Non-perturbative hadronisation corrections with the ARES method

Andrea Banfi[1], Basem Kamal El-Menoufi[2], Ryan Wood[1]

[1]Department of Physics and Astronomy
University of Sussex

[2]Consortium for Fundamental Physics, School of Physics and Astronomy
University of Manchester

QCD Master Class 2021

Introduction

Broader Context

- Event shape observables (infrared and collinear safe) in e^+e^- annihilation are often used to perform precise determinations of the strong coupling constant, α_s.

Why do we wish to perform precise determinations of α_s?

- α_s is the least well-known coupling in the gauge sector of the Standard Model.
- 2019 Particle Data Group world average of α_s had an uncertainty of about 1%, ($\alpha_s = 0.1179 \pm 0.0010$).
- The uncertainty on α_s is becoming increasingly critical for precision collider phenomenology.

Why use these event shape observables?

- In principal they are the ideal testing ground for perturbative QCD.
- They can be computed order-by-order in perturbation theory.
- α_s small due to $Q \sim M_{\mathrm{Z}}$, therefore a well-behaved perturbation series.
- Non-perturbative (hadronisation) effects should be suppressed by inverse powers of Q.

Problems:

- Hadrons (in Measurements) vs Quarks/Gluons (in Theoretical Calculations).
- Non-perturbative effects turn out to be significant even at $Q \sim M_{\mathrm{Z}}$.
- How do we estimate these hadronisation effects?

- In the 1990's it was discovered that Hadronisation Corrections to collinear and infrared-safe two-jet event shape observables in e^+e^- annihilation could be described in terms of an expansion in negative powers of the centre of mass energy, Q.

- The leading correction in this sense leads to a shift of the cumulative distribution of the form:

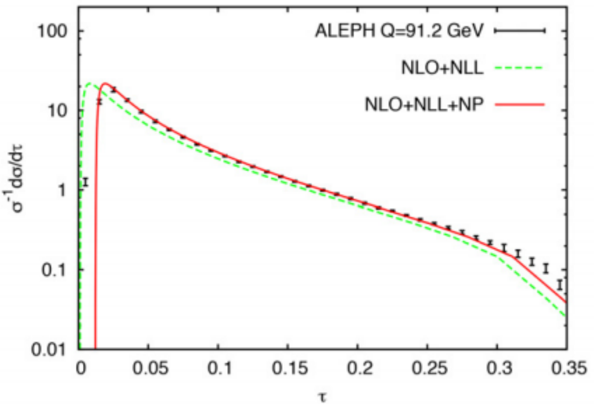
$$\Sigma(V) = \Sigma^{\mathrm{PT}}(V - \langle \delta V \rangle)$$

with:

$$\langle \delta V \rangle \propto \frac{1}{Q}$$

- The mean change in the observable's value is due to the emission of soft non-perturbative radiation.

- Illustrated by way of an example ($\tau = 1 - T$):



- To reiterate - Leading Hadronisation Corrections provide a shift of the perturbative event-shape distribution.

- In 2019, a collaboration of theorists and experimentalists presented state-of-the-art extractions of the strong coupling based on $N^3LO + NNLL$ accurate predictions for the two-jet rate in the Durham clustering algorithm in e^+e^- annihilation.

Owing to the high accuracy of the predictions used, the perturbative uncertainty is considerably smaller than that due to hadronization. Our best determination at the Z mass is $\alpha_s(M_Z) = 0.11881 \pm 0.00063(exp.) \pm 0.00101(hadr.) \pm 0.00045(ren.) \pm 0.00034(res.)$, which is in agreement with the latest world average and has a comparable total uncertainty.

[arXiv: 1902.08158v1]

- This was the first time in history that hadronisation uncertainties for these types of observables were larger than QCD uncertainties.

- Our aim is to introduce a new method to compute leading hadronisation corrections to two-jet event shapes in e^+e^- annihilation.

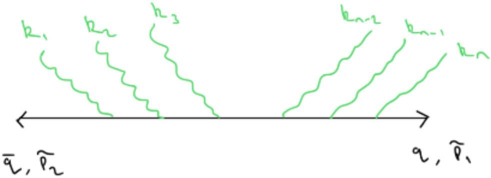
Set-up

- We begin by considering a generic recursive infrared and collinear safe (rIRC) observable in e^+e^- annihilation:

$$V\left(\{\tilde{p}\}, k_1, \dots, k_n\right) \geqslant 0$$

where:

  - $\{\tilde{p}\} = \{\tilde{p}_1, \tilde{p}_2\}$ are the momenta of a hard quark-antiquark pair
  - $k_1, \dots, k_n$ are the subsequent emissions



- In the Born limit $V(\{\tilde{p}\}) = 0$, whereas in general $V(\{\tilde{p}\}) \geqslant 0$.
- We shall consider the region in which:

$$V(\{\tilde{p}\}, k_1, ..., k_n) = v \ll 1$$

- In this limit all secondary emissions are soft &/or collinear.
- We shall only consider soft & collinear secondary emissions widely separated in angle.
- The observable cumulant is given by:

$$\Sigma(v) = \Sigma_{\mathrm{PT}}(v) + \delta\Sigma_{\mathrm{NP}}(v)$$

- At NLL accuracy we can write:

$$\Sigma_{\mathrm{PT}}(v) = e^{-R_{\mathrm{NLL}}(v)} \mathcal{F}_{\mathrm{NLL}}(v)$$

where:

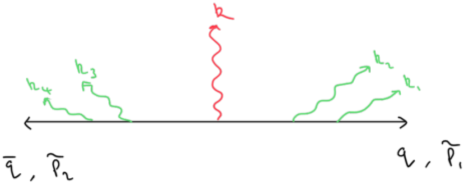
  - $R_{\mathrm{NLL}}(v)$ is the Radiator (can be calculated analytically)
  - The transfer function is given by:

$$\mathcal{F}_{\mathrm{NLL}}(v) = \left\langle \Theta \left(1 - \frac{V_{\mathrm{sc}}\left(\{\tilde{p}\}, k_1, \ldots, k_n\right)}{v} \right) \right\rangle$$

and can be computed by a Monte Carlo method.

Approach

- Leading hadronisation corrections are due to the contribution of a very soft gluon (aka gluer):



- We define:

$$\delta V_{\mathrm{NP}} \equiv V\left(\{\tilde{p}\}, k, \{k_i\}\right) - V\left(\{\tilde{p}\}, \{k_i\}\right)$$

- We find that:

$$\delta\Sigma_{\mathrm{NP}} = -\left\langle \delta V_{\mathrm{NP}} \right\rangle \frac{d\Sigma_{\mathrm{PT}}}{dv}$$

- This gives us that:

$$\begin{aligned} \Sigma(v) &= \Sigma_{\mathrm{PT}}(v) + \delta\Sigma_{\mathrm{NP}}(v) \\ &= \Sigma_{\mathrm{PT}}(v) - \left\langle \delta V_{\mathrm{NP}} \right\rangle \frac{d\Sigma_{\mathrm{PT}}}{dv} \\ &= \Sigma_{\mathrm{PT}}\left(v - \left\langle \delta V_{\mathrm{NP}} \right\rangle\right) \end{aligned}$$

- The gluer produces a shift in the perturbative event-shape distribution

- We shall restrict ourselves to event shapes for which:

$$\delta V_{\mathrm{NP}}\left(\{\tilde{p}\}, k, \{k_i\}\right) = \frac{k_t}{Q} f_V\left(\eta, \phi, \{k_i\}\right)$$

with $\eta = -\ln\tan\frac{\theta}{2}$

- Therefore:

$$\langle \delta V_{\rm NP}\left(\{\tilde{p}\}, k, \{k_i\}\right)\rangle = \frac{\langle k_t \rangle}{Q} c_V$$

with:

$$\boxed{c_{\rm V} = \frac{\left\langle f_{\rm V}\left(\eta, \phi, \{k_i\}\right)\delta\left(1 - \frac{V_{\rm sc}(\{\tilde{p}\},\{k_i\})}{v}\right)\right\rangle}{\left\langle \delta\left(1 - \frac{V_{\rm sc}(\{\tilde{p}\},\{k_i\})}{v}\right)\right\rangle}}$$

- The denominator may be written as $R'\mathcal{F}(R')$ with:

$$R' \equiv -v\frac{dR}{dv}$$

Additive Observables

- For additive observables we have:

$$V\left(\{\tilde{p}\}, k_1, ..., k_n\right) = \sum_{i=1}^{n} \zeta_i \quad \text{with} \quad \zeta_i \equiv V\left(\{\tilde{p}\}, k_i\right)$$

- Therefore we trivially find that:

$$\delta V_{\rm NP}\left(\{\tilde{p}\}, k, \{k_i\}\right) = V\left(\{\tilde{p}\}, k\right)$$

  and therefore:

$$f_{\rm V}\left(\eta, \phi, k_1, ..., k_n\right) = f_{\rm V}\left(\eta, \phi\right)$$

- This results in a trivial cancellation, giving:

$$c_{\rm V} = \int d\eta \, \frac{d\phi}{2\pi} \, f_{\rm V}\left(\eta, \phi\right)$$

- Let us consider the following examples (Thrust, C-parameter, Heavy Jet Mass)

Thrust

- The Thrust is given by:

$$T \equiv \max_{\vec{n}} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{Q} \quad , \quad \tau \equiv 1 - T$$

- We find that:

$$f_\tau(\eta, \phi) = e^{-|\eta|}$$

- Therefore:

$$\begin{aligned} c_\tau &= \int d\eta \, \frac{d\phi}{2\pi} \, f_\tau(\eta, \phi) \\ &= \int_{-\infty}^{\infty} d\eta \, e^{-|\eta|} \\ &= 2 \end{aligned}$$

C-parameter

- The C-parameter is given by:

$$C \equiv 3 \left(1 - \frac{1}{2} \sum_{i,j} \frac{\left(p_i \cdot p_j\right)^2}{\left(p_i \cdot Q\right)\left(p_j \cdot Q\right)} \right)$$

- We find that:

$$f_{\mathrm{C}}\left(\eta, \phi\right) = \frac{3}{\cosh \eta}$$

- Therefore:

$$\begin{aligned} c_{\mathrm{C}} &= \int d\eta \, \frac{d\phi}{2\pi} \, f_{\mathrm{C}}\left(\eta, \phi\right) \\ &= \int_{-\infty}^{\infty} d\eta \, \frac{3}{\cosh \eta} \\ &= 3\pi \end{aligned}$$

Heavy Jet Mass

- The Heavy Jet Mass is given by:

$$\rho_{\mathrm{H}} \equiv \max_{i=1,2} \frac{M_i^2}{Q^2} \quad , \quad M_i^2 \equiv \left(\sum_{j \in \mathcal{H}^{(i)}} p_j \right)^2$$

- A non-zero hadronisation correction to the heavy-jet mass arises only when the NP gluon is emitted in the heavier hemisphere.
- We find that:

$$f_{\rho_{\mathrm{H}}}(\eta, \phi, k_1, \ldots, k_n) = e^{-\eta}\Theta(\eta)\Theta(\rho_1 - \rho_2) + e^{\eta}\Theta(-\eta)\Theta(\rho_2 - \rho_1)$$

- Therefore:

$$\begin{aligned} c_{\rho_{\mathrm{H}}} &= \int d\eta \, \frac{d\phi}{2\pi} \, f_{\rho_{\mathrm{H}}}(\eta, \phi) \\ &= \frac{1}{2} \int_{-\infty}^{0} d\eta \, e^{\eta} + \frac{1}{2} \int_{0}^{\infty} d\eta \, e^{-\eta} \\ &= 1 \end{aligned}$$

Non-additive Observables

- For non-additive observables life becomes more tricky.
- To illustrate, let us consider Broadening-like observables:
  - The Single Jet Broadenings are given by:

$$B_{\mathrm{L}} \equiv \sum_{i \in \mathcal{H}^{(1)}} \frac{|\vec{p_i} \times \vec{n_{\mathrm{T}}}|}{2Q} \quad \& \quad B_{\mathrm{R}} \equiv \sum_{i \in \mathcal{H}^{(2)}} \frac{|\vec{p_i} \times \vec{n_{\mathrm{T}}}|}{2Q}$$

  - The Wide Jet Broadening is given by:

$$B_{\mathrm{W}} \equiv \max\{B_{\mathrm{L}}, B_{\mathrm{R}}\}$$

  - The Total Jet Broadening is given by:

$$B_{\mathrm{T}} \equiv B_{\mathrm{L}} + B_{\mathrm{R}}$$

- The difficulty presents itself when the shape-functions are calculated:
- We find that:

$$
\begin{aligned}
&f_{B_W}(\eta, \phi, k_1, \ldots, k_n) \\
&= \Theta(\eta^{(1)}) \left[\sqrt{1 + 2e^{\eta^{(1)}} \frac{p_{t,1}}{Q} \cos\phi_1 + e^{2\eta^{(1)}} \left(\frac{p_{t,1}}{Q}\right)^2} - e^{\eta^{(1)}} \frac{p_{t,1}}{Q} \right] \Theta(B_1 - B_2) \\
&\quad + 1 \leftrightarrow 2
\end{aligned}
$$

and

$$
\begin{aligned}
&f_{B_T}(\eta, \phi, k_1, \ldots, k_n) \\
&= \sum_\ell \Theta(\eta^{(\ell)}) \left[\sqrt{1 + 2e^{\eta^{(\ell)}} \frac{p_{t,\ell}}{Q} \cos\phi_\ell + e^{2\eta^{(\ell)}} \left(\frac{p_{t,\ell}}{Q}\right)^2} - e^{\eta^{(\ell)}} \frac{p_{t,\ell}}{Q} \right]
\end{aligned}
$$

- Analytical expressions for c_{B_W} and c_{B_T} have been computed [hep-ph/9812487v3]:

$$c_{B_{\mathrm{W}}} = \frac{1}{2}\left[-2 - \psi(1) - \ln B + \eta_0 + \chi\left(\frac{R'}{2}\right) - \rho\left(\frac{R'}{2}\right) + \psi\left(1 + \frac{R'}{2}\right)\right]$$

$$c_{B_{\mathrm{T}}} = 2c_{B_W} - \psi\left(1 + \frac{R'}{2}\right) + \psi\left(1 + R'\right) + \frac{1}{R'}$$

- N.B $c_{B_{\mathrm{T}}}$ holds in the limit $R' \gg \sqrt{2R''}$

- We have successfully been able to reproduce these results both analytically and numerically using our method.

Thrust Major

- Having used the Broadening-like Observables as an ideal 'test of concept' the next step is to apply our approach to suitable observables for which no analytic calculation for the shift has been computed (nor is currently believed to be possible).

- The Thrust Major is given by:

$$T_{\mathrm{M}} \equiv \max_{\vec{n} \cdot \vec{n_T}} \frac{\sum_i |\vec{p_i} \cdot \vec{n}|}{Q}$$

- We find that:

$$f_{T_{\mathrm{M}}}(\eta, \phi, k_1, \ldots, k_n)$$
$$= \sum_{\ell} \Theta(\eta^{(\ell)}) \left[\left| \cos \phi_\ell + \frac{e^{\eta^{(\ell)}}}{Q} |p_{y,\ell}| \right| - \frac{e^{\eta^{(\ell)}}}{Q} |p_{y,\ell}| \right]$$

Next Steps

- We are currently completing our computation for $c_{\mathrm{T_M}}$
- On completion we shall compare our results to parton-shower event generators.
- We shall use results for the shifts of Broadening-like observables and the Thrust Major to perform a simultaneous fit for α_s and α_0.
- Thank you for listening