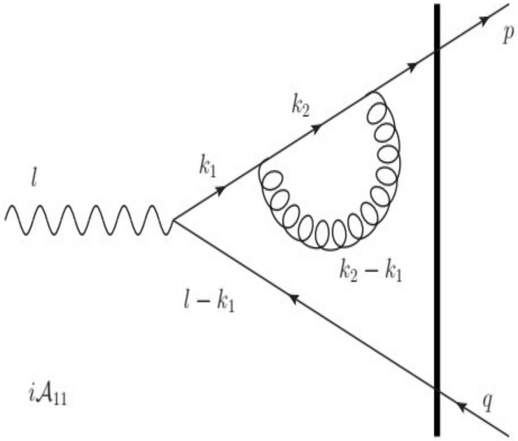


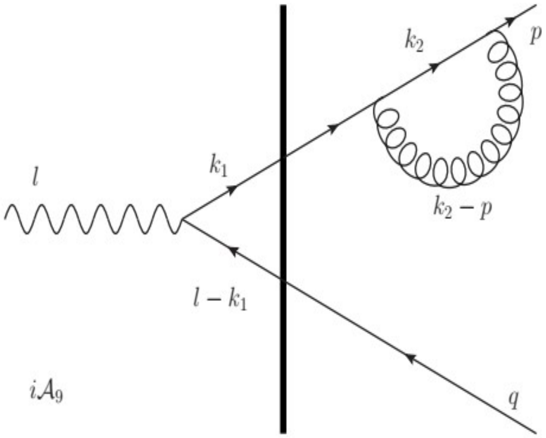


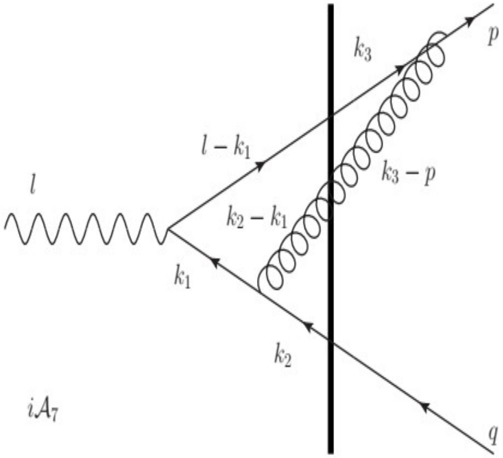
3-parton production: Ayala, Hentschinski, JJM, Tejeda-Yeomans
PLB 761 (2016) 229 and NPB 920 (2017) 232

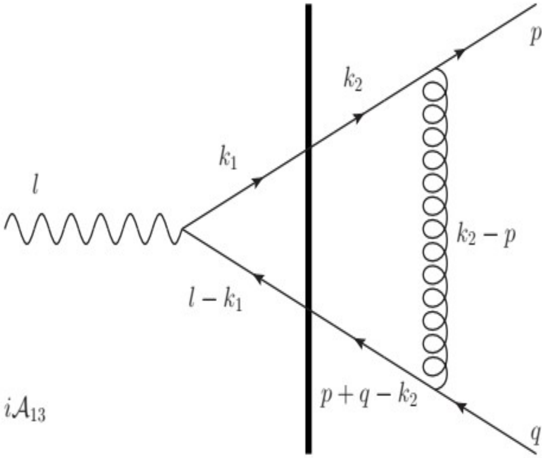
One loop corrections – virtual diagrams

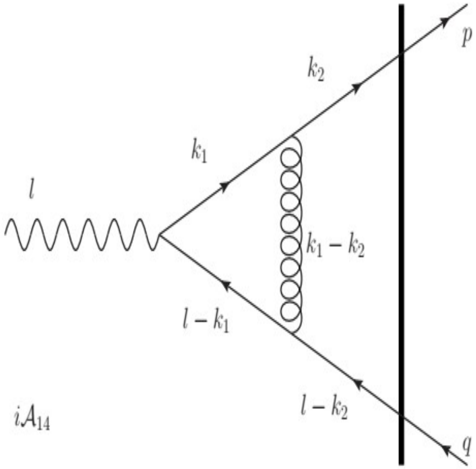












F. Bergabo and JJM, dihadrons, 2207.03606
P. Taels et al., dijets, 2204.11650
P. Caucal et al., dijets, 2108.06347

Cancellation of divergences

- ***Ultraviolet:***

  Real corrections are UV finite
  UV divergences cancel among virtual corrections

- ***Soft:***

  Soft divergences cancel between real and virtual corrections

- ***Colinear***

  Collinear divergences are absorbed into hadron fragmentation functions

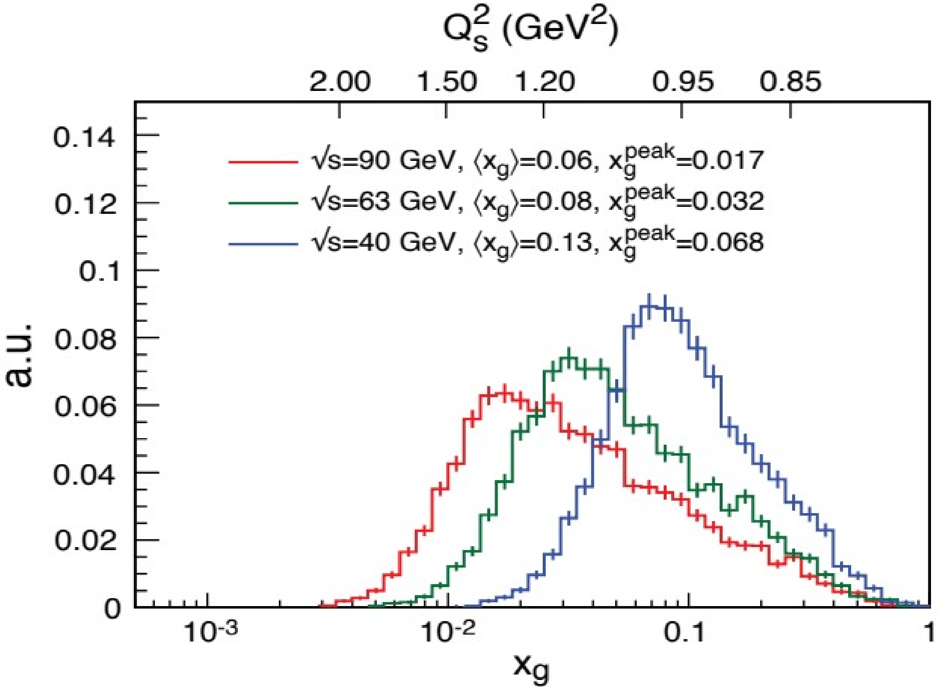
- ***Rapidity***

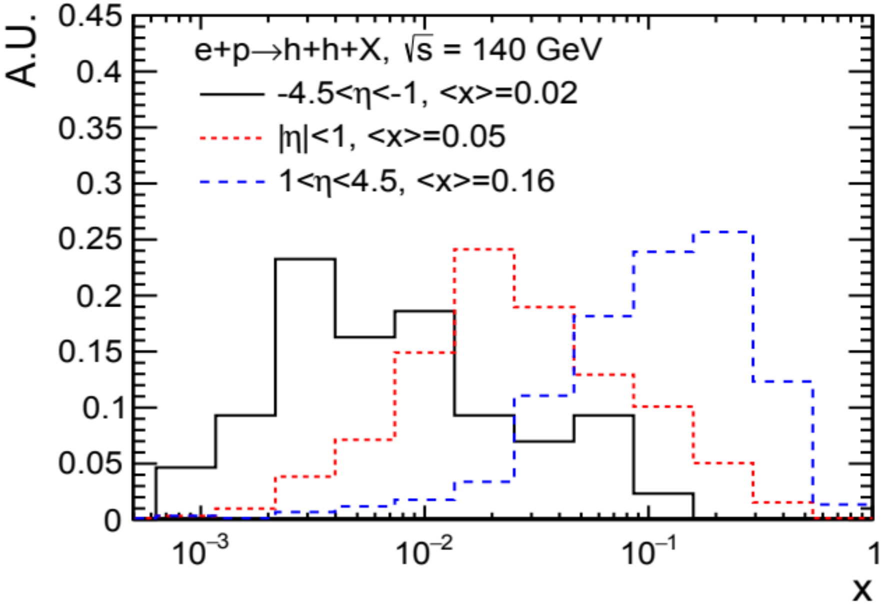
  rapidity divergences are absorbed into JIMWLK evolution of dipoles, quadrupoles

$$\sigma^{\gamma^* A \to h_1 h_2 X} = \sigma_{LO} \otimes \mathrm{JIMWLK} + \sigma_{LO} \otimes D_{h_1/q}(z_1, \mu^2) D_{h_2/\bar{q}}(z_2, \mu^2) + \sigma_{NLO}^{\mathrm{finite}}$$

EIC

kinematics of double inclusive hadron production



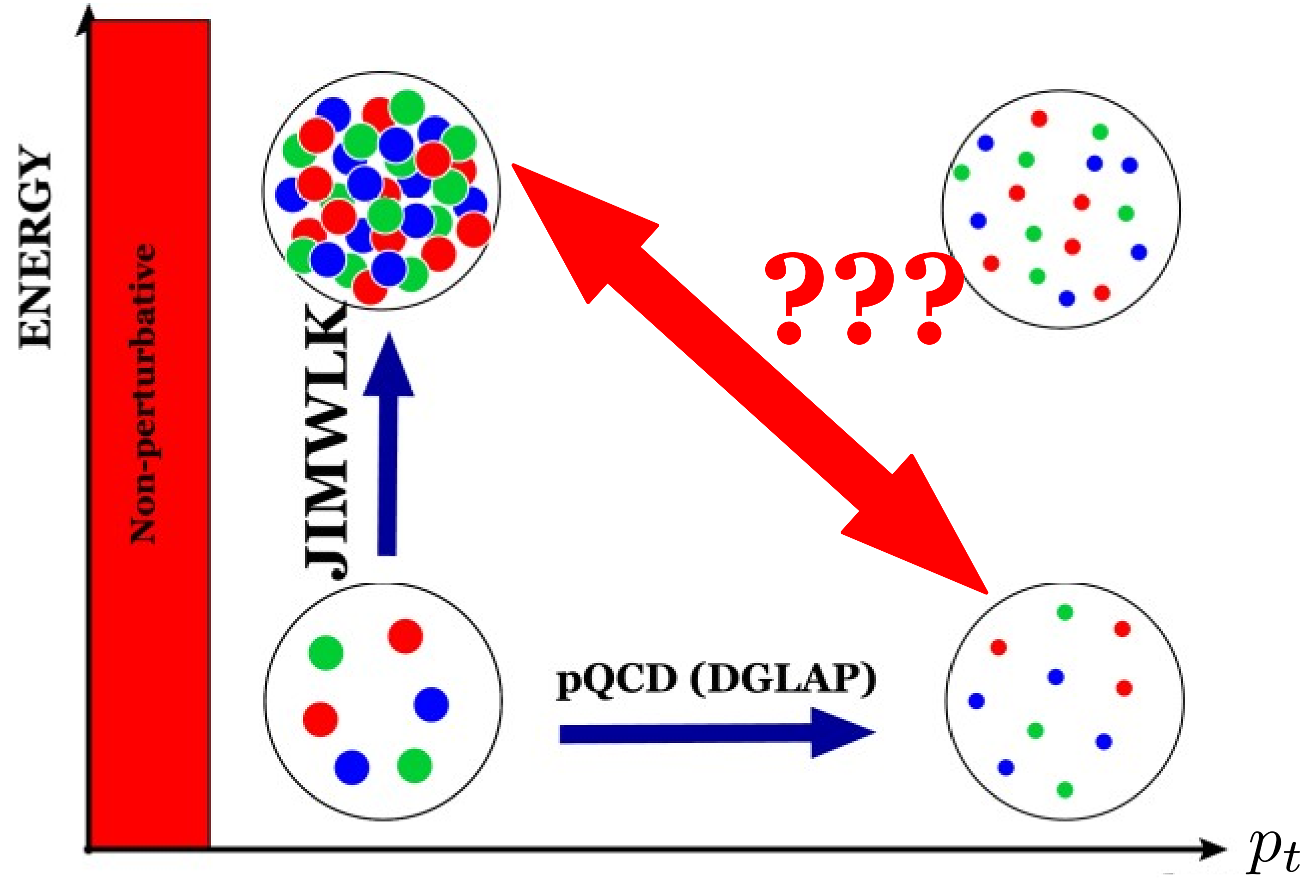


Aschenauer et al. arXiv:1708.01527

Fig. courtesy of Xiaoxuan Chu

rapidity dependence

QCD kinematic phase space



unifying saturation with high p_t (large x) pnysics?

kinematics of saturation: *where is saturation applicable?*
structure functions at all Q^2
high p_t and forward-backward correlations,
spin physics, early time e-loss in heavy ion collisions, …….

Beyond eikonal approximation: longitudinal momentum exchange

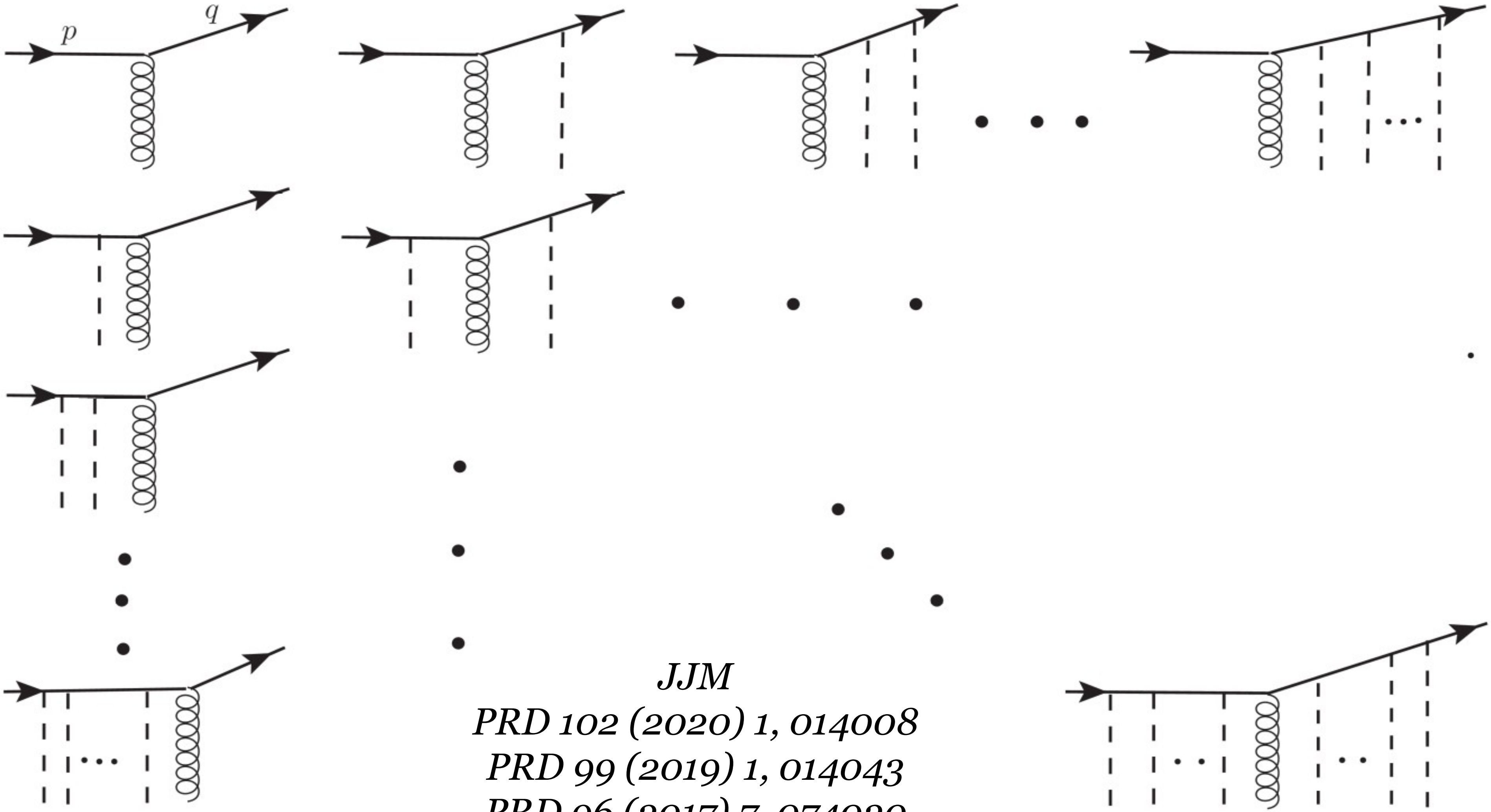
$$\mathcal{A}^{\mu} = \mathbf{S}^{\mu} + \mathbf{A}^{\mu}$$

single scattering from
large x gluons of target

$$\mathbf{A}^{\mu} = (\mathcal{A}^{\mu} - \mathbf{S}^{\mu})$$

multiple scatterings from
small x gluons of target (CGC)

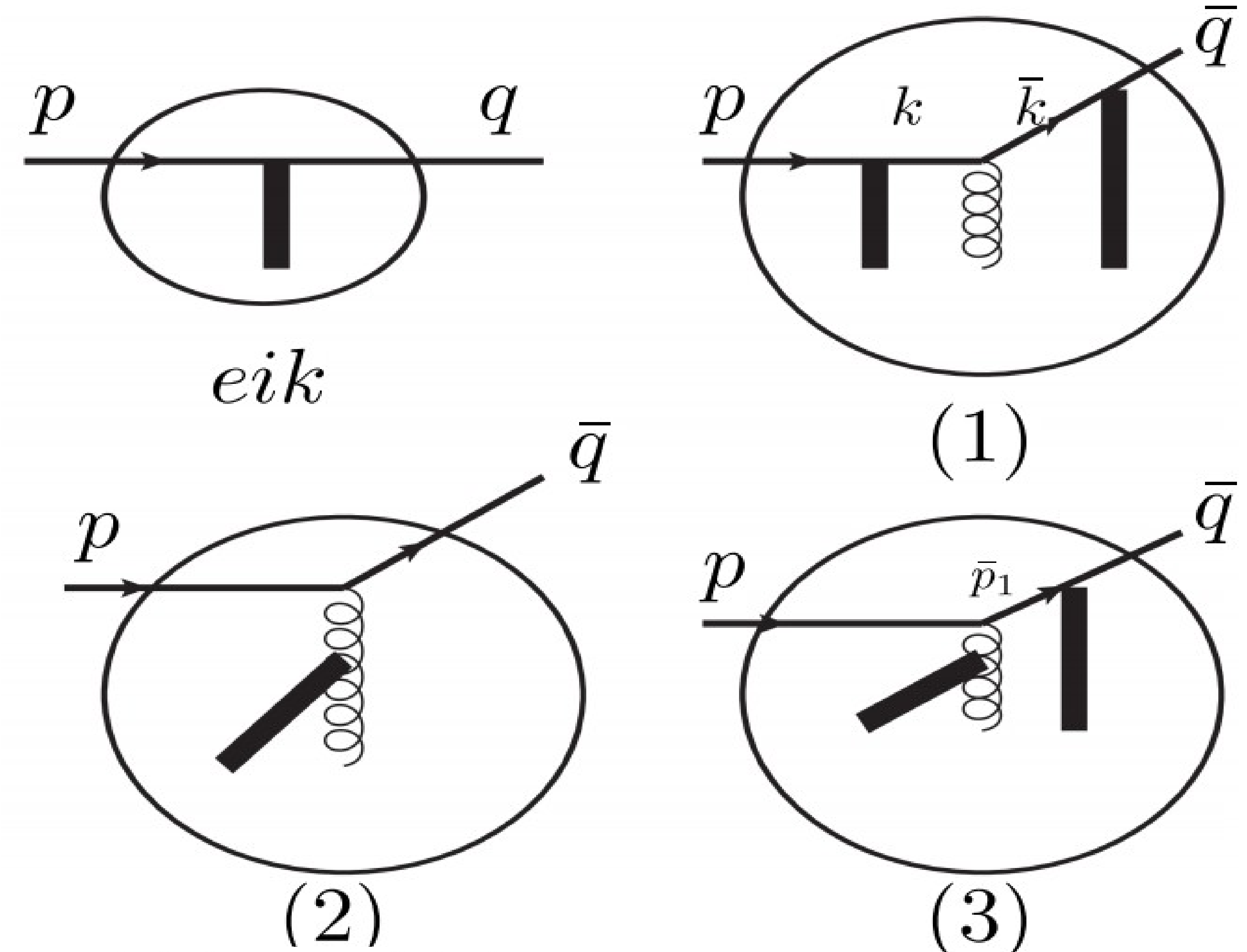
$$\mathbf{S}^{\mu}$$

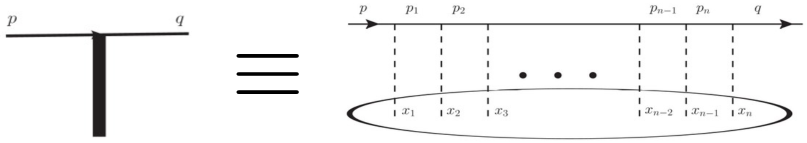


JJM
PRD 102 (2020) 1, 014008
PRD 99 (2019) 1, 014043
PRD 96 (2017) 7, 074020

Quark scattering: beyond small x approximation

large x partons of target can cause a large-angle deflection of the projectile





Including large x partons of the target leads to:

longitudinal double spin asymmetries (A_{LL})

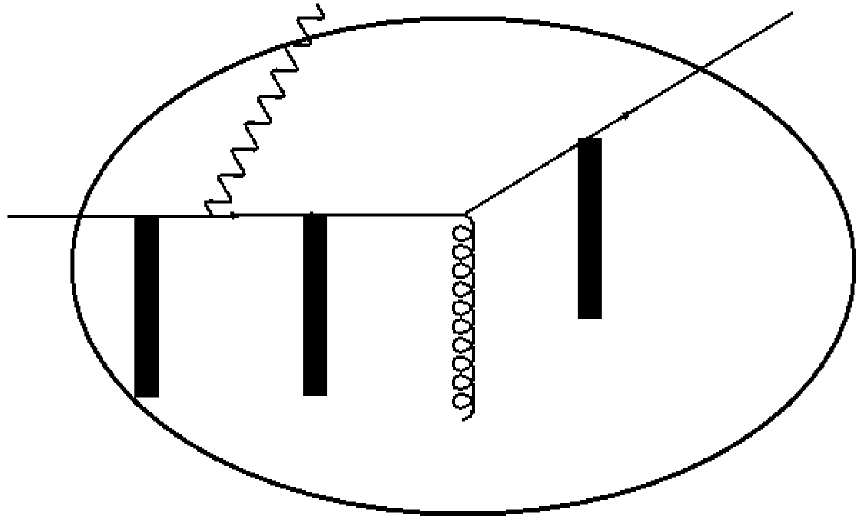
baryon transport (beam rapidity loss), ……

Photon production at all x

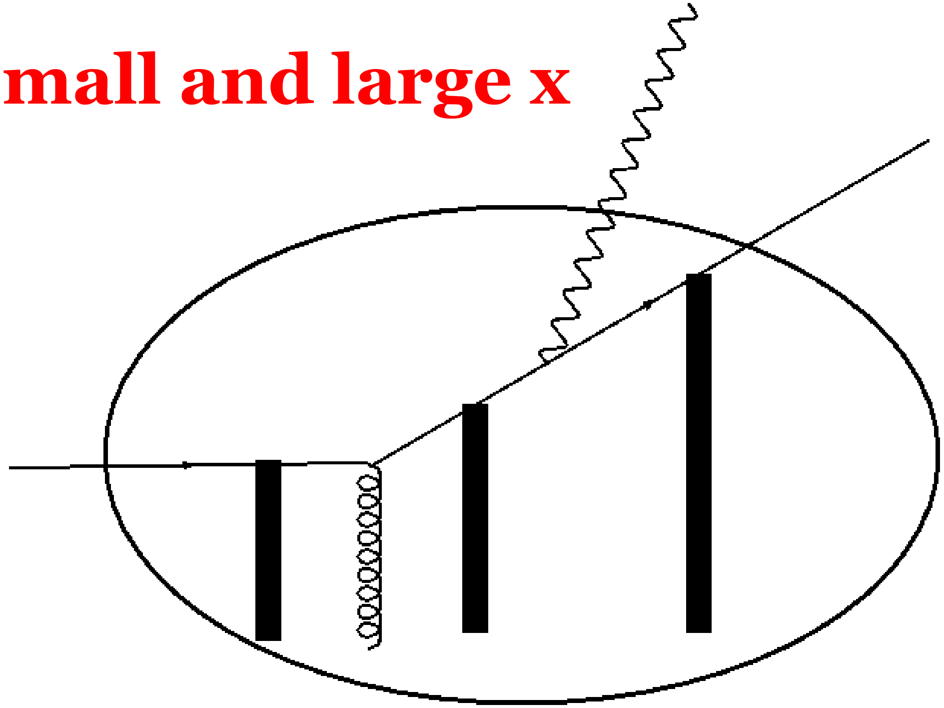
photon-hadron correlations:

- azimuthal angular correlations from low to high p_t
- forward-backward rapidity correlations

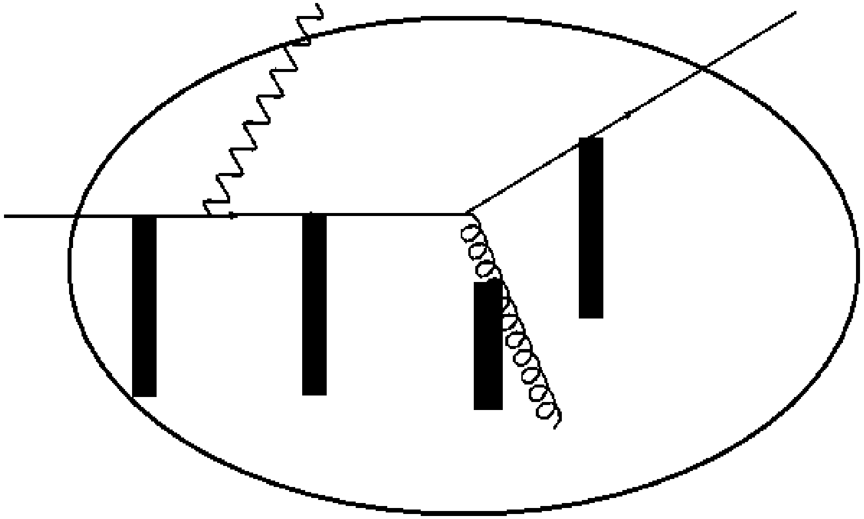
photon production: both small and large x

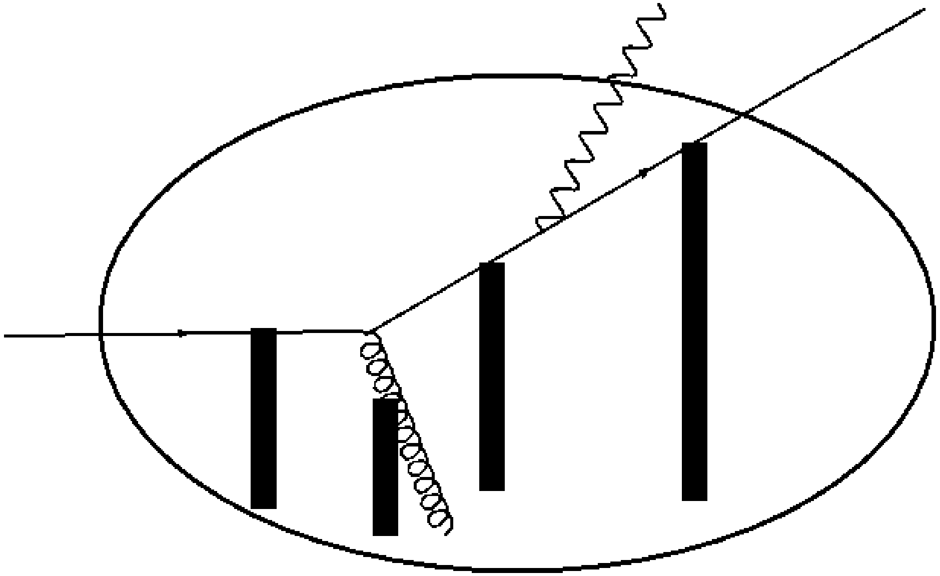


before hard scattering

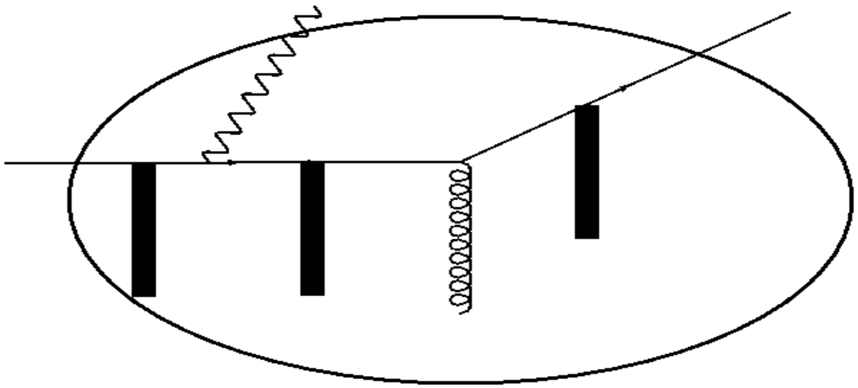


after hard scattering





photon production: both small and large x



$$\mathcal{N}_{1-1} = \bar{u}(\bar{q})\, \frac{\not{\bar{n}}\, \not{\bar{k}}_1}{2\bar{n}\cdot\bar{q}}\, \not{A}(x)\, \frac{\not{k}_3\, \not{n}\, \not{k}_2 \not{\epsilon}(l)\, \not{k}_1\, \not{n}}{2n\cdot p\, 2n\cdot(p-l)\, 2n\cdot(p-l)}\, u(p)$$

$$\mathcal{N}_{1-2} = \bar{u}(\bar{q})\, \frac{\not{\bar{n}}\, \not{\bar{k}}_1}{2\bar{n}\cdot\bar{q}}\, \not{A}(x)\, \frac{\not{n}\, \not{\epsilon}(l)\, \not{k}_1\, \not{n}}{2n\cdot p\, 2n\cdot(p-l)}\, u(p)$$

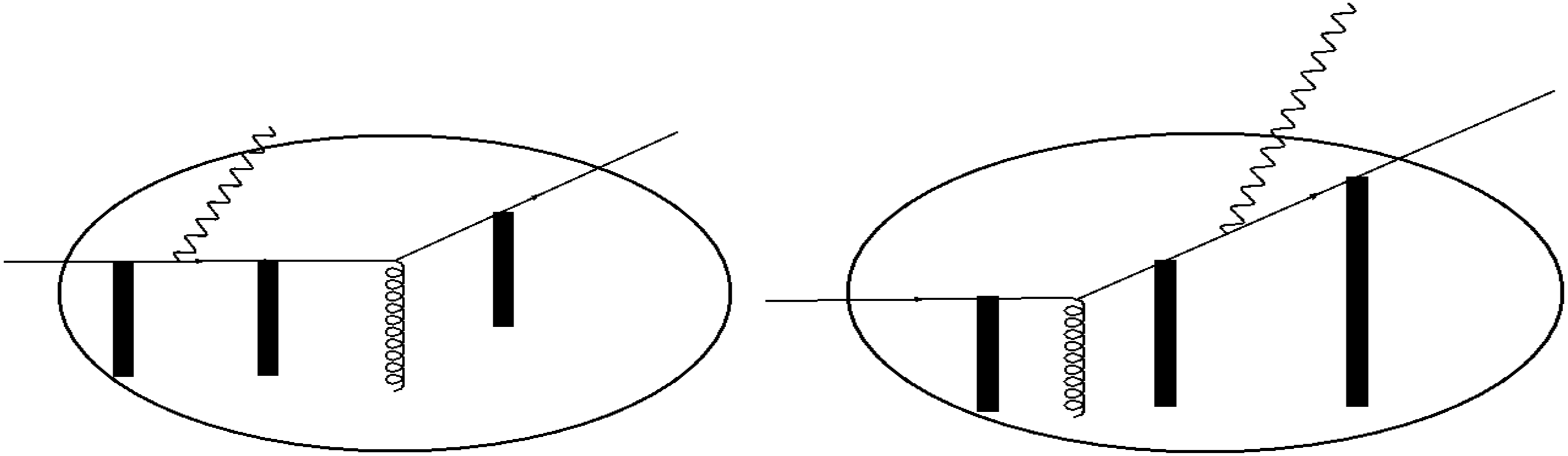
$$\mathcal{N}^{++}_{1-1} = (\mathcal{N}^{--}_{1-1})^\star = -\sqrt{\frac{n\cdot p}{n\cdot(p-l)}}\, \frac{[n\cdot l\, k_{2\perp}\cdot\epsilon^\star_\perp - n\cdot(p-l)\, l_\perp\cdot\epsilon^\star_\perp]}{n\cdot l\, n\cdot(p-l)}\, \langle\bar{k}_1^+|\not{A}(x)|k_3^+\rangle$$

$$\mathcal{N}^{++}_{1-2} = (\mathcal{N}^{--}_{1-2})^\star = -\sqrt{\frac{n\cdot p}{n\cdot(p-l)}}\, \langle\bar{k}_1^+|\not{A}(x)|n^+\rangle$$

$$\mathcal{N}^{+-}_{1-1} = (\mathcal{N}^{-+}_{1-1})^\star = -\sqrt{\frac{n\cdot p}{n\cdot(p-l)}}\, \frac{[n\cdot p\, l_\perp\cdot\epsilon_\perp - n\cdot l\, k_{1\perp}\cdot\epsilon_\perp]}{n\cdot p\, n\cdot l}\, \langle\bar{k}_1^+|\not{A}(x)|k_3^+\rangle$$

$$\mathcal{N}^{+-}_{1-2} = \mathcal{N}^{-+}_{1-2} = 0$$

pQCD limit (large x: gluon PDF X partonic cross section):



V = U = 1

SUMMARY

CGC is a systematic approach to high energy collisions

strong hints from RHIC, LHC,...

to be probed extensively at EIC

toward precision: NLO, sub-eikonal corrections, ...

CGC breaks down at large x (high p_t)

a significant part of EIC/RHIC/LHC phase space is at large x

transition from large x physics (pQCD) to small x (CGC)

Toward inclusion of large x physics:

spin asymmetries

beam rapidity loss

particle production in both small and large p_t kinematics

two-particle correlations: from forward-forward to forward-backward

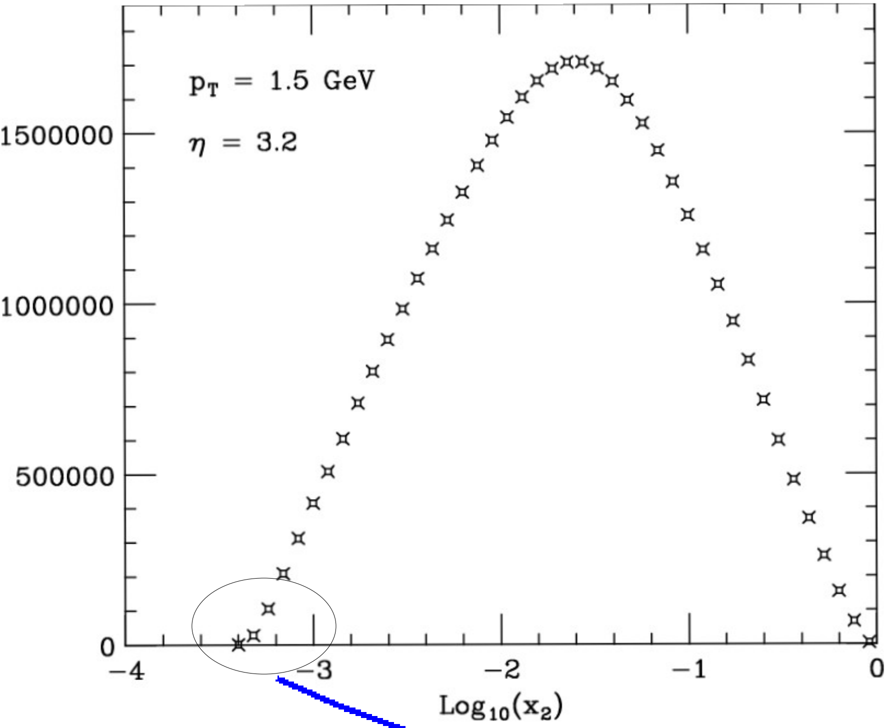
one-loop correction: both collinear and CGC factorization limits

need to clarify/understand: gauge invariance, initial conditions, ...

Single inclusive pion production in pp at RHIC

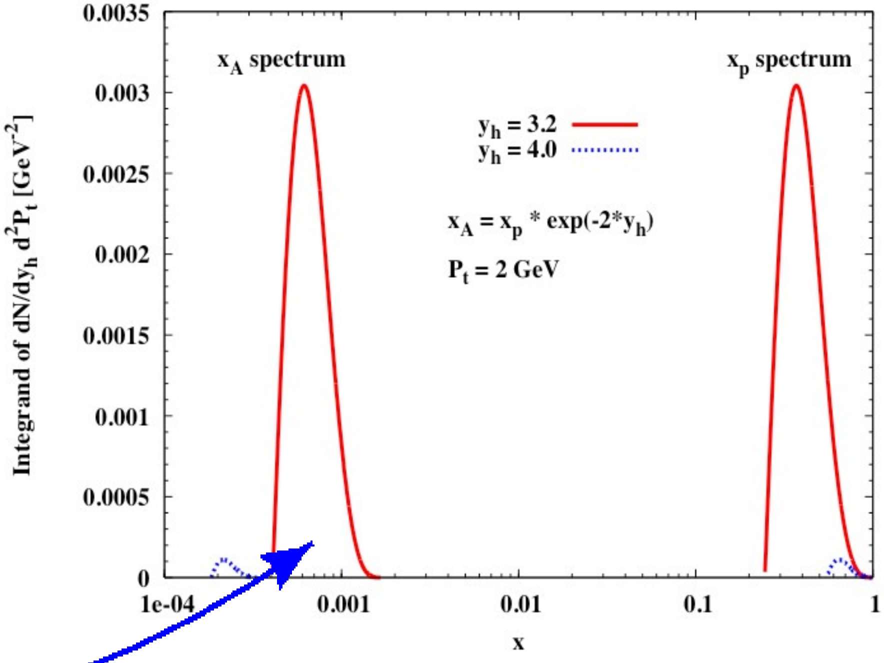
collinear factorization

GSV, PLB603 (2004) 173-183



CGC

DHJ, NPA765 (2006) 57-70



$$\int_{x_{min}}^{1} dx\, xG(x, Q^2)\cdots\cdots \longrightarrow x_{min}G(x_{min}, Q^2)\cdots$$

which kinematics are we in?