TMDs At Small-x: Quark Sivers Function and the Spin Dependent Odderon

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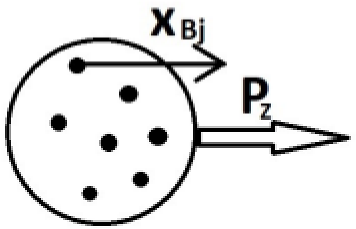
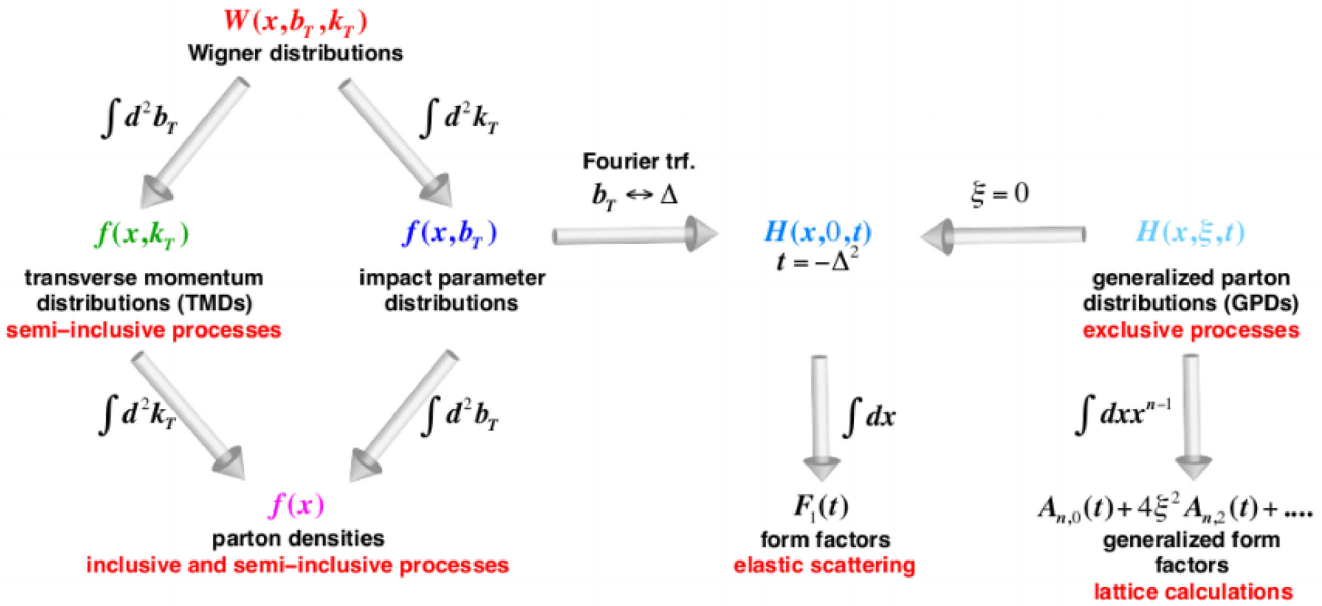
Why TMDs? 3D Hadron Structure

- Parton Distribution Functions (PDFs) give us information on the nonperturbative structure of hadrons and have known equations for both Q^2 and small-x, allowing us to probe them in extreme regimes
- But they only know about longitudinal momentum, only one dimension of information!

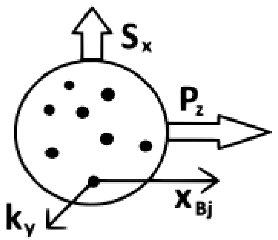


- In order to probe the 3D structure of hadrons we need to go beyond PDFs and take transverse motion of quarks and gluons into account

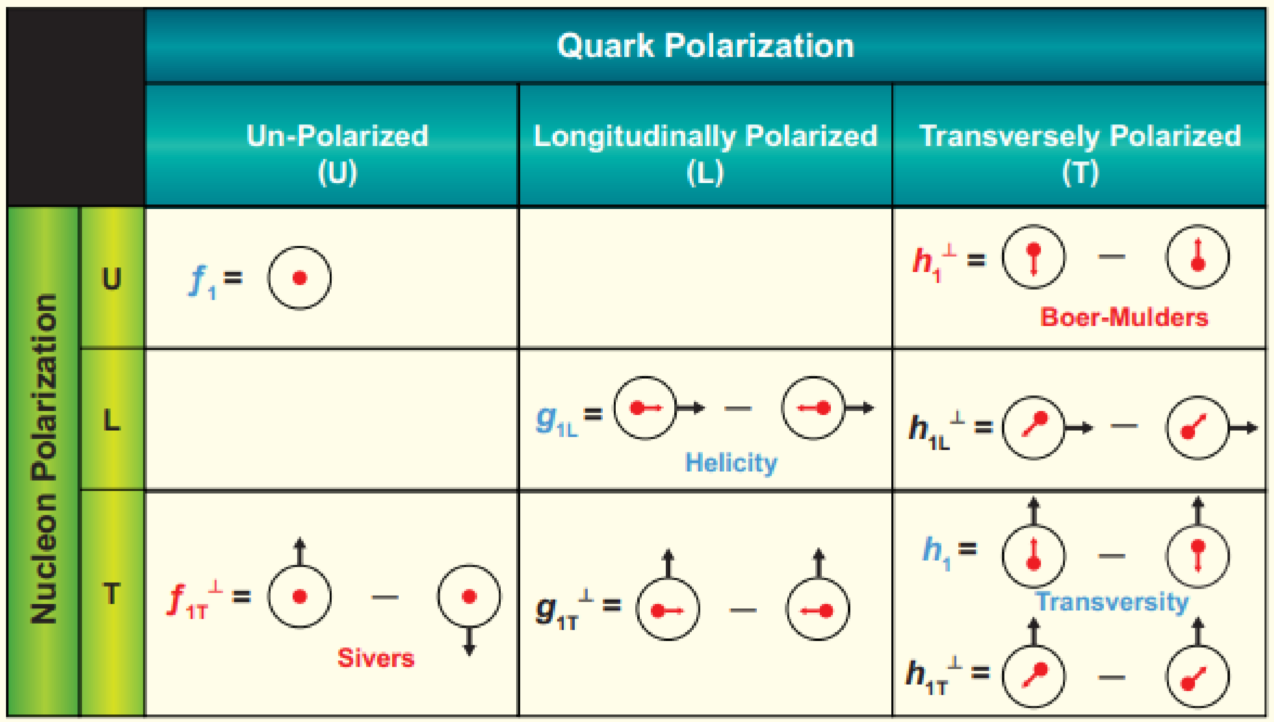
Full 3D Hadron Structure



- The full phase space treatment of hadrons requires Wigner functions due to the uncertainty principle
- For 3D momentum space information, we turn to TMDs



Quark TMDs

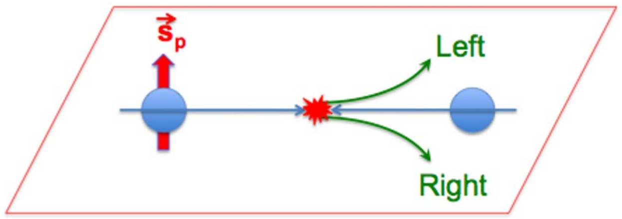


- The leading twist quark TMDs give various correlations between the transverse momentum and polarizations of the quarks within a hadron with the polarization of the parent hadron
- Their scale evolution in Q^2 is given by the CSS equations, but the small-x evolution is an ongoing effort

Quark Sivers Function

- The Sivers function gives the correlation between the transverse momentum of unpolarized quarks
- It captures orbital angular momentum and spin-orbit coupling as seen in single spin asymmetries

$$p(\vec{s}_\perp) + p \to h(\pi^\pm, \pi^0, \ldots) + X$$



$$\propto (S_P \times \vec{p}) \cdot \vec{k}_T$$

- It is one of the two time-reversal odd TMDs, the other being the Boer-Mulders function. This translates to process dependence and a sign change between semi-inclusive DIS (SIDIS) and Drell-Yan (DY) pair production

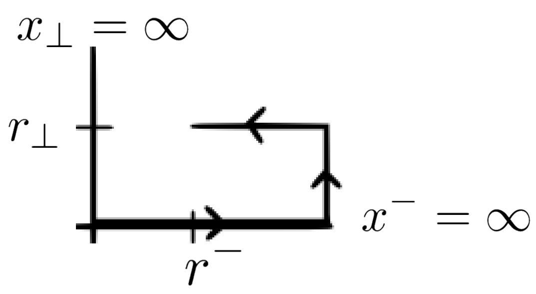
$$f_{1T\,\mathrm{SIDIS}}^{\perp q} = -f_{1T\,\mathrm{DY}}^{\perp q}$$

Definition of TMDs

- Quark TMDs are defined by the non-local operator product in the hadron state

$$\Phi^{[\Gamma]} = \int \frac{\mathrm{d}r^- \, \mathrm{d}^2 r_\perp}{2\,(2\pi)^3} e^{ik\cdot r} \langle P, S | \bar{\psi}(r) \mathcal{U}[r,0] \Gamma \psi(0) | P, S \rangle$$

$$\mathcal{U}[r,0] = \mathcal{P}\mathrm{exp}\left[ig \int_0^r \mathrm{d}x_\mu \, A^\mu(x) \right]$$



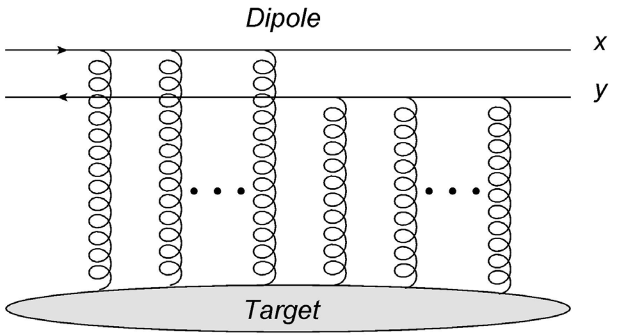
- Different linear combinations of TMDs come from different choices of the Dirac matrix Γ, for example the unintegrated quark density f_1^q and the Sivers function $f_{1T}^{\perp q}$ are given by the taking the matrix to be $\gamma^+/2$

$$f_1^q(x, k_T^2) - \frac{\underline{k} \times \underline{S}_P}{M_P} f_{1\,T}^{\perp\,q}(x, k_T^2) = \int \frac{\mathrm{d}r^- \, \mathrm{d}^2 r_\perp}{2\,(2\pi)^3} e^{ik\cdot r} \langle P, S | \bar{\psi}(r) \mathcal{U}[r,0] \frac{\gamma^+}{2} \psi(0) | P, S \rangle$$

Small-x TMDs from polarized Wilson lines

- *Kovchegov and Sievert* (2019) constructed the helicity and quark transversity TMDs using a high-energy scattering operator formalism
- The strategy is to rewrite the TMD operator definitions as dipole correlators

$$\mathrm{tr}[V_x V_y^\dagger] =$$



$$V_x[b^-, a^-] = \mathcal{P}\exp\left[ig\int_{a^-}^{b^-} \mathrm{d}s^-\, A^+(s^-, x_\perp)\right]$$

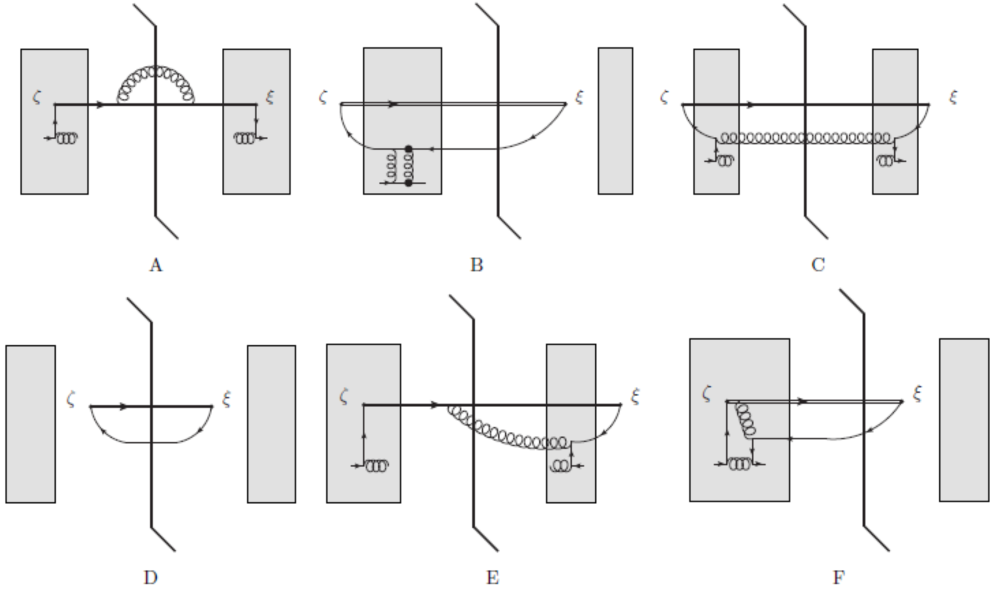
*in $A^- = 0$ or $\partial_\mu A^\mu$ gauge

$$V_x = V_x[\infty, -\infty]$$

Small-x TMDs from polarized Wilson lines

- By inserting a complete set of states, one can write the operator product as a sum over cut diagrams for the scattering of a quark on the shockwave of a target hadron

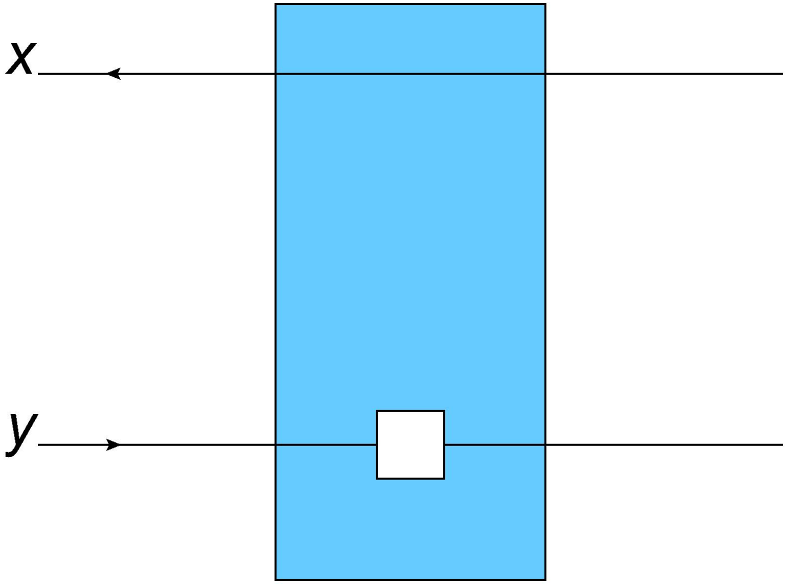
$$f_1^q(x,k_T^2) - \frac{\underline{k}\times \underline{S}_P}{M_P} f_{1T}^{\perp q}(x,k_T^2) = \frac{2p_1^+}{2(2\pi)^3}\sum_X \int \mathrm{d}\xi^- \, \mathrm{d}^2\xi_\perp \, \mathrm{d}\zeta^- \, \mathrm{d}^2\zeta_\perp \, e^{ik\cdot(\zeta-\xi)} \left[\frac{\gamma^+}{2}\right]_{\alpha\beta} \left\langle \bar\psi_\alpha(\xi) V_{\underline{\xi}}[\xi^-,\infty]|X\rangle\langle X|V_{\underline{\zeta}}[\infty,\zeta^-]\psi_\beta(\zeta)\right\rangle$$



Small-x TMDs from polarized Wilson lines

- Define new polarized dipoles by adding a spin-dependent interaction

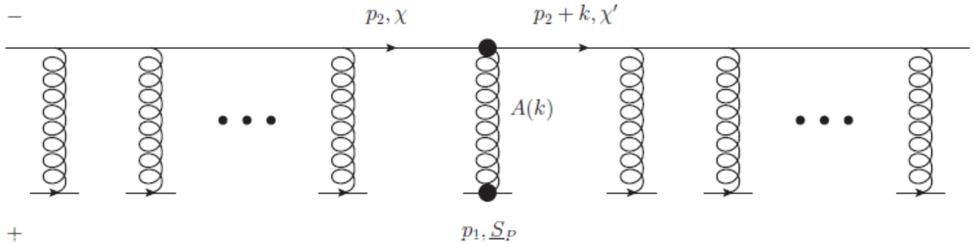
$$\mathrm{tr}[V_x V_y^{pol\,\dagger}] =$$



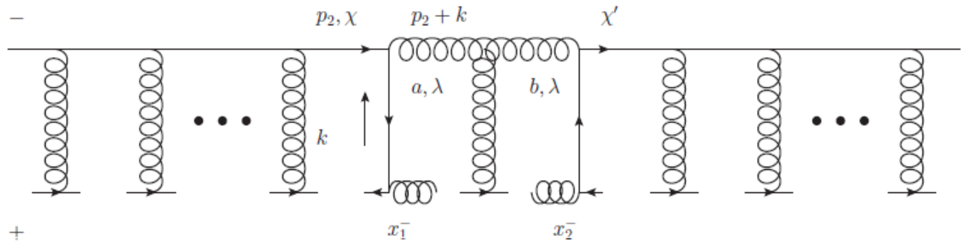
- Helicity interaction comes at sub-eikonal level, transversity at sub-sub-eikonal
- Define evolution for new polarized dipoles to get TMD evolution!

General polarized Wilson line

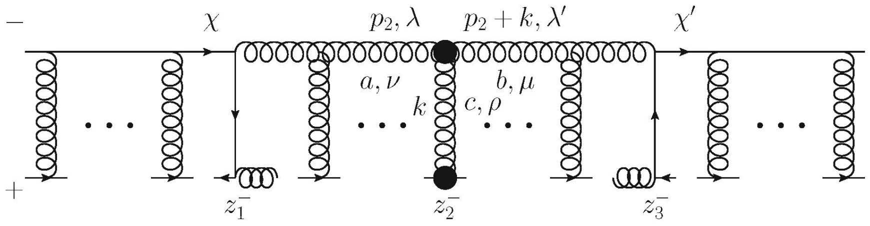
- Insert sub- or sub-sub-eikonal operator into ordinary Wilson lines



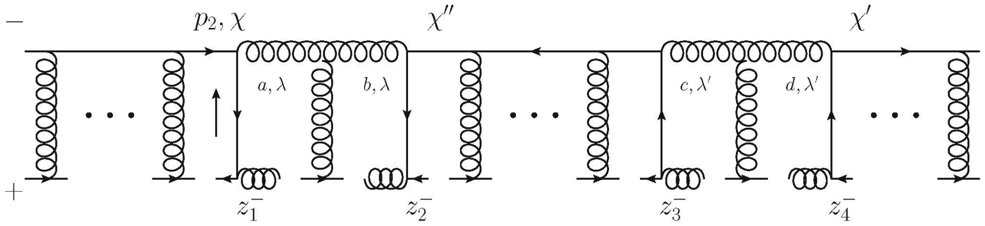
$\mathcal{O}^{\text{pol G}}_{\chi,\chi'}$



$\mathcal{O}^{\text{pol q}\overline{\text{q}}}_{\chi,\chi'}$



$\mathcal{O}^{\text{pol qG}\overline{\text{q}}}_{\chi,\chi'}$

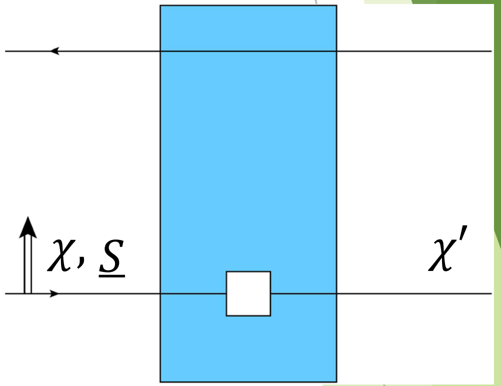


$\mathcal{O}^{\text{pol qq}\overline{\text{qq}}}_{\chi,\chi'}$

General polarized Wilson line

- Full sub-sub-eikonal polarized fundamental Wilson line for TMDs which depend on the proton's transverse spin

$$
\begin{aligned}
V_{\underline{x},\underline{y};\chi',\chi} &= V_{\underline{x}}\, \delta^2(\underline{x}-\underline{y})\, \delta_{\chi,\chi'} + \int\limits_{-\infty}^{\infty} \mathrm{d}z^-\, d^2z\; V_{\underline{x}}[\infty, z^-]\, \delta^2(\underline{x}-\underline{z})\, \mathcal{O}^{\mathrm{pol\ G}}_{\chi',\chi}(z^-, \underline{z})\, V_{\underline{y}}[z^-, -\infty]\, \delta^2(\underline{y}-\underline{z}) \\
&+ \int\limits_{-\infty}^{\infty} \mathrm{d}z_1^-\, d^2z_1 \int\limits_{z_1^-}^{\infty} \mathrm{d}z_2^-\, d^2z_2 \sum_{\chi''=\pm 1} V_{\underline{x}}[\infty, z_2^-]\, \delta^2(\underline{x}-\underline{z}_2)\, \mathcal{O}^{\mathrm{pol\ G}}_{\chi',\chi''}(z_2^-, \underline{z}_2)\, V_{\underline{z}_1}[z_2^-, z_1^-]\, \delta^2(\underline{z}_2-\underline{z}_1) \\
&\times \mathcal{O}^{\mathrm{pol\ G}}_{\chi'',\chi}(z_1^-, \underline{z}_1)\, V_{\underline{y}}[z_1^-, -\infty]\, \delta^2(\underline{y}-\underline{z}_1) + \int\limits_{-\infty}^{\infty} \mathrm{d}z_1^- \int\limits_{z_1^-}^{\infty} \mathrm{d}z_2^-\; V_{\underline{x}}[\infty, z_2^-]\, \mathcal{O}^{\mathrm{pol\ q\bar{q}}}_{\chi',\chi}(z_2^-, z_1^-; \underline{x}, \underline{y})\, V_{\underline{y}}[z_1^-, -\infty] \\
&+ \int\limits_{-\infty}^{\infty} \mathrm{d}z_1^- \int\limits_{z_1^-}^{\infty} \mathrm{d}z_2^- \int\limits_{z_2^-}^{\infty} \mathrm{d}z_3^- \int\limits_{z_3^-}^{\infty} \mathrm{d}z_4^-\, d^2z \sum_{\chi''=\pm 1} V_{\underline{x}}[\infty, z_4^-]\, \mathcal{O}^{\mathrm{pol\ q\bar{q}}}_{\chi',\chi''}(z_4^-, z_3^-; \underline{x}, \underline{z})\, V_{\underline{z}}[z_3^-, z_2^-]\, \mathcal{O}^{\mathrm{pol\ q\bar{q}}}_{\chi'',\chi}(z_2^-, z_1^-; \underline{z}, \underline{y})\, V_{\underline{y}}[z_1^-, -\infty] \\
&+ \int\limits_{-\infty}^{\infty} \mathrm{d}z_1^- \int\limits_{z_1^-}^{\infty} \mathrm{d}z_2^- \int\limits_{z_2^-}^{\infty} \mathrm{d}z_3^- \int\limits_{z_3^-}^{\infty} \mathrm{d}z_4^-\; V_{\underline{x}}[\infty, z_4^-]\, \mathcal{O}^{\mathrm{pol\ qq\bar{q}\bar{q}}}_{\chi',\chi}(z_4^-, z_3^-, z_2^-, z_1^-; \underline{x})\, V_{\underline{y}}[z_1^-, -\infty] \delta^2(\underline{x}-\underline{y}) \\
&+ \int\limits_{-\infty}^{\infty} \mathrm{d}z_1^- \int\limits_{z_1^-}^{\infty} dz_2^- d^2z_2 \int\limits_{z_2^-}^{\infty} dz_3^-\; V_{\underline{x}}[\infty, z_3^-]\, \delta^2(\underline{z}_2-\underline{x}) \mathcal{O}^{\mathrm{pol\ \ qG\bar{q}}}_{\chi',\chi}(z_1^-, z_2^-, z_3^-; \underline{x}, \underline{z}_2) \delta^2(\underline{z}_2-\underline{y}) V_{\underline{y}}[z_1^-, -\infty] \\
&+ \int\limits_{-\infty}^{\infty} \mathrm{d}z_1^- \int\limits_{z_1^-}^{\infty} \mathrm{d}z_2^- \int\limits_{z_2^-}^{\infty} \mathrm{d}z_3^-\, d^2z \sum_{\chi''=\pm 1} V_{\underline{x}}[\infty, z_3^-]\, \delta^2(\underline{x}-\underline{z})\, \mathcal{O}^{\mathrm{pol\ G}}_{\chi',\chi''}(z_3^-; \underline{z})\, V_{\underline{z}}[z_3^-, z_2^-]\, \mathcal{O}^{\mathrm{pol\ q\bar{q}}}_{\chi'',\chi}(z_2^-, z_1^-; \underline{z}, \underline{y})\, V_{\underline{y}}[z_1^-, -\infty] \\
&+ \int\limits_{-\infty}^{\infty} \mathrm{d}z_1^- \int\limits_{z_1^-}^{\infty} \mathrm{d}z_2^- \int\limits_{z_2^-}^{\infty} \mathrm{d}z_3^-\, d^2z \sum_{\chi''=\pm 1} V_{\underline{x}}[\infty, z_3^-]\, \mathcal{O}^{\mathrm{pol\ q\bar{q}}}_{\chi',\chi''}(z_3^-, z_2^-; \underline{x}, \underline{z})\, V_{\underline{z}}[z_2^-, z_1^-]\, \mathcal{O}^{\mathrm{pol\ G}}_{\chi'',\chi}(z_1^-; \underline{z})\, V_{\underline{y}}[z_1^-, -\infty]\, \delta^2(\underline{y}-\underline{z})
\end{aligned}
$$



TMD Small-x Evolution: General Strategy

- Rewrite operator product as a scattering process with Wilson lines
- Insert relevant pieces of the Polarized Wilson line
- Working order by order in eikonality, express the TMD in terms of new polarized dipoles
- Expand the field operators in the polarized dipoles and sum over contractions corresponding to gluon or quark emissions to obtain a step in evolution
- Identify logarithmic and double logarithmic contributions and resum
- Solve the evolution equation for the dipole and plug back into the TMD

Eikonal Small-x Sivers function

- Apply the formalism to rewrite the Sivers function in terms of polarized transverse Wilson lines
- The quark-quark correlator which yields the leading order Sivers function is

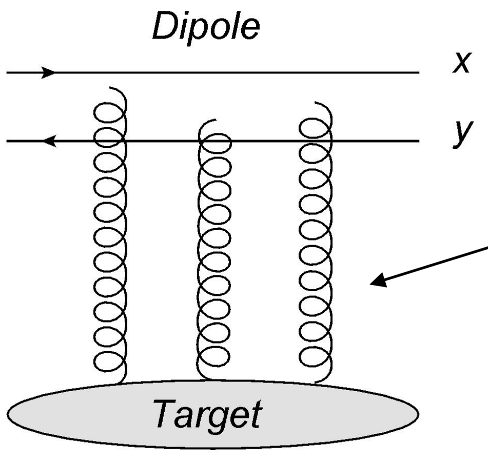
$$\left[f_1^q(x, k_T^2) - \frac{\underline{k} \times \underline{S}_P}{M_P} f_{1\,T}^{\perp\,q}(x, k_T^2) \right]_{\text{eikonal}} = \frac{4p_1^+}{(2\pi)^3} \int \mathrm{d}^2\zeta_\perp\, \mathrm{d}^2 w_\perp\, \frac{\mathrm{d}^2 k_{1\perp}\, \mathrm{d}k_1^-}{(2\pi)^3} e^{i(\underline{k}_1 + \underline{k})\cdot(\underline{w} - \underline{\zeta})} \theta(k_1^-)$$
$$\times \left\{ \frac{\underline{k} \cdot \underline{k}_1}{(xp_1^+ k_1^- + \underline{k}_1^2)(xp_1^+ k_1^- + \underline{k}^2)} \left\langle \mathrm{T}\, \mathrm{tr}\left[V_{\underline{\zeta}}\, V_{\underline{w}}^\dagger \right] + \bar{\mathrm{T}}\, \mathrm{tr}\left[V_{\underline{\zeta}}\, V_{\underline{w}}^\dagger \right] \right\rangle + \frac{\underline{k}_1^2}{(xp_1^+ k_1^- + \underline{k}_1^2)^2} \left\langle \mathrm{T}\, \mathrm{tr}\left[V_{\underline{\zeta}}\, V_{\underline{w}}^\dagger \right] \right\rangle \right\}$$

- By symmetry, the Sivers function must come from the imaginary part of the Wilson line correlators

Dipole Odderon

- In the color dipole picture, it is the antisymmetric, imaginary piece of a dipole correlator

$$\mathcal{O}(x,y) \sim$$



$$d^{abc} = \mathrm{tr}[t^a\{t^b, t^c\}]$$

$$\frac{1}{N_c}\mathrm{tr}\left[V_x V_y^\dagger\right] = \mathcal{S}(x,y) + i\mathcal{O}(x,y)$$

Spin-Dependent Odderon

- *Boer et al* (2016) showed that at small-x the T-odd gluon TMDs are generated by a gauge link which couples an odderon exchange to the proton's spin

$$f_{1T}^{\perp g} = h_{1T}^{g} = h_{1T}^{\perp g} \propto \mathcal{O}_{1T}^{\perp} \ , \ \ x << 1$$

- The odderon is an elusive C-odd three gluon exchange
- The D0 and TOTEM collaborations recently announced odderon in the asymmetry between pp and $p\bar{p}$ collisions [2012.03981 [hep-ex]]
- *Zhou et al* (2019) showed that the quark Sivers function also has a spin-dependent odderon contribution

Small-x Sivers = Odderon

- The imaginary correlator in the Sivers function is exactly the odderon amplitude, so we have

$$-\frac{\underline{k} \times \underline{S}_P}{M_P} f_{1\,T}^{\perp\,q}(x, k_T^2)\Big|_{\text{eikonal}} = \frac{4i\, N_c\, p_1^+}{(2\pi)^3} \int \mathrm{d}^2\zeta_\perp\, \mathrm{d}^2 w_\perp\, \frac{\mathrm{d}^2 k_{1\perp}\, \mathrm{d}k_1^-}{(2\pi)^3} e^{i(\underline{k}_1 + \underline{k})\cdot(\underline{w} - \underline{\zeta})} \theta(k_1^-) \\ \times \left[\frac{2\, \underline{k} \cdot \underline{k}_1}{(x p_1^+ k_1^- + \underline{k}_1^2)(x p_1^+ k_1^- + \underline{k}^2)} + \frac{\underline{k}_1^2}{(x p_1^+ k_1^- + \underline{k}_1^2)^2} \right] \mathcal{O}_{\underline{\zeta}\underline{w}}$$

- Agreement with the results of *Zhou et al* (2019) as well as the small-x gluon T-odd TMDs, spin-dependent odderon!
- The odderon is known to remain exactly eikonal under small-x evolution in the linear regime, possibility to see the spin-dependent odderon

Sub-Eikonal Quark Sivers Function

- Extract sub-eikonal terms from polarized Wilson line which couple proton transverse spin to unpolarized quarks
- Write Sivers function in terms of new dipoles

$$-\frac{\underline{k} \times \underline{S}_P}{M_P} f_{1\,T}^{\perp\, q}(x, k_T^2)\Big|_{\text{sub-eikonal}} = \frac{4}{(2\pi)^3} \int \mathrm{d}^2\zeta_\perp\, \mathrm{d}^2 w_\perp \frac{\mathrm{d}^2 k_{1\perp}}{(2\pi)^3} e^{i(\underline{k}+\underline{k}_1)\cdot(\underline{w}-\underline{\zeta})} \frac{\underline{k}_1 \cdot \underline{k}}{\underline{k}_1^2\, \underline{k}^2} \int\limits_{\frac{\Lambda^2}{s}}^{1} \frac{dz}{z} \left[(k_1 - k)^i\, F^i_{\underline{w},\underline{\zeta}}(z) + i\, F^{[2]}_{\underline{w},\underline{\zeta}}(z) \right]$$

$$F^i_{\underline{w},\underline{\zeta}}(z) \equiv \frac{1}{2N_c} \mathrm{Re} \left\langle\!\!\left\langle \mathrm{T\ tr}\left[V_{\underline{\zeta}} V_{\underline{w}}^{i\,\mathrm{pol}\,\dagger} \right] + \mathrm{T\ tr}\left[V_{\underline{w}} V_{\underline{\zeta}}^{i\,\mathrm{pol}\,\dagger} \right] \right\rangle\!\!\right\rangle(z),$$

$$F^{[2]}_{\underline{w},\underline{\zeta}}(z) \equiv \frac{1}{2N_c} \mathrm{Im} \left\langle\!\!\left\langle \mathrm{T\ tr}\left[V_{\underline{\zeta}} V_{\underline{w}}^{[2]\,\mathrm{pol}\,\dagger} \right] - \mathrm{T\ tr}\left[V_{\underline{w}} V_{\underline{\zeta}}^{[2]\,\mathrm{pol}\,\dagger} \right] \right\rangle\!\!\right\rangle(z)$$

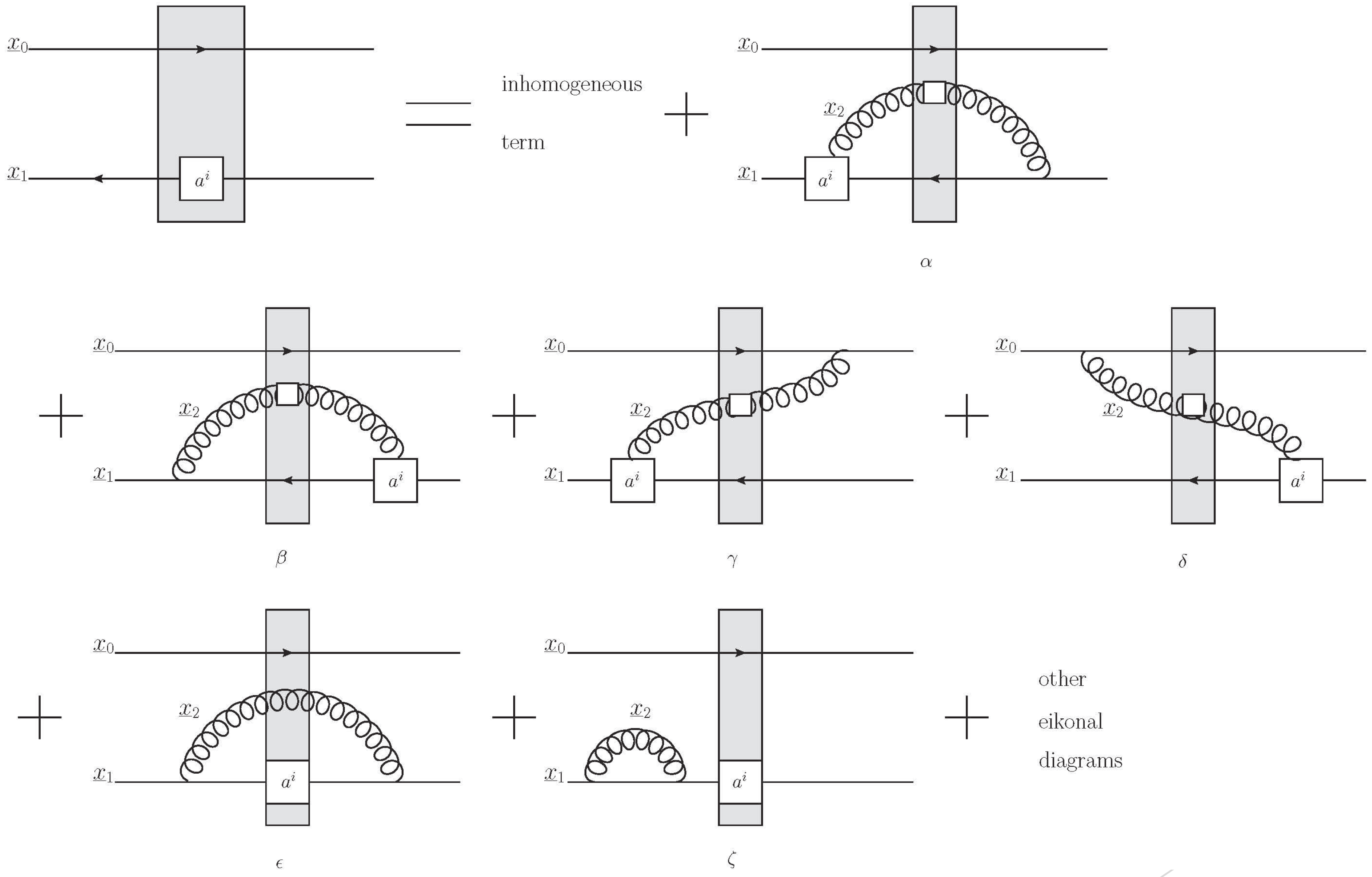
- Here $\langle\langle \ldots \rangle\rangle = zs\langle \ldots \rangle$ with z the internal longitudinal momentum fraction and s the center of mass energy squared

Large-N_c Small-x Evolution

- Quark exchange operators are suppressed
- At large N_c the dipoles do not mix
- The initial condition for $F^{[2]}_{\underline{w},\underline{\zeta}}(z)$ is zero, so we just need to evolve $F^{i}_{\underline{w},\underline{\zeta}}(z)$

Large-N_c Small-x Evolution

- Diagrams for one step in evolution



Large-N$_c$ Small-x Evolution

- Summing the sub-eikonal contributions from each diagram we obtain a closed set of double logarithmic (DLA) evolution equations

$$F^i(x_{10}^2, z) = F^{i(0)}(x_{10}^2, z) - \frac{\alpha_s N_c}{4\pi} \int\limits_{\frac{\Lambda^2}{s}}^{z} \frac{dz'}{z'} \left\{ \int\limits_{x_{10}^2}^{\frac{z}{z'} x_{10}^2} \frac{dx_{21}^2}{x_{21}^2} F^i(x_{21}^2, z') + 3 \int\limits_{\frac{1}{z' s}}^{x_{10}^2} \frac{dx_{21}^2}{x_{21}^2} \Gamma(x_{10}^2, x_{21}^2, z') \right\},$$

$$\Gamma(x_{10}^2, x_{21}^2, z') = F^{i(0)}(x_{10}^2, z') - \frac{\alpha_s N_c}{4\pi} \int\limits_{\frac{\Lambda^2}{s}}^{z'} \frac{dz''}{z''} \left\{ \int\limits_{x_{10}^2}^{\frac{z'}{z''} x_{21}^2} \frac{dx_{32}^2}{x_{32}^2} F^i(x_{32}^2, z'') + 3 \int\limits_{\frac{1}{z'' s}}^{\min\{x_{10}^2, \frac{z'}{z''} x_{21}^2\}} \frac{dx_{32}^2}{x_{32}^2} \Gamma(x_{10}^2, x_{32}^2, z'') \right\}$$

- Here Γ is an auxiliary 'neighbor' dipole and x_{ij} is the dipole size $x_i - x_j$

Sub-Eikonal Small-x Asymptotics

- Solving the large-N_c equations and plugging the dipole back into the Sivers function yields the sub-eikonal small-x asymptotics

$$-\frac{\underline{k} \times \underline{S}_P}{M_P} f_{1\,T}^{\perp\,q}(x, k_T^2)\Big|_{\text{sub-eikonal}} \approx \frac{4}{(2\pi)^3} \int \mathrm{d}^2\zeta_\perp \, \mathrm{d}^2 w_\perp \frac{\mathrm{d}^2 k_{1\perp}}{(2\pi)^3} e^{i(\underline{k}+\underline{k}_1)\cdot(\underline{w}-\underline{\zeta})} \frac{\underline{k}_1 \cdot \underline{k}}{\underline{k}_1^2 \, \underline{k}^2} \int\limits_{\frac{\Lambda^2}{s}}^{1} \frac{dz}{z} (k_1 - k)^i \, F^i_{\underline{w},\underline{\zeta}}(z)$$

$$\sim \left(\frac{1}{x}\right)^0$$

- The sub-eikonal correction has no power correction from evolution, similar to the odderon!

Summary and Outlook

- Fundamental Polarized Wilson line can be used to find the small-x asymptotics for all the leading twist quark TMDs

- Small-x Sivers function is dominated by eikonal spin-dependent odderon with a sub-eikonal, energy independent correction

$$f_{1\,T}^{\perp\,q}(x, k_T^2) = C_O(k_T^2, x)\,\frac{1}{x} + C_1(k_T^2)\,\left(\frac{1}{x}\right)^0 + \ldots\,, \quad x << 1$$

- Opens the possibility to see spin-dependent odderon in spin asymmetries of SIDIS (and DY) with future colliders such as EIC!

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Thank You

Rewriting the Sivers Function Operator

- The quark correlator can be rewritten in terms of Wilson lines

$$f_1^q(x,k_T^2) - \frac{k_T \times \underline{S}_P}{M_P} f_{1\,T}^{\perp\,q}(x,k_T^2) = \frac{2p^+}{(2\pi)^3} \sum_{\bar{q}} \int\limits_{-\infty}^{0} \mathrm{d}\zeta^- \int\limits_{0}^{\infty} \mathrm{d}\xi^- \int \mathrm{d}^2\zeta_\perp\, \mathrm{d}^2\xi_\perp\, e^{ik\cdot(\zeta-\xi)}$$

$$\times \left[\frac{\gamma^+}{2}\right]_{\alpha\beta} \left\langle \bar{\psi}_\alpha(\xi) V_{\underline{\xi}}[\xi^-,\infty] |\bar{q}\rangle \langle \bar{q}| V_{\underline{\zeta}}[\infty,\zeta^-] \psi_\beta(\zeta) \right\rangle + c.c.$$

$$= -\frac{2p^+}{(2\pi)^3} \int \mathrm{d}^2\zeta_\perp\, \mathrm{d}^2 w_\perp \frac{\mathrm{d}^2 k_{1\perp}\, \mathrm{d}k_1^-}{(2\pi)^3} e^{i(\underline{k}_1+\underline{k})\cdot(w-\zeta)} \theta(k_1^-) \sum_{\chi_1,\chi_2} \bar{v}_{\chi_2}(k_2) \frac{\gamma^+}{2} v_{\chi_1}(k_1)$$

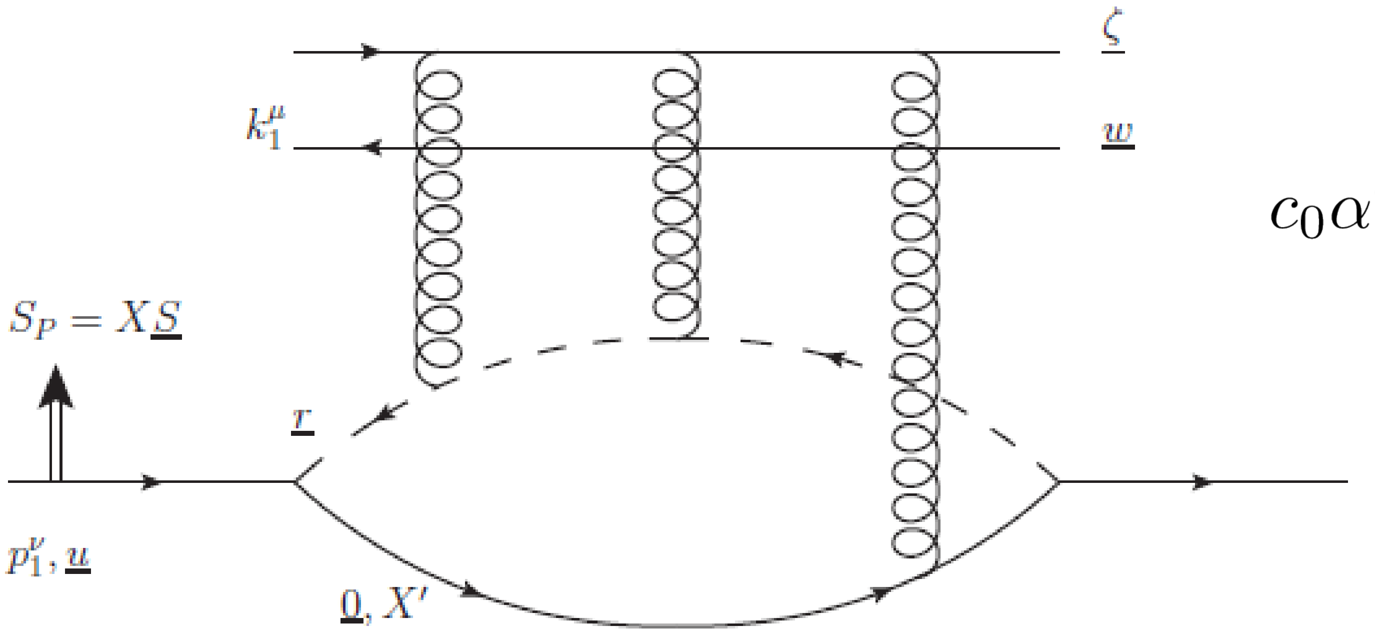
$$\times \left\langle \mathrm{T}\, V_{\underline{\zeta}}^{ij}[\infty,-\infty] \bar{v}_{\chi_1}(k_1) V_{\underline{w}}^{\dagger\,\mathrm{pol,\ T}\,ji} v_{\chi_2}(k_2) \right\rangle \frac{1}{(2xp^+k_1^- + \underline{k}_1^2)(2xp^+k_1^- + \underline{k}^2)} + c.c.$$

- The polarized Wilson line $V_{\underline{w}}^{\dagger\,\mathrm{pol,\ T}}$ makes the correlator a transverse polarized dipole

Diquark model calculation

- We can estimate the small-x Sivers function in the diquark model, where the interaction between the point-particle proton ψ_P, the quark field ψ, and the scalar diquark φ is

$$\mathscr{L}_{int} = G\,\varphi^{*\,i}\,\bar{\psi}^i_q\,\psi_P + \text{c.c.}$$



$$c_0 \alpha_s^3 \ln^3\left(\frac{|\underline{\zeta} - \underline{0}||\underline{w} - \underline{r}|}{|\underline{w} - \underline{0}||\underline{\zeta} - \underline{r}|}\right)$$

Diquark model calculation

- Plugging the diquark model odderon into our Sivers function result gives

$$f_{1\,T}^{\perp\,q}(x, k_T^2)\Big|_{\text{eikonal}} = \frac{1}{x}\,\frac{N_c G^2\, c_0 \alpha_s^3}{2(2\pi)^5}\,\frac{M_P^2}{\underline{k}^2 \Lambda^2}$$

- Nonzero spin-dependent odderon!
- Interesting behavior as $M_P, \Lambda, \Lambda_{QCD} \to 0$, Sivers function does not vanish