Lattice and PDFs overview

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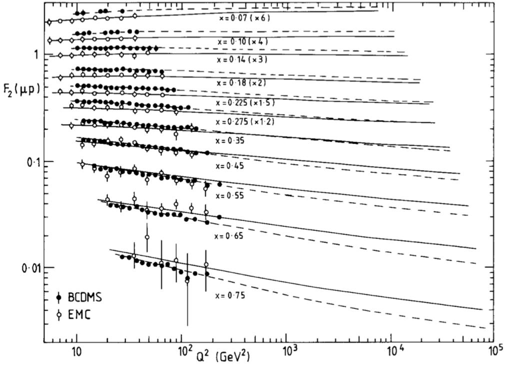
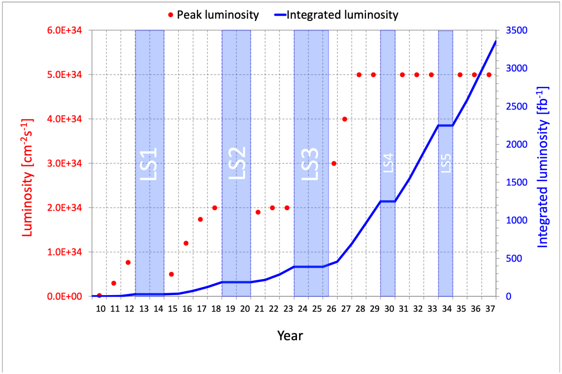


in collaboration with M Wilson, T Giani, C Monahan, K Cichy, J Karpie, K Orginos, A Radyushkin, S Zafeiropoulos

plan

1. PDFs/TMDs/GPDs in QFT
2. lattice observables
3. inverse problems

run 3/hi-lumi LHC: setting the target for theory predictions



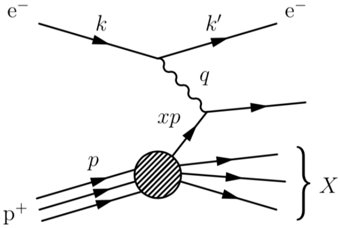
theory predictions need to match the experimental precision
$\hookrightarrow$ increased precision and accuracy of *parton distribution functions*

EIC and strong interactions

- hadron *tomography*
- understanding the nucleon spin
- understanding nucleons/nuclear physics from QCD
- current understanding of nucleon structure:

$$W(x, k_\perp, b_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\Delta_\perp \cdot b_\perp} \, \frac{1}{2} \int \frac{dz^- d^2 z_\perp}{(2\pi)^3} e^{i\left[(xp^+)z^- - k_\perp \cdot z_\perp\right]} \times \langle p + \frac{\Delta_\perp}{2} | \bar{\psi}(-z/2) \, \Gamma \lambda_A \, \mathcal{U} \, \psi(z/2) | p - \frac{\Delta_\perp}{2} \rangle$$

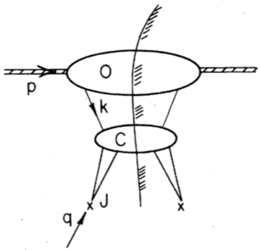
DIS



$$\boxed{\mathcal{M} \propto \bar{u}' \gamma^\mu u \frac{1}{q^2} \langle X | J_\mu(0) | p \rangle \quad \Longrightarrow \quad d\sigma \propto \left(\frac{\alpha}{Q^2} \right)^2 L^{\mu\nu} H_{\mu\nu}}$$

$$H_{\mu\nu} = \sum_X (2\pi)^D \delta\left(p - p_X - q\right) \, \langle p | J_\mu(0) | X \rangle \langle X | J_\nu(0) | p \rangle$$

hadronic tensor – factorization



$$H_{\mu\nu} = \int d^D y \, e^{iq\cdot y} \langle p | J_\mu(y) J_\nu(0) | p \rangle$$

nucleon dynamics encoded in Lorentz-invariant form factors

$$H_{\mu\nu} = F_1(x, Q^2) \left(\frac{q_\mu q_\nu}{q^2} - g_{\mu\nu} \right) + F_2(x, Q^2) \left(p + \frac{1}{2x} q \right)_\mu \left(p + \frac{1}{2x} q \right)_\nu$$

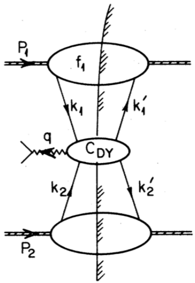
expressed in terms of PDFs – physical, finite quantities

$$F_i(x, Q^2) = \int_x^1 \frac{d\xi}{\xi} \, C_i(\xi, Q^2, \mu^2) f_R(x/\xi, \mu^2) + \mathcal{O}(1/Q^2)$$

$$f(x) = \int \frac{dz^-}{2\pi} e^{i(xp^+)z^-} \, \langle p | \bar{\psi}(-z^-/2) \, \Gamma \lambda_A \, \mathcal{U} \, \psi(z^-/2) | p \rangle$$

universality

dominant diagrams for Drell-Yan processes



$$\frac{d\sigma}{dQ^2dy} \propto \int dx_1 dx_2\, f_R^i\left(x_1, Q^2\right) f_R^j\left(x_2, Q^2\right) D_{ij}\left(x_1, x_2, y\right)$$

traditionally PDFs are extracted from fits to experimental data

lattice QCD for PDFs?

- cannot compute light-cone quantities in Euclidean field theory
- Xi. Ji (2013): compute the spatial correlator [ji 20]

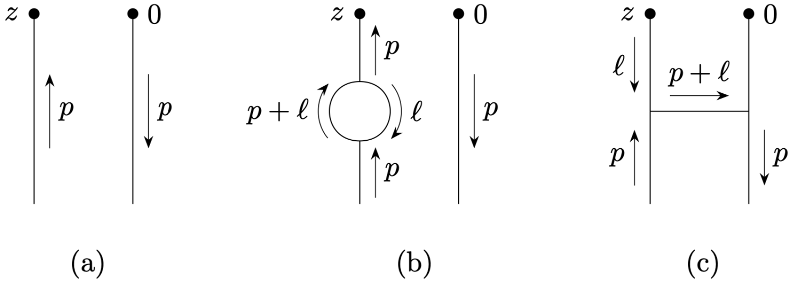
$$\langle p|\bar{\psi}(-z_3/2)\,\Gamma\lambda_A\,\mathcal{U}\,\psi(z_3/2)|p\rangle$$

- spun a lot of activity in LQCD community: **quasi-PDF**, **pseudo-PDF**, **Ioffe Time Distributions** – see recent reviews at Lattice conferences and dedicated workshops [constantinou et al 20]

- a factorization formula relates correlators in the spatial direction with the light-cone quantities (after renormalization)

toy-model computation

$$\widehat{\mathcal{M}}\left(\nu, z^2\right) = \langle p|\phi(z)\phi(0)|p\rangle$$



at tree level

$$\widehat{\mathcal{M}}_a(\nu, z^2) = \exp[-ip \cdot z] = \exp[-i\nu] = \widehat{\mathcal{M}}^{(0)}(\nu, 0)$$

one-loop example

- light-cone separation

$$\widehat{\mathcal{M}}_R\left(\nu;\mu^2\right) = \left[1+\frac{\alpha}{6}\left(\log\frac{m^2}{\mu^2}+b\right)\right]\widehat{\mathcal{M}}^{(0)}\left(\nu,0\right) + \alpha\int_0^1 dx\,\left(1-x\right)\,\log\frac{\mu^2}{m^2\left(1-x+x^2\right)}\widehat{\mathcal{M}}^{(0)}\left(x\nu,0\right)$$

- spatial separation

$$\widehat{\mathcal{M}}_R\left(\nu,z_3^2;\mu^2\right) = \left[1+\frac{\alpha}{6}\left(\log\frac{m^2}{\mu^2}+b\right)\right]\widehat{\mathcal{M}}^{(0)}\left(\nu,0\right) + \alpha\int_0^1 dx\,\left(1-x\right)\,2K_0\left(mz_3\right)\widehat{\mathcal{M}}^{(0)}\left(x\nu,0\right)$$

factorization theorem

for $mz_3 \ll 1$ (small distances, large momenta)

$$2K_0\left(Mz_3\right) = -\log\left(m^2 z_3^2\right) + 2\log\left(2e^{-\gamma_E}\right) + \mathcal{O}\left(m^2 z_3^2\right)$$

and therefore

$$\widehat{\mathcal{M}}_R\left(\nu, -z_3^2;\, \mu^2\right) = \int_{-1}^{1} d\xi\, \tilde{C}\left(\xi\nu, \mu^2 z_3^2\right) \hat{f}_R\left(\xi, \mu^2\right) + O(m^2 z_3^2)$$

$$\tilde{C}\left(\xi\nu, \mu^2 z_3^2\right) = e^{i\xi\nu} - \alpha \int_0^1 dx\ (1-x)\log\left(\mu^2 z_3^2 \frac{e^{2\gamma_E}}{4}\right) e^{ix\xi\nu}$$

Wilson coefficient is IR-safe

z^2 dependence only at $\mathcal{O}(\alpha)$

QCD matrix elements

$$\boxed{\mathcal{M}_{\Gamma,A}(z) = \bar{\psi}(z)\Gamma\lambda_A \,\mathrm{P}\exp\left(-ig\int_0^z d\eta\, A(\eta)\right)\psi(0)}$$

Ioffe time distributions

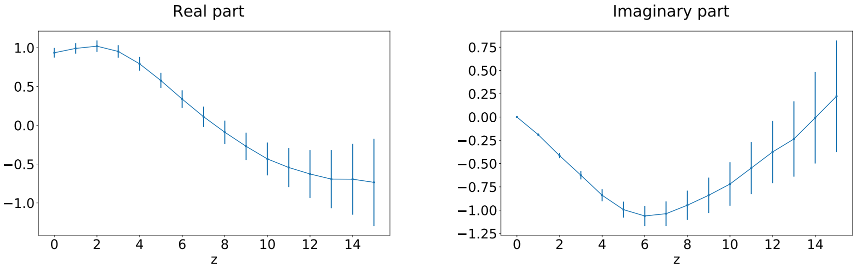
$$M_{\gamma^\mu,A}(z,P) = \langle P|\mathcal{M}_{\gamma^\mu,A}(z)|P\rangle$$

Lorentz covariance

$$M_{\gamma^\mu,A}(z,P) = P^\mu h_{\gamma^\mu,A}(z\cdot P, z^2) + z^\mu h'_{\gamma^\mu,A}(z\cdot P, z^2)$$

lattice observables - 1

$$\mathcal{O}^{\text{Re}}_{\gamma^0}\left(zP_z, z^2\right) \equiv \text{Re}\left[\text{h}_{\gamma^0,3}\left(zP_z, z^2\right)\right] \qquad \mathcal{O}^{\text{Im}}_{\gamma^0}\left(zP_z, z^2\right) \equiv \text{Im}\left[\text{h}_{\gamma^0,3}\left(zP_z, z^2\right)\right]$$



[C Alexandrou et al 18]

systematic errors

- cut-off effects
- finite volume effects
- excited states contamination

- truncation effects
- higher-twist terms
- isospin breaking

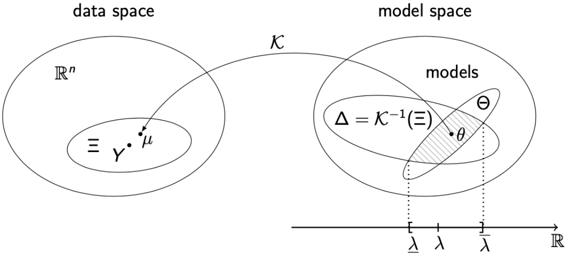
$\hookrightarrow$ **renormalized lattice observables at the continuum limit**

lattice QCD yields a discrete set of datapoints to be included in PDF fits

inverse problems

unknown function: $\theta \in \Theta$

Data: $Y = K\theta + \varepsilon$, where: $Y \in \mathcal{Y}$, $\varepsilon \sim \mathcal{N}(0, \Sigma)$



stochastic solution

$$\theta \in \Theta \quad \Longrightarrow \quad \mathrm{Prob}(y|\theta) = f(y - K\theta)$$

f is the probability distribution of ε

Bayes theorem

$$\mathrm{Prob}(\theta|y) = \frac{\mathrm{Prob}(\mathrm{y}|\theta)\mathrm{Prob}(\theta)}{\mathrm{Prob}(y)}$$

choose a parametrization, find the posterior distribution

inverse problem for PDFs

data:

$$\boxed{F = K \otimes f[\theta]}$$

- PDFs extracted from data
- there is no difference between expt data and lattice data
- complementary information
- lattice data implemented in the nnpdf framework

replicas

data: $y_i, i = 1, \ldots N_{\rm dat}$ are treated as stochastic variables $\sim \mathcal{N}(Y, C)$

bootstrap: replicas simulate the fluctuations of y

$$y^{(k)} = Y + \varepsilon^{(k)}, \;\; k = 1, \ldots N_{\rm rep}$$

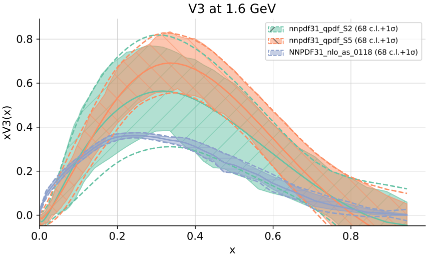
for each replica we minimize:

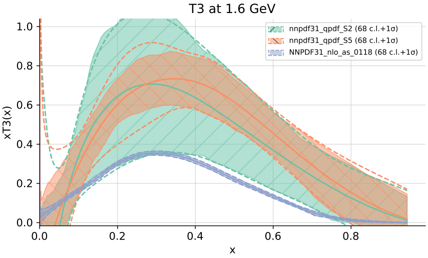
$$\chi^{2\,(k)}[\theta] = \frac{1}{N_{\rm dat}} \sum_{ij} \left(g[\theta]^{(k)} - y^{(k)} \right)_i C^{-1}_{ij} \left(g[\theta]^{(k)} - y^{(k)} \right)_j + \mathrm{priors}$$

$\left\{ \theta^{(k)}, k = 1, \ldots, N_{\rm rep} \right\}$ yields the probability distribution in model space

fit results

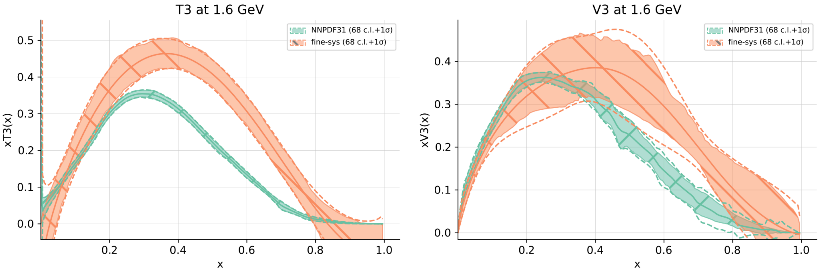
lattice data can be included in NNPDF fits, just like any other data!





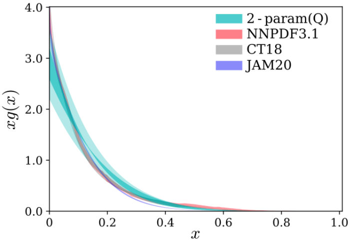
[ldd et al 19, using ETMC data]

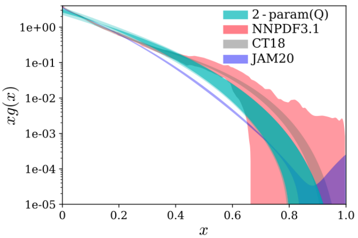
more fits



[ldd et al 20, using ITD data]

HadStruc

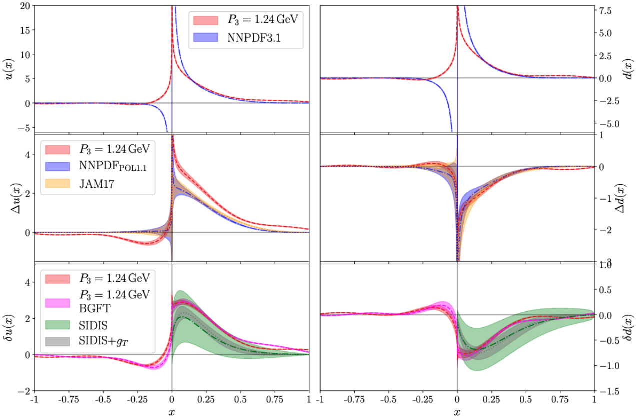




$$m_\pi = 358 \text{ MeV}$$

[Khan et al 21]

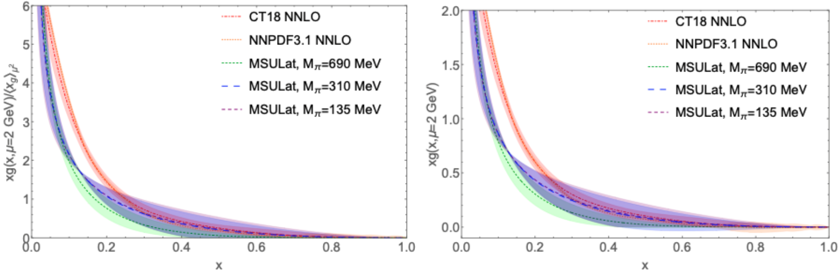
ETMC



$$m_\pi = 260 \text{ MeV}$$

[Alexandrou et al 21]

MSULat

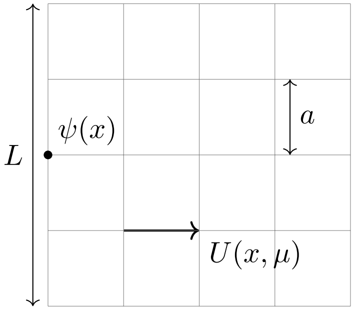


[Fan et al 21]

outlook

- light-cone PDFs + factorization describe the structure of the proton
- current extraction from data is very precise + improving
- lattice quantities needs to be properly renormalised and extrapolated to the continuum limit
  $\hookrightarrow$ are on the same footing as experimental data
- lattice data provide complementary information, can be included in global fits like any other data (inverse problem)
- identify the areas where a significant phenomenological impact from lattice QCD is possible

lattice QCD



$$Z = \int \mathcal{D}U \mathcal{D}\psi \mathcal{D}\bar{\psi}\; e^{-S[U,\psi,\bar{\psi}]}$$

$$\langle \phi(x_1)\ldots\phi(x_n)\rangle = \frac{1}{Z}\int \mathcal{D}U \mathcal{D}\psi \mathcal{D}\bar{\psi}\; e^{-S[U,\psi,\bar{\psi}]}\,\phi(x_1)\ldots\phi(x_n)$$